Recent Advances in Superconducting-Mixer Simulations

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March 22, 1992

1 Introduction

Over the last few years, considerable progress has been made in the development of techniques for fabricating high-quality superconducting circuits, and this success, together with major advances in the theoretical understanding of quantum detection and mixing at millimetre and submillimetre wavelengths [1], has made the development of CAD techniques for superconducting nonlinear circuits an important new enterprise. For example, arrays of quasioptical mixers are now being manufactured, where the antennas, matching networks, filters and superconducting tunnel junctions are all fabricated by depositing niobium and a variety of oxides on a single quartz substrate. There are no adjustable tuning elements on these integrated circuits, and therefore, one must be able to predict their electrical behaviour precisely. This requirement, together with a general interest in the generic behaviour of devices such as direct detectors and harmonic mixers, has lead us to develop a range of CAD tools for simulating the large-signal, small-signal, and noise behaviour of superconducting tunnel junction circuits.

2 Large-signal analysis

To model the behaviour of a quasiparticle mixer, it is first necessary to simulate the large-signal steady-state dynamics of the local-oscillator circuit. Once the large-signal operating point is known, it is then possible to perturb, either numerically or analytically, the underlying system of equations to gain information about the linear relationships between signal and noise variables.

The main problem is how does one calculate the periodic current that flows through a tunnel junction when a periodic voltage is applied? For semiconductor devices this calculation is almost always carried out in the time domain, and fast Fourier transforms are used to interface the terminal waveforms to the frequency-domain description of the embedding circuit.

Classical resistive mixer diodes are relatively easy to simulate because the induced current is an instantaneous function of the terminal voltage. Quantum mixer diodes on the other hand are difficult to simulate because the tunnelling current depends on the voltage that was across the junction at very long times in the past. In the time domain

the current is calculated through an integral which is similar to the convolution integral of linear systems theory, and the tunnel junction is characterised by a response function which oscillates at the gap frequency with an envelope which decays inversely with time at large times. To evaluate the tunnelling current it is necessary to sample the terminal voltage at a rate greater than the gap frequency , and to integrate beyond a limit which is inversely related to the voltage width of the dc nonlinearity. Time-domain simulations are useful for studying the switching behaviour of tunnel junctions, but they are inappropriate for studying the steady-state behaviour of RF circuits.

When a sinusoidal potential is applied to a superconducting tunnel junction, the wave-functions associated with the quasiparticle states on one side of the barrier are coherently phase modulated. The spectrum of the phase factor is a comb of delta functions whose coefficients are the elements of a Bessel-function sequence. The Bessel functions have the same argument, determined by the voltage drive level, and consecutive orders ranging from some large positive integer to the same negative integer. The trick is to recognize that when a periodic potential is applied, the spectrum of the overall phase factor is the convolution of the spectra associated with the individual harmonic contributions. Once the spectrum of the overall phase factor is known, it is possible to calculate the harmonic phasors of the tunnelling current from the dc I-V curve and its Hilbert transform [2].

The above procedure describes a way of calculating the periodic current that flows in a tunnel junction when a periodic potential is applied. In a real circuit the tunnel junction is embedded in a linear network and the problem of determining the various voltages and currents is complex. Applying the method of harmonic balance [3] to a generic circuit comprising a tunnel junction and a Thévenin voltage source, leads to a system of coupled nonlinear algebraic equations. Mathematically, the problem then consists of finding the roots of these equations; electrically, the problem is equivalent to searching for a waveform that simultaneously satisfies the circuit equations at every harmonic frequency. In nonlinear CAD terminology, the scheme is a frequency-domain spectral-balance method, however, unlike other versions, the spectral decomposition is based on device physics, rather than on expanding the terminal behaviour in a set of basis functions.

The set of algebraic equations that results from applying the method of harmonic balance to a tunnel-junction circuit must, in general, be solved numerically. By repeatedly analyzing, in different ways, a large pseudo-random set of tunnel-junction circuits, we have investigated the speeds and stabilities of a range of iterative root-finding techniques. A comparison of the techniques is shown in Fig. 1, where we have plotted the percentage of circuits that converge, and the mean number of iterations taken, as a function of the damping factor. The damping factor is a coefficient between 0 and 1 which determines the degree to which the result of an iteration influences the next guess. A small value improves stability at the expense of reducing the rate of convergence. The solid and dashed lines in Fig. 1 correspond to two different quality characteristics. It should be appreciated that the plots represent a total of around 20,000 circuit simulations.

The tunnel junction is a nonlinear admittance in the sense that it is most easy to calculate the current in terms of the terminal voltage. However, fixed-point voltage-update methods [4] are inappropriate for analyzing tunnel-junction circuits, especially in a common-user environment, because they fail to converge when the large-signal harmonic admittances of the tunnel junction are much greater than those of the embedding circuit. This problem is clearly demonstrated in Fig. 1 where it is seen that the routine will only converge if the system is heavily damped. A slightly more sophisticated way of finding the roots is

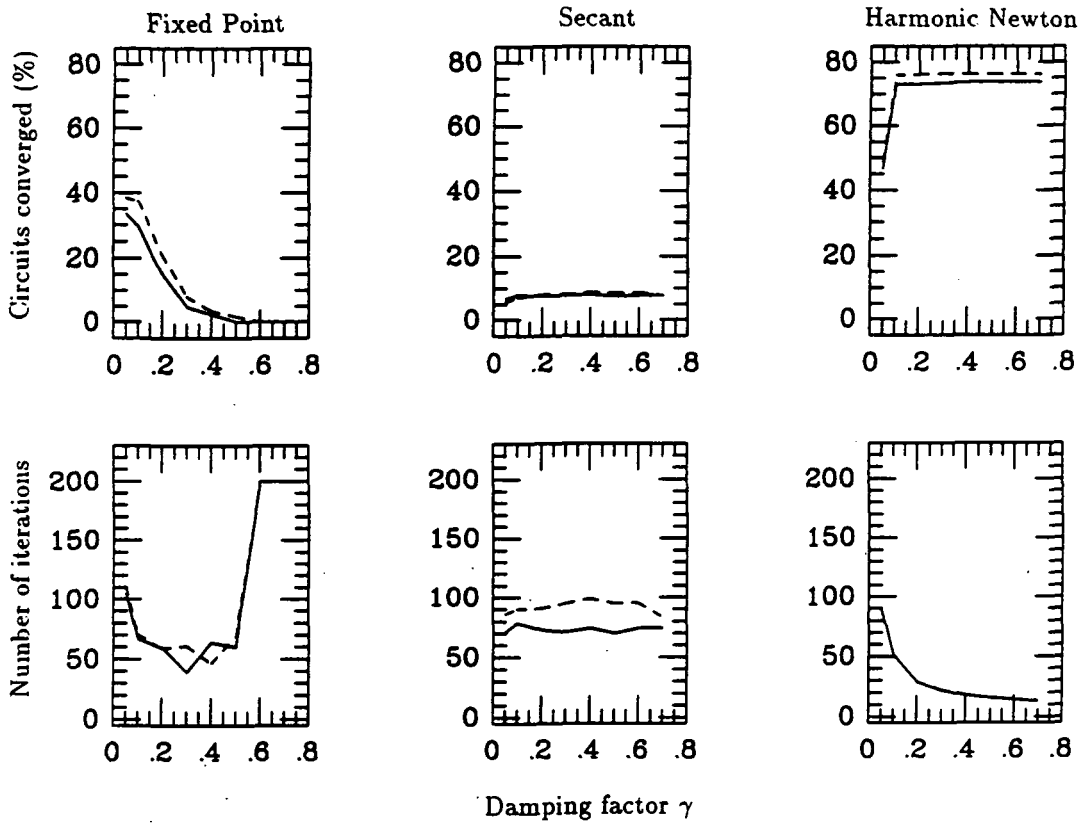

Figure 1: Comparison of various techniques for calculating the large-signal quantum behaviour of superconducting tunnel-junction circuits.

to use a multi-dimensional variant of the secant method [5]. This method is similar to the fixed-point method, in the sense that it is only necessary to calculate the tunnelling current once per iteration, however, because coarse derivative information is included one might expect the routine to behave more reasonably. Somewhat surprisingly, the routine is significantly worse despite the additional information. The problem is caused by the fact that, effectively, only the terms on the leading diagonal of the Jacobian matrix are included, and coupling between harmonics relies on the current calculations. As long as the current at a given harmonic is most strongly influenced by the voltage at the same harmonic then the routine will work well. In a highly nonlinear tunnel-junction circuit, however, there is strong coupling between harmonics and the routine is inadequate. Fig. 1 shows the behaviour of a harmonic-Newton [6] [7] scheme where the full Jacobian matrix is used. It is possible to calculate the Jacobian matrix analytically, however, we prefer to calculate the Jacobian matrix using finite differences. Harmonic Newton results in the least number of failures; in fact, it finds a solution for 75 % of the circuits studied, and to a large extent the stability of the method is independent of the quality of the junction being investigated. The fact that the convergence parameter has little effect on this fraction, together with the almost reciprocal dependence of the mean number of iterations, shows that if it is possible for the method to find a solution then it will eventually do so. Reducing the damping factor

simply reduces the size of the voltage steps taken at each iteration, however, these steps are usually in the correct direction.

We have now performed a very large number of real circuit simulations, and despite the fact that 25 % of the randomly generated circuits failed to converge, we have never come across a real circuit that has not converged. We have investigated this problem in some detail, and we have found that many of the circuits that do not converge are behaving in a non-periodic manner. This behaviour usually requires that the embedding circuit impedances are very much larger than the normal-state resistance.

An alternative approach to finding a root in many dimensions is to recast the problem into a multidimensional optimization. To do this change, the error function is used to construct a scalar quantity that has a global minimum at the required root. One of the attractions of optimization is that uninteresting variables, such as the local oscillator drive level, can be eliminated from the analysis by making the variable part of the objective function. In general, we have found that optimization methods are slow and should not be used unless there is a particular reason to do so. Unfortunately, there is insufficient time to discuss this more advanced topic in this short paper.

The results of a typical large-signal analysis are shown in Fig. 2. The sequence of plots shows how the pumped dc I-V curve of a typical Nb-AlOx-Nb tunnel junction evolves as the ωCR product is changed. As the capacitance decreases the subgap current increases, and non-classical negative differential resistance is induced on a number of photon steps. Notice that large capacitances are required before the characteristic relaxes to its constant sinusoidal-voltage-driven form. Also shown, for comparison, is an analysis where the harmonic feedback is turned off. Curiously, it seems as if internal harmonic pumping can enhance the negative differential resistance induced on high-order photon steps—for this reason it is possible, in certain circumstances, for the small-signal behaviour of a mixer to be very sensitive to harmonic impedance levels.

3 Small-signal analysis

Once the large-signal behaviour of a mixer has been established, it is possible to calculate the small-signal and noise performance. The admittance and noise-current correlation matrices are determined, in the usual way, through quantum-mechanical generalizations of commonly used classical concepts. We then use a selection of linear transforms to reduce the admittance parameters to two-port impedance and scattering parameters, and the current correlation matrix to a noise-temperature matrix from which the standard two-port noise parameters can be deduced [8] [9]. It transpires that the whole scheme, both signal and noise, can be very elegantly normalized to the gap voltage and gap current of the tunnel barrier. The advantages of our generalized approach are that one does not have to specify before hand which ports are to be used for the input and output, and one can easily calculate the two-port small-signal and noise parameters which can then be loaded into proprietary microwave circuit simulators for further analysis. For example, we are interested in designing mixers that have the first stage of low-noise IF amplification in the mixer block. A further advantage of our scheme is that the noise performance is described in terms of correlated travelling noise waves, and this approach is an elegant way of considering a mixer as an integral part of a quasioptical system along which noise waves propagate.

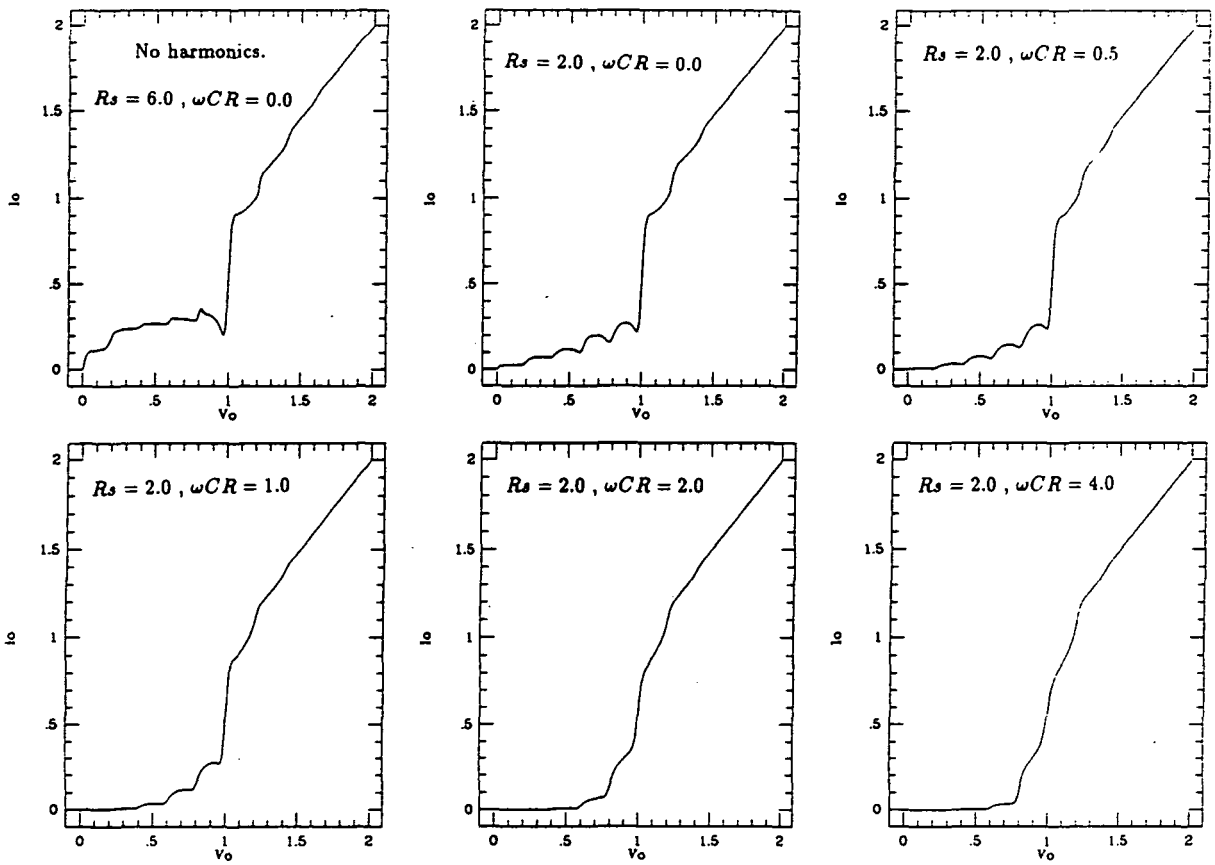

Figure 2: The dc I-V curve of a pumped Nb-AlOx-Nb tunnel junction for different values of ωCR product. The first plot does not include internal harmonic pumping.

4 Mixer simulations

To date we have studied the large-signal, small-signal, and noise behaviour of mixers by adopting the design procedure suggested by Kerr [10]. That is to say, the mixer is operated in a double-sideband mode, and the source and load impedances are assumed to be real. The ratio of the source and load impedances, which in practice is determined by the geometry of the mount, is set at some fixed value. In general, we use a value of unity as a higher value tends to degrade the input return loss of the mixer. It is interesting to note, however, that it may be possible to choose the ratio so as to minimize the sensitivity of the gain to variations in the source resistance. The free parameter, as far as the design process is concerned, is the normal-state resistance. Although we assume that the capacitance of the tunnel junction is tuned out at the fundamental, we assume that the impedance at the harmonics is given by the capacitance of the tunnel junction alone. The effects of junction capacitance are considered in companion paper [11], here we simply demonstrate the procedure by plotting, in Fig. 3, the transducer gain, noise temperature, input return loss, and normalized output impedance of a typical Nb-AlOx-Nb mixer as a function of the normalized source resistance; the various curves are for different normalized frequencies (normalized to the gap frequency). It is interesting to note that the overall performance is

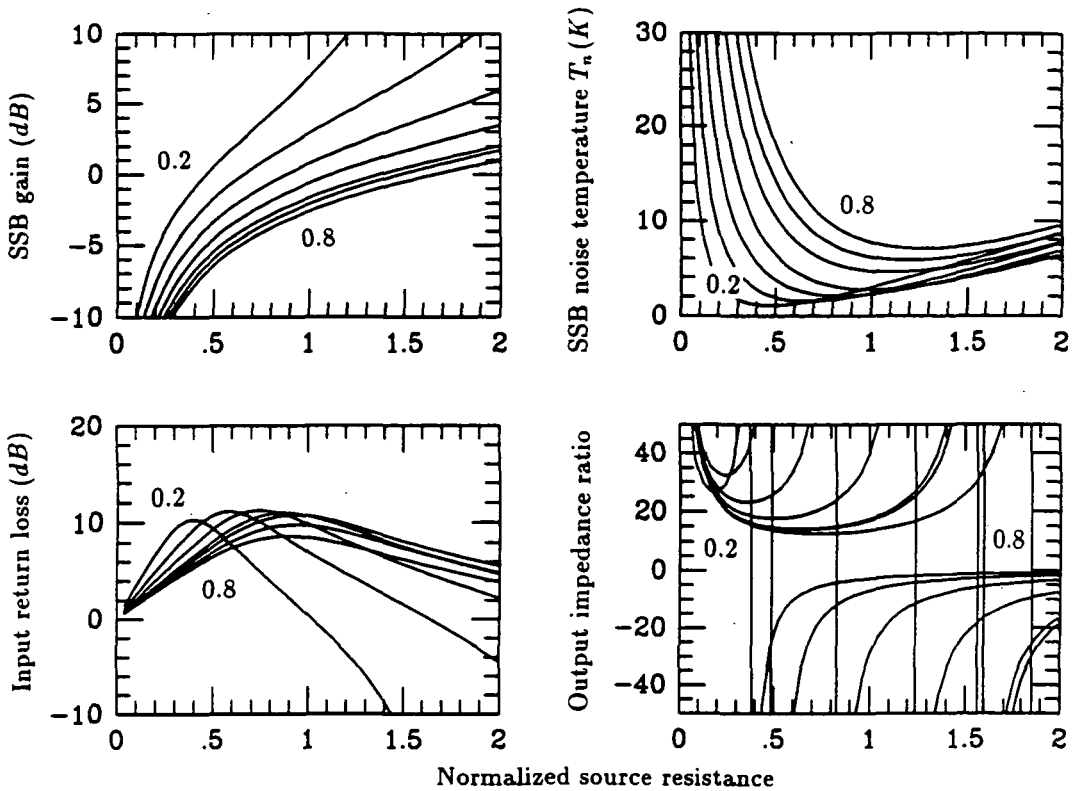

Figure 3: The transducer gain, noise temperature, input return loss, and normalized output impedance of a typical Nb-AlOx-Nb mixer as a function of the normalized source resistance.

very poor for low values of source resistance, and this is probably the single most important reason why mixer performances improve significantly when integrated tuning elements are used.

A useful normalized expression can easily be derived for the source resistance at which unity gain, good input match, and minimum noise temperature can be achieved. The expression is

$$\frac{R_n}{R_s} = 0.5V_{PH}^{-0.92} = 78\left[\frac{f(GHz)}{V_g(mV)}\right]^{-0.92} \qquad (1)$$

and it applies for frequencies between 0.2 and 0.8 of the gap frequency. The exponent is slightly different from that given by Kerr and Pan, because we have taken into account the fact that the optimum bias point does not remain in the middle of the first photon step below the gap for frequencies greater than about 0.5 of the gap frequency.

It is well known that if one plots the conversion gain, at a given frequency, as a function of the ωCR product, at some point the conversion gain becomes depressed. This can be regarded as the frequency at which harmonic effects become significant; or equivalently, the frequency at which the five-port, rather than the three-port model should be used. Using the above value for the source resistance, we have investigated this behaviour and generated

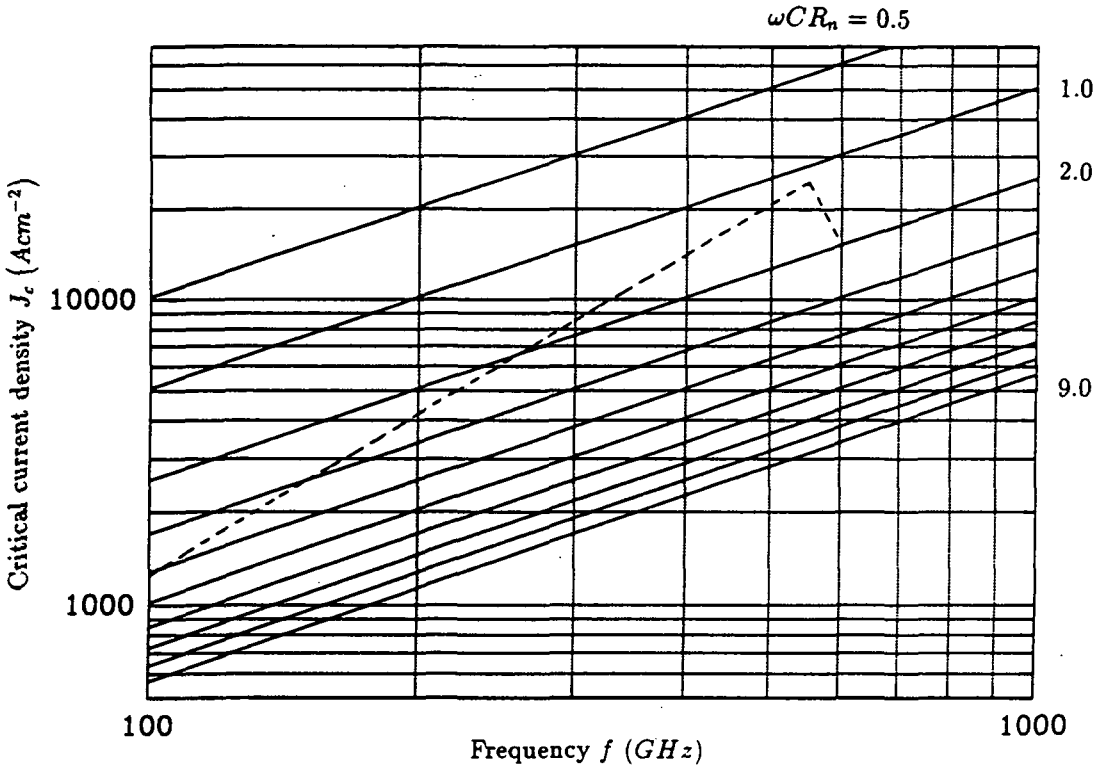

Figure 4: Critical current density against frequency for different values of ωCR product. The dotted line shows the optimum current density as a function of frequency.

the following expression for the optimum ωCR product

$$\omega CR\,|_{opt} = V_{PH}^{-0.75} = 61\left[\frac{V_g\,(mV)}{f\,(GHz)}\right]^{0.75}, \qquad (2)$$

and this in turn generates the following expression for the optimum value of the critical current density: $J_c\,(Acm^{-2}) = 0.4f\,(GHz)^{1.75}$. We have assumed $I_cR_n = 1.8mV$ and a specific capacitance of $45fF\mu m^{-2}$. Once again the above requirement is less severe than that published by Kerr. Finally, in Fig. 4 we plot the optimum critical current density as a function of frequency, and we show lines of constant ωCR product. Above the dotted line, harmonic effects are important, whereas below the dotted line harmonic effects can be ignored.

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