N93-27341

GALILEAN COVARIANT HARMONIC OSCILLATOR

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Abstract

A Galilean covariant approach to classical mechanics of a single particle is described. Within the proposed formalism we reject all non-covariant force laws defining acting forces which become to be defined covariantly by some differential equations. Such an approach leads out of the standard classical mechanics and gives an example of non-Newtonian mechanics. It is shown that the exactly solvable linear system of differential equations defining forces contains the Galilean covariant description of harmonic oscillator as its particular case. Additionally we demonstrate that in Galilean covariant classical mechanics the validity of the second Newton law of dynamics implies the Hooke law and vice versa. We show that the kinetic and total energies transform differently with respect to the Galilean transformations.

1 Introduction

Recently we have proposed a new approach to classical mechanics which leads to a manifestly Galilean covariant models of mechanics for a single interacting particle [1]. Our main goal was to construct a self-consistent and complete scheme avoiding all relations of standard classical mechanics which break the Galilean covariance. It is easy to see that all such relations belong to the class of the so-called "constitutive relations" [2] and in order to achieve our goal we had to reexamine the role of these relations in mechanics. The relation between momentum and velocity is an example of the Galilean covariant constitutive relation [3] while all explicit expressions of the mechanical forces in terms of positions and velocities, called force laws, obviously break this covariance. Hence, in a Galilean covariant formulation of classical mechanics of a single particle we have to reject all known force laws. To keep the formalism as predictive as the usual one we propose to determine all mechanical quantities from the set of differential equations of the evolution type.

Our program leads us to a broader than Newtonian formalism model of classical mechanics in which more than one vector-valued measure of mechanical interaction is introduced. The time evolution of these measures is described by a set of differential equations called the equations of the environment which are used to determine the interaction of the particle with its environment in a fully covariant way. The simplest version of such a scheme contains two measures of interaction:

the customary force $\vec{F}(t)$ measuring the momentum non-conservation and a new quantity which we have called the influence $\vec{I}(t)$ governing the time evolution of the acceleration. We do not assume *a priori* the Galilean covariant Newton's second law of dynamics in the form

$$M\vec{a}(t) = \vec{F}(t) \tag{1.1}$$

where M denotes the inertial mass of the particle because this equation is not of the evolution type for the acceleration and contains a physical constant. According to our general philosophy [2] we avoid to use any such constants unless we really need to introduce them as phenomenological parameters. In our case this will happen only for the equations of the environment for which without any doubt we are forced to use in the theory some information of the phenomenological character. All the remaining equations describing the particle are universal, interrelate only basic theoretical concepts and do not contain any phenomenological constant. In our theory the experimental input is used therefore only for the description of the environment and we consider this fact as a big advantage of our formalism. The relation between classical Newtonian mechanics based on the equation (1.1) and our scheme is established using (1.1) as a constraint put on the set of solutions of the differential equations. It is also a constraint put on solutions of the equation

$$\frac{d\vec{F}(t)}{dt} = \frac{1}{M}\vec{I}(t) \tag{1.2}$$

which in the framework of the Newton's mechanics follows from the definition of $\vec{I}(t)$. The solutions of our model which satisfy (1.1) we shall call Newtonian solutions while solutions satisfying the relation (1.2) only will be called the generalized Newtonian solutions.

2 Linear model

The aim of this talk is to illustrate our approach on a simple example of linear evolution equations for the force and influence. We shall show that such a model includes, as its particular case, the Newtonian mechanics of the material point which motion is defined by the force provided by a linear in position and velocity force law.

In the case under consideration the complete set of differential equations describing the system consists of two purely kinematical equations of motion

$$\frac{d\vec{x}(t)}{dt} = \vec{v}(t) \tag{2.1}$$

$$\frac{d\vec{v}(t)}{dt} = \vec{a}(t) \tag{2.2}$$

one dynamical equation of motion

$$\frac{d\vec{a}(t)}{dt} = \vec{I}(t) \tag{2.3}$$

one equation of balance

$$\frac{d\vec{p}(t)}{dt} = \vec{F}(t) \tag{2.4}$$

and the system of two equations of environment

$$\begin{aligned}\frac{d\vec{F}(t)}{dt} &= \alpha\vec{F}(t)+\beta\vec{I}(t)\\ \frac{d\vec{I}(t)}{dt} &= \gamma\vec{F}(t)+\delta\vec{I}(t)\end{aligned} \tag{2.5}$$

where $\vec{x}(t), \vec{v}(t), \vec{a}(t)$ and $\vec{p}(t)$ are the trajectory function of the particle, its velocity, acceleration and momentum, respectively. The meaning of $\vec{F}(t)$ and $\vec{I}(t)$ has been explained above and the parameters α, β, γ and δ represent dimensional coupling constants specifying the model.

The model is covariant with respect to the Galilean transformations parametrized by a rotation R, a boost $\vec{u}$, and a space-time translation $(\vec{a}, b)$ if all mechanical quantities used obey the following transformation rules

$$\vec{x}(t) \rightarrow \vec{x}'(t') = R\vec{x}(t) + \vec{u}t + \vec{a} \tag{2.6}$$

$$\vec{v}(t) \rightarrow \vec{v}'(t') = R\vec{v}(t) + \vec{u} \tag{2.7}$$

$$\vec{a}(t) \rightarrow \vec{a}'(t') = R\vec{a}(t) \tag{2.8}$$

$$\vec{p}(t) \rightarrow \vec{p}'(t') = R\vec{p}(t) + m\ \vec{u} \tag{2.9}$$

$$\vec{F}(t) \rightarrow \vec{F}'(t') = R\vec{F}(t) \tag{2.10}$$

$$\vec{I}(t) \rightarrow \vec{I}'(t') = R\vec{I}(t) \tag{2.11}$$

where

$$t \rightarrow t' = t + b \tag{2.12}$$

and m is the Galilean mass of the particle [3] which we shall not assume to be equal to the inertial mass.

As we stressed in the Introduction the only external parameters characterizing the model are coupling constants in the equation of the environment (2.5). The mutual relation between them defines the shape of general solution of (2.1) - (2.5). Denoting by A the matrix of coupling constants

$$A = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \tag{2.13}$$

and their following combinations by $\lambda_\pm$

$$\lambda_\pm = \frac{1}{2}\left[trA \pm \sqrt{(TrA)^2 - 4detA}\right] \tag{2.14}$$

we may write down for $4detA > (TrA)^2$ the general solution of the equations (2.1) - (2.5) in the form

$$\vec{x}(t) = \vec{A} + \vec{B}t + \vec{C}t^2 + \vec{D}\ exp(\lambda_+ t) + \vec{E}\ exp(\lambda_- t) \tag{2.15}$$

$$\vec{v}(t) = \vec{B} + 2\vec{C}t + \vec{D}\ \lambda_+\ exp(\lambda_+ t) + \vec{E}\ \lambda_-\ exp(\lambda_- t) \tag{2.16}$$

$$\vec{a}(t) = 2\vec{C} + \vec{D}\ \lambda_+^2\ exp(\lambda_+ t) + \vec{E}\ \lambda_-^2\ exp(\lambda_- t) \tag{2.17}$$

$$\vec{p}(t) = \vec{P} - \frac{\delta - \lambda_+}{\gamma}\ \lambda_+^2\ \vec{D}\ exp(\lambda_+ t) - \frac{\delta - \lambda_-}{\gamma}\ \lambda_-^2\ \vec{E}\ exp(\lambda_- t) \tag{2.18}$$

$$\vec{F}(t) = -\frac{\delta - \lambda_+}{\gamma} \lambda_+^3 \vec{D}\, exp(\lambda_+ t) - \frac{\delta - \lambda_-}{\gamma} \lambda_-^3 \vec{E}\, exp(\lambda_- t) \tag{2.19}$$

$$\vec{I}(t) = \lambda_+^3 \vec{D}\, exp(\lambda_+ t) + \lambda_-^3 \vec{E}\, exp(\lambda_- t) \tag{2.20}$$

Vector-valued constants $\vec{A}, \vec{B}, \vec{C}, \vec{D}, \vec{E}$ and $\vec{P}$ are the integration constants of the system of differential equations (2.1) - (2.5) and in order to satisfy the transformation rules (2.6) - (2.11) they have to transform in the following way

$$\vec{A} \to \vec{A}' = R\vec{A} - bR\vec{B} + b^2 R\vec{C} - b\vec{u} + \vec{a} \tag{2.21}$$

$$\vec{B} \to \vec{B}' = R\vec{B} - 2bR\vec{C} + \vec{u} \tag{2.22}$$

$$\vec{C} \to \vec{C}' = R\vec{C} \tag{2.25}$$

$$\vec{D} \to \vec{D}' = R\vec{D}\, exp(-\lambda_+ b) \tag{2.26}$$

$$\vec{E} \to \vec{E}' = R\vec{E}\, exp(\lambda_- b) \tag{2.27}$$

$$\vec{P} \to \vec{P}' = R\vec{P} + m\vec{u} \tag{2.28}$$

which explicitly show how their values depend on the choice of the reference frame.

Here we should like to stress the difference between our approach, demanding the Galilean covariance as the most fundamental feature of the theory and standard expositions of mechanics which treat it almost always as a branch of the theory of ordinary differential equations. There is no principle of relativity in the theory of differential equations and, consequently, there is no problem of transformation properties of the solutions and integration constants. In contradistinction to mathematics, this subject is of primary interest to physics and we have to realize that the integration constants take the whole responsibility for the transformation properties of all physical quantities. This means that the original preparation of physical system already contains almost the whole information on the symmetries of this system. The time evolution of the system has only to preserve the original symmetries. It should not be unexpected that in our scheme which is an example of a non-Newtonian mechanics (and, as a matter of fact, its generalization) the careful analysis of the properties of integration constants and their relation to the initial conditions may lead out of the framework of standard classical mechanics.

There is a lot of different initial conditions which may be imposed on the solution (2.13 - 2.18). For instance, we may use the values of the first four derivatives of the function $\vec{x}(t)$ at the same instant of time t_0 to fix the values of the constants $\vec{A}$ to $\vec{E}$. It remains in obvious contradiction to the widely spread opinion that in mechanics only the initial position and velocity are needed for the unique determination of the trajectory. This is the property of Newtonian mechanics only in which the relation (1.1) is always satisfied. In our formalism the acceleration $\vec{a}(t)$ and the force $\vec{F}(t)$ are *a priori* independent as determined from independent equations and the relation (1.1) imposed on these quantities reduces the number of degrees of freedom for initial conditions. It enables us to calculate some parameters of a model in terms of the other. We shall see below that it may be used for determination of the inertial mass M in terms of the coupling constants given by elements of the matrix A.

The analysis of the model depends on the mutual relation between TrA and $detA$. In order to concentrate the attention on the harmonic oscillator problem we shall omit the case $TrA \geq 4detA$ because it does not describe oscillatory motion. The complete analysis of the problem will be found in [4].

3 Oscillatory motion

It is immediately seen from (2.14) and (2.15) that the trajectory (2.15) oscillates if the inequality $(TrA)^2 < 4detA$ holds and the oscillations may be damped or not depending on the value of $Re\lambda_\pm$. The reality of all mechanical quantities requires that the constants $\vec{D}$ and $\vec{E}$ are complex valued and they must be complex conjugated

$$\vec{D} = \vec{E}^* \tag{3.1}$$

which is the first condition restricting the arbitrariness of integration constants $\vec{A}, \vec{B}, \vec{C}, \vec{D}, \vec{E}$. The Newtonian condition (1.1), as well as its generalization of the form

$$\vec{F}(t) = M\,[\vec{a}(t) - 2\vec{C}] \tag{3.2}$$

are satisfied provided

$$M = -\frac{\delta - \lambda_+}{\gamma}\,\lambda_+ = -\frac{\delta - \lambda_-}{\gamma}\,\lambda_- \tag{3.3}$$

supplemented additionally in the Newtonian case by the Galilean invariant relation

$$\vec{C} = 0 \tag{3.4}$$

Relation (3.4) fixes invariantly one of the parameters of the solution and we shall use it as a criterion of the Newtonian character of the solution considered.

Substituting in (3.3) the values $\lambda_\pm$ from (2.14) we come to the conclusion that the equality (3.3) may be satisfied only for $\alpha = 0$ which gives

$$M = \beta \tag{3.5}$$

The value of the Galilean mass m remains arbitrary because it is a parameter which identifies the particle and has nothing to do with its possible interactions.

Taking into account (2.15) and (2.19) it is easy to see that the famous Hooke force law

$$\vec{F}(t) = -k\vec{x}(t) \tag{3.6}$$

may be satisfied in a selected reference frame for which $\vec{A} = \vec{B} = \vec{C} = 0$ i.e. only for reference frames satisfying the criterion (3.3) of the Newtonian character of mechanics. This means, because of the invariance of this relation, that the Newtonian condition (1.1) is equivalent to the requirement of the existence of the Hooke law. This fact has a far going consequences because in all treatments of the foundations of mechanics the forces are measured by dynamometers which operate on the principle of the Hooke law. Therefore any mechanics using such an operational definition of forces must be Newtonian. The Newton laws of mechanics follow thus from the adopted operational definition of force. In order to detect any violation of these laws we should first invent a new operational definition of the force not based on the Hooke law. It is indeed a very surprising conclusion which however uniquely follows from our more general approach to mechanics.

The above conclusion is less surprising after observing that the linear relation between momentum and velocity

$$\vec{p}(t) = M\vec{v}(t) \tag{3.7}$$

is possible also in the Newtonian mechanics only. Therefore the almost always assumed relation (3.7) prevent to observe any deviation from the Newton's laws. We have to conclude that many fundamental assumptions of standard mechanics are interrelated and their possible interrelations may be found only if the analysis is performed in the framework of the approach to mechanics more general than the standard one. Our method is just an example of such a scheme.

For the non-Newtonian mechanics we may replace the Hooke law by the relation

$$\vec{F}(t) = -k\left[\vec{x}(t) - \vec{A} - \vec{B}t - \vec{C}t^2\right] \tag{3.8}$$

which for $\vec{A} = \vec{B} = \vec{C} = 0$ reduces to (3.6). Using again the solutions (2.15) and (2.19) we come to the conclusion that (3.8) may be satisfied only if

$$k = \frac{\delta - \lambda_+}{\gamma}\,\lambda_+^3 = \frac{\delta - \lambda_-}{\gamma}\,\lambda_-^3 \tag{3.9}$$

Together with (3.3) and (2.14) it implies that

$$(TrA)\sqrt{(TrA)^2 - 4detA} = 0 \tag{3.10}$$

The square root must be different from 0 due to the condition $4detA > (TrA)^2$ assumed and therefore we must have

$$TrA = 0 \tag{3.11}$$

Since we already have got $\alpha = 0$ this condition gives $\delta = 0$. The frequency of oscillations is given by

$$\omega^2 = detA \tag{3.12}$$

and because of (3.5) and $\alpha = \delta = 0$ we have

$$\omega^2 = -\beta\gamma = -M\gamma \tag{3.13}$$

We may therefore conclude that in the framework of Galilean covariant approach to classical mechanics the non-Newtonian generalization of the standard harmonic oscillator is given by linear evolution equations for the force and the influence and that the matrix of the coupling constants has the form

$$A = \begin{pmatrix} 0, & M \\ -\frac{\omega^2}{M}, & 0 \end{pmatrix} \tag{3.14}$$

The most general Galilean covariant linear relation between the force, the position and the velocity is the non-Newtonian generalization of the superposition of Hooke and linear friction ($\eta < 0$) forces

$$\vec{F}(t) = \eta\left(\vec{v}(t) - \vec{B} - \vec{C}t\right) - \kappa\left(\vec{x}(t) - \vec{A} - \vec{B}t - \vec{C}t^2\right) \tag{3.15}$$

which, after substitution of (2.15), (2.16) and (2.19) into it leads to the following relations between parameters of the model

$$\begin{aligned} M\lambda_+^2 &= \eta\lambda_+ - \kappa \\ M\lambda_-^2 &= \eta\lambda_- - \kappa \end{aligned} \tag{3.16}$$

if additionally the generalized Newtonian condition (3.2) is demanded. We are still restricted to the case $4 det A > (Tr A)^2$ which gives the only solution of (3.16) in the form

$$\delta = \frac{\eta}{M} \tag{3.17}$$

$$\gamma = -\frac{\kappa}{M^2} \tag{3.18}$$

and it immediately follows from it and (2.15) that the matrix of coupling constants

$$A = \begin{pmatrix} 0, & M \\ -\frac{\kappa}{M^2}, & \frac{\eta}{M} \end{pmatrix} \tag{3.19}$$

describes damped oscillatory motion with frequency given by

$$\omega = \frac{1}{2M}\sqrt{\eta^2 - 4\kappa M} \tag{3.20}$$

and an amplitude damping exponentially according to the factor $exp\frac{\eta}{2M}t$.

4 Kinetic and total energies

In the standard approach to classical mechanics the kinetic energy is defined by one of the equivalent expressions

$$k(t) = \frac{\vec{p}^{\,2}(t)}{2M} = \frac{M\ \vec{v}^{\,2}(t)}{2} = \frac{1}{2}\,\vec{p}(t)\cdot\vec{v}(t) \tag{4.1}$$

where M is the inertial mass of the particle. Relations (4.1) are a straightforward consequence of the Newtonian relation between momentum and velocity (3.7) which in Galilean covariant scheme proposed should be treated as additional assumption only. Discarding (3.7) as *a priori* valid we cannot identify the inertial mass present in second law of dynamics and the mass parameter appearing in the momentum transformation rule (2.9). The general relation between momentum and velocity written down with Galilean mass introduced into it has now, according to [3], the form

$$\vec{p}(t) = (m - M)\vec{v}(t_0) + M\vec{v}(t) \tag{4.2}$$

where $\vec{v}(t_0)$ is an integration constant having the meaning of an initial velocity which has to be specified from initial conditions.

We define the kinetic energy as bilinear form of momentum and velocity satisfying two fundamental conditions put on it:

i.) the balance equation

$$\frac{dk(t)}{dt} = \vec{F}(t)\cdot\vec{v}(t) \tag{4.3}$$

and

ii.) the Galilean transformation rule

$$k(t) \rightarrow k'(t') = k(t) + R\vec{p}(t)\cdot\vec{u} + \frac{1}{2}m\vec{u}^{\,2} \tag{4.4}$$

According to these conditions [3] the kinetic energy is given by

$$k(t) = \frac{m-M}{2}\vec{v}^{\,2}(t_0) + \frac{M}{2}\vec{v}^{\,2}(t) \tag{4.5}$$

which, in notation introduced by (2.15) - (2.20) and in Newtonian regime $\vec{C} = 0$, may be written down as

$$k(t) = \frac{\left(\vec{P} - M\vec{B}\right)^2}{2(m-M)} + \frac{M}{2}\vec{v}^{\,2}(t) \tag{4.6}$$

To obtain the correct formula for the kinetic energy in non-Newtonian regime we shall start with the general expression

$$\begin{aligned} k(t) &= A\vec{p}^{\,2}(t) + B\vec{v}^{\,2}(t) + C\vec{p}(t)\cdot\vec{v}(t) + \\ &\quad + \vec{\lambda}\cdot\vec{x}(t) + \mu t + \nu t^2 + \Delta \end{aligned} \tag{4.7}$$

The transformation rule (4.4) implies the following conditions and transformation properties which parameters in (4.7) have to obey

$$\begin{gathered} A' = A, \qquad B' = B, \qquad C' = C \\ B = -\frac{m}{2}(1 - 2mA) \\ C = 1 - 2mA \\ \nu' = \nu, \\ \vec{\lambda}' = R\vec{\lambda} \\ \mu' = \mu - 2\nu b - R\vec{\lambda}\cdot\vec{u} \\ \Delta' = \Delta + \nu b^2 - \mu b - R\vec{\lambda}\cdot\vec{a} + R\left(\vec{\lambda}\cdot\vec{u}\right)b \end{gathered} \tag{4.8}$$

while the balance equation (4.3) gives

$$\begin{gathered} A = \frac{1}{2(m-M)}, \qquad B = \frac{mM}{2(m-M)}, \qquad C = -\frac{M}{m-M} \\ \vec{\lambda} = -2M\vec{C} \\ \mu = \frac{2M}{m-M}\left(\vec{P} - M\vec{B}\right)\cdot\vec{C} \\ \nu = -\frac{2M^2}{m-M}\vec{C}^{\,2} \end{gathered} \tag{4.9}$$

It is obvious that the balance equation (4.3) cannot fix the value of the constant Δ in (4.7) which remains arbitrary but has to satisfy the transformation rule listed in (4.8) as the last. For example, we may represent Δ in the following form

$$\Delta = -\vec{\lambda} \cdot \vec{x}_0 - \mu t_0 - \nu t_0^2 \tag{4.10}$$

with $(\vec{x}_0, t_0)$ denoting the space-time coordinates of an arbitrary event. They may be chosen as coordinates of an event for which the momentum and the velocity of the particle simultaneously vanish. Such a choice guarantees that the kinetic energy also vanishes at this point which we consider the most natural condition possible to demand.

Substituting all values of coefficients (4.9) into (4.7) we obtain

$$\begin{aligned} k(t) &= \frac{(\vec{P} - M\vec{B})^2}{2(m-M)} + \frac{M}{2}\vec{v}^{\,2}(t) - 2M\vec{C} \cdot [\vec{x}(t) - \vec{x}_0(t)] - \\ &- \frac{2M}{m-M}\left(\vec{P} - M\vec{B}\right) \cdot \vec{C}\, t_0 + \frac{2M^2}{m-M}\vec{C}^{\,2} t_0^2 \end{aligned} \tag{4.11}$$

and comparing it with the expression obtained for the Newtonian case (4.5) we see that the only parameter which controls the Newtonian character of mechanics is $\vec{C}$ the vanishing of which is equivalent to vanishing of $\vec{\lambda}$, μ, ν and Δ in any reference frame.

In contradistinction to the kinetic energy the definition of the total energy E for conservative system cannot be based on the above listed basic properties of the kinetic energy. The balance equation for the total energy

$$\frac{dE}{dt} = 0 \tag{4.12}$$

does not give any hint on the transformation rule of E. This rule cannot be of the same shape as for the kinetic energy since this immediately leads to a contradiction. Indeed, if we suppose

$$E \to E' = E + R\vec{p}(t) \cdot \vec{u} + \frac{1}{2}m\vec{u}^{\,2} \tag{4.13}$$

the conservation law (4.12) implies that

$$\frac{d\vec{p}(t)}{dt} = 0 \tag{4.14}$$

which is true for free particles only. To construct the correct expression for the total energy we shall start from the general bilinear form of $\vec{v}, \vec{p}, \vec{x}, t, \vec{F}, \vec{I}$ which satisfies the following two conditions:

i.) it reduces to the expression for $k(t)$ if $\vec{F} = \vec{I} = 0$,

ii.) it satisfies the conservation law (4.12).

After straightforward but tedious calculations it can be shown that the only form which obeys these two conditions is given by

$$\begin{aligned} E &= k(t) + \frac{\gamma(\delta^2 - \gamma M)}{2(\gamma M)^3}\vec{F}^{\,2} + \frac{1}{2(\gamma^2}\vec{I}^{\,2} - \frac{\delta}{(\gamma M)^2}\vec{F} \cdot \vec{I} + \\ &+ \frac{\delta}{\gamma M}\vec{F} \cdot \vec{v} - \frac{1}{\gamma}\vec{I} \cdot \vec{v} + \frac{2}{\gamma M}\vec{F} \cdot \vec{C} \end{aligned} \tag{4.15}$$

and consequently we shall take it as the definition of the total energy for the Galilean covariant harmonic oscillator.

It is now easy to see that under Galilean transformations the total energy changes according to the following rule

$$E \to E' = E + R\left(\vec{p}(t) + \frac{1}{\gamma M}\left(\delta\vec{F} - M\vec{I}\right)\right)\cdot\vec{u} + \frac{1}{2}m\vec{u}^{\,2} \tag{4.16}$$

The most important point is connected with the quantity in the second term. It shows that the momentum associated with the total energy E is not the momentum $\vec{p}(t)$ but

$$P = \vec{p}(t) + \frac{1}{\gamma M}\left(\delta\vec{F}(t) - M\vec{I}(t)\right) \tag{4.17}$$

which is conserved in time because of the fundamental equations (2.4) - (2.5). The difference in the transformation rules for the kinetic and total energies is a new fact in mechanics which without our Galilean covariant approach to mechanics could not be derived. Here we would like to remark this so important fact is not specific for the non-Newtonian case only. As we have mentioned several times the Newtonian case which is equivalent to the Galilean invariant choice $\vec{C} = 0, \delta = 0$. However it must not be taken directly by putting these values into (4.15) because such a choice corresponds to the singular system of algebraic equations used to determine coefficients in (4.15). The correct result is given by

$$E_N = \frac{M}{2}\left(\vec{v}^{\,2} + \frac{2\vec{I}\cdot\vec{v}}{\omega^2} + \frac{\vec{I}^{\,2}}{\omega^4}\right) + \chi\left(\vec{F}^{\,2} + \left(\frac{M}{\omega}\vec{I}\right)^2\right) \tag{4.18}$$

where $\omega^2 = -\gamma M$ according to (3.13) and χ is arbitrary parameter. It remains in full agreement with our previous result obtained in [1] within less general approach and gives for the total energy the Galilean transformation rule

$$E_N \to E'_N = E_N + R\left(\vec{p} + \frac{M}{\omega^2}\vec{I}\right)\cdot\vec{u} + \frac{1}{2}m\vec{u}^{\,2} \tag{4.19}$$

which means again that the total energy transforms differently from the kinetic energy and that its transformation properties are associated with a conserved quantity

$$P = \vec{p}(t) + \frac{M}{\omega^2}\vec{I}(t) \tag{4.20}$$

and not with ordinary momentum $\vec{p}(t)$.

5 Conclusions

We have demonstrated that the requirement of the Galilean covariance of classical mechanics leads to a formalism broader than the standard Newtonian one. The new formalism enlarges the class of mechanical systems including those with some unusual properties. In particular, in the next talk we shall discuss the application of the formalism obtained to description of the so-called confined systems.

6 Acknowledgments

This work was supported in part by funds provided by Polish State Committee for Scientific Research (KBN) under grant 2 0342 91. AH is also very grateful to KBN for financial support which enables him to participate in the "Workshop on Harmonic Oscillators" and to Dr.W. Broniowski for his hospitality at College Park.

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V. THERMODYNAMICS AND STATISTICAL MECHANICS