HOW TO COMBINE PROBABILISTIC AND FUZZY UNCERTAINTIES IN FUZZY CONTROL

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ABSTRACT

Fuzzy control is a methodology that translates natural-language rules, formulated by expert controllers, into the actual control strategy that can be implemented in an automated controller. In many cases, in addition to the experts' rules, we have additional statistical information about the system. In the present paper, we explain how to use this additional information in fuzzy control methodology.

INTRODUCTION

There are two main methodologies that lead to automated control. If we have a mathematical description of the system that we are going to control (either in deterministic, or in statistical terms), then we can apply methodology of traditional control theory. If we do not have such a description, but we have experts who are good in controlling this kind of objects, then we can ask the experts to formulate the rules that they use in whatever fuzzy, natural-language terms they can, and then apply fuzzy control methodology (see, e.g., [1, 2, 4]) to translate these rules into the actual control strategy.

Both methodologies work fine. If we have enough rules, then we can apply fuzzy control. If we have a sufficient amount of statistics, we can build a mathematical model of the controlled object, and then apply traditional control methodology.

Usually, when we start controlling some complicated object, we first do not have a mathematical model, so the only information we have is the experience of the expert controllers. Then, gradually, we get more and more statistical information about this object, and eventually, we become able to apply traditional control methods. During this transition period, we do not yet have a precise mathematical model, but we already have some statistical information about the object. While controlling the system, in course of time we get some experience, and we can extract some statistical information from our experience. Since we now know more about the controlled system, we would like to use this additional statistical knowledge to improve the control strategy. How to do it?

At present there are no known ways to do it, and the only suggestion is to wait until we have enough information for applying traditional control theory, and then find an optimal control and switch to this control.

So, we need a method to "translate" probabilistic knowledge into fuzzy terms. In the present report, we propose and justify such a method.

FORMULATION OF A PROBLEM: A REALISTIC EXAMPLE

To give the reader a better understanding of what we are talking about, let us give a simple example of this kind of a situation. Let us consider a control system whose purpose is to stabilize the value of some parameter x at some desired value x_0, and this parameter is difficult to measure directly (e.g., the temperature inside the nuclear or chemical reactor). We will consider the simplest situation, when it is possible to apply the direct control u that changes the value of x in the desired direction: $dx/dt = u$. For such systems, the optimal control can be described as a function $u(\Delta x)$, where $\Delta x = x - x_0$. When $\Delta x = 0$, we do not need

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any control at all, so $u(0) = 0$. For Δx close to 0 (i.e., for the situations in the vicinity of equilibrium), we can neglect quadratic and higher terms in the Taylor expansion of $u(\Delta x)$, and thus approximate this function $u(\Delta x)$ by a linear expression $u = -k\Delta x$.

Since we assumed that the parameter x is difficult to measure directly, we can have two kinds of information about its value: first, we can apply indirect (and therefore, not very precise) measurements. Second, we can rely on the ability of the experts to control such systems and thus to estimate Δx. Suppose that for some situation (some combination of observable parameters), an indirect measurement resulted in an approximate value $\tilde{x} \approx x$, and that the standard deviation of this estimate (i.e., the mean square value of the difference $x - \tilde{x}$) equals σ (e.g., $\tilde{x} = 1.0$, and $\sigma = 0.5$). We can assume that the probabilities of different errors are normally distributed (this is more or less standard assumption in measurement theory). Suppose that for this same situation, an expert uses his experience to estimate the actual value of x as "approximately X, with precision $\approx \varepsilon$" for some values X and ε (e.g., "approximately 1.5, with precision ≈ 0.5"). Using known methods of fuzzy theory, we can describe this statement by a membership function $\mu(x)$ whose maximum corresponds to $x = X$ (e.g., a triangular membership function).

If we use only the statistical information (i.e., the result of the measurements), then it is reasonable to apply the value of the control u that corresponds to the most probable value of x, i.e., $u = -k\tilde{x}$. If we use only the expert's estimate, then it is reasonable (according to well known defuzzification techniques) to apply a control that corresponds to the most possible values of x, i.e., in this case, a control $u = -kX$.

Both estimates of x are not very precise: expert's estimates are practically never precise, and about the result of the measurement, we specifically assumed that it is not precise. Therefore, both control values $-k\tilde{x}$ and $-kX$ are far from being ideal. So, it is desirable to combine these two types of knowledge and design a better control strategy.

But how to do it? If we use statistical methods, then we do not know how to use fuzzy estimates. Besides, even if we invent some methods to translate fuzzy estimates into probabilities, these fuzzy estimates will still remain subjective expert's estimates. Calling them probabilities will be misleading: if this "probability" of an error is 0.5, it does not mean (as for usual probabilities) that this kind of an error occurs in half of the cases. So, statistical methods are out of question. Hence, we must somehow use fuzzy methods to handle both fuzzy estimates and probabilities. But how?

So, what we need is a method to translate probabilities into fuzzy terms.

BASIC IDEA OF TRANSLATING PROBABILITIES INTO FUZZY TERMS

Fuzzy estimates of degree of belief: where do they come from? In fuzzy control, we start with the uncertainty values that characterize our degree of belief that, say, 0.3 is small, or that 10 is big. Where do we get these degrees of belief from? One of the standard ways to do so is to ask an expert to quantify his degree of belief, say, on a scale from 0 to 10, and then, if he chooses some value D, to estimate his degree of belief as $D/10$ (e.g., if he chooses $D = 6$, then his degree of belief is 60%). The readers who ever answered any polls or sociological tests will easily recognize the standard way to quantify such vague notions as "degree of satisfaction with the service", etc.

In applying this methodology, one has to be very accurate in choosing a scale (10? 5? 100?). On one hand, the bigger the scale, the better estimates we get. On the other hand, an expert cannot distinguish between too many possible degrees of certainty, so there is no sense in using extremely long scales.

Optimal decisions are based on probabilistic estimates. In decision making, it is well known since [3] that if decisions of a decision-maker are consistent (in some reasonable sense), then they have to be based on some probabilistic estimates.

It sounds reasonable to assume that experts (whose decisions we are analyzing) are consistent decision-makers (else they would not have been successful in control, and would not have been experts). So, it is reasonable to assume that the decision-making process that is going on inside their brains is based on some probabilities, i.e., is based on some statistical estimates.

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Let us apply optimal decision theory to experts estimating their degree of belief. How does an expert get these probabilities? Suppose, for example, that someone asks an expert to estimate his degree of belief in some statement A (e.g., that 0.3 is negligible). To give such an estimate, an expert recollects (consciously or subconsciously) all the cases in which 0.3 (or a value that is close to 0.3) was tested, and figures out when 0.3 proved to be really negligible, and when the difference of 0.3 caused important changes. Suppose that totally, he recalls n cases, and in m of them 0.3 was negligible. Then a reasonable estimate for the probability $p(A)$ (i.e., the probability that 0.3 is negligible) is $f = m/n$.

The actual probability p can be different from f and, therefore, different values of m/n can correspond to one and the same probability. Indeed, if we have a sequence of n independent events with probability p, then the mathematical expectation of m/n is p, and the standard deviation σ of m/n is $\sqrt{p(1-p)/n}$. We can now apply a "$3\sigma-$ rule" from mathematical statistics, and conclude that for a given f, all the values p such that $|f - p| \leq 3\sqrt{p(1-p)/n}$ are possible. Therefore, the estimates m/n and m'/n can correspond to one and the same probability p if there exists a probability p such that $|m/n - p| \leq 3\sqrt{p(1-p)/n}$ and $|m/n - p| \leq 3\sqrt{p(1-p)/n}$. We say that the estimates are *different* if there is no such p. Now, we are ready to form a scale: we take 0 as the first element f_0 of this scale; for the second element f_1, we take the smallest estimate that is different from 0; for f_2, we take the smallest estimate that is different from both 0 and f_1, etc. Suppose that there are totally k elements on this scale. Then, when we must estimate our degree of belief on a scale from 0 to k, we recall n cases, estimate $f = m/n$, and produce k for which $f_k \leq m/n < f_{k+1}$.

Let us denote by $f(p)$ the value of this scale that corresponds to a probability p. This is not a uniform scale, because the distance between two consequent elements p and $p + \Delta p$ on this scale is proportional to $\sqrt{p(1-p)}$. In other words, $\Delta p \sim \sqrt{p(1-p)}$ leads to $\Delta f(p) = const$. For small Δp, we get $\Delta f(p) \approx f'(p)\Delta p$. Therefore, from $\Delta f(p) = const$ and $\Delta p \sim \sqrt{p(1-p)}$, we conclude that the unknown function $f(p)$ must satisfy the differential equation $f'(p)\sqrt{p(1-p)} = const$.

From the definition of $f(p)$, we can easily conclude that $f(0) = 0$ and $f(1) = 1$. The solution of the above-given differential equation with these boundary conditions is $f(p) = 1/2 + 1/\pi \, arcsin(2p - 1)$. So, we arrive at the following conclusion:

RECOMMENDATIONS

If we know the probability $p(A)$ of some event A, and we want to use this information in the fuzzy knowledge base, then we must to this knowledge base that we know A with degree of belief $f(p(A))$, where $f(p) = 1/2 + 1/\pi \, arcsin(2p - 1)$.

APPLYING THESE RECOMMENDATIONS TO THE ABOVE EXAMPLE

Let us follow these recommendations on the above realistic example. In that example, it was necessary to translate the following statistical information into the fuzzy language: that x is distributed according to the Gaussian law, with the average value $\bar{x}$, and the standard deviation σ.

For this information, the most probable value of x is $\bar{x}$, and the bigger the difference between x and $\bar{x}$, the less probable this value x. Hence, it is reasonable to translate this information into a membership function $\mu(x)$ that would attain its maximal value for $x = \bar{x}$, and would monotonically decrease to 0 as x starts decreasing or increasing. So, we are looking for a membership function of the type $\mu(x) = g(|x - \bar{x}|/\sigma)$, where $g(z)$ is a decreasing function from $(0, \infty)$ to $[0, 1]$.

A reasonable interpretation of a membership function $\mu(x)$ is as follows: for every value v from 0 to 1, our degree of belief that it is possible for x to belong to the set $\{x : \mu(x) \geq v\}$, is equal to v. This means that our degree of belief that it is impossible for x to belong to that set is equal to $1 - v$. But the fact that it is impossible for x to belong to this set means that x necessarily belongs to its complement $\{x : \mu(x) < v\}$. According to our expression of μ in terms of g, the inequality $\mu(x) < v$ is equivalent to $|z| > g^{-1}(v)$, where g^{-1} means an inverse function, and $z = (x - \bar{x})/\sigma$. So, our degree of belief that $|z| > g^{-1}(v)$ is equal to $1 - v$. If we denote $w = g^{-1}(v)$, we conclude that $v = g(w)$, and so our degree of belief that $|z| > w$ is equal to $1 - g(w)$.

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Now, z has a standard normal distribution, and hence, the probability that $|z| > w$, is equal to $2F(-w)$, where by $F(z)$ we denoted a (cumulative) distribution function of a standard normal distribution. According to our recommendations, this means that our degree of belief that $|z| > w$, is equal to $f(2F(-w))$, where $f(p)$ is the above-described function. So, $1 - g(w) = f(2F(-w))$, hence $g(w) = 1 - f(2F(-w))$, and $\mu(x) = g(|x - \bar{x}|/\sigma)$.

So, we have translated the statistical information into a membership function. Now, both parts of our knowledge are expressed in fuzzy terms: statistical one on terms of this function $\mu(x)$, and the original fuzzy one in terms of some other fuzzy function $\mu_0(x)$.

Now, we can apply an &-operation to combine these two pieces of knowledge into a combined membership function $\mu_c(x)$ that expresses both parts of this knowledge. E.g., if we use min as &, then $\mu_c(x) = \min(\mu(x), \mu_0(x))$. If we use product as &, then $\mu_c(x) = \mu(x)\mu_0(x)$. To this resulting function $\mu_c(x)$, we can apply a defuzzification procedure and determine the appropriate value of x_c (e.g., the value for which $\mu_c(x)$ attains its maximum), and then apply the control $u = -kx_c$.

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