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SUMMARY

In this work we utilize fuzzy sets theory to evaluate and make predictions of flexural strength and density of NASA 6Y silicon nitride ceramic. Processing variables of milling time, sintering time, and sintering nitrogen pressure are used as an input to the fuzzy system. Flexural strength and density are the output parameters of the system. Data from 273 Si_3N_4 modulus of rupture bars tested at room temperature and 135 bars tested at 1370 degrees Celsius are used in this study. Generalized mean operator and Hamming distance are utilized to build the fuzzy predictive model. The maximum test error for density does not exceed 3.3%, and for flexural strength 7.1%, as compared with the errors of 1.72% and 11.34% obtained by using neural networks, respectively.

These results demonstrate that fuzzy sets theory can be incorporated into the process of designing materials, such as ceramics, especially for assessing more complex relationships between the processing variables and parameters, like strength, which are governed by randomness of manufacturing processes.

INTRODUCTION

High engine operating temperatures made possible by ceramics will result in energy savings, reduced weight, and environmental benefits. Estimates of potential efficiency improvements for automotive engines with structural ceramic components range from 30 to 50 percent over current engine technology. Structural ceramics such as silicon carbide

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and silicon nitride are leading candidates for engine hot-section components because of their relatively light weight, excellent oxidation and thermal shock resistance, and good high-temperature strength. They consist of nonstrategic materials that can be inexpensive when mass produced. However, their wide variation in strength properties and relatively low fracture toughness have precluded a large-scale move toward replacing many metal engine parts with ceramics. The scatter in strength and low toughness are generally attributed to discrete defects such as voids, inclusions, and cracks introduced during processing. Current cost-effective fabrication procedures also frequently produce ceramics containing bulk density variations and microstructural anomalies that can adversely affect performance [1]. Therefore it is essential to have non-destructive evaluation (NDE) methods available that can not only detect discrete flaws but also provide information regarding the material's physical and mechanical properties and uniformity. By incorporating NDE technology into a ceramic materials technology development program, information may be obtained regarding the source of defects so that steps can be taken to minimize their occurence through improved fabrication procedures. Thus, NDE may help to reduce the effort expended in a materials research program and shorten the time needed to develop strong, reliable structural ceramics.

Scatter in mechanical properties of ceramics is a great drawback from a design/reliability standpoint. This scatter is attributed to defects and inhomogeneities occuring during processing of silicon nitride powder compositions and during fabrication. From research work on a silicon nitride composition at the National Aeoronautics and Space Administration Lewis Research Center (NASA LeRC) it was evident that density gradients were strongly dependent upon sintering conditions [2, 3]. Based upon the sintered silicon nitride preliminary X-radiographic characterization work at NASA LeRC, a program was undertaken to systematically investigate density gradient-flexural strength relationships as affected by sintering and powder processing variables. The sintering variables were temperature, nitrogen overpressure, time, setter contact, and furnace position. The powder processing variables were grinding time, and inclusion or exclusion of powder wet-sieving procedures. In [4], the results of an extensive investigation of one silicon nitride composition involving sintering trials of 21 batches of material were described. Sintering/processing conditions were varied based on feedback from radiography to obtain high-density uniform structures with concomitant improved strength and reduced scatter. These previous results,

in turn, were used in neural systems [5], and are used in this paper to study the viability of using fuzzy sets for predicting strength and density, speeding the optimization of the manufacturing process, and for comparing fuzzy systems with neural systems.

In this work we are interested in finding whether it is possible to utilize fuzzy systems to help in the material development process of advanced ceramics. Fuzzy systems are good in function approximation, and if the trend could be easily noticed as to which variable contributes most for the increase of a desired output parameter, say strength, then this may help in speeding up the process of manufacturing and optimizing a new material. Material developers can easily notice such changes for a few variables but it becomes very difficult to do so for a large number of variables. From the data collected by Sanders and Baaklini [4], we selected only three input variables, namely, milling time of the composition powder, the sintering time of the modulus of rupture test bars, and the nitrogen pressure employed during sintering. From the output variables, flexural strength and density were selected. The rationale for using only the above mentioned variables is that there were not enough training pairs (outputs associated with inputs) for all the variables used in the experimental work [4]. In this paper, relationships between the milling time, sintering time and nitrogen pressure and the resultant strength and density are established by using fuzzy systems. Fuzzy set results are compared with those obtained using radial basis function neural network [5].

BASICS OF FUZZY SET THEORY

Randomness is not the only form in which uncertainty reveals itself. In the sixties a mathematical tool was developed to formulate and deal with other forms of uncertainty and became to be known as fuzzy set theory. It was first introduced and published by Zadeh [10, 11]. Essentially, fuzzy set theory provides a natural approach to manipulating problems in which the transition between membership and nonmembership of the classes of objects is gradual rather than abrupt, and the source of imprecision is the absence of sharply defined class membership rather than the presence of random variables. In other words, it renders a methodology for dealing with phenomena that are vague, imprecise, too complex or too ill defined to be susceptible to analysis by conventional strict mathematical approaches [7, 9]. In the next sections we provide basic definitions of the fuzzy set theory which are used in this paper.

Definition of Fuzzy Sets

Let R be the set of reals and U be the universe of discourse (crisp set) which is a collection of items of interest. Let u be a generic element of U. A fuzzy subset A of U is defined by a membership function μ_A : U -> [0, 1], where [0, 1] denotes the closed unit interval on the real line. Then the fuzzy subset A of U can be expressed as:

$$A = \{ \mu_A(u)/u ; u \varepsilon U , \mu_A(u) \varepsilon [0,1] \}.$$

In this case, the value μ_A is referred to as the degree or grade of membership of u in A. Note that a classical non-fuzzy set B can be considered as a binary characteristic function U -> {0, 1}, where {0, 1} is the set of values 0 and 1 rather than an interval.

Support

Let A be a fuzzy subset of U. The support of A, Supp(A), is the set of elements in U whose memberships in A $\mu_A(u)$, are positive. That is,

$$\text{Supp(A)} = \{ u/u \varepsilon U , \mu_A(u) > 0 \}.$$

Normality

A fuzzy subset A is normal if and only if $Sup\ (\mu_A(u)) = 1$.

That is, the supremum over U is unity; otherwise, it is subnormal.

As an example let us consider a crisp set U, where U = 1 + 2 + ... + 10, where 1, 2, ..., 10 are elements of U, and the sign " + " is used here to express membership (union) in the set. Then a fuzzy subset of U, called "much less than 5", may be expressed as:

Much less than 5 = 1/1 + 1/2 + 1/3 +1/4 + 0.6/5 + 0.4/6 + 0.3/7 + 0.1/8.

Note that the items which have zero value in the grade of membership have been ignored in the above itemizing expression.

Fuzzy operations

Let A and B be fuzzy subsets of a crisp set U, with membership functions μ_A and μ_B . Major operations for fuzzy sets follow:

Complement

The complement of a fuzzy subset A of a crisp set U, denoted A', is defined by

$$A' = \sum_{U} (1 - \mu_A(u))/u, u \varepsilon U,$$

where $\sum$ is used as a convenient notational form.

Union

The union of A and B, denoted $A \cup B$, is defined by

$$A \cup B = \sum_{U} (\text{Max}[\mu_A(u), \mu_B(u)])/u, u \varepsilon U.$$

Intersection

The intersection of A and B, denoted $A \cap B$, is defined by

$$A \cap B = \sum_{U} (\text{Min}[\mu_A(u), \mu_B(u)])/u, u \varepsilon U.$$

The union corresponds to the connective "OR", while the intersection corresponds to the connective "AND", and the operation of complementation corresponds to negation.

Examples of the operations defined above follow:

Let

U = { 1, 2, 3, 4, 5, 6, 7 },

and A = 0.8/3 + 1/4 + 1/5 + 0.6/6,

and B = 0.7/3 + 1/4 + 0.5/6.

Then

$A \cup B$ = 0.8/3 + 1/4+ 1/5 + 0.6/6,

$A \cap B$ = 0.7/3 + 1/4 + 0.5/6,

A' = 1/1 + 1/2 + 0.2/3 + 0.4/6 + 1/7.

AGGREGATION OF FUZZY SETS

Aggregation operations on fuzzy sets are operations by which several fuzzy sets are combined into a single set. In general, any aggregegation operation is defined by the function

$$h : [0,1]^n \rightarrow [0,1]$$

for some n ≥ 2. When applied to n fuzzy sets defined on U, h produces an aggregate fuzzy set A by operating on the membership grades of each element of U in the aggregated sets.

In order to qualify as an aggregation function, h must satisfy at least the following two axiomatic requirements, which express the essence of the notion of aggregation:

Axiom 1. Boundary conditions

$$h(0, 0, ..., 0) = 0 \quad \text{and} \quad h(1, 1, ..., 1) = 1.$$

Axiom 2. For any pair a_i , b_i , where $a_i \in [0, 1]$ and $b_i \in [0, 1]$, if $a_i \geq b_i$ for all i, then $h(a_i) \geq h(b_i)$, that is, h is monotonic nondecreasing in all its arguments.

Two additional axioms are usually employed to characterize aggregation operations despite the fact that they are not essential:

Axiom 3. h is a continuous function.

This axiom guarantees that an infinitesimal variation in any argument of h does not produce a noticeable change in the aggregate.

Axiom 4. h is a symmetric function in all its arguments, that is, the aggregated sets are equally important.

We can easily see that fuzzy unions and intersections qualify as aggregation operations on fuzzy sets. Although they are defined for only two arguments, their property of associativity provides a mechanism for extending their definition to any number of arguments. Hence, fuzzy unions and intersections can be viewed as special aggregation

operations that are symmetric, usually continuous, and required to satisfy some additional boundary conditions. As a result of these additional requirements, the standard max and min operations represent boundaries between the averaging operations and the fuzzy unions and intersections, respectively.

There are several classes of averaging operations [8, 9]. One of them that covers the entire interval between the min and max operations consists of generalized means. This class of operations will be used in this paper. It is defined as folows:

$$h_\alpha (a_1 , a_2 , \dots , a_n) = \left(\frac{a_1^\alpha + a_2^\alpha + \dots + a_n^\alpha}{n} \right) ^{\frac{1}{\alpha}}$$

where α is a parameter by which different means are distinguished, and $\alpha \in R \left(\alpha \neq 0 \right)$. In our application we used $\alpha = 2$. Function h_α clearly satisfies Axioms 1 through 4, and, consequently, it represents a parameterized class of continuous and symmetric aggregation operations.

A METHOD FOR PREDICTION

The fuzzy system was built using the data from 273 Si_3N_4 modulus of rupture bars which were tested at room temperature and 135 bars which were tested at 1370 C. For the room temperature, 18 different combinations of milling time, sintering time, and nitrogen pressure yielded the composition strengths and densities which are listed in Table I. Also listed in Table I are the strengths and densities for 9 combinations at 1370 ^{o}C.

In order to determine our confidence in the fuzzy system predictions, we needed to test the system using known test vectors and then evaluate the error of the predictions. We were particularly interested in the ability of the network to predict the output values for batch number 6Y25, as this batch number represents the optimium combination for the processing variables from the available data set [4]. Thus, batch number 6Y25 was first removed from the data. The data were then pseudo-randomly divided into a ratio of approximately 70 % for training and 30 % for testing. Batch number 6Y25 was then inserted into the test data set. This was repeated for 5 times in order to have 5 different pairs of training and test data sets which were labeled as combinations A through E (Table

Room temp Batch #	Number of specimen	Milling time, [hr]	Sintering time, [hr]	Nitrogen pressure, [MPa]	Actual strength, [MPa]	Actual density, $[g/cm^3]$
6Y1B	30	24	1	2.5	556	3.12
6Y2B	30	24	1	2.5	532	3.18
6Y11	15	100	1	2.5	490	3.23
6Y12	15	300	1	2.5	579	3.25
6Y13	15	100	1	2.5	684	3.24
6Y14	14	300	1	2.5	746	3.24
6Y1516	19	24	2	5	664	3.22
6Y17	10	100	2	5	646	3.23
6Y18	10	100	1.5	5	608	3.21
6Y19	10	100	1.5	5	570	3.22
6Y20	10	100	2	5	650	3.22
6Y23	15	100	1.25	5	631	3.24
6Y24A	15	100	1.25	3.5	586	3.26
6Y24B	15	100	2	3.5	619	3.26
6Y25	10	300	2	5	714	3.28
6Y26A	15	100	1	3.5	479	3.20
6Y26B	15	100	1	5	503	3.18
6Y28	10	100	2	5	671	3.21
1370C Batch #						
6Y9B	29	24	1	2.5	383	3.12
6Y11	13	100	1	2.5	444	3.23
6Y12	14	300	1	2.5	416	3.25
6Y13	15	100	1	2.5	406	3.24
6Y14	14	300	1	2.5	425	3.24
6Y1516	20	24	2	5	402	3.22
6Y17	10	100	2	5	439	3.23
6Y18	10	100	1.5	5	458	3.21
6Y25	10	300	2	5	467	3.28

Table I: Strength/Density at room temperature and 1370 C for different processing and sintering conditions.

Table II: Selected batch numbers for 70% and 60% training sets A through E

Room Temp. Batch no.	Training Sets 70%					Training Sets 60%				
	A	B	C	D	E	A	B	C	D	E
6Y1B	*		*	*	*			*	*	*
6Y2B		*		*	*				*	*
6Y11	*	*	*		*	*	*			*
6Y12		*	*	*	*		*	*	*	*
6Y13	*	*		*		*				
6Y14	*	*	*	*		*	*	*		
6Y15 6Y16	*		*		*					*
6Y17		*	*	*	*		*	*	*	*
6Y18		*		*			*		*	
6Y19	*		*	*		*		*	*	
6Y20	*	*	*		*	*	*	*		*
6Y23	*	*	*	*	*	*	*	*	*	*
6Y24A			*	*	*			*	*	*
6Y24B	*		*		*	*		*		
6Y25										
6Y26A	*	*	*	*		*	*	*	*	
6Y26B	*	*			*	*	*			*
6Y28	*	*		*	*	*	*		*	*
1370°C Batch no.										
6Y9B	*	*		*	*	*			*	*
6Y11		*	*	*	*		*	*	*	
6Y12	*	*	*		*	*	*			*
6Y13	*		*	*	*		*	*	*	*
6Y14	*	*			*	*				*
6Y15 6Y16		*	*	*			*	*		
6Y17	*	*	*	*		*	*	*	*	
6Y18	*		*	*	*	*		*	*	*
6Y25										

II). This entire process was then repeated using a ratio of approximately 60 % data used for training and 40 % for testing.

Next, a training data set consisting of all the batch numbers (100 %) except 6Y25 was created. Batch number 6Y25 was placed in the test data set as the sole vector. Finally, all the batch numbers were placed in a training data set and we made predictions for the input vectors for which we do not know the outputs.

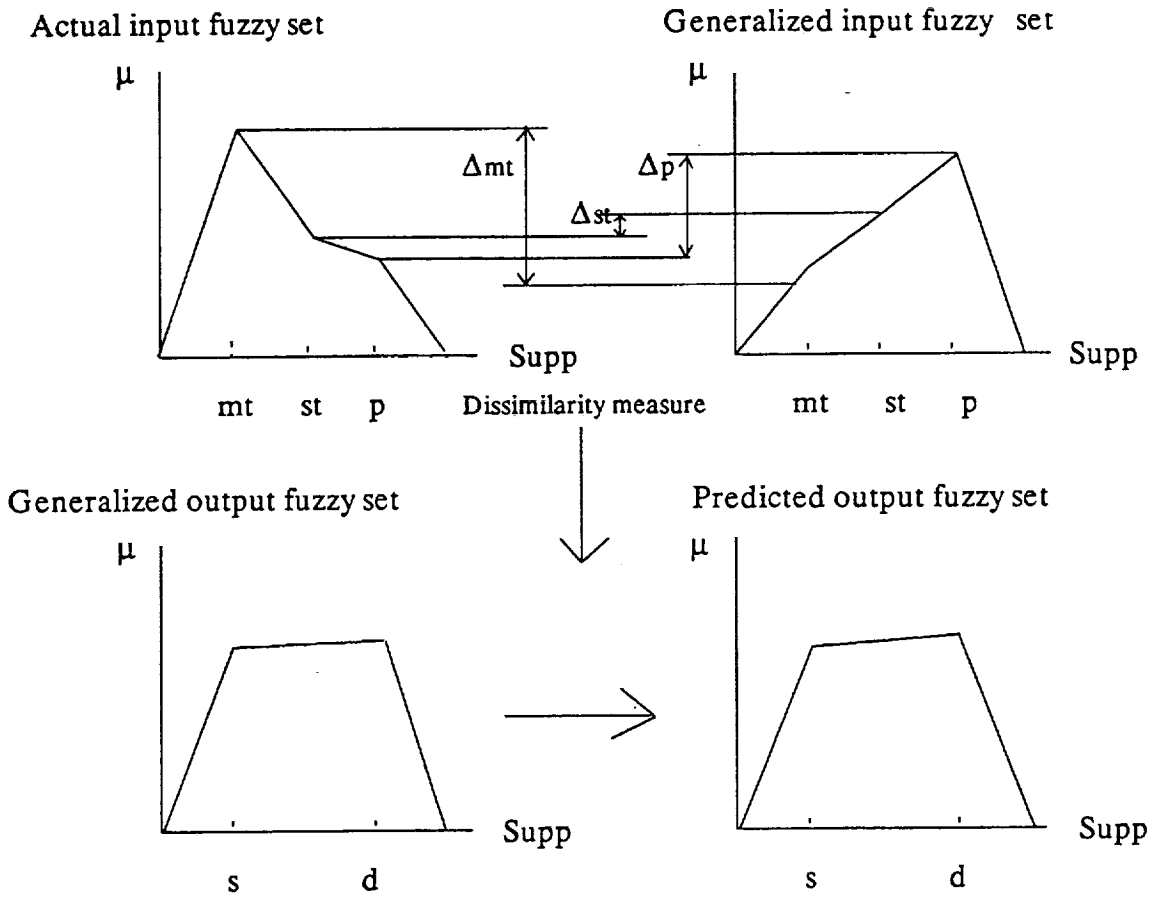
The collected data representing sintering and processing variables of 21 batches of silicon nitride composition was formed to establish 42 fuzzy sets for each batch of the material (21 input and 21 output fuzzy sets). The input fuzzy sets are defined for three values of support (nitrogen pressure, sintering time, and grinding time) while the output ones have their support of two elements (flexural strength and density). Different fuzzy sets were formed for the room temperature and 1370 C. The grades of memberships were normalized elementwise, and the normalization was repeated for every step of prediction. The resulting membership grades were combined by means of generalized mean operation to produce the resulting fuzzy sets. The fuzzy sets for 70% and 60% training data for the room temperature are shown in Tables III and IV, respectively. The grades of membership for 1370 C are shown in Tables V and VI. Thus, appropriate models were formed for the flexural strength and density as the output parameters, and for nitrogen pressure, sintering time and grinding time as the input parameters.

After that a measure similar to Hamming distance is proposed to calculate the difference between the actual and generalized fuzzy sets of input parameters. The measure is defined by:

$$D\left(A, B / S\right) = \sum_{u \varepsilon S} \left(\mu_A (u) - \mu_B (u) \right)$$

where S is an interval in R, and A and B are fuzzy subsets on S. In the case when S = R, then D (A, B / R) = D (A, B).

Next, the *k*-fraction of the measure, where k ε (0, 1), is either added to or subtracted from the generalized grades of memberships of the output parameters. This results in the values for flexural strength and density; k was chosen to be 0.1. The graphical explanation of the method is depicted on Fig. 1. As an example let us consider 6Y12 test batch from combination A with 70% training at the room temperature. The generalized input fuzzy set



The generalized input fuzzy set consists of grades of membership obtained by generalized mean operation performed on normalized values of input parameters: milling time, sintering time, and pressure (note that these fuzzy sets differ for different combinations of training data). The actual input fuzzy set represents normalized values of a particular input. The dissimilarity measure based on Hamming distance concept is then employed, that is, the elementwise differences are summed up. k- fraction of the measure is then added to the grades of membership of the generalized output fuzzy set (a fuzzy set obtained by generalized mean operation performed on normalized values of output parameters: strength and density), resulting in the predicted output fuzzy set. The grades of memberships of the latter are compared then with the actual ones obtained by normalization of the values of a particular output.

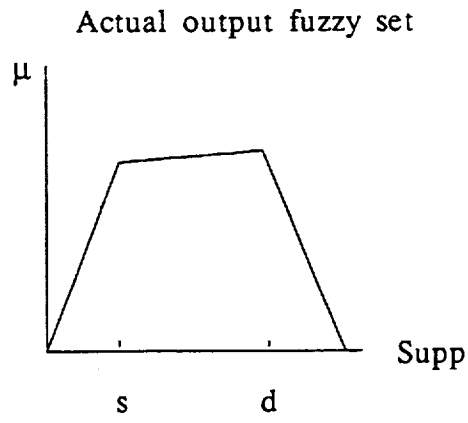


Fig. 1. A graphical explanation of the method used for fuzzy set prediction.

consists of grades of membership obtained by generalized mean operation performed on normalized values of input parameters: milling time (mt), sintering time (st), and pressure (p). In this particular case the fuzzy set with the support of (mt, st, p) has grades of membership 0.44, 0.73, and 0.82, respectively (Table III). Note that generalized input fuzzy sets differ for different combinations of training data. The actual input fuzzy set represents normalized values of the 6Y12 batch input, that is, a fuzzy set with support of (mt, st, p) with grades of membership 1, 0.5, 0.5, respectively. The dissimilarity measure based on Hamming distance is then employed, that is, the elementwise differences between grades of membership of actual and generalized input fuzzy sets are summed up. In this particular case the dissimilarity measure is 0.01. The k-fraction of the measure, which is 0.001 when k=0.1, is then added to the grades of membership of the generalized output fuzzy set. The generalized output fuzzy set, obtained by generalized mean operation performed on normalized values of output parameters, strength (s) and density (d), has in this case grades of membership 0.82 and 0.99, respectively, as shown in Table III. Addition of the k-fraction of dissimilarity measure results in the predicted fuzzy set with grades of membership 0.821 and 0.991, respectively. The latter are then compared with the actual grades of membership obtained by normalization of the values of the 6Y12 batch output 0.781 and 0.991, resulting in error of 4% for the strength and perfect prediction of density for the batch under consideration.

RESULTS AND DISCUSSION

The method described above was used to predict randomly chosen values of the flexural strength and density batch samples in room temperature and 1370 C, as well as for prediction of the 6Y25 batch. The training sets are listed in Table II. Table VII shows detailed results for the 30% test data set, for the first combination (A). The overall results for combinations A through E are shown in Table VIII for 70% training, and in Table IX for 60% training. Table X shows the results obtained to predict 6Y25 strength and density using 100% of the data. Table XI shows predictions made for selected new combinations of processing and sintering variables. Resultant strengths and densities are lower than the ones for the optimum batch 6Y25. The reason being is that fuzzy systems are bounded as

was proven in our other work [6].

With 40% test data at room temperature, the strength and density values were predicted with an average percentage error of less than or equal to 7.1% and 2.8%, respectively. When the slightly smaller test set of 30% was used, the average percentage errors for strength and density dropped slightly to less than or equal to 5.7% and 2.4%, respectively. Similar results were obtained for the 1370 ^{o}C data (Tables XII-XV). With 40% test data the average percentage errors for strength and density were less than or equal to 5.4% and 3.3%, respectively. With 30% test data these values were 4.6% and 2%, respectively. For 1370^{o}C, prediction of the 25th batch was perfect (0% error) for all combinations of 30% test data and even for all combinations of 40% test data, for both strength and density. So, even using only 60% of the data for training, the model was able to exactly predict 25th batch output parameters. The reason for that is that the fuzzy set representing the 25th batch is actually a crisp set with all membership functions equal 1 (it reaches maximum in all input parameters).

The information in Tables XI and XVI suggests that there may be other combinations of sintering and processing variables that will produce material almost as strong and dense as that obtained for 6Y25 where shorter grinding time or lower nitrogen pressure and lower sintering time can be used. For example, in Table XVI, using a milling time of 250 hours, a sintering time of 2 hours, and a nitrogen pressure of 5 MPa, the fuzzy sets predict that a strength of 462.86 MPa can be obtained. This is only slightly less than the value for 6Y25 of 467 MPa, but with a reduction in milling time of 50 hours. This also makes sense from the materials processing viewpoint. Namely, shorter milling time results in a coarser powder, which when sintered will yield a microstructure with large, randomly-oriented columnar grains, which can make the ceramics of higher fracture toughness and more resistant to high temperature creep. Long sintering times may lead to exaggerated grain growth and actual material loss due to evaporation, and thus were not tried. Nitrogen pressure beyond that pressure required to prevent decomposition was not found to have great influence on either strength or density.

The system was also used to find the optimal combination of input parameters which turned out to be the same as for 6Y25 for both room and 1370 C temperatures. In addition, for the 1370 C some other combinations of input parameters gave the maximal output parameters. They were the following triples (mt, st, p): (300, 1.75, 5), (300, 2, 4.5), and

Table III: The resulting fuzzy sets, after generalized mean operation, for the room temperature for 70% training.

	Strength	Density	Milling time	Sintering time	Pressure
A	0.82	0.99	0.44	0.73	0.82
B	0.82	0.99	0.50	0.69	0.80
C	0.82	0.99	0.49	0.75	0.79
D	0.83	0.99	0.49	0.67	0.77
E	0.79	0.99	0.39	0.76	0.79

Table IV: The resulting fuzzy sets, after generalized mean operation, for the room temperature for 60% training.

	Strength	Density	Milling time	Sintering time	Pressure
A	0.82	0.99	0.45	0.72	0.82
B	0.81	0.99	0.53	0.72	0.85
C	0.82	0.99	0.53	0.73	0.79
D	0.79	0.99	0.42	0.70	0.82
E	0.79	0.99	0.41	0.75	0.77

TableV: The resulting fuzzy sets, after generalized mean operation, for 1370 C temperature for 70% training.

	Strength	Density	Milling time	Sintering time	Pressure
A	0.92	0.99	0.61	0.65	0.71
B	0.91	0.99	0.60	0.71	0.71
C	0.93	0.99	0.47	0.74	0.79
D	0.92	0.98	0.24	0.74	0.79
E	0.92	0.99	0.62	0.55	0.61

Table VI: The resulting fuzzy sets, after generalized mean operation, for 1370 C temperature, for 60% training.

	Strength	Density	Milling time	Sintering time	Pressure
A	0.93	0.98	0.65	0.68	0.74
B	0.93	0.99	0.65	0.74	0.74
C	0.94	0.99	0.26	0.78	0.84
D	0.93	0.98	0.26	0.68	0.74
E	0.91	0.98	0.67	0.56	0.63

Table VII: Predicted room temperature strength with 70% training, Combination A

Batch Number	Actual Strength, MPa	Predicted Strength, MPa	% Error	Actual Density, g/cm^3	Predicted Density, g/cm^3	% Error
6Y2B	532	537	1	3.12	2.93	6
6Y12	579	602	4	3.25	3.25	0
6Y17	646	633	2	3.23	3.26	1
6Y18	608	614	1	3.21	3.27	2
6Y24A	586	604	3	3.26	3.23	1
6Y25	714	686	4	3.28	3.28	0
Average Error			3			2

Table VIII: Overall results for room temperature, 70% training

Combination	Strength - average % error for all test vectors	Strength - % error for 6Y25	Density - average % error for all test vectors	Density - % error for 6Y25
A	2.2	4	2.0	0
B	3.0	4	2.8	0
C	7.0	4	3.6	0
D	6.2	3	1.6	0
E	10.0	7	2.0	0
Combined Average % error	5.7	4.4	2.4	0

Table IX: Overall results for room temperature, 60% training

Combination	Strength - average % error for all test vectors	Strength - % error for 6Y25	Density - average % error for all test vectors	Density - % error for 6Y25
A	2.7	4	2.5	0
B	6.0	6	4.3	0
C	7.3	4	3.6	0
D	10.5	7	2.2	0
E	9.0	7	1.3	0
Combined Average % error	7.1	5	2.8	0

Table X: Prediction for 6Y25 density and strength at room temperature, 100% training

Batch Number	Actual Strength, MPa	Predicted Strength, MPa	% Error	Actual Density, g/cm^3	Predicted Density, g/cm^3	% Error
6Y25	714	716	0.3	3.28	3.28	0

Table XI: Prediction of selected processing and sintering variables for optimum room temperature strength and density, 100 % plus 6Y25 training

Milling Time, hr	Sintering Time, hr	Nitrogen Pressure, MPa	Predicted Strength, MPa	Predicted Density, g/cm^3
150	1.5	3	596	3.15
175	1.5	3	604	3.18
200	1.5	3	611	3.21
200	1.75	4	634	3.28
250	1.5	3	619	3.25
250	1.5	4	634	3.28
250	1.75	4	649	3.28
300	1.5	4	649	3.28
300	1.75	4	656	3.28
300	2	5	686	3.28

Table XII: Predicted strength at 1370 °C with 70% training, Combination A

Batch Number	Actual Strength, MPa	Predicted Strength, MPa	% Error	Actual Density, g/cm^3	Predicted Density, g/cm^3	% Error
6Y11	444	395	11	3.23	3.04	6
6Y156Y16	402	426	6	3.22	3.25	1
6Y25	467	467	0	3.28	3.28	0
Average Error			5.7			2.3

Table XIII: Prediction for 6Y25 density and strength at 1370 °C with 100% training

Batch Number	Actual Strength, MPa	Predicted Strength, MPa	% Error	Actual Density, g/cm^3	Predicted Density, g/cm^3	% Error
6Y25	467	467	0	3.28	3.28	0

Table XIV: Overall results for 1370 °C, 70% training

Combination	Strength - average % error for all test vectors	Strength - % error for 6Y25	Density - average % error for all test vectors	Density - % error for 6Y25
A	8.5	0	3.5	0
B	6.5	0	1.5	0
C	1.0	0	3.0	0
D	2.5	0	0.5	0
E	4.5	0	1.0	0
Combined Average % error	4.6	0	2.0	0

Table XV: Overall results for 1370 °C, 60% training

Combination	Strength - average % error for all test vectors	Strength - % error for 6Y25	Density - average % error for all test vectors	Density - % error for 6Y25
A	6.3	0	5.7	0
B	4.3	0	4.3	0
C	3.3	0	3.0	0
D	6.3	0	0.7	0
E	6.7	0	2.7	0
Combined Average % error	5.4	0	3.3	0

Table XVI: Prediction of selected processing and sintering variables for optimum density and strength at 1370 °C with 100 % plus 6Y25 training

Milling Time	Sintering Time	Nitrogen Pressure	Predicted Strength	Predicted Density
150	1.5	4	425	3.18
175	1.5	4	430	3.21
200	1.5	4	434	3.25
200	1.5	5	444	3.28
200	1.75	5	448	3.28
250	2	5	462	3.28
300	1.5	4	453	3.28
300	1.5	5	462	3.28
300	1.75	5	467	3.28
300	2	5	467	3.28

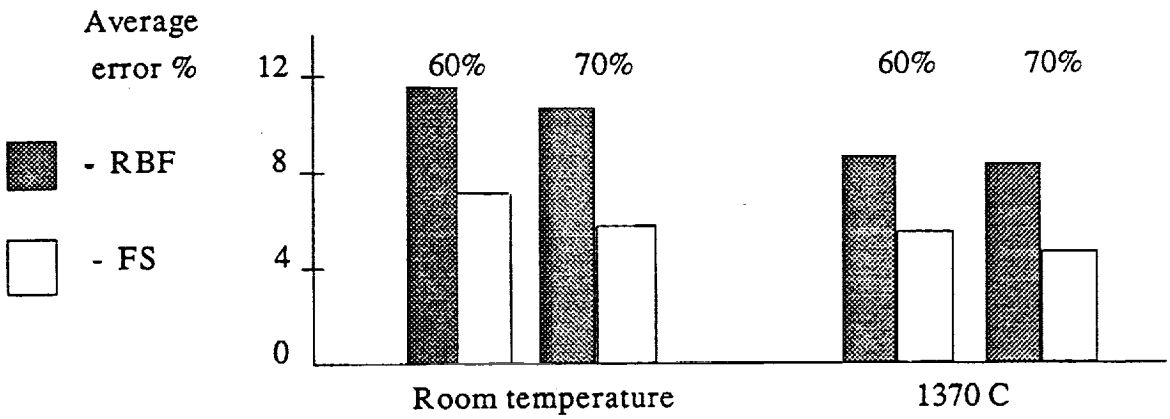


Fig. 2 Average errors in predicting strength using radial basis function neural network (RBF) and fuzzy sets (FS).

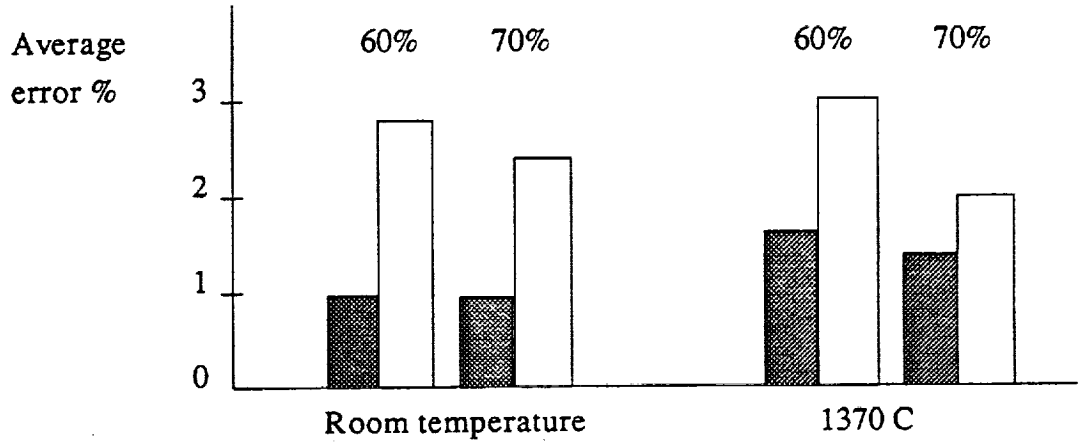


Fig. 3 Average errors in predicting density using radial basis function neural network (RBF) and fuzzy sets (FS).

(270, 2, 5).

The results show that fuzzy set theory can be a powerful tool for both process modeling and property control. Fuzzy logic should help optimize and speed the development and processing of emerging ceramic materials. The fuzzy system was found to be applicable for learning the host processing parameters and consequently predicting strength and density based on three processing variables as input features for silicon nitride.

In general, predicting bulk density was more successful than predicting strength. This was due to the fact that bulk density is directly related to milling time, sintering time and pressure, whereas the flexural strength is additionally dependent on pore morphology, on microstructure, and on the presence of failure causing defects.

Comparison of the results obtained by using fuzzy sets with those obtained previously with neural networks on the same data [5] indicates that fuzzy sets are superior in modeling less precise relationships existing between the processing variables and strength (Fig. 2 and Fig. 3) which are due to statistical variations in the manufacturing process. The more precise relationship between the processing variables and density was modelled better (in terms of error) using neural networks.

Developers of ceramics and composite structures can achieve better strength and density, and shorten the processing by utilizing fuzzy sets and neural networks in tandem. The former will help to capture imprecise relationships which are due to unavoidable variations in a manufacturing process, the latter to capture more precise, although still very complex, relationships. This can be seen as the alternative to the Taguchi method [12].

CONCLUSIONS

Fuzzy sets theory was applied to learn the relationships that exist between the strength and density, and the three processing variables: milling time, sintering time, and nirogen pressure. The learned relationships were used for predicting strength and density for new combinations of the processing variables. The reliability of these predictions was validated by calculating the errors on test data encompassing either 30% or 40% of available data. The maximum error for strength was 7.1%, and for density it was 3.3%. It was found that fuzzy sets are superior to neural networks in capturing vague relationships between the

processing variables and strength. However, for density which is more directly related to the input variables, neural networks gave better results.

Summarizing, developers of structural ceramics and ceramic composites, may utilize computational paradigms of fuzzy sets and neural networks to optimize the desired parameters and to shorten the processing and manufacturing cycle.

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REFERENCES

[1] Klima S. J. and Baaklini G. Y., Nondestructive Characterization of Structural Ceramics, SAMPE Quarterly, Vol. 17., No. 3, 1986, pp. 13-19.

[2] Klima S. J., NDE for Heat Engine Ceramics, NASA TM-86949, 1984.

[3] Baaklini, G.: NDE Reliability and Process Control for Structural Ceramics. Journal of Engineering for Gas Turbines and Power, Vol. 109, 1987, pp. 263-266.

[4] Sanders W. A. and Baaklini G. Y., Correlation of Processing and Sintering Variables with the Strength and Radiography of Silicon Nitride, Ceramic Engineering and Science Proceedings, Vol. 7, No. 7-8, pp. 839-860, 1986.

[5] Cios K. J., Baaklini G. Y., Vary A. and Tjia R. E., Radial Basis Function Network Learns Ceramic Processing and Predicts Related Strength and Density, International Advances in Nondestructive Testing, Gordon and Breach, 1992, submitted.

[6] Cios K. J. and Sztandera L. M., Boundness and Equivalence of Fuzzy Systems and Feed Forward Artificial Neural Networks, IEEE Transactions on Fuzzy Systems, 1992, submitted.

[7] Cios K. J., Shin I., and Goodenday L., Using Fuzzy Sets to Diagnose Artery Coronary Stenosis, IEEE Computer Magazine, Vol. 24:3, 1991, pp. 57-63.

[8] Klir G. J. and Folger T. A., Fuzzy Sets, Uncertainty, and Information, Prentice Hall, Englewood Cliffs, 1988.

[9] Sztandera L.M., Spatial Relations Among Fuzzy Subsets of an Image, M.S. Thesis, Department of Electrical and Computer Engineering, University of Missouri-Columbia, Columbia, MO, 1990.

[10] Zadeh L., A Theory of Approximate Reasoning, Machine Intelligence, 9, in Hayes J. K., Michie D., and Mikulich L. I., Eds., John Willey, New York, 1979, pp. 149-194.

[11] Zadeh L., Fuzzy sets, Information Control, 8, pp. 338-353, 1965.

[12] McEntire B. J., Taglialavore A. P., Heichel D. N., Bright E., Yeckley R. L., Holowczak J. E., and Quackenbush C. L., Silicon Nitride Component Development for Advanced Gas Turbine Engines, Proceedings of the 27th Automotive Technology Development Contractors' Coordination Meeting, Society of Automotive Engineers, Inc., Warrendale, PA, 1990, pp. 341-349.

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13. ABSTRACT (*Maximum 200 words*)

In this work we utilize fuzzy sets theory to evaluate and make predictions of flexural strength and density of NASA 6Y silicon nitride ceramic. Processing variables of milling time, sintering time, and sintering nitrogen pressure are used as an input to the fuzzy system. Flexural strength and density are the output parameters of the system. Data from 273 Si_3N_4 modulus of rupture bars tested at room temperature and 135 bars tested at 1370 degrees Celsius are used in this study. Generalized mean operator and Hamming distance are utilized to build the fuzzy predictive model. The maximum test error for density does not exceed 3.3%, and for flexural strength 7.1%, as compared with the errors of 1.72% and 11.34% obtained by using neural networks, respectively. These results demonstrate that fuzzy sets theory can be incorporated into the process of designing materials, such as ceramics, especially for assessing more complex relationships between the processing variables and parameters, like strength, which are governed by randomness of manufacturing processes.

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