## Determination of Design and Operation Parameters

## for Upper Atmospheric Research Instrumentation

to Yield Optimum Resolution with Deconvolution

FINAL REPORT

## APPENDIX 2

Dr. George E. Ioup, Principal Investigator
Dr. Juliette W. Ioup, Principal Investigator
Department of Physics
University of New Orleans
New Orleans, LA 70148

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## FINAL REPORT

## APPENDIX 2

Dr. George E. Ioup, Principal Investigator Dr. Juliette W. Ioup, Principal Investigator Department of Physics
University of New Orleans
New Orleans, LA 70148

# Expanded Analysis of Combined Window Spectral Estimate Channel Clearing 

Submitted to the Graduate Faculty of the University of New Orleans in Partial fulfillment of the Requirement for the degree of Master of Science in the<br>Department of Physics

by
James Lester Kreamer
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## ABSTRACT

This thesis reviews the technique established to clear channels in the Power Spectral Estimate by applying linear combinations of well known window functions to the autocorrelation function. The need for windowing the autocorrelation function is due to the fact that the true autocorrelation is not generally used to obtain the Power Spectral Estimate. When applied, the windows serve to reduce the effect that modifies the autocorrelation by truncating the data and possibly the autocorrelation has on the Power Spectral Estimate.

It has been shown in previous work that a single channel has been cleared, allowing for the detection of a small peak in the presence of a large peak in the Power Spectral Estimate.

The utility of this method is dependent on the robustness of it on different input situations. We extend the analysis in this paper, to include clearing up to three channels. We examine the relative positions of the spikes to each other and also the effect of taking different percentages of lags of the autocorrelation in the Power Spectral Estimate.

This method could have application wherever the Power Spectrum is used. An example of this is beam forming for source location, where a small target can be located next to a large target. Other possibilities extend into seismic data processing. As the method becomes more automated other applications may present themselves.

## CHAPTER 1

## INTRODUCTION


#### Abstract

Autocorrelation estimates are adversely effected when truncated data sets are used (Blackman and Tukey, 1958). To overcome the bias due to the data truncation, the autocorrelations are themselves often truncated, producing sidelobes around peaks and the corresponding power spectral estimates. Figures 1.1 thru 1.3 demonstrate this effect. Figure 1.1 is a power spectrum with a single spike residing in channel 31. When inverse Fourier transformed this spike produces a function domain time series shown in Figure 1.2. This function domain time series is modified by taking only a limited number of samples from the original series corresponding to the multiplication by a rectangular window. This modified or truncated time series is Fourier transformed so that we witness the effect of this modification (Figure 1.3). The spike remains prevalent in channel 31 but now the "ringing" sidelobes pervade throughout the spectrum. These side lobes can override and mask smaller peaks in the vicinity of the dominant peak. The application of windows also has the general effect of smoothing the power spectral estimate which may also have a deleterious effect on a small peak in the vicinity of a larger peak in the spectrum.


There has been much work done with the utilization of window functions to reduce the effects of truncation. Harris (1978) details the use of windows in harmonic analysis, and describes a number of variables with which to measure the performance of each. Specific application of these windows has been used to modify an autocorrelation
function prior to power spectral estimation. This technique is described by Blackman and Tukey (1958) and Robinson (1980). The autocorrelation being windowed is modified such that when transformed the effects of this modification are seen in the power spectrum. These effects are typically seen as reduced sidelobes associated with dominant peaks in the power spectrum. Still there remains sidelobes, although they are much smaller than when no window used.

Because of these effects, Smith (1985) developed a technique in which a linear combination of window functions were implemented to clear a single channel in the power spectral estimate. Although the technique was established, its implementation was limited to two input data situations, which will be reviewed. As a consequence, this thesis expands on the work done by Smith by examining the performance of this method on different input data sets. A review of the mathematical background and theory of the technique is given in order to establish a foundation for this work. To understand the procedures involved in utilizing this method, it is beneficial to understand the theory it is based on. In light of this, a review of the techniques and algorithms used in performing these calculations is presented. Finally, the input data and results of the calculations performed on these data are presented and analyzed.

This method has potential applicability in a number of fields which involve power spectral estimation. Among these is beam forming, where the objective is to locate a small target in the presence of a larger target. In seismic data processing we will, in future research, investigate the influence of a dominant signal in the power spectrum,
specifically 60 Hz noise. The purpose of this is to determine by channel clearing whether we can detect reflection signals near a dominant event such as 60 Hz noise.

The mathematical notations used in this paper will be similar to those Smith (1985) used in his work. The symbol w with or without subscripts will refer to a weighting function or a window in the function domain ( $t$ ). The Fourier transform of $w(t)$ is represented by $W(s)$ where ( $s$ ) is the transform domain variable. Likewise, the function domain autocorrelation is represented by $g(t)$ and its Fourier transform by $G(s)$.

## TECHNICAL BACKGROUND

In order to understand the need for channel clearing in the Power Spectral Estimate, we need to review the concept and purpose of the autocorrelation function and windows. Given a discrete input time series with $M$ terms

$$
(x(0), x(1), x(2), \ldots, x(m-1)
$$

where

$$
x(j)=0 \text { for } j<0 \text { and } j>M-1,
$$

the autocorrelation is defined as follows:

$$
\begin{equation*}
g(j T)=\sum_{k=0}^{M-j-1} x(k+j) x(k) \tag{1}
\end{equation*}
$$

where $j$ is a lag index and $T$ is the sample increment. The autocorrelation function, as described by equation (1), is a shift, multiply and add operation which has the property of being non-negative definite (Robinson 1980). It is also commutative, which is expressed by the following equality:

$$
\begin{equation*}
\sum_{k=0}^{M-j-1} x(k+j) x(k)=\sum_{k=0}^{M-j-1} x(k) x(k-j) \tag{2}
\end{equation*}
$$

We can show this easily by expanding the summation operation as follows. We write the input series

$$
\begin{align*}
& x(0) x(1) x(2) x(3) \ldots \ldots . . x(m-1) \\
& x(0) x(1) x(2) x(3) \ldots \ldots . \times x(m-1) \tag{3}
\end{align*}
$$

where in this case $\mathrm{j}=0$, meaning there is no time shift or we are calculating the zero lag value of the autocorrelation. The result of applying (2) to (3) gives

$$
\begin{equation*}
g(0)=x^{2}(0)+x^{2}(1)+x^{2}(2)+{ }^{2}(3)+\ldots \ldots \ldots x^{2}(m-1) \tag{4}
\end{equation*}
$$

We recognize equation (4) as the total energy of the series given in (3), because by definition (Robinson (1980) the total energy of a signal is given by

$$
\begin{equation*}
E=\sum_{n=0}^{m-1}|x|^{2} \tag{5}
\end{equation*}
$$

The lag value will now be shifted by 2 units to the right.

$$
x(0) x(1) x(2) x(3) \ldots \ldots \ldots x(m-1)
$$

$$
\begin{equation*}
x(0) x(1) \ldots \ldots . x(m-3) x(m-2) x(m-1) \tag{6b}
\end{equation*}
$$

or

$$
\begin{equation*}
g(2)-(x(0) x(2))+(x(1) x(3))+\ldots(x(m-1)) \tag{7}
\end{equation*}
$$

Similarly a shift of 2 units to the left yields

$$
\begin{equation*}
x(0) x(1) \ldots \ldots x(m-3) x(m-2) x(m-2) \tag{8a}
\end{equation*}
$$

$$
\begin{equation*}
x(0) x(1) x(2) x(3) \ldots \ldots x(m-1) \tag{8b}
\end{equation*}
$$

therefore

$$
\begin{equation*}
g(-2)-(x(2) x(0))+(x(3) x(1))+\ldots .(x(m-1) x(m-3)) \tag{9}
\end{equation*}
$$

Now we can see from equations (7) and (9) that

$$
g(-2)-g(2)
$$

and in general,

$$
\begin{equation*}
g(j)=g(-j) \tag{10}
\end{equation*}
$$

showing the autocorrelation function is symmetric about the zero lag (j-0).

It was shown earlier that the zero lag value of the autocorrelation of a signal was equivalent to the total energy of the signal. Another quantity calculated from the autocorrelation is based on the interrelation of the autocorrelation function and the power spectral estimate and the effects windowing has on each. Bracewell (1978), through the use of the Autocorrelation Theorem and associated derivation, defines the relationship between the autocorrelation function and the power spectrum. These two quantities are Fourier transform pairs where the Fourier transform of a function $f(x)$ is defined as

$$
\begin{equation*}
F(s)=\int_{-\infty}^{\infty} f(x) e-i 2 \pi 5 x d x \tag{11}
\end{equation*}
$$

Similarly, the Fourier transform of $F(s)$ is defined as

$$
\begin{equation*}
f(x)=\int_{-\infty}^{\infty} F(s) e^{i 2 \pi s X} d s \tag{12}
\end{equation*}
$$

The quantity given in equation 11 is the result of what Harris (1978) refers to as a decomposition of a signal over a basis set. The Fourier transform specifically decomposes the signal set ( $f(x)$ ) into the sums of sines and cosines. This complex arithmetic transform is defined for the discrete functions by the following:

$$
\begin{equation*}
F(s)=T / T(0) \sum f(x) \cos (2 \pi s x / M) \tag{13}
\end{equation*}
$$

$$
\mathrm{iT} / \mathrm{T}(0) \sum \mathrm{f}(\mathrm{x}) \sin (2 \pi s x / \mathrm{M})
$$

where

$$
\begin{aligned}
& M=\text { the total number of samples } \\
& T \text { - the sample interval } \\
& s \text { - a multiple of the harmonic frequency counter, a multiple of } \\
& 1 / T(0)
\end{aligned}
$$

$T(0)=$ the length of the data set

```
        x = the sample counter
    f(x) = input data samples
```

Using the Euler relation

$$
\begin{equation*}
e^{i \theta}=\cos \theta-i \sin \theta \tag{14}
\end{equation*}
$$

we can rewrite equation 13 as

$$
\begin{equation*}
F(s)=\sum f(x) \exp (-2 \pi s x / M) \tag{15}
\end{equation*}
$$

A discussion of the applicability of equations 11 and 15 will be given later in this chapter. The sums in equation 13 are over the aforementioned basis set. Through this decomposition or Fourier transform we obtain a representation of the signal in terms of frequency based on the component of each monotonic cosine and sine. The $f(x)$ and $F(s)$ in equations 11 and 12 are referred to as Fourier transform pairs and are represented by the following notation

$$
f(x) \supset F(s)
$$

Therefore, in agreement with the notation given in the introduction, the Fourier pair, autocorrelation and power spectrum, are represented by the following.

$$
g(x) \supset G(s)
$$

The calculation of the Fourier transform as defined in equation 11 is for a continuous function of $x$, evaluated form minus infinity to plus infinity. Due to the reality of not being able to measure an infinite signal, we are confined to analyzing a finite data set. Also, we are not interested here in evaluating a continuous function but rather, a sampled function. The above discussion suggests the relationship between equations 11 and 15 , i.e., equation 15 is the finite sampled function domain equivalent of equation 11. Brigham (1974) extensively discusses sampling theory as related to the Fourier transform and also discusses the algorithms developed for the Fast Fourier Transform, which is implemented in the work presented in this paper. Inherent in the transform of this sampled-finite data set, are artifacts of the numeric process. These artifacts of the transform process can cause the deleterious consequences shown in Figure 1.3. Specific to this, Harris
(1978) discusses the concept of spectral leakage. When sampling a finite data set with $T$ sample interval, we obtain a total length $n T$, where $n$ is the number of samples. If there are frequencies contained in the signal not contained in the basis set then these signals will not be periodic in the total sample interval. These signals which are not commensurate with the natural period exhibit discontinuities at their periodic extension. These discontinuities contribute to the spectral content of the signal calculated by the Fourier transform. The phenomenon just described is a particular case of Gibbs phenomenon (Bracewell 1978), which refers to the sidelobe or ripple effect in the function produced by the discontinuity in the transform. It is this behavior in the transform, due to truncation of the autocorrelation, which prompts us to perform windowing.

Because of the reasons stated above, Blackman and Tukey (1958) developed a technique whereby an autocorrelation function is modified by multiplying a weighting function in order to obtain a Power Spectral Estimate. This weighting function, which multiplies the autocorrelation in the function domain, is equivalent to the convolution of the Fourier transforms of the weighting function and the autocorrelation in the transform domain. The Fourier transform of the weighting function is called a window. The expression for the above relationship is the following:

$$
\begin{equation*}
W(s)-\int_{-\infty}^{\infty} w(x) \exp (-i 2 \pi s x) d x . \tag{16}
\end{equation*}
$$

To avoid a convolution in the power spectral estimate calculation, Blackman and Tukey presented the method for windowing in the continuous case by multiplying the autocorrelation with the weighting function. The product of the above was then Fourier transformed to obtain the power spectral estimate. For the discrete case the convolution in the transform domain is not difficult. Therefore convolution of the window with the transform of the autocorrelation is an alternative. Robinson (1980) also elaborates on the above method. The specific methodology used in this work will be reviewed later.

A number of windows have been introduced in the literature (Geckinli and Yavuz 1983). Harris (1978) elaborates on several of these and presents quantitative measurements which measure the performance of each. Table 1 of Harris's paper lists these parameters. As with the truncated input autocorrelation, the windows themselves will exhibit spectral leakage. This leakage is measured in Harris's work by the peak sidelobe level and the asymptotic rate of falloff of the sidelobes. These quantities are illustrated in Harris's Figure 5. Though the ratings of these quantities given by Harris are a measure of the performance quality of the window, they may not accurately reflect the usefulness of the windows in this work. The sidelobes play a critical role in the work presented here, because through these we will be determining the proper combinations to clear channels in the power spectral estimate.

## CHAPTER 3

## CHANNEL CLEARING: THEORETICAL BACKGROUND AND INITIAL RESULTS

In 1985 smith introduced the technique of channel clearing. This method is based on the ability to apply weighting functions alone or in linear combinations to the autocorrelation function in order to clear channels or domain segments in the power spectral estimate.

A specific problem as stated by Smith is the location of a small spike near a large spike in the power spectrum. In this case application of a window along with its sidelobes effect on the power spectrum has masked the smaller spike. Figure 3.1 shows the spectrum with a spike in channel 16 and a spike of $1 / 20$ th amplitude in channel 20. Figure 3.2 shows the same spectrum with a single window applied. The influence of the larger spike combined with the window has masked the smaller spike in channel 20. The idea behind the technique is to combine windows linearly and apply to the spectrum containing the large spike only in order to clear or bring to zero the channel in which the smaller spike resides. When this combination is established, it is applied to the spectrum which contains both spikes. Subsequent to this the smaller spike would be detected.

This combination of windows is called a hybrid window. We are allowed to make this construction because scaler addition and multiplication by a constant are preserved through the Fourier transform. This is shown in the following relationship

$$
\sum a_{i} w_{i}(t) J a_{i} w_{i}(s)
$$

where a is a coefficient such that the sum of all a's for a particular hybrid window equals unity. This normalization is recommended due to the fact that the zero lag of the autocorrelation is a maximum and amplification of this value can occur if the weighting is nor normalized. Another result that is related to this normalization is that the hybrid window $w(x)$ is bounded by the windows of which it is composed. A consequence of this bounding is that it prevents amplification in any of the sidelobes. The proof of this bounding is given in Smith's thesis (1985). This proof extends to the case where the hybrid window is constructed of $n$ component windows. In this case an envelope is constructed with an upper limit window comprised of greatest components of all the input windows and a lower limit window comprised of the least magnitude components of the input windows. The analogy here results in the following statement: that for any $x$ there are two component windows such that

$$
w_{1}(x) \leq w_{h}(x) \leq w_{u}(x)
$$

where $u$ = the upper bound window, 1 - the lower bound window and

$$
\sum_{i=1}^{n} a(i)-1
$$

and

$$
0 \leq a(i) \leq 1 .
$$

The last property discussed in this section is the combination process and the use of hybrid windows as component windows in subsequent hybrid windows. It was shown that a linear combination of any number of windows, with normalized coefficients, produces a hybrid window. This hybrid window can be used in the construction of a new hybrid window, again with proper normalization of the coefficients. This property is important in the calculation of hybrid windows when attempting to clear more than one channel simultaneously. The technique used in determining the coefficients for the windows will demonstrate this.

The initial results of this technique showed that a single channel can be cleared within $10^{-7}$ of the normalized peak value. The use of double precision may result in clearing within $10^{-15}$. The experiment was conducted by putting a unit spike in channel 31 and a spike of $1 / 20$ th amplitude in channel 35. The purpose of the experiment was to see if a combination of windows could be calculated to eliminate the effects of the spike in channel 31 at channel 35 . A combination of three windows were used in order to eliminate channel 35 from the spectrum with only the spike in channel 31 present. Windows 3,6 and 9 of Table 4.1 with respective coefficients of .0000016, .0000074, . 9999910 were the choices. This hybrid window was then applied to the spectrum with the two spikes and clearly the spike in channel 35 was detected (Figure 5.3 and 5.4). The exact procedure for calculating the coefficients will be detailed in the next chapter.

The second experiment involved replacing the spikes with Gaussians centered at channels 31 and 35. Again the maximum amplitude of each was 1.0 and .05 respectively. The purpose of this experiment is to


#### Abstract

attempt to clear a finite extent or more than one channel and to treat peaks which are not spikes in the power spectral estimate. In this experiment a combination of two hybrid window were used to clear channels 35 and 36 . The final hybrid window was constructed from windows 2 , 6 and 9 from Table 4.1. The respective coefficients were . 3533488, .0501631 , and . 5965287. The clearing of channels 35 and 36 was successful (Figure 5.6). A similar result is achieved in this work and is shown in chapter 5.


## METHOD AND ALGORITHMS FOR CALCULATIONS

The method, at this time, is based on the assumption that we have some knowledge of the type of power spectral estimate and that portion relative to a large peak in which we need to consider the effects of windowing. For example, in the results reviewed in chapter 3 we set up the spectrum such that we knew channel 31 contained a dominant peak. We can model this response independently and design the appropriate hybrid windows to clear certain channels relative to the peak from the effect of this response.

The exact method employed here utilizes a combination of hand and computer calculation. The first step in the method involves creating the model spectrum. For example, in the first experiments we placed a unit spike in a channel. Upon the creation of the model input each window is scaled to unity and applied to the model data. This is done by a program of the type listed in Appendix 1. These windows are listed in Table 4.1 and graphically represented in Figures 4.1 thru 4.11. The numeric output is listed for each window result. The actual calculation by the algorithm is as follows:

1) Input window number (based on Table 4.1)
2) Input of coefficients
3) Algorithm multiplies windows by coefficients
4) Algorithm inverse Fourier Transform of input
5) Algorithm multiplies windows by inverse transformed input

## 6) Algorithm Fourier transform result to frequency domain.


#### Abstract

The output represents the numeric value for each frequency domain channel. From this a certain channel will be selected to extinguish. This selection requires for the window combination that there is a negative and a positive result in that particular channel. This restriction is due to the fact we are limiting the value of the coefficients to being positive and less than unity (Smith 1985). With the window and the associated values in the specific channel chosen we can use the following formula to calculate the coefficients.


$$
\begin{equation*}
w(\text { hybrid })=(1 /(a(1)+a(2)))(a(1) W(2)+a(2) W(1)) \tag{17}
\end{equation*}
$$

where

$$
(1 /(a(1)+a(2))) \text {-normalization }
$$

Therefore the coefficient applied to window
$W(1)-a(2) /(a(1)+a(2))$, and for
$W(2)-a(1) /(a(1)+a(2))$.
The following is a numeric example of this calculation. Suppose from the initial output we have values for channel 35 of -.0004536 and .0658214 from windows 1 and 2 respectively. Then in the formula $a(1)=.0004536$ and $a(2)=.0658214$. These values can be inserted into equation 17. The normalization factor is equal to

```
1/(.0004536+.0658214)-15.0886457.
```

Therefore the coefficient for window 1 is equal to $a(2) 15.0886457-(.0004536)(15.0886457)-.0068442$
and for window 2 is equal to

$$
a(1) 15.0886457=(.0658214)(15.0886457)=.9931558 .
$$

These coefficients can then be applied to a spectrum with a response in the channel cleared other than the response resulting from the model input. This hybrid window along with another hybrid window clearing the same channel may be utilized in the construction of new hybrid windows to extinguish further channels. We are able to do this because we know that for the first channel cleared there are zeroes or relatively small numbers resident in this channel. So when we add and then apply by multiplying these hybrid windows, the first channel will remain extinguished. The same process for clearing as outlined above, in which we can utilize the numeric output from the first hybrids to identify a channel with a positive and negative value, is used.

## CHAPTER 5

COMPUTATIONAL RESULTS

The first experiment conducted was to duplicate the results obtained by Smith (1985) in his work, which was reviewed in Chapter 3. The purpose for this exercise was twofold. 1) The algorithm utilized by Smith was transported from a mainframe environment to a personal computer. The Fortran used by the two systems is different. As a consequence of this, some conversion was necessary. 2) The plots produced in the original work displayed the resultant Power Spectral Estimate with the normalized amplitude values from the calculation. In keeping with the literature, a dB-down representation of the results is generated for this work. We wanted to present Smith's results in this manner also.

Figure 5-1 duplicates the result of the window combination used to extinguish channel 35 with the unit spike in channel 31 only. Notice from this scale the extinction of channel 35 is not observable. The rescaling to dB -down (Figure 5.2) clearly shows the extinction at channel 35 . Figures 5.3 and 5.4 show the results of the applied hybrid window to the spectrum consisting of spikes in both channels 31 and 35 . The spike located in channel 35 is $1 / 20$ th the amplitude of the spike in channel 31. Restating the conclusions from the original results, it appears that the spike in channel 35 suffered no adverse effects generated by the spike in channel 31 subsequent to windowing.

Figures 5.5 through 5.8 duplicate the results for the Gaussian input introduced in chapter 3. The numeric objective in this experiment was to clear channels 35 and 36. Again in the amplitude plot of Figure 5.5 , the extinction of these channels is not detectable. In Figure 5.6 channels 35 and 36 are obviously down from their adjacent channels. Figures 5.7 and 5.8 show the results of application of the same hybrid window to the two Gaussian input. Notice the increase of amplitude in channel 34 when comparing Figures 5.6 and 5.8. This increase in amplitude is attributed to the application of the hybrid window to clear 35 and 36 but not 34 and presence of the smaller Gaussian centered in channel 35.

As explained in the methodology in chapter 4 , it is necessary to examine the actual numeric output in order to calculate the coefficients for the windows. In the subsequent experiments these lists of values will be given as tables.

The main purpose of the experimental work is to introduce new data input to this technique in order to examine the robustness thereof.

The first computation is to investigate, for the same magnitude input spikes used in the above, whether the channel clearing is affected by the location of the spikes in the spectrum. The first data set was constructed by placing the large spike in channel 16 and the smaller spike in channel 20. Application of the same scaled windows were applied to these data with the intent of clearing channel 20 from the effects of the spike in channel 16. The results are displayed in figures 5.9 through 5.12. From analyzing figure 5.10 it is evident that the extinction at channel 20 is not significant relative to its two
adjacent channels. This is borne out in the output run listed in table 5.1. Although the extinction was not as complete as it was in channel 31-35 case, the sidelobe level of the spike in channel 16 generated by the windowing did not mask the smaller spike in channel 20 (Figure 5.11 and 5.12). The technique was then applied to these same input data with windows 3 and 9 applied with respective coefficients of . 000002279 and .999997736. Table 5.2 lists the results of the application of these windows to the input of a spike in channel 16 only. Comparing channels 19 through 21 in tables 5.1 and 5.2 demonstrates that the new calculated windows performed better. Figures 5.13 and 5.14 also show that channel 20 has been better cleared relative to its adjacent channels, but once again, it does not appear that the sidelobe effect of the spike in channel 16 is greatly influential out to channel 20. Because of this the application of the windows to the input of spikes in both channels 16 and 20 was not performed, knowing the smaller spike in 20 would be detectable.

The second experiment involves taking different percentages of possible lag values of the autocorrelation as represented in the input power spectral estimate. By placing spikes in the spectrum, as we have in the first experiments, we have represented an autocorrelation function from a very small percentage of total possible lags (re. Chapter 2). Typically in the use and application of the autocorrelation function a greater percentage of lags are used to represent the function. In this work we will examine the effect of taking 20 and 100 percent of the lags. The effect of taking different percentages of lags in itself is a type of windowing. Figures 5.15 and 5.16 show the function domain
weightings representing 20 and 100 percent lags respectively. These weightings are multiplied times the input similar to the other function domain windows to give the effect of using $20 \%$ or $100 \%$ of available lags.

The first analysis was performed with a single spike in channel 31 using 20 percent of the lags. Again the windows will be designed to extinguish channel 35 . Figures 5.17 and 5.18 show the smoothing of the spectrum from a larger number lags. It should be noted here that, when adding the smaller spike to the spectrum, the smoothing effect of taking 20 percent of the lags is present as a result of the presence of the smaller spike also. Consequently, there is a larger effect in channel 35 (Table 5.3). The clearing of channel 35 was accomplished twice using two different combinations of windows. At this point, for the remaining analysis, the $d B$-down representations will only be shown with the exception of Figure 5.24. The first result used windows 1 and 6 (Table 4.1) with respective coefficients of . 312745355 and .687254645 . Figure 5.19 and Table 5.14 give the results of this calculation. The extinction of channel 35 was accomplished along with channel 27 . It should be stated that the calculations were not intended to clear channel 27 , but the clearing appears to be the result of symmetry. Figure 5.20 shows the result of applying the above window combination with the input consisting of the smaller spike in channel 35 . The spike in channel 35 is now detectable apparently with the correct relative amplitude. The second combination of windows applied to this data set was 3 and 7 with respective coefficients of . 450843611 and . 549156389. Figures 5.21 and 5.22 along with Table 5.5 show a similar result as for window
combination 1 and 6. The last part of this experiment was to attempt to clear a channel closer to the main peak. The input is the 20 percent lags of the autocorrelation with the spectrum containing the large spike in channel 31 and the smaller spike in channel 35 . In the previous part of this experiment two window combinations were given to clear channel 35. In order to clear the second channel the method of chapter 4 will be used in conjunction with the fact that the combination of existing hybrid windows can be used as a new hybrid window. An important aspect of methodology stems from this analysis and will be shown in detail in another experiment. It is important when calculating the coefficients for the extinction of the second channel that the presence of the other spike, which may be detected due to the clearing of the first channel, be taken into account. The two hybrid windows used consisted of windows 1, 6, 3 and 7 with coefficients $.1325286, .291230213, .259794658$, and .316446529 respectively. Figure 5.23 and Table 5.6 show that this combination cleared channel 33 and channel 35 , resulting in the preservation of the small spike.

The second experiment involves the use of 100 percent of the lags of the input data. This input is shown in Figures 5.24, 5.25, and Table 5.7. The smoothing effect from using all the lags is greater. Because of this broadening of the main lobe, this data set was used more extensively in this analysis. It is felt that this broadening effect can better demonstrate the robustness of the method. The experiments performed this on this data set required the use of many different window combinations. Therefore, in like manner to the 20 percent lag case, several window combinations were used to clear a single channel.

Table 5.8 gives a list of window combinations and their respective coefficients which clear channel 35 . This table also provides the Figure and Table numbers associated with the results of these calculations. The five window combinations utilized were all successful in clearing channel 35 to at least -70 db . The first four combinations show symmetry in their spectra. The fifth window combination is anomalous in this regard. This variability in symmetry should be investigated but was not in this analysis.

The second a part of this analysis was to clear two consecutive channels or in effect clear a small domain segment. For this we utilized the window combinations of $(3,8)$ and $(7,10)$. The coefficients are as follows: $\quad a(3)=.201309972, \quad a(8)=.193117857, \quad a(7)=.494649063$, a(10).110862395. Figure 5.31 and Table 5.14 demonstrate the successful results of this combination. The concept of clearing domain segments or successive channels has more importance in considering broad lobed input spectra (e.g. broad Gaussians, Smith 1985).

The third part of this analysis refers back to the first experiment. It demonstrates the importance of the methodology of the technique when clearing more than one channel after detecting an event in the first channel cleared. The combination of hybrid windows ( 1,6 ) and $(4,5)$ will be used to clear channel 44 after clearing of channel 35 . Figure 5.32 and Table 5.15 show the extinction of channels 35 and 44 in the presence of a spike in channel 31 with 100 percent of lags. The following coefficients were used: $a(1)=.517188658, a(6)=.007515903$, $a(4)=.307604029, a(5)=.16769141$. Figure 5.33 and Table 5.16 show the results of using the same windows and coefficients applied to the input
spectrum including the smaller spike in channel 35 . The presence of this spike has adversely affected the ability of these specific coefficients to clear channel 44. Recalculation of the coefficients, utilizing the same window combinations, produces the results shown in figure 5.34 and Table 5.17. These recalculated coefficients are $a(1)=.5290774$, $a(6)=.007688673, a(4)=.29979798, a(5)=.163435915$. This procedure has more importance as the second channel to be cleared comes nearer to the secondary peak. Also, the magnitude of the secondary peak influences this result proportionally. The next experiment extends this technique to clearing three channels simultaneously without the presence of a secondary or tertiary response in any of the previously cleared channels. The hybrid window combinations of (1,6) (4,5) (7,10) are used in this case with the following coefficients:

$$
\begin{aligned}
& a(1)=.661751704 \\
& a(6)=.009616726 \\
& a(4)=.198866131 \\
& a(5)=.108412565 \\
& a(7)=.017443401 \\
& a(10)=.003909473
\end{aligned}
$$

Figure 5.35 and Table 5.18 show the success of clearing channels 35 , 39, and 44. Any further extension of this to more channels or larger domain segments will necessitate the automation of the method. This is due to the number of computations to be made.

The last experiment considers three large spikes in the input spectrum at channels 30,35 , and 40 . In this case we considered a small percentage of lags. Using windows 3 and 8 with respective coefficients .785292479 and .214707521 , the attempt to clear channel 33 was made. Figure 5.36 and Table 5.19 show the extinction of channel 33 greater than -70 dB . Figure 5.37 and Table 5.20 show the results of the same window combination with a $1 / 20$ th amplitude spike placed in channel 33. The smaller spike is detected despite the proximity of the two large spikes.

## CHAPTER 6

## CONCLUSIONS AND FUTURE CONSIDERATIONS

The purpose for and technique of channel clearing, in the power spectral estimate by application of linearly combined window functions, has been reviewed. It has been established in previous work, that this method is effective in clearing up to two channels. This work extends the applicability of this method to different input data.

The first calculation demonstrated that the location of the spike in the input spectrum varied the result of channel clearing calculations relative to an equivalent valued spike used in the previous work. Part of this effect could be the result of the transform wrap-around.

The second investigation involved the use of varied percentages of lags relative to the input autocorrelation and its associated power spectral input. It was demonstrated that the effect of broadening the main lobe did not effect the channel clearing capabilities of the method. Several calculations were performed using the greater percentage lag input. It was shown that three channels could be cleared simultaneously. Two consecutive channels or a small domain segment were cleared. The importance of the methodology in the calculations was demonstrated by clearing two channels simultaneously with and without the presence of a smaller spike in the first channel cleared. The final calculation demonstrated the effectiveness of the technique when three large spikes are relatively close together. A single channel between two of the large spikes was cleared.

The method of channel clearing in the power spectral estimate needs further evaluation. Specific to this is the need to examine varying input data, different windows, more arithmetic precision and further clearing of more channels or larger domain segments. In light of the latter it is important that the technique become more automated. This automation could include the capability of channel marching, whereby each channel is cleared successively. Lastly, the implementation of this technique to solve real data situations needs to be investigated.

TABLE 4.1

| WINDOW NUMBER | w(x) for $-1 / 2<x<+1 / 2$ |
| :--- | :--- |
| 1 | 1 |
| 2 | $1-4 x^{2}$ |
| 3 | $\cos (\pi x)$ |
| 4 | $\operatorname{Sin}(2 \pi x) / 2 \pi x$ |
| 5 | $1+2\|x\|$ |
| 6 | $.5+.5 \operatorname{Cos}(2 \pi x)$ |
| 7 | $.54+.46 \operatorname{Cos}(2 \pi x)$ |
| 9 | $1-2\|x\| \operatorname{Cos}(2 \pi x)+(1 / \pi)\|\operatorname{Sin}(2 \pi x)\|$ |
| 10 | $.42+.5 \operatorname{Cos}(2 \pi x)+.08 \operatorname{Cos}(4 \pi x)$ |
| 11 | $1-2\|x\|$ |
| 9 | $.5+.5 \operatorname{Cos}(2 \pi(\|x\|-.15)) / .7$ |

TAELE 5.1

| CHANINEL | ArFFlitule | CHANINEL | AMFLITUDE |
| :---: | :---: | :---: | :---: |
| 1 | 0.0000000168 | 35 | 0.000000015 |
| 2 | 0.6000000108 | 34 | 0.0000000086 |
| $\pm$ | 0.0000000044 | 5.5 | 0.0000000255 |
| 4 | -0.0000000199 | 36 | C. 00000000008 |
| 5 | 0.0000000009 | 37 | 0.0000000061 |
| 6 | 0.0000000029 | 38 | 0.0000000138 |
| 7 | -0.000000017 | 39 | 0.0000000143 |
| 8 | -0.0000000442 | 40 | 0.0000000207 |
| 9 | -0.00000000046 | 41 | -0.0000000074 |
| 10 | -0.0000000176 | 42 | 0.0000000047 |
| 11 | 0.0000000126 | 43 | 0.0000000039 |
| 12 | -0.0000000458 | 44 | 0.0000000121 |
| 13 | 0.0000000557 | 45 | C.0000000048 |
| 14 | 0.079999164 | 45 | 0.0000000050 |
| 15 | 0.49999979 | 47 | 0.0000900153 |
| 16 | 0.8409017 | 4 E | -0.000000037 |
| 17 | 0.49999982 | 49 | 0.0000000298 |
| 18 | 0.079999164 | 50 | 0.0000000596 |
| 19 | 0.0000001012 | 51 | O. 00000000447 |
| 20 | 0.0000000184 | 52 | 0.0000000075 |
| 21 | 0.0000000308 | 53 | -0.0000000054 |
| 22 | -0.00000001ة7 | 54 | 0.0000000006 |
| 23 | 0.0000000534 | 55 | 0.0000000175 |
| 24 | 0.0000000551 | 56 | 0.0000000086 |
| 25 | 0.0000000201 | 57 | 0.0000000068 |
| 20 | 0.0000000086 | 5 | 0.0000000138 |
| 27 | 0.0000000143 | 59 | 0.0000000102 |
| 28 | 0.0000000107 | 60 | 0.0000000103 |
| 29 | 0.0000000075 | 61 | 0.0000000167 |
| צ0 | -0.0000000085 | 62 | -0.0000000102 |
| 31 | -0.0000000024 | 63 | 0.0000000017 |
| 32 | -0.0000000034 | 64 | -0.0000000201 |

TABLE 5．2

| CHinluivel | AMFLITULE | CHANIVEL | AMFLITULE |
| :---: | :---: | :---: | :---: |
| Chindid | 0.0000000022 | E3 | 0.0000000008 |
| 2 | －0．00000 | 34 | －0．00c0000119 |
| $\vdots$ | 0．0000001 | 35 | 0.0000000155 |
| 4 | － 0.0000158 | 30 | －0．0000000606 |
| 5 | 9．E6SE2EE－11 | $\Xi 7$ | 0.0000000132 |
| 6 | －0．0000000035 | 38 | 0.0000000119 |
| 7 | －0．0000006124 | 39 | 0.0000000210 |
| 8 | －0．060000004 | 40 | 0.00000000132 |
| 9 | O． 00000000 | 41 | 0.0000000066 |
| 10 | －0．0000000174 | 42 | －0．000000004 |
| 11 | 0.0000000004 | 43 | 0.0000000079 |
| 12 | －0．0000nose | 44 | 0.0000000118 |
| 15 | 0.00000051 | 45 | 0.0000000102 |
| 14 | 0.079949643 | 46 | 0.0000000946 |
| 15 | 0.49 ¢イテッフio | 47 | 0.0000000123 |
| 10 | 0.64000061 | 48 |  |
| 17 | 0.49959570 | 45 | 0.0000000298 |
| 15 | 0.079995611 | 50 | 0 |
| 19 | $0.00000010 t$ | 51 | 0.000000258 |
| 20 | 0.0000000042 | 52 |  |
| 21 | 0.0000006395 | 5 | 0.0000000021 |
| 22 | －0．0000000152 | 54 | 0.0000000006 |
| 23 | 0.0000000415 | 55 | 0.0000000212 |
| 24 | 0.0000000408 | 56 | 0.0000000042 |
| 25 | 0.0000000204 | 57 | 0.000000117 |
| $2 \cdot$ | 0.00000000044 | 58 | 0.000000097 |
| 27 | 0.000000016 | 59 | －0．0000000013 |
| 28 | 0.0000000065 | 60 | 0.0000000147 |
| $2 \cdot$ | －0．0000000057 | t1 | 0.0000006142 |
| 30 | －0．000000011 | 62 | 0． 00000000019 |
| 31 | －0．0000006135 | 63 | －0．0000000091 |
| 32 | 0． 0000000005 | 64 | －0．0000000174 |

TABLE 5.3

| Chanivel | AMFLITUDE | CHANINEL | AMPLITUDE |
| :---: | :---: | :---: | :---: |
| CHANT | O | S | 0.0032085 |
| 2 | 0 | 34 | 0.0080555 |
| $\cdots$ | 0 | 35 | 0.0082575 |
| 4 | 0 | 36 | 0.0049645 |
| 5 | 0 | 37 | 0.0032585 |
| 6 | 0 | 38 | 0.0041285 |
| 7 | 0 | 55 | 0.0032575 |
| 8 | 0 | 40 | 0.0037895 |
| 9 | 0 | 41 | 0.0032575 |
| 10 | 0 | 42 | 0.0036195 |
| 11 | 0 | 43 | 0.0032585 |
| 12 | 0 | 44 | 0 |
| 13 | 0 | 45 | 0 |
| 14 | 0 | 46 | 0 |
| 15 | 0 | 47 | 0 |
| 16 | 0 | 48 | 0 |
| 17 | 0 | 49 | 0 |
| 18 | 0 | 50 | $\bigcirc$ |
| 19 | 0.008208. | 51 | 0 |
| 20 | 0.036195 | 52 | 0 |
| 21 | 0.0032575 | 53 | 0 |
| 22 | 0.0087895 | 54 | 0 |
| 23 | 0.0082575 | 55 | 0 |
| 24 | 0.0041285 | 56 | 0 |
| 25 | 0.0052585 | 57 | 0 |
| 26 | 0.0049645 | 58 | 0 |
| 27 | 0.0032575 | 59 | 0 |
| 28 | 0.008055 | 60 | 0 |
| 25 | 0.0052585 | 61 | 9 |
| 30 | 0.0473235 | 62 | 9 |
| 31 | 1 | 63 | 0 |
| 32 | 0.0478235 | 64 | 6 |

$\because$ AELE 5.4

| CHAANINEL | AFIFLI TUUE |
| :---: | :---: |
| 1 | －0．0087503614 |
| 2 | 0．0087505623 |
| $\pm$ | －9．087503213 |
| 4 | 0.0087505625 |
| 5 | －0．008750359 |
| 6 | 0.006750306 |
| 7 | －0．0087505 57 |
| 8 | $0.008750 \times 577$ |
| 9 | －0．0087505418 |
| 10 | 0.008750 .688 |
| 11 | －9．008750558 |
| 12 | 9．0087503023 |
| 13 | －0．0087503716 |
| 14 | 0.008750595 |
| 15 | －0．008750345 |
| 16 | 0.0087502605 |
| 17 | －0．0087505577 |
| 15 | 6． 0098700728 |
| 19 | －0．0052250181 |
| 20 | 0.015740527 |
| 21 | －0．001428166 |
| 22 | $0.01546 \leq 722$ |
| 23 | －0．0175ج10？ |
| 24 | 0.0164096 |
| ご | －© OUS4E1E84 |
| 20 | 0.01750654 |
| 27 | －0．00000075 |
| 28 | 0．021564264 |
| 25 | 0.014556956 |
| 30 | 0.41562104 |
| 3 1 | 1．336518 |
| 32 | 0.41562104 |


| CHANNEL | ArFFITIUDE |
| :---: | :---: |
| E．3 | 0． 014550956 |
| $\pm 4$ | 0.02156420 |
| ミ5 | 0.0000000596 |
| E6 | 0.017506585 |
| こ | －0．0013481798 |
| 38 | 0.016495135 |
| 39 | －0．0017531442 |
| 40 | 0.015963767 |
| 41 | －0．0019281527 |
| 42 | 0.015740524 |
| 43 | －0．0032289964 |
| 44 | 0.0098700784 |
| 45 | －0．008750．511 |
| 46 | 0.0087503547 |
| 47 | －0．0087503772 |
| 48 | 0.008750352 |
| 49 | －0．0087503797 |
| 50 | 0.008750805 |
| 51 | －0．00875036玉 |
| 52 | 0.0087503625 |
| 53 | －0．0087503614 |
| 54 | 0.008750351 |
| 55 | －6．087503881 |
| 56 | 亿． 008750367 |
| 57 | －0．00E750さ62E |
| 58 | 0.0087503521 |
| 59 | －0．0087503642 |
| 69 | 0.0087505688 |
| 61 | －0．0087503502 |
| 62 | 0.0087503726 |
| 6.5 | －0．00E75056E8 |
| 64 | 0.008750 .5688 |

[^0]TAELE 5．5

| CHAINNEL | Al＇iF＇L I $T$ UDE |
| :---: | :---: |
| 1 | －0．0015\％20413 |
| 2 | 0.0015730328 |
| $\pm$ | －0．001576548日 |
| 4 | 0.0015815215 |
| 5 | －0．00158858． |
| 6 | 0.0015780169 |
| 7 | －6． 01016948884 |
| 8 | O．0016247729 |
| 9 | －0．001642806 |
| 10 | 0.91604705 |
| 11 | －0．001691038？ |
| 12 | O．00172288日1 |
| 13 | －0．0017615802 |
| 14 | 0.001804100 C |
| 15 | －0．00185 08 ¢ |
| 10 | －101547007\％ |
| 17 | －0．0020815721 |
| 18 | $0.003439 \% 191$ |
| 15 | $0.003100463 \%$ |
| 20 | G．0093984こここ |
| 21 | 0.0042746561 |
| 22 | $0.01001584 \%$ |
| $2 \square$ | $0.003820 .24 \%$ |
| 24 | 0.0113945 |
| 25 | $0.002802565 \%$ |
| 26 | 0.01481205 |
| 27 | －0．00000ミ1461 |
| 28 | 0.027011868 |
| 29 | －0．010478E45 |
| 30 | 0.49984768 |
| $\checkmark 1$ | 1.2078052 |
| 32 | 0.49984694 |

CHÁNIVEL．
ショ
34
5
30
37
38
84
40
41
42
43
44
45
46
47
48
45
50
51
52
53
54
55
56
$5 \%$
58
59
60
61
62
6.3

64
áMFLITUDE
－0．01047701E
0.027605567
$0.0148082=8$ 0.002607214 0.011588922 0.0038268475 0.010008026 0.0042828671 0.0093892235 0.0031100449 0.0054286149
$-0.0020051007$
0.0018559596 $-0.001854112$ 0．0017920926
－．O1745e622
0.0017033712
$-0.0016697475$
0.0016414967
$-0.0016175528$
0.001597214
$-0.0015799081$
0.015652351
－0．001552757！
0.0015423286
$-0.001533656$
0.015265713
－0． 015 －0け
0.0015106529
$-0.0015156404$
0.0015118601

TAELE 5.6

|  | CHAMNIEL | AMFLI TUDE | CHANNEL | AMFLITUDE |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 | -0.0048152702 | ES | -0.000000021 |
|  | 2 | O. 6448158318 | 34 | 6. 0.45085905 |
| - | E | -0.0048175207 | Ef | 0.055507839 |
|  | 4 | 0.09482003 | 36 | 0.056541209 |
|  | 5 | -0.0048:46735 | 37 | 0.0011855346 |
| - | 6 | 0.0648202528 | 38 | 0.014625136 |
|  | 7 | -0.0048575439 | 39 | 0.0014689919 |
|  | 8 | 0.004346074 | 40 | 0.013229766 |
|  | 9 | -0.0048567597 | 41 | 0.0016927826 |
| - | 10 | 0.0048696799 | 42 | 0.01266754 |
|  | 11 | -0.0048852339 | 43 | 0.0004212931 |
|  | 12 | 0.0049040862 | 44 | 0.0064487387 |
| - | 13 | -0.0049268907 | 45 | -0.0051310528 |
|  | 14 | 0.0049549919 | 40 | 0.0050476063 |
|  | 15 | -0.0049908756 | 47 | -0.0049959356 |
| - | 16 | 0.005058044 | 48 | 0.004957155 |
|  | 17 | -0.0051168.761 | 49 | -0.00492026.99 |
|  | 18 | 6.0064271176 | 50 | 0.0049009654 |
|  | 19 | 0.0004415253 | 51 | -0.004 0799003 |
| - | 20 | 0.012593273 | 52 | 0.0048622494 |
|  | 21 | 0.0017247929 | 5. | -0.004847E151 |
|  | 22 | 0.013049007 | 54 | 0.0048346845 |
| - | 23 | 0.00155426 .34 | 5 | -0.004825852S |
|  | 24 | 0.014043696 | 50 | 0.0048148078 |
|  | 25 | 0.0011129705 | 5.7 | -0.004807068 |
|  | 20 | 0.016499601 | 50 | 0.0048006219 |
|  | 27 | 0.0000582112 | 59 | -0.004795194t |
|  | 2 E | 0.02568553 | 60 | 0.0047908537 |
|  | 24 | 0.0001712512 | 61 | -0.0047873775 |
| - | 30 | 0.46480489 | 62 | 0.0047847191 |
|  | $\pm 1$ | 1.2619438 | 63 | -0.0047829049 |
|  | 32 | $0.46447 E 45$ | 64 | 0.0047817547 |

TAELE 5.7

| CHANTVEL | AMFLitude | CHANNEL | AMFLITUDE |
| :---: | :---: | :---: | :---: |
| 1 | 0 | 53 | 0.0000915 |
| 2 | 0 | 34 | 0.0456595 |
| $\pm$ | 0 | Ј5 | 0.0000935 |
| 4 | 0 | 36 | 0.0163905 |
| 5 | 0 | 37 | 0.0000945 |
| 6 | 0 | 30 | 0.0088305 |
| 7 | 0 | 59 | 0.0000955 |
| 8 | 0 | 40 | 0.0050195 |
| 9 | 0 | 41 | 0.0000965 |
| 10 | 0 | 42 | 0.0033475 |
| 11 | 0 | 43 | 0.0000975 |
| 12 | 0 | 44 | 0 |
| 13 | 0 | 45 | 0 |
| 14 | 0 | 45 | 0 |
| 15 | 0 | 47 | 0 |
| 16 | 0 | 48 | 0 |
| 17 | 0 | 49 | 0 |
| 16 | 0 | 50 | 0 |
| 19 | 0.0000975 | 51 | 0 |
| 20 | 0.0038475 | 52 | 9 |
| 21 | 0.0000965 | 53 | 0 |
| 22 | $0.00501 \% 5$ | 5.4 | $\because$ |
| 23 | 0.0009955 | 55 | $\dot{0}$ |
| 24 | 0.0053305 | 50 | 0 |
| 25 | 0.0000945 | 57 | 0 |
| 26 | 0.0163905 | 58 | 0 |
| 27 | 0.0000935 | 59 | 0 |
| 20 | 0.0456595 | 60 | 0 |
| 24 | 0.0000915 | 61 | 0 |
| 3 | 0.4115925 | 62 | 0 |
| 31 | 1 | 63 | 0 |
| 32 | 0.4115925 | 64 | O |

$$
\begin{array}{cc}
\text { HYERID WINDOW } & \text { WINDOW WUFIEEF } \\
1 & 1 \\
2 & 6 \\
& 7 \\
\vdots & 7 \\
& 4 \\
4 & 5 \\
& 3 \\
5 & 7
\end{array}
$$

TAELE 5.9

| CHFINNEL | AIFIFL I TUDE |
| :---: | :---: |
| 1 | －0．000030172 |
| 2 | $0.0006 \pm 0194$ |
| $\Xi$ | －0．0006300242 |
| 4 | 0.0006301751 |
| 5 | －0．0006301632 |
| 6 | 0.0006301354 |
| 7 | －0．0006S01847 |
| 8 | 0.000631642 |
| 9 | －6．0006301193 |
| 10 | $0.000630170 \leq$ |
| 11 | －0．0006301042 |
| 12 | 0．0006501827 |
| 13 | －0．0006301479 |
| 14 | 0.0006501561 |
| 15 | －0．0006501106 |
| 16 | 0.0006301499 |
| 17 | －0．0006S0165 |
| 18 | 0.0006308665 |
| 14 | －0．000412513 |
| 20 | 0.0072785877 |
| 21 | －0．0003786093 |
| 22 | 0.010598552 |
| 25 | －0．00§45い了55 |
| 24 | O．017173154 |
| 25 | －0．0002654459 |
| 26 | 0.03517773 |
| 27 | －0．000000130 |
| 28 | 0.0412964 .4 |
| 25 | 0.002 E 264024 |
| 50 | 0.82508206 |
| § 1 | 1.990941 |
| 32 | 0.82508206 |

CHANINEL
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$-0.0003786093$
0.010598552
$-0.000345055$
0.017173154
$-0.0002054459$
0.03517775
$-0.000000136$
0.041296424
0.82508206
0.82508206
$64 \quad 0.0006301457$

Cratworn
Of Pax sinnery

TABLE 5.10

| CHANINEL | AMflitude | CHANNEL | AMFLitule |
| :---: | :---: | :---: | :---: |
| 1 | -0.0000301595 | 3 S | 0.0028265982 |
| 2 | 0.0006301828 | 34 | 0.091256464 |
| $\pm$ | -0.000030054 | 35 | -0.0000001172 |
| 4 | 0.0006501713 | S6 | 0.035177735 |
| 5 | -0.0006301547 | $\Xi 7$ | -0.000265445 |
| 6 | $0.0006301 \geq 91$ | 38 | 0.017173231 |
| 7 | -0.0006301939 | 39 | -0.0003448291 |
| 8 | 0.0006301592 | 40 | 0.01059808 |
| 9 | -6.0006301218 | 41 | -0.0003786082 |
| $10^{\circ}$ | 0.0006801683 | 42 | 0.0072786187 |
| 11 | -0.0006300929 | 43 | -0.0004125502 |
| 12 | 0.0000301932 | 44 | 0.0006308747 |
| 13 | -0.0006301431 | 45 | -0.0006-61298 |
| 14 | 0.0006301478 | 46 | 0.000630154 |
| 15 | -0.0006.301464 | 47 | -0.0006 0.01718 |
| 16 | 0.0006501695 | 48 | 0.0006301422 |
| 17 | -0.000630149 | 49 | -0.0006301503 |
| 12 | 0.0006 .308007 | 50 | 0.0006301635 |
| 19 | -0.0004125077 | 51 | -0.0006 0.00571 |
| 20 | 0.0072785805 | 52 | 0.0006301824 |
| 21 | -0.0003786188 | 53 | -0.0006301414 |
| 22 | 0.010598598 | 54 | 0.0006301624 |
| 23 | -0.0005450203 | 55 | -0.0000301602 |
| 24 | $0.01717 \pm 145$ | 56 | 0.0000301712 |
| 25 | -0.0002054413 | 57 | -0.0006301574 |
| 20 | 0.033177722 | 58 | 0.0006301671 |
| 27 | -0.0000001421 | 54 | -0.0000301196 |
| 28 | 0.091250449 | 60 | 0.0006301573 |
| 29 | 0.0028268789 | 61 | -0.0006301436 |
| 30 | 0.82508206 | 62 | 0.0006301451 |
| $\bigcirc 1$ | 1.990941 | 63 | -0.0000301208 |
| 5 | 0.82509200 | 64 | 0.0006301316 |

TAELE E． 11.

| CHANNEL | AMFLITUUE | CHANINEL | AMFLI TUDE |
| :---: | :---: | :---: | :---: |
| 1 | －0．0008384196 | 3． | 0.030994404 |
| 2 | 0.0000694477 | 34 | 0.063495096 |
| $\pm$ | －0．0008 76982 | ミ5 | 9 |
| 4 | $0.000060 \leq 489$ | こ6 | 0．022702038 |
| 5 | －0．000851211 | 57 | －0．00005306 14 |
| 6 | 0.00041145 | 58 | 0.011076647 |
| 7 | －0．0008749869 | 59 | －0．0006569907 |
| 9 | O．0000989249 | 40 | 0.0071865907 |
| 4 | －0．0009114152 | 41 | －0．0005918291 |
| 10 | －0．0000571538 | 42 | 0.0049963823 |
| 11 | －0．0009047381 | 4さ | －0．000898215t |
| 12 | －0．0001006394 | 44 | －0．0006964591 |
| $1 \Xi$ | －0．0010418404 | 45 | －0．0013124065 |
| 14 | －0．0002110194 | 46 | －0．003632295 |
| 15 | －0．0011546231 | 47 | －0．0011379927 |
| 10 | －6． 000 ¢78375 | 48 | －0．0001928792 |
| 17 | －6．001こ2628．7 | 49 | －0．0010219279 |
| 18 | －6．0007090171 | 50 | －0．0000847636 |
| 19 | －0．0004096047 | 51 | －0．0004410232 |
| 20 | O．O049861227 | 52 | －0．000011332 |
| 21 | －6．0006009785 | 5.3 | －0．00088こ285－ |
| 22 | 9．00717E45＇1 | 54 | 0.0000400222 |
| 23 | －0．00664264 | 55 | －0．0008414645 |
| 24 | 0.01107044 | 56 | Q． 0000777696 |
| 25 | －0．0006962649 | 57 | －0．000811511 |
| 26 | 0.022697762 | 58 | 0.0001041191 |
| 27 | －0．00000．4742 | 59 | －0．0007900024 |
| 28 | 0.063493118 | 60 | 0.0001222219 |
| 29 | 0.030992692 | 61 | －0．0007757854 |
| 30 | 0.88186356 | 62 | 0.000155511 |
| 31 | 1．9435923 | 63 | －0．0007676826 |
| 32 | 0.89186419 | 64 | 9．0001390325 |

TAELE 5．12

| CHANNJEL | ArIFLITULE | CHANNEL． | AHFLITUDE |
| :---: | :---: | :---: | :---: |
| 1 | －0．0006こ75484 | $3:$ | －0．0041795401 |
| 2 | 0.006629549 | 34 | $0.10516 \leq 8$ |
| $\pm$ | －0．0006271469 | 55 | 0．0006011\％2 |
| 4 | 0．0066292728 | 36 | 0.034970 .65 |
| 5 | －6．000626 80 | 37 | －0．0001949549 |
| 6 | 0.000628546 | 38 | 0.017589815 |
| 7 | －0．0000240361 | $\pm 5$ | －0．0003095506 |
| 8 | 0.000526748 | 40 | 0.010751481 |
| 4 | －0．0006214118 | 41 | －0．0003590322 |
| 10 | 0.000628291 | 42 | 0.0075338859 |
| 11 | －0．0000154042 | 43 | －0．0003448787 |
| 12 | $0.000617 ヲ 120$ | 44 | 0.0005664784 |
| 15 | －0．0000037591 | 45 | －0．0005002590 |
| 14 | 6.000057019 | 46 | 0.000577 .3117 |
| 15 | －0．0065\％8－07 | 47 | －0．00057785； |
| 16 | O．005774596 | 48 | 0.0006055641 |
| 17 | －6．0006030 | 49 | －0．0060ごs96 |
| 18 | 0.00056585 | 50 | 0．00061777 |
| 15 | －0．0008449187 | 51 | －0．0006154683 |
| 20 | 0.0073 .59711 | 5． | 0.00062 －6124 |
| 21 | －0．00039908：4 | 5. | －0．00621489\％ |
| 22 | O． 0107514 E | 54 | 0.0066266517 |
| 2 | －0．000こ0ヶ80́ | 55 | － 60062479 |
| 24 | G．017589815 | 56 | O．OOOS2E：113 |
| 25 | －6．001949752 | 57 | －0．0006267225 |
| 26 | 0.054776389 | 58 | 0.0006292514 |
| 27 | －0．0000001007 | $5 \%$ | －0．0006278247 |
| 28 | 0.10816382 | 60 | O．O006297429 |
| 29 | －0．0041795578 | 61 | －0．0006284985 |
| 30 | 0.60595249 | 62 | 0.00063008 |
| $\pm 1$ | 2.0041552 | 63 | －0．006287865 |
| 32 | 0.80595243 | 64 | 0.00035025 |

TAELE 5.13

| CHANNEL | AMFILITUDE | CHANIVEL | AMFLITUDE |
| :---: | :---: | :---: | :---: |
| 1 | -0.01612797 | 35 | 0.30148342 |
| 2 | -0.013706565 | 34 | 0.0743815 |
| $\pm$ | -0.0089507475 | 35 |  |
| 4 | -0.006047863 | St | -0.022786975 |
| 5 | -0.00805510t: | 37 | -0.01013275 |
| 6 | -0.012974271 | 38 | 0.01276414 |
| 7 | -0.016506637 | 35 | 0.020240535 |
| $\varepsilon$ | -0.01492344 | 40 | 0.0096400874 |
| 4 | -0.0096571175 | 41 | -0.01097194 |
| 10 | -0.0052116001 | 42 | -0.02322383: |
| 11 | -0.0001881421 | 43 | -0.021978475 |
| 12 | -0.011894779 | 44 | -0.0098132491 |
| 13 | -0.617562045 | 45 | 0.001688645 |
| 14 | -0.017429573 | 46 | 0.0017857002 |
| 15 | -0.011094769 | 47 | -0.008\%619368 |
| 16 | -0.008588052 | 48 | -0.020197598 |
| 17 | -0.0021740822 | 49 | -0.023196142 |
| 18 | -0.0089588. 14 | 50 | -0.0156.6812 |
| 15 | -0.017194647 | 51 | -0.0048406710 |
| 20 | -0.019569034 | 52 | 0.0002371874 |
| 21 | -0.010338039 | 53 | -0.0046547656 |
| 22 | 0.0057766885 | 54 | -0.014672683 |
| 23 | 0.015224451 | 55 | -0.02139 139 |
| 24 | 0.010702341 | 56 | -0.019007541 |
| 25 | -0.007413416t | 5.7 | -0.010122581 |
| 20 | -0.017665148 | 58 | -0.0022506194 |
| 27 | 0.0032037748 | 59 | -0.002312514 |
| 28 | 0.072982423 | $6{ }^{\circ}$ | -0.0098404614 |
| 29 | 0.29067765 | 61 | -0.018509429 |
| 30 | 1.0420661 | 62 | -0.020614322 |
| $\Xi 1$ | 1.618788 | 63 | -0.01469534 |
| 32 | 1.0462196 | 64 | -0.0056453291 |

TAELE 5.14

| CHANINEL | AmFlitude | CHANAJEL | AMFLITUDE |
| :---: | :---: | :---: | :---: |
| 1 | -0.610013235 | 33 | 0.1809028 E |
| 2 | -0.0080511672 | 34 | 0.087708324 |
| S | -0.0056072352 | 35 | -0.0000000596 |
| 4 | -0.0034138164 | 36 | -0.0000000298 |
| 5 | -0.0051245769 | 37 | -0.0062124208 |
| 6 | -0.0076082028 | 38 | $0.0146 \pm 151$ |
| 7 | -0.010241375 | 39 | 0.012164063 |
| 8 | -6.0087890709 | 40 | 0.010078502 |
| 9 | -0.0060926247 | 41 | -0.0068010194 |
| 10 | -0.0029090408 | 42 | -0.01116678 |
| 11 | -0.0059897766 | 43. | -0.013444284 |
| 12 | -0.0069586977 | 44 | -0.0057185046 |
| 15 | -0.010872209 | 45 | 0.0008251369 |
| 14 | -0.01031487 | 46 | 0.0013090054 |
| 15 | -0.0069459821 | 47 | -0.005533443 |
| 16 | -0.001944795s | 48 | -0.01199099 |
| 17 | -6.0015138001 | 49 | -0.014285.719 |
| 18 | -0.0052011646 | 50 | -0.0092434743 |
| 19 | -0.01054764 | 51 | -0.0031738635 |
| 20 | -0.0083482042 | 52 | 0.0003896514 |
| 21 | -0.0064172242 | 53 | -0.0030636846 |
| 22 | 0.0077392290 | 54 | -0.0086372625 |
| 23 | 0.0090963 EE | 55 | -0.013200269 |
| 24 | 0.013419379 | 50 | -0.011261439 |
| 25 | -0.0045658275 | 57 | -0.006:766045 |
| 25 | 0.0081013507 | 58 | -6.0011674292 |
| 27 | 0.00153966\% | 59 | -0.0016479371 |
| 28 | 0.086801067 | 60 | -0.00557100602 |
| 29 | 0.17799297 | 61 | -0.011455595 |
| 80 | 0.94892192 | 62 | -0.012233058 |
| 31 | 1.7708108 | 63 | -0.009146275 |
| 32 | 0.951437 | 64 | -0.0051696965 |

TAELE 5．15

| CHANNEL | AMFLI TUDE |
| :---: | :---: |
| 1 | －0．0907267897 |
| 2 | 0.000363213 |
| E | －0．0007288065 |
| 4 | 0.000359521 |
| 5 | －0．0007352171 |
| 6 | 0.0005502349 |
| 7 | －0．0007465136 |
| 8 | 0.00035581 |
| 9 | －0．0007638502 |
| 10 | $0.000 \% 129798$ |
| 11 | －0．0007891749 |
| 12 | 0.0002799801 |
| 13 | －0．0008258244 |
| 14 | 0.000280844 |
| 15 | －0．0006754409 |
| 16 | 0.00150803 |
| 17 | －0．0909610341 |
| 18 | －0． 000009596 |
| 17 | －0．000648750t |
| 20 | 0.0061890148 |
| 21 | －0．0048428こ1 |
| 22 | 9．00947 0 ¢66 |
| 23 | －0．0004567655 |
| 24 | $0.0145577 \%$ |
| 25 | －0．0004711586 |
| 26 | 0.02815664 |
| ご7 | －0．0000016817 |
| 28 | 0.078081675 |
| 29 | 0.016213726 |
| 30 | 0.85206997 |
| 31 | 1.4684565 |
| 32 | 0.85207033 |

CHANNEL AMFLITUDE
玉3． 0.016214548
340.078082889
550.0000000596
$36 \quad 0.028198689$
玉7－0．0004686741
0.014560707
$-0.00049 .2102$
0.0089769382
$-0.0004749348$
0.006193867
－0．00064E3616
0.0000000065
－0． 0009544246
0.0001579961
$-0.0008715 .94$
0.0002369747
－0．000 63.554
0.0002902792
$-0.000779046$
0.000 .252768
$-0.0007504617$
0.0003499581
$-0.0007305867$
0.000567618
$-0.0007162648$
0.0003801139
$-0.0007061326$
0.0005887051
$-0.006944074$
0.0005940 .842
$-0.006955029$ 0.0005967483

TAELE 5．10

| CHANISEL | AMFLITUDE |
| :---: | :---: |
| 1 | －0．0007717．16 |
| 2 | O．0003937155 |
| $\pm$ | －0．00077ミ800 |
| 4 | 0.0005893094 |
| 5 | －0．0007604 27 |
| 6 | 0.000579650 |
| 7 | －0．0907920204 |
| $\varepsilon$ | 0.00545715 |
| 9 | －0．0008054162 |
| 10 | 0．00941583 |
| 11 | －0．0006259\％29 |
| 12 | 0.000976256 |
| 13 | －0．0098755864 |
| 14 | 9． 0002506777 |
| 15 | －0．0009285468 |
| 16 | 0.0001751472 |
| 17 | －0．0019121401 |
| 18 | 0.000147671 |
| 19 | －0．000910545 |
| 20 | $0.00035565 \%$ |
| 21 | －0．005\％2466： |
| 22 | 0.0091902679 |
| 25 | －0．0005552072 |
| 24 | 0.014859481 |
| 25 | －0．00051194c\％ |
| ごo | 0．028640091 |
| 27 | －0． 0 00404716 |
| 28 | 0.07880754 |
| 29 | 0.016175529 |
| 30 | 0.85347605 |
| $\leq 1$ | 1．9684162 |
| 32 | 0.85546057 |

CHFINIVEL
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$\pm 6$
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$\therefore 9$
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520.0003476705
$53-0.000801898$
540.0003746022
$55-0.0007801768$
$56 \quad 0.000395804$
57 －0．0007646017
$58 \quad 0.000407564$
57 －0．0以うESE4E
600.0004100675
$61-0.0007463661$
020.0004224689
$63-0.0007421259$
640.00942531 Ó

TAELE 5.17

| CHFININEL | AMFL I TUDE |
| :---: | :---: |
| 1 | －0．0007692082 |
| 2 | 0.000400869 |
| $\leq$ | －0．000771243 |
| 4 | 0.0003965298 |
| 5 | －0．00077770．5 |
| 6 | 0.000387824 |
| 7 | －0．0007890451 |
| 8 | 0．0003724316 |
| 9 | －0．00080t4367 |
| 10 | 0.00035002 |
| 11 | －0．0008 17996 |
| 12 | 0．000こ1690\％ |
| 13 | －0．008 $65 \times 5$ |
| 14 | 0.0002672450 |
| 15 | －0．0009220875 |
| 16 | 0.0001877875 |
| 17 | －0．0010025612 |
| 18 | 0.0000514850 |
| 19 | －0．000684932 |
| 20 | 0.0065850107 |
| 21 | －0．0005197326 |
| 22 | 0.007225596 |
| 23 | －0．0005312714 |
| 24 | 0.014927295 |
| 25 | －0．0005066058 |
| 26 | 0.02876861 |
| 27 | －0．0000407096 |
| 28 | 0.079146251 |
| 25 | 0.015836435 |
| $\because 0$ | 0.85279759 |
| こ1 | 1.968987 |
| 32 | 0.85529252 |

## CHANNEL

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[^1]TAELE G． 10

| CHANNEL | AMF＇LITUDE |
| :---: | :---: |
| 1 | －0．00102ge4y |
| 2 | 0.0001517812 |
| צ | －0．0008715268 |
| 4 | 0． 0001258 |
| 5 | －0．0008565409 |
| 6 | 0． 0001567075 |
| 7 | －0．0010444112 |
| 8 | 0.0001074481 |
| 9 | －0．0009093．4E |
| 10 | 0.0005008 |
| 11 | －0．0008516007 |
| 12 | 0.000156 |
| 13 | －6．0011182111 |
| 14 | －0．00001 59704 |
| 15 | －0．00101475E． |
| 10 | 勺．002361898 |
| 17 | －0．0008770221 |
| 18 | 0.0000143998 |
| 19 | －0．0009285\％ |
| 20 | 0.006622498 |
| 21 | －0．0006596057 |
| 22 | 0.0094446745 |
| ご | －0．0001106963 |
| 24 | 0.015 .344099 |
| 25 | －0．00551095c |
| 26 | 0.028871785 |
| 27 | 0.0000672724 |
| 28 | 0.082362017 |
| 24 | 0.017755866 |
| 30 | 0.84716290 |
| $\bigcirc 1$ | 1．468445\％ |
| 32 | 0.84725189 |

CHANNINL
3. 34 $\pm 5$ ふ6 3 38 39 40
41
42
4.3

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AMFLITUDE
0． 017855034
0.082592663
0.028763741
$-0.0006075463$
6．015388817
$-6.000001227$
0．0095こ9754
－0．00067035．6
0.0059259652
$-0.001022295$
$-0.0000000149$
$-0.0007902887$
0.0003495654
$-0.009548459$
－0． 000107477
$-0.0012 こ 23714$
0． 000062424
$-0.0008154805$
0.0004247915
$-0.00075384$
0.0001222414
$-0.001138442$
0.0000411075
－O．OOBEEE15e
0.0004072665
－0． 0007151581
0.0002504764
$-0.0010566916$
0.0000239295
－0．00097こ657
0．0003452759

TAELE 5．15

| CHAINNEL | AIP1FLITUDE | CHATNNEL． | Aliflitude |
| :---: | :---: | :---: | :---: |
| 1 | O． 01402750 ， | ご | －0．O00000612 |
| 2 | －0．0149282\％7 | 34 | 0.190 .610 |
| 3 | 0.01403409 | 35 | 1．6371415 |
| 4 | －0．014041785 | 36 | $0.1903598=$ |
| 5 | 0.01405165 | $\underline{3}$ | 0.00000 0 064 |
| 6 | －0．014 076 GE | 38 | －0．03625989 |
| 7 | 0.014092386 | 39 | 0.22756406 |
| 8 | －0．014115675 | 40 | 1.597275 |
| 9 | 0.014149866 | 41 | 0．23479062 |
| 10 | －0．01418541 | 42 | －0．057561494 |
| 11 | 0.014234834 | 43 | $0.0 \pm 9612744$ |
| 12 | －0．014288335 | 44 | －0．026918054 |
| 13 | O．014359578 | 45 | 0.0225711 |
| 14 | －0．01443日513 | 46 | －0．019405698 |
| 15 | 0.014545067 | 47 | 0.017918076 |
| 16 | －0．014665774 | 48 | －0．016925864 |
| 17 | 0.61483250 | 49 | 0.01618080 .4 |
| 18 | －0．01502596 | 50 | －0．01557018 |
| 19 | 0.015 .34765 | 51 | 0.015 .38654 |
| 20 | －0．0156－9521 | 52 | －0．015065119 |
| 21 | O．016152E7 | 53 | 0.014673256 |
| 22 | －0．016800822 | 54 | －0．014710286 |
| 23 | 0.0178958 | 55 | 0.01459406 |
| 24 | －0．019385ここ9 | 56 | －0．01449209\％ |
| 25 | 0.022 .52934 | 57 | 0.014418521 |
| 26 | －0．026902139 | 58 | －0．014352719 |
| 27 | $0.03979801 \%$ | 59 | 0.01430596 |
| 28 | －0．057549351 | 60 | －0．014264372 |
| 29 | 0.23478037 | 61 | 0.0142 －615 |
| 36 | 1.597302 | 62 | －0．01421174 |
| $\pm 1$ | G． 2275579 | $6 \leq$ | 0.014198057 |
| 32 | －0．9ごこち5151 | 04 | －0．014187949 |

TAELE 5.20

| CHANNEL | AMFLITUDE | CHANNEL | AMFLITUDE |
| :---: | :---: | :---: | :---: |
| 1 | 0.01532643 .4 | S3 | 0.07948482 |
| 2 | -0.013227075 | 34 | 0.20238938 |
| S | 0.013532912 | 35 | 1.0540545 |
| 4 | -0.01334006 | 36 | 0.19246924 |
| 5 | 0.013552912 | $\Xi 7$ | -0.0014325902 |
| 6 | -0.013367363 | 38 | -0.055075802 |
| 7 | 0.013588529 | 39 | 0.226541 |
| 8 | -0.013411701 | 40 | 1.5982311 |
| 9 | 0.013443629 | 41 | 0.23591546 |
| 10 | -0.013478592 | 42 | -0.056725346 |
| 11 | 0.01352516 | 43 | 0.059006829 |
| 12 | -0.013576454 | 44 | -0.026132951 |
| 12 | 0.013644875 | 45 | 0.021602865 |
| 14 | -0.013720875 | 46 | -0.018649761 |
| 15 | 0.013823727 | 47 | 0.017172426 |
| 16 | -0.013939913 | 48 | -0.01608807 |
| 17 | 0.014161580 | 49 | 0.015449688 |
| 18 | -0.014268158 | 50 | -0.0149443E |
| 19 | 0.014559092 | 51 | 0.014617355 |
| 20 | -0.014883511 | 52 | -0.014345434 |
| 21 | 0.015384625 | 53 | 0.014158753 |
| 22 | -0.016015731 | 54 | -0.013998868 |
| 23 | 0.017087425 | 55 | 0.013884398 |
| 24 | -0.018549023 | 56 | -0.013784283 |
| 25 | 0.021477759 | 57 | 0.013712084 |
| 26 | -0.025964499 | 58 | -0.013647805 |
| 27 | 0.038774937 | 59 | 0.013602111 |
| 28 | -0.056363199 | 60 | -0.013561386 |
| 29 | 0.23354444 | 61 | 0.013535884 |
| 30 | 1.5994115 | 62 | -0.013510015 |
| 3.1 | 0.22451079 | 63 | 0.013476719 |
| 32 | -0.024277404 | 64 | -0.013485905 |











( $x$ ) $m$




(x)M


(x) $m$


umop-ap









UMOP-EP







30nunawy

30nınaw



C-2






FIGURE 5.26













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```
    DIMENS10NA(E0),G(20),W(257), F(E13), FT(S13), X(257)
    F'FINT 10
    FOFMHT; ENTEFF NU. OF FAFITITIUNS ;
        FEAL * N
        FFINT 2G
        FOF|MAT(ENTEF NF ',
        FEEAD *, TM
        FI=3.1413%2%
        T-O.
        FFiNT O
    SO FOFMGT: ENTEF NÖ. OF WJNLOLWS ;
        FiEABM *, M
        [0.50, 1-1,M
        FFINT * 'WINDOU NLNMER, I
        FFINT 4S
        FEFT1AT( ENTER COEF. )
        FEF+1,* A(i)
        FFIINT 44
        FEHNHAT: ENTEF SUESCFIFTSS;
        FEFD *,G(1)
        NF=%
        NF=NFF-2
        FFINT *,NF-NF
        IF(NF.LT.N) GOTO 52
        NFこここ**Vi゙
        DQ シG K=1,NF
        W(k)=0.
        IF(K.GT.N) GO TO 9%
        [DGG II=1,M
        IF(T.EQ.G.) W(K)=1.
        IF(T.EG.%.) GU TEGO
        IF(GCII).GT.1.) GO TG %O
        IF(T.LE.NF) W(F)=W(F)+G(II)
    W2=(1.-4.*(T**2))
    IF(C!(II).EG.Z.) W(*)=W(K)+A(II)*WZ
    IF(O(II).EQ.J.)W(K)=W(r)+A(II)*COS(FI*T)
    IF(G!(IIj.EG.4.) W(R)=W(ト)+&(II)* (SIN(2.*FI*T)/(2.*FI*T))
    IF(O(II).EG.E.; W(r)=W(k)+A(II)*(1.+Z.*(ABS(T)))
    IF(O(II).EU.6.; W(F)=W(%)+A(II)*(.5+.5*COS(2.*FI*T))
    W1=1.
    IF(G(II).EE.7.) W(F)=W(忄)+A(II)*(.54+.4S*COS(2.*F.I*T))
    IF(O(II).EO.1.) W(F)=W(F)+A(IJ)*WI
    WB=(1./F.I)*GES(SIN(2.*FI|T)
```

```
        IF(Q(II).EGI.8.) W(t.)=W(F.)+G(II;*WE
        WO=.42+.5*LOS(2.*F゙丁*T)+.0も*[OS(4.*FI*T,
        IF(O(II) EE!.%.; W(&)=W(&)+A(II)*WQ
        IF(T.GI.(.2G)) GD 1G 71
        IF(O(II).G7.1U.) GOTL 8%
        W1O-1.-(ご.*AES(T)**2)*(1.-2.*HES(T))
        IF(C(Il).EU, 1O; W(K)=W(1:)+WlO
        71
        IF(E(II).LT.10., GO TO 90
        IF(D(IJ).GT.1G.) GO TO E#
        IF(7.GE. & 25:) W(r)-山(t.)+A(II)*2*(1.-2.*AES(T))**S
        #* IF:O(11!.EO.11.) W(V.)=W(F)+A(11)*(1.-2.*(AES(T)}
        IF(T.Gt.(.15;) GO TO &E
        W(k):=W(t)+A(Ij)#Wl
        GO)}7[19
        E# W12=(.5+.5*COS,(2*FI*(AES(T)-.15)) i.7,)
            IF(G(I1).EO.1こ.) W(&゙)=W(ト)+H(II)*W12
        9G COIVTIINJE
        IF(E.EO.1) GO TO 9%
        W(NFQ-K+2)=W(F)
        T=T+1./(TVF._)
        LO BO L=1,NF=
        W(NF+1)=0.
        FT(ごR.)=-0.
```



```
        DU1 101 N,-1,2*NF
        10] 人(%2)=0.
            x(37)=.00005%5
            x(39)=.6052475
            x(41)= - MNOGEE,
            x(4马)=-005019',
            x(45)=.0000955
            x(47)=.00日S.05
            x(45) =.0000945
            x(51)=.016390G
            x(5゙シ)=.00009こE
            X(55)=.045059%
            x(5.7)=.0000515
            *(5G)=.4115925
            x(\epsilon1)=1.0
            x(65) =x (5%)
            X(65)}=x(57
            X(67) =x(55)
            x(69)=x(5)
            x(71)=x(51)
            x(73)}=x(44
```

```
    x(75)=x(47)
    x(77)=x(45)
    x(79)=x(43)
    x(81) =x(41)
    x(8S)=x(39)
    X(85)}=x(37
    x(221)=x(37)
    x(217)=x(39)
    x(217) = x(41)
    x(こ15)=x(4S)
    x(21\Xi)=x(45)
    x(211)=x(47)
    x(20%)=-x(49)
    x(207)=x(51)
    x(205)=x(5.)
    x(20)
    x(201)=x(57)
    x(197) =x(55)
    x(19%)=x(Є1)
x(195) =x(65)
x(19%)=x(05)
x(191) =x(67)
x(189)=x(67)
xi18#7)=\lambda(71)
x(19G)=x(73)
x(1夏き)=人(75)
x(1E1) =x(77)
x(179) =x(79)
x(177)=x(81)
x(175)=x(ES!
x(175)=x(85)
SOE CONTINUE
    CALL FFT (NF2,1,X)
    DU 102 FZ=1,NF2
FT(2*FS)=0.
102 FT (2*N-1)=W(kS)* X(2*)S-1)
WF:ITE(75,405) (t:,W(k),k=1,nP2)
40S FUFMAT(14, 2X,F16.E)
    WFITE(7ち,*) (FT(r),F=1,2*NF2)
    CALL FFT (NF2,-1,FT)
        DO 4O1 IK=1,2*NF2
    FT(IK)=FT(IF)/も4.
401
205
206
    FORMAT (F4.1,2X,E1O.E)
    DD 206 IA = 1,2*NFこ-1,2
    X(IA)=(1A+1)/2
```

```
            WFITE(5G,2OS) (X(IA),FT(IA),IA=1,12`,Z)
```

            WFITE(5G,2OS) (X(IA),FT(IA),IA=1,12`,Z)
            WFITE(51,*) {FT(IA),IA-2,2*NF, 2)
            WFITE(51,*) {FT(IA),IA-2,2*NF, 2)
            WFITE(55,*) ((1A+1),2,IA-6.,7%,2), (FT(IG),IA=65,77,2)
            WFITE(55,*) ((1A+1),2,IA-6.,7%,2), (FT(IG),IA=65,77,2)
                    FFIINT 140
    ```
                    FFIINT 140
```




```
                    STOF
```

                    STOF
            END
            END
            SUEFIOUTINE FFT (NN, ISIGN,DATA)
            DIMENSION DATA(OOO)
            N=2*NN
            J=1
            LO E I=1,N,2
            IF(I-J)1,2,2
    TEMFF=DAGA(3)
                            TEMFI= DATA (J+1)
                            DATA(J)=DGTA(I)
                            DATA(J+1)= DFTA(I+1)
                    DATA(I)=TEMFF
                            DATA(I+1)=TEMFI
    M=N/2
If (J-M,5,5,4
J=.7-T=1
M=M/二
IF (M-2)E, S,
M1/14A=2
IF (|||AX-N) T, 10, 10
ISTEF=2*MMAX
THETA=0. 2ES185S/FLOAT (ISIGN*MF|AX)
SINTH=SIN(THETA/2.)
WSTFK---2.*EINTH*SINTH
WSTFI=SINUTHFTA,
WF:=1
WI=0
DO }9M=1,M|\mp@code{M},
DO }8\textrm{I}=1\cdot1,N, ISTEF
J=j +MINA
TEMFF:WF*DATA(J)-W]*D)TA (J +1)
TEMFI=WF**DFTA(J+1)+WI*DATA(J)
DATA(I) =DATA(I)-IEMFH:

```

```

                            DATA(I)=DATA(I) + TEMFFi
    DATA(I+1)=DATA(I+1)+TEMFI
TEMIFF=WF:
WF:=WF:*WOTF\cdotF:WI *WSTF'I +WFi

```

The author, James Lester Kreamer, was born April 24, 1955. His undergraduate work was done at Southern Illinois University in Carbondale and Northern Illinois University in Dekalb, where he was awarded a Bachelor of Science Degree in Geology in December, 1980. He was employed by the U.S.G.S., Woods Hole. He is currently employed by Mobil Oil Corporation as a geophysicist and is a candidate for a Master of Science Degree from the University of New Orleans in Applied Physics.```


[^0]:    Q－at．：
    C？Wat dumhy

[^1]:    AMFLITUDE O． $016646=66$ 0.12092602 0.098412037 0.076831031 0.0002950743 0.018493826 $-0.0005306772$
    0.010366028
    $-0.0005469068$
    0.0068696984
    －0． 0007297598
    －0．0000000289
    $-0.019197775$ 0.0001753424
    －0．0009290504
    0.0002635152
    －0．0008692722
    0.0003185 .329
    $-0.00082797$
    0.0003559383
    $-0.000798621$
    0.0003822021
    $-0.0007774544$
    0.00104009211
    $-0.0007625264$
    0.0004141744
    $-0.001515688$ 0.0004232136
    －0．0007444701
    0.006428871
    $-0.0007403672$ 0.0004316319

