CHOICE OF ANTENNA GEOMETRY FOR MICROWAVE POWER TRANSMISSION FROM SOLAR POWER SATELLITES

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Abstract

A comparison is made between square and circular transmitting antennas for solar power satellite microwave power transmission. It is seen that the exclusion zone around the rectenna needed to protect populations from microwaves is smaller for a circular antenna operating at 2.45 GHz than it is for a square antenna at that frequency. If the frequency is increased, the exclusion zone size remains the same for a square antenna, but becomes even smaller for a circular antenna. Peak beam intensity is the same for both antennas if the frequency and antenna area are equal. The circular antenna puts a somewhat greater amount of power in the main lobe and somewhat less in the sidelobes. Since rain attenuation and atmospheric heating remain problems above 10 GHz, it is recommended that future solar power satellite work concentrate on circular transmitting antennas at frequencies roughly 10 GHz.

Introduction

In order to maximize the safety and efficiency of microwave power transmission from a solar power satellite (SPS) to the Earth, the shape of the satellite's transmitting antenna must be optimized. Previous work at New York University[1,4] has considered a square transmitting antenna, typically 1 kilometer across. The width of the main beam lobe at the ground, and thus, the width of an economical rectifying antenna (rectenna), varies as the inverse of the frequency. It was shown that, to a good approximation, the size of the exclusion zone needed to protect people from microwave exposure is independent of frequency (see *Figures 2* and *3*). Furthermore, rain and atmospheric attenuation become significant for frequencies above 10 to 15 GHz. This was considered to be the maximum usable frequency range. Since many SPS studies have considered circular transmitting antennas[2], it is instructive to examine such antennas in a manner analogous to Reference 1.

Square Transmitting Antennas

It was shown that for a square phased array of side D_t = 1 km, at geostationary altitude h = 35,786 km, with N X N isotropically radiating elements uniformly spaced less than one-half wavelength apart beaming power to a rectenna at the equator, the microwave beam intensity at the rectenna, with boresight peak I_o is expressible as the product of the x and y distributions. Two-dimensional intensity is given by[1,4]:

$$I(\hat{x},\hat{y}) = I_o \times \left\{ \frac{\sin \frac{\pi \hat{x}}{2}}{N \sin \frac{\pi \hat{x}}{2N}} \right\}^2 \left\{ \frac{\sin \frac{\pi \hat{y}}{2}}{N \sin \frac{\pi \hat{y}}{2N}} \right\}^2 \qquad \text{(Equation 1)}$$

where

$$\hat{x} = x\left\{\frac{2D_t}{\lambda h}\right\}, \quad \hat{y} = y\left\{\frac{2D_t}{\lambda h}\right\} \qquad \text{(Equations 2)}$$

and x, y = distances from the center of the diffraction pattern at the Earth's surface in the x and y directions. Note that $N > 2D_t/\lambda$, so $N \approx 20{,}000$ at a frequency of 2.45 GHz (see *Figure 1* for a plot of Equation 1 at this frequency). Since $\hat{x}$ and $\hat{y}$ are small, Equation 1 can be approximated by:

$$I(\hat{x},\hat{y}) = I_o \times \left\{ \frac{\sin \frac{\pi \hat{x}}{2}}{\frac{\pi \hat{x}}{2}} \right\}^2 \left\{ \frac{\sin \frac{\pi \hat{y}}{2}}{\frac{\pi \hat{y}}{2}} \right\}^2 \qquad \text{(Equation 3)}$$

By integrating this in two dimensions from negative infinity to positive infinity, it is seen that

$$I_o = \frac{P_t}{4}\left\{\frac{2D_t}{\lambda h}\right\}^2 = \frac{P_t D_t^2}{\lambda^2 h^2} = \frac{P_t A_t}{\lambda^2 h^2}$$ (Equation 4)

where P_t is the total transmitted power and $A_t = D_t^2$ = transmitting antenna area. If Equation 3 is integrated in two dimensions over the area of the main lobe and divided by the integral over all (two-dimensional) space, it is seen that 81.5% of the energy of the main lobe is in the central maximum. The remaining 18.5% of the energy is spread out in the form of sidelobes. These sidelobes represent energy that is spread too thinly to be economically rectifiable, but, as seen in Reference 1, may be a hazard to surrounding populations. For P_t = 5 GW, this wasted energy is 925 MW. It is instructive to investigate if this can be improved upon with the use of a circular transmitting antenna. In addition, the independence of exclusion zone size with frequency discussed in Reference 1 is a somewhat counterintuitive result. If the exclusion zone size decreases significantly with frequency for a circular antenna, then it may pay to increase the frequency beyond 10 to 15 GHz, despite the atmospheric and rain attenuation that occurs at higher frequencies.

Circular Transmitting Antennas

The intensity of a microwave beam transmitted from a circular antenna array is given by[5,6]:

$$I(\hat{r}) = I_o \times \left\{\frac{2J_1\left(\frac{\pi\hat{r}}{2}\right)}{\frac{\pi\hat{r}}{2}}\right\}^2$$ (Equation 5)

where J_1 is the first order Bessel function of the first kind, and $\hat{r}$ is the dimensionless distance from the center of the diffraction pattern and is analogous to $\hat{x}$ and $\hat{y}$ given by Equations 2. Thus,

$$\hat{r} = r\left\{\frac{2D_t}{\lambda h}\right\}$$ (Equation 6)

and r = radial distance from the center of the diffraction pattern at the Earth's surface. Here, D_t is the diameter of the circular transmitting antenna. Equation 5 is a two-dimensional intensity, since, for a transmitter directly overhead, the diffraction pattern is azimuthally symmetric.

By integrating Equation 5, it is seen that

$$I_o = \frac{\pi P_t}{16}\left\{\frac{2D_t}{\lambda h}\right\}^2 = \frac{P_t A_t}{\lambda^2 h^2}$$ (Equation 7)

Here, D_t and A_t are, respectively, the diameter and area of the circular transmitting antenna, and $A_t = \pi D_t^2/4$. By comparing Equations 4 and 7, it is seen that, for square and circular transmitting antennas of equal areas, equal peak beam intensities will result. Thus, to facilitate a comparison of antenna geometries, calculations will be done for a consistent value of A_t, specifically, 1 square kilometer (used in Reference 1). Therefore, D_t = 1000 meters for the square antenna and 1128 meters for the circular antenna. The beam intensity at the Earth's surface for a circular antenna of this size transmitting 5 GW of power at a frequency of 2.45 GHz is plotted in *Figure 4*. The peak beam intensity is 26 mW/cm^2. In order to investigate the dependence of exclusion zone size on frequency for a circular transmitting antenna, an approximation to the Bessel function can be used. For large z,

$$J_1(z) \approx \frac{1}{\sqrt{\pi z}}[\sin(z) - \cos(z)] \qquad \text{(Equation 8)}$$

$$\textit{Beam intensity} \propto \left\{\frac{J_1(z)}{z}\right\}^2 \approx \frac{1}{\pi z^3}[1 - 2\cos(z)\sin(z)] \qquad \text{(Equation 9)}$$

In order to eliminate the oscillating term in square brackets on the right side of Equation 9, its upper bound will be used. Since the upper bound is 2, Equation 9 becomes:

$$Max\left[\left\{\frac{J_1(z)}{z}\right\}^2\right] = \frac{2}{\pi z^3} \qquad \text{(Equation 10)}$$

The dimensionless argument of Equation 10 is given by:

$$z = \frac{\pi r'}{2} = \frac{\pi}{2} r\left\{\frac{2D_t}{\lambda h}\right\} = \frac{\pi r D_t}{\lambda h} \qquad \text{(Equation 11)}$$

Thus,

$$I(z) = I_o \times Max\left[\left\{\frac{2J_1(z)}{z}\right\}^2\right] = I_o \times \frac{8}{\pi z^3} \qquad \text{(Equation 12)}$$

Substituting Equations 7 and 11 into Equation 12 gives:

$$I(r) = \frac{2P_t\lambda h}{\pi^3 r^3 D_t} \qquad \text{(Equation 13)}$$

If c is the speed of light and f is the frequency, then $\lambda = c/f$. I(r) can be set equal to the microwave safety threshold I_s, and r can be set equal to the exclusion zone radius r_s. If these substitutions are made in Equation 13, and the equation is rearranged, then the approximate radius of an exclusion zone for a circular transmitting antenna is given by:

$$r_s = \frac{1}{\pi}\left\{\frac{2P_t ch}{I_s f D_t}\right\}^{1/3} \qquad \text{(Equation 14)}$$

Thus, for a circular transmitting antenna, the radius of the exclusion zone r_s is proportional to the frequency to the minus one-third power.

The exclusion zone plots shown in *Figures 5* and *6* were done using Equation 5; however, Equation 14 yields values at or near the upper bounds of the curves shown in these figures. By comparing *Figures 2* and *3* with *Figures 5* and *6*, it can be seen that at a given frequency and microwave safety threshold, the circular transmitting antenna yields a smaller exclusion zone than the square transmitting antenna. Furthermore, exclusion zone size does depend on frequency for a circular antenna, though the dependence is not strong. Thus, circular antennas represent an improvement over square antennas. With the use of a circular antenna, it would be advantageous to increase the frequency from 2.45 GHz to approximately 10 GHz. The frequency should not be increased significantly more than this, because the rain attenuation and atmospheric heating problems that affect the square antenna system would have the same effect on the circular antenna system. The exclusion zone decrease would not be worth the price paid in attenuation and heating. For example, for $A_t = 1$ km², $P_t = 5$ GW, and $I_s = 1$ mW/cm², the square antenna exclusion zone half-width is 7118 m for all frequencies; its area is thus (2 x 7118 m)² or 203 km². The exclusion zone area for a circular antenna with the same parameters is given by πr_s^2, where r_s is given by Equation 14. This area is 79 km² at 2.45 GHz, 31 km² at 9.8 GHz, 13 km² at 35 GHz, and 7 km² at 94 GHz. The latter two frequencies are subject to significant rain attenuation, as well as some atmospheric heating.[1] This is true regardless of antenna geometry, since square and circular transmitting antennas of equal areas, wavelengths, and total transmitted powers have the same peak beam intensity (Equations 4 and 7).

By comparing the integral of the circular antenna pattern over the main lobe with the integral over all two-dimensional space[5], it is seen that 83.8% of the energy is concentrated in the main lobe. Since this figure is 81.5% for the square aperture, this represents another advantage of using a circular antenna. This 2.3% difference may not seem like much, but it amounts to 115 MW if the total transmitted power is 5 GW. One hundred fifteen MW of additional energy distributed among the sidelobes is what makes the square aperture exclusion zones so much larger than those of the circular aperture. Furthermore, the size of the main lobe can be found by letting $\hat{x} = \hat{y} = 2$ in Equations 2, and $\hat{r} = 2.440$ (Reference 5) in Equation 6, and solving for the dimensionalized distance in each case. This results in exclusion zone sizes of:

$$x = y = \frac{\lambda h}{\sqrt{A_t}} \quad \textit{for the square aperture}$$

and (Equations 15)

$$r = 1.081\frac{\lambda h}{\sqrt{A_t}} \quad \textit{for the circular aperture.}$$

Since x and y are the half-widths of a square lobe and r is the radius of a circular lobe, the areas of the main lobe can be found as follows:

$$\textit{Area of square lobe} = 4xy = 4\frac{\lambda^2 h^2}{A_t}$$

$$\textit{Area of circular lobe} = \pi r^2 = 3.671\frac{\lambda^2 h^2}{A_t} \qquad \text{(Equations 16)}$$

It is thus seen that for a given A_t, the circular aperture places [(83.8% - 81.5%) / 81.5%] x 100% or 2.8% more power into a main lobe that is 8.2% smaller in area, while maintaining the same peak beam intensity. The use of circular transmitting antennas is thus more economical than the use of square transmitting antennas.

Conclusions

For square transmitting antennas, exclusion zone width is independent of frequency. For circular transmitting antennas, exclusion zone radius is proportional to frequency to the minus one-third power. The width of the main lobe is inversely proportional to frequency for both geometries (see Equations 15). At a given frequency, the circular aperture allows for somewhat smaller rectennas, much smaller exclusion zones, and somewhat higher amounts of power in the main lobe than the square aperture. Furthermore, it does so while maintaining the same peak beam intensity. It is therefore recommended that future work on solar power satellites concentrate on circular apertures. In order to take advantage of the decrease of exclusion zone and main lobe size with frequency, the frequency should be increased from 2.45 GHz to roughly 10 GHz. Rain attenuation and atmospheric heating continue to be problems above that frequency. Further attempts to concentrate more power in the main lobe and reduce power to the sidelobes should therefore concentrate on beam tapering.

References

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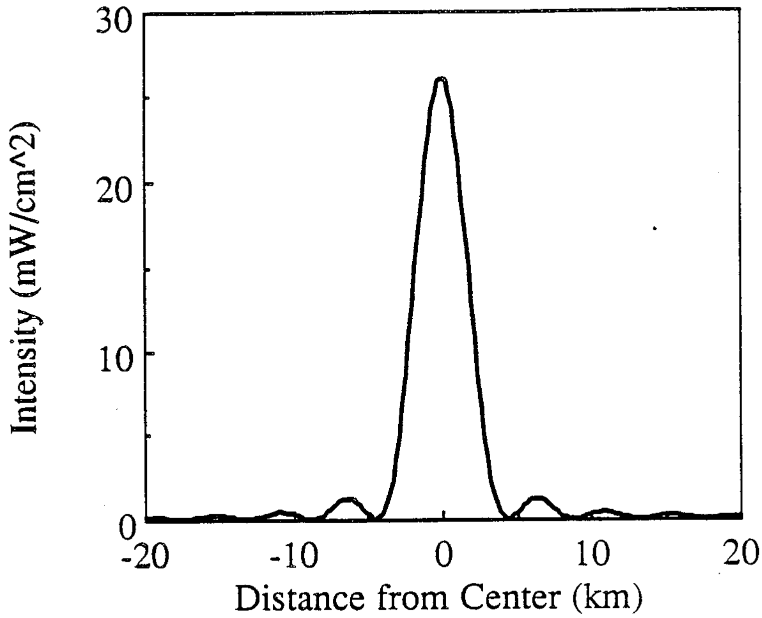

FIGURE 1. Microwave beam intensity at the earth's surface for a geostationary SPS beaming 5 gigawatts of power from a square antenna with 1000 meter sides to the equator at a frequency of 2.45 GHz. (Based on Reference 1, Figure 2.)

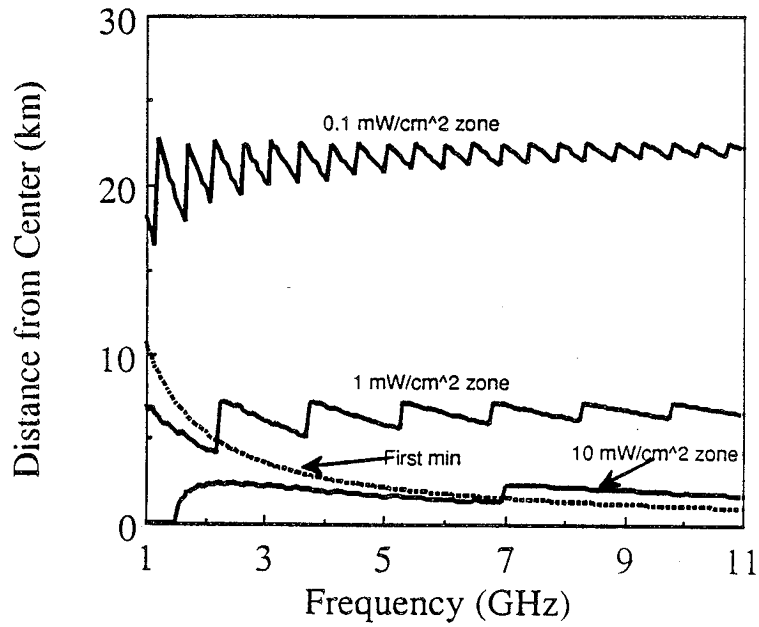

FIGURE 2. Microwave beam exclusion zones at the earth's surface for a geostationary SPS beaming 5 gigawatts of power from a square antenna with 1000 meter sides to the equator for frequencies of 1 to 11 GHz. Exclusion zones for 0.1, 1, and 10 mW/cm^2 safety thresholds are shown, as well as the location of the first diffraction minimum. (Based on Reference 1, Figure 3.)

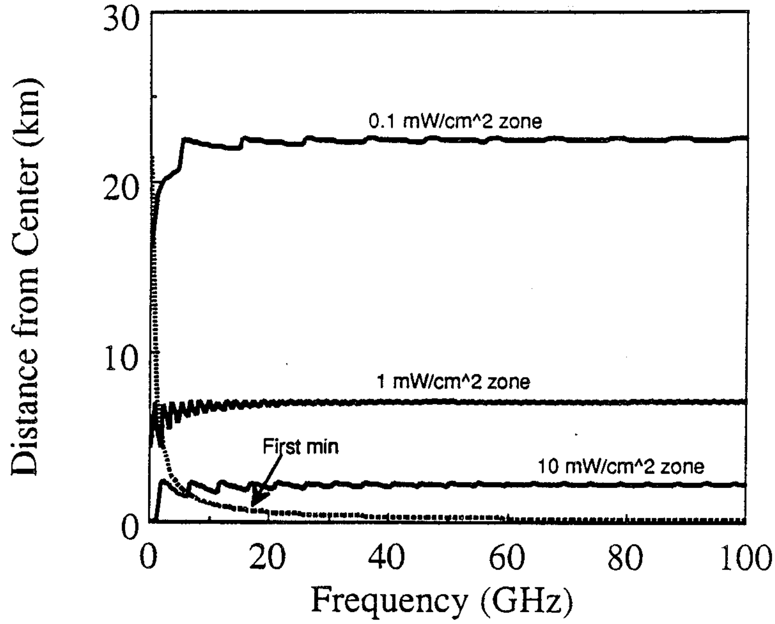

FIGURE 3. Microwave beam exclusion zones at the earth's surface for a geostationary SPS beaming 5 gigawatts of power from a square antenna with 1000 meter sides to the equator for frequencies of 1 to 100 GHz. Exclusion zones for 0.1, 1, and 10 mW/cm^2 safety thresholds are shown, as well as the location of the first diffraction minimum. (Based on Reference 1, Figure 4.)

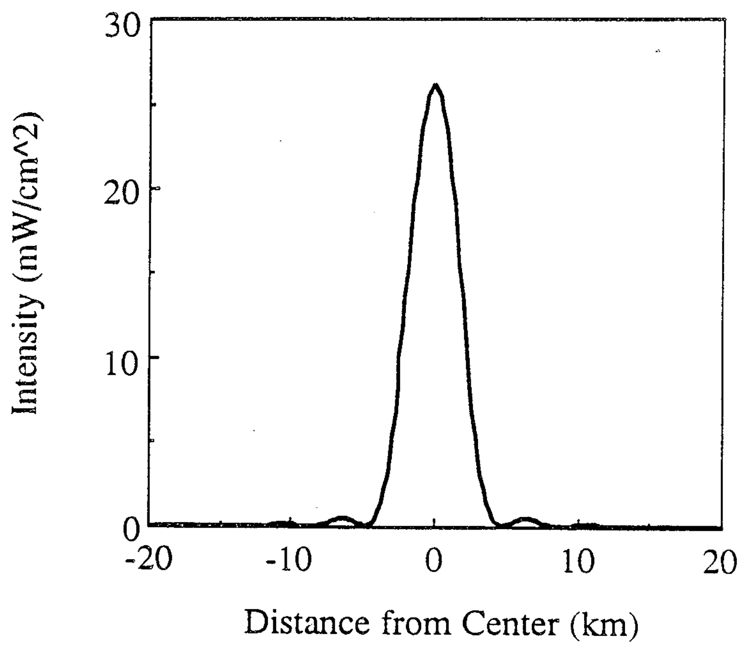

FIGURE 4. Microwave beam intensity at the earth's surface for a geostationary SPS beaming 5 gigawatts of power from a 1128 meter diameter circular antenna to the equator at a frequency of 2.45 GHz.

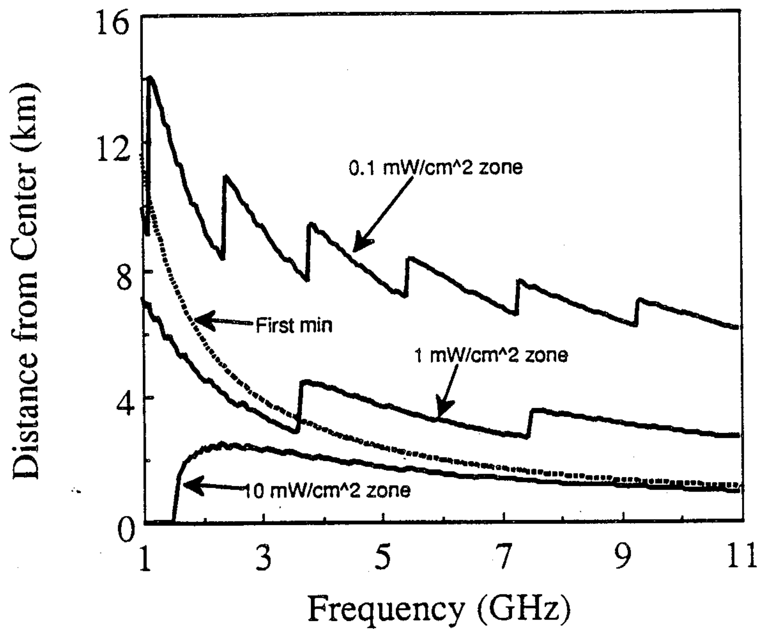

FIGURE 5. Microwave beam exclusion zones at the earth's surface for a geostationary SPS beaming 5 gigawatts of power from a 1128 meter diameter circular antenna to the equator for frequencies of 1 to 11 GHz. Exclusion zones for 0.1, 1, and 10 mW/cm^2 safety thresholds are shown, as well as the location of the first diffraction minimum.

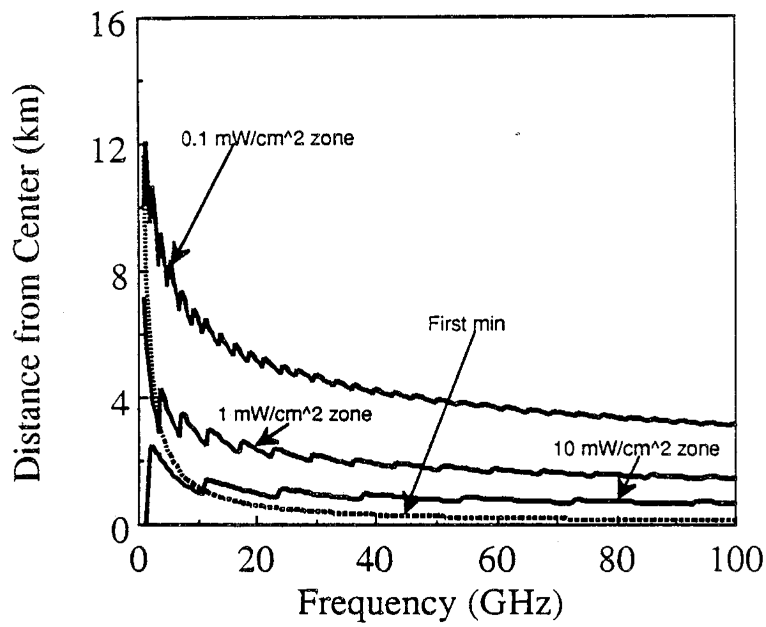

FIGURE 6. Microwave beam exclusion zones at the earth's surface for a geostationary SPS beaming 5 gigawatts of power from a 1128 meter diameter circular antenna to the equator for frequencies of 1 to 100 GHz. Exclusion zones for 0.1, 1, and 10 mW/cm^2 safety thresholds are shown, as well as the location of the first diffraction minimum.