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ROBUST CONTROL FOR UNCERTAIN STRUCTURES

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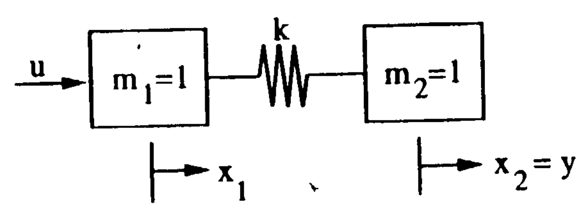
APPROACH

- Assume full-state feedback
- Try to guarantee stability and performance robustness of classical LQR design
  - Guaranteed stability
  - Reasonable guaranteed robustness (gain and phase margin properties)
- Apply to benchmark problem to see interesting properties



$.5 \leq k \leq 2$

ROBUST LQR FORMULAS

- Standard LQR design when there is no uncertainty

$$J = \int_0^\infty (x^T(t) Q_0 x(t) + \rho u^T(t) u(t)) dt$$

$$P A_0 + A_0^T P + Q_0 - \frac{1}{\rho} P B B^T P = 0$$

- Apply Petersen-Hollot bounds to derive robust Riccati Equation

$$A = A_0 + \sum_{i=1}^{p} q_i E_i \quad |q_i| \leq 1$$

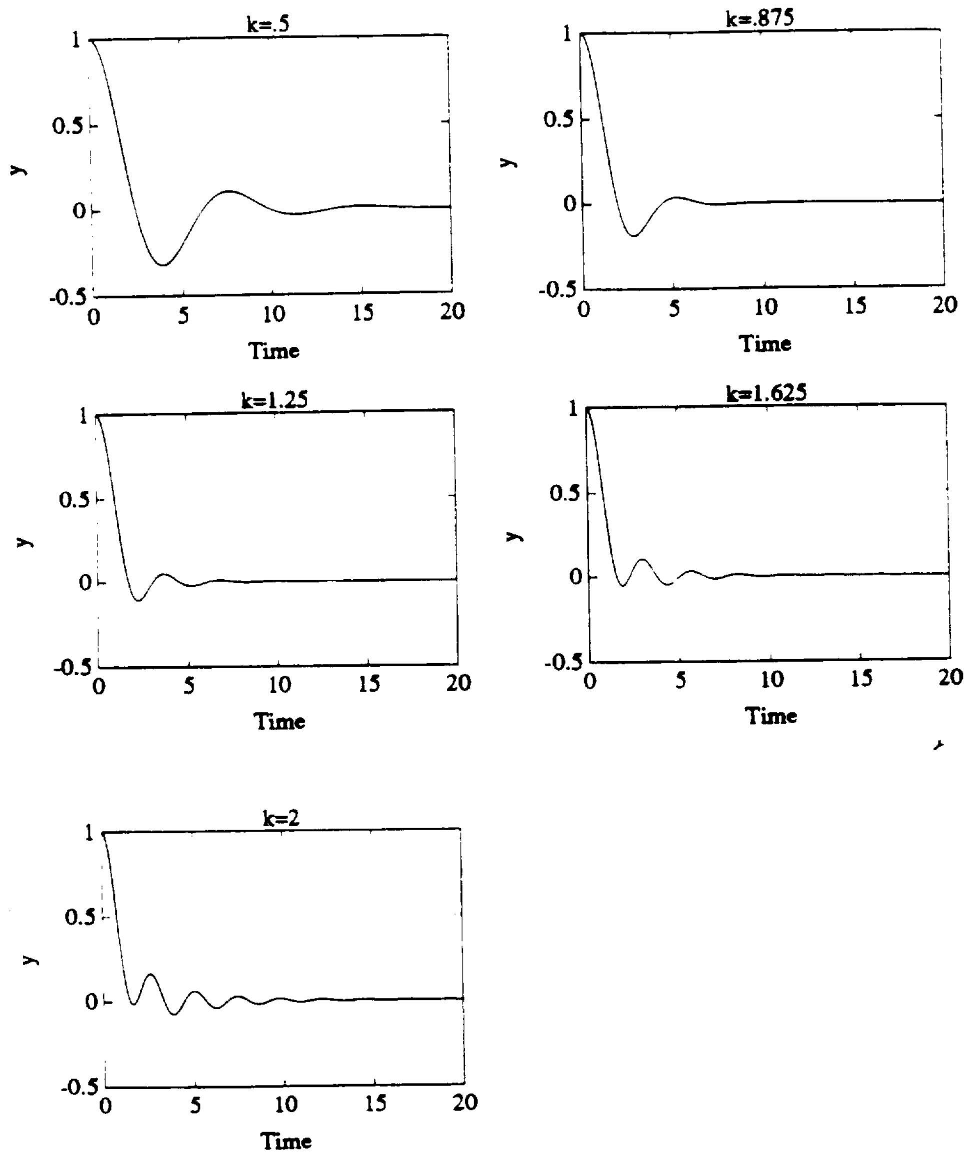
$$E_i = l_i n_i^T \qquad L = [l_1\ l_2\ l_3 \ldots]; \qquad N = [n_1\ n_2\ n_3 \ldots]$$

$$P A_0 + A_0^T P + (Q_0 + \gamma N N^T) - P(\frac{1}{\rho} B B^T - \frac{1}{\gamma} L L^T) P = 0$$

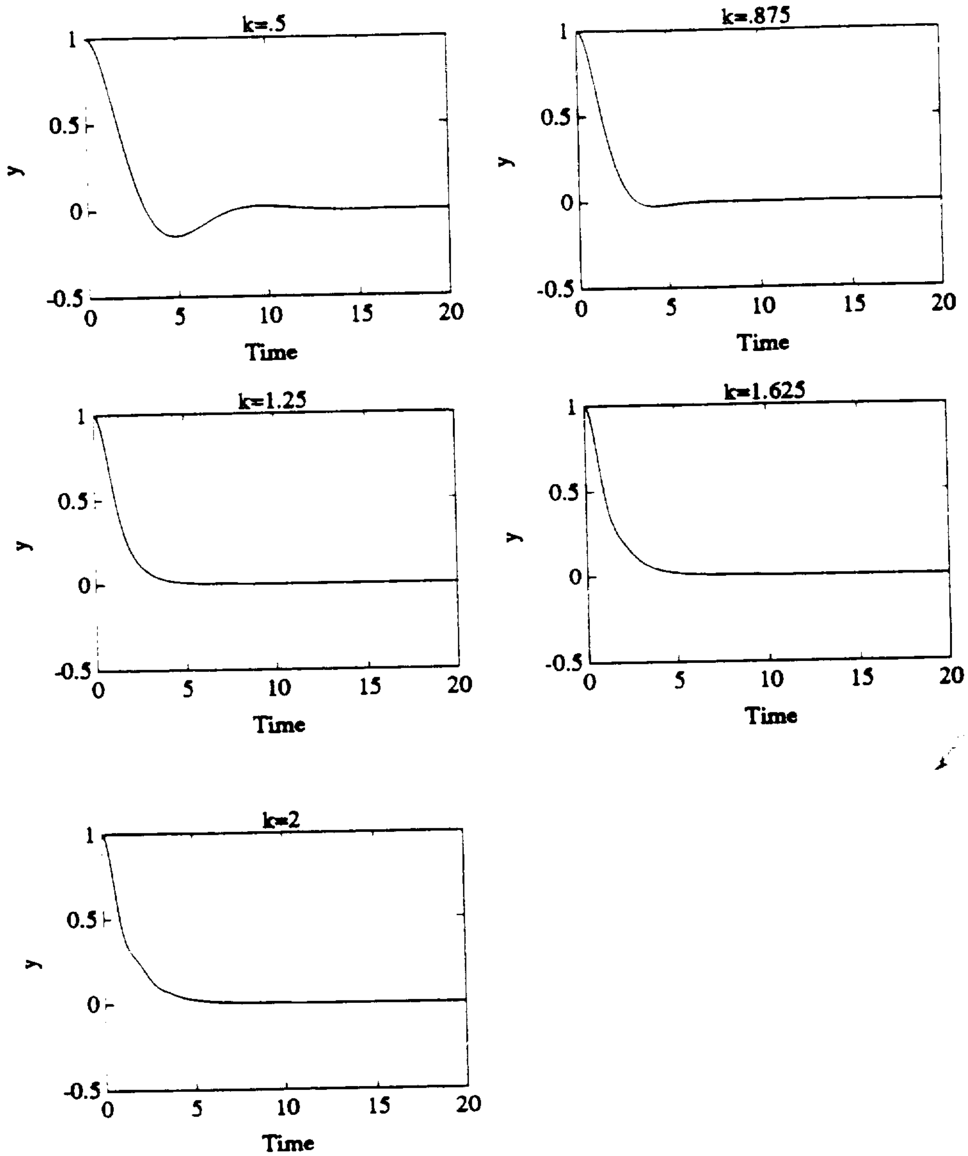
- Control

$$G = \frac{1}{\rho} B^T P \qquad u = -Gx$$

MISMATCHED LQR DESIGN



RLQR DESIGN



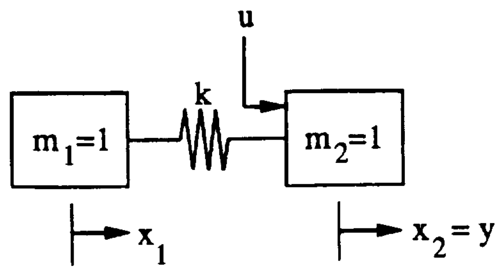
INTERPRETATIONS OF RLQR DESIGN

- Equivalent to an optimal design where we minimize the cost functional

$$J = \int_0^\infty (x^T(t)Q_0x(t) + x^T(t)\gamma NN^Tx(t) + \underbrace{x^T(t)\frac{1}{\gamma}PLL^TPx(t)}_{-\beta d^T(t)d(t)} + \rho u^T(t)u(t))dt$$

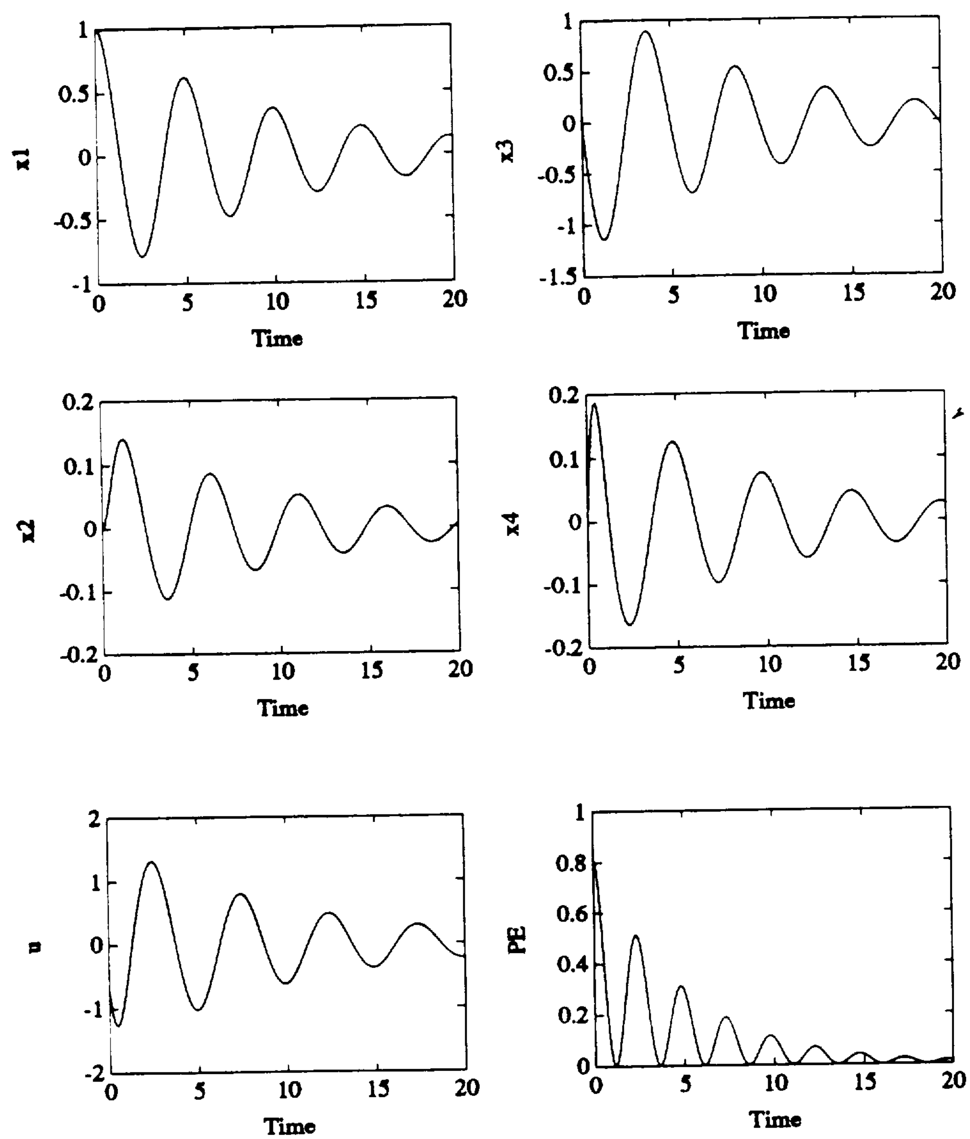
  - $x^T(t)Q_0x(t)$ is the state weighting
  - $x^T(t)NN^Tx(t)$ has been shown to be uncertain potential energy of an uncertain spring (or rate of dissipation for a damper)
  - $x^T(t)PLL^TPx(t)$ is an equivalent $\mathcal{H}_\infty$ term.

- Parameter γ is therefore a tradeoff between minimizing unknown uncertain energy and worst case disturbance arising from forces due to parameter errors.

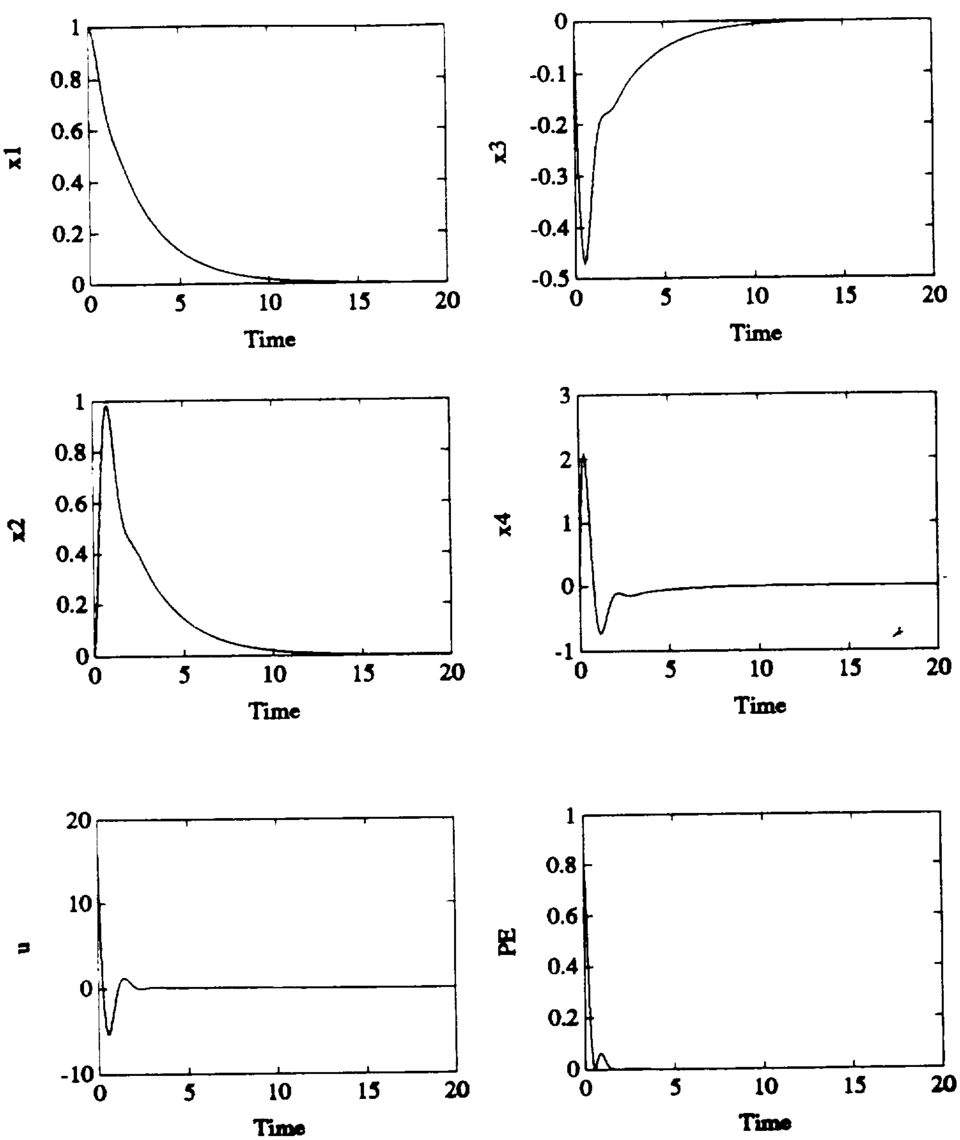


$.5 \leq k \leq 2$

K = 1.625

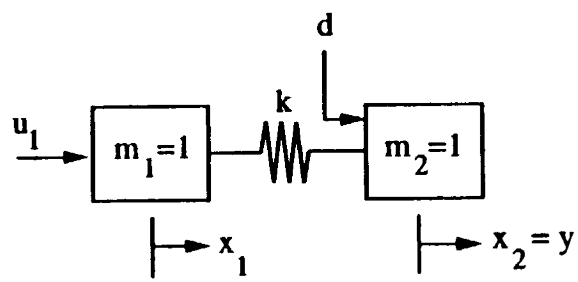


K = 1.625 (RLQR)



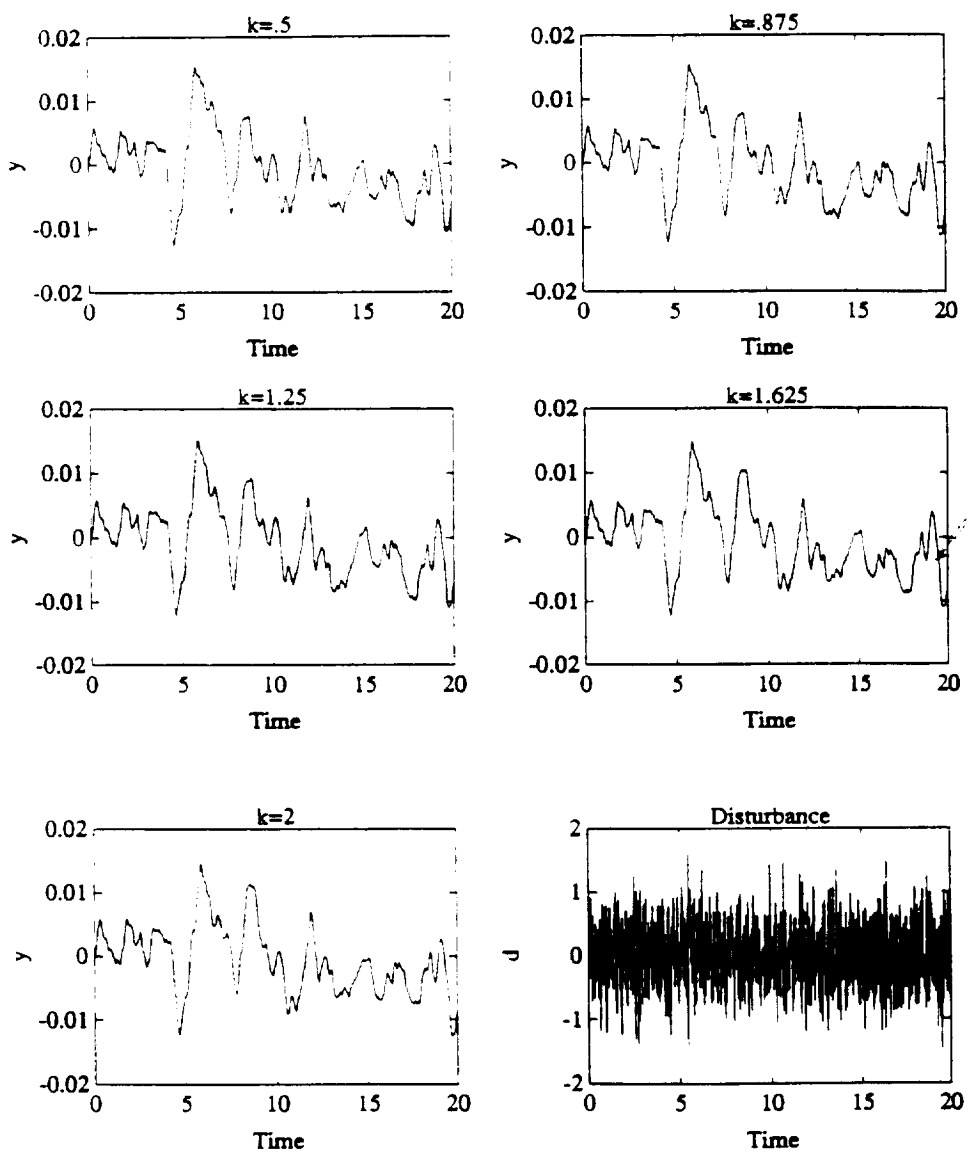
DISTURBANCE REJECTION

- Does the RLQR controller reject disturbances?
- Add a white noise disturbance at the output
- Apply both mismatched LQR and RLQR designs

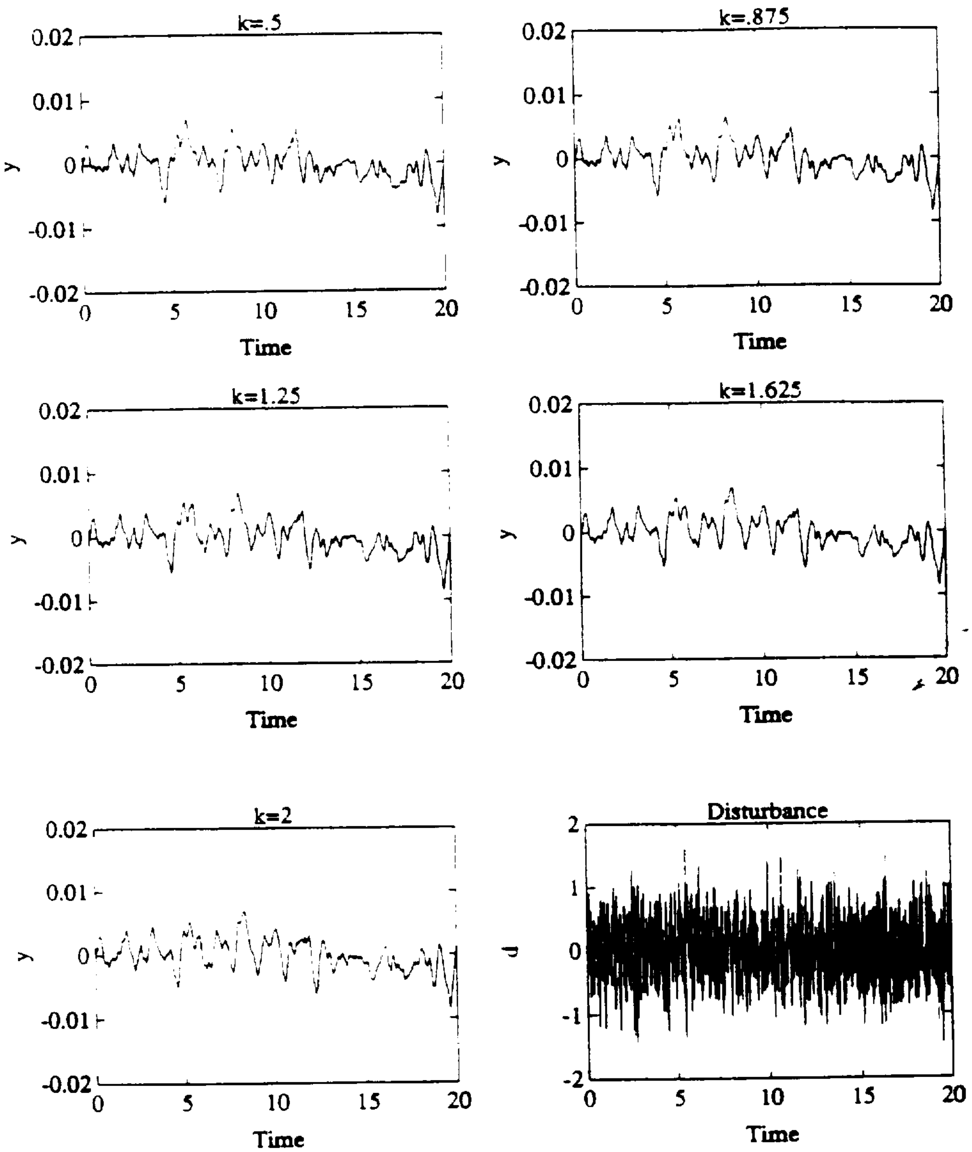


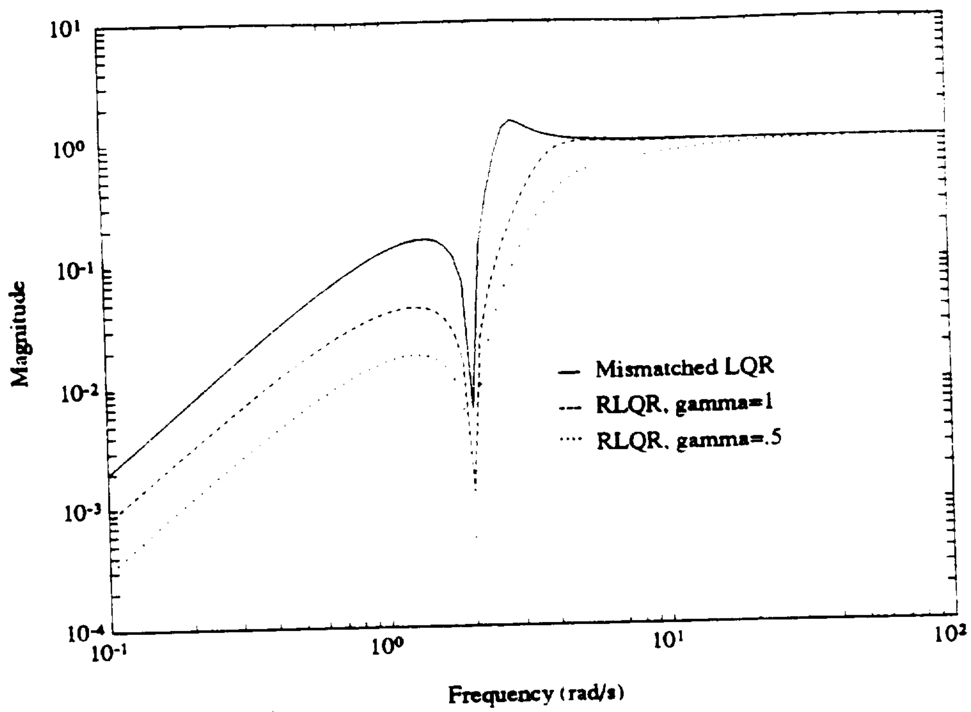
$.5 \leq k \leq 2$

MISMATCHED LQR DESIGN

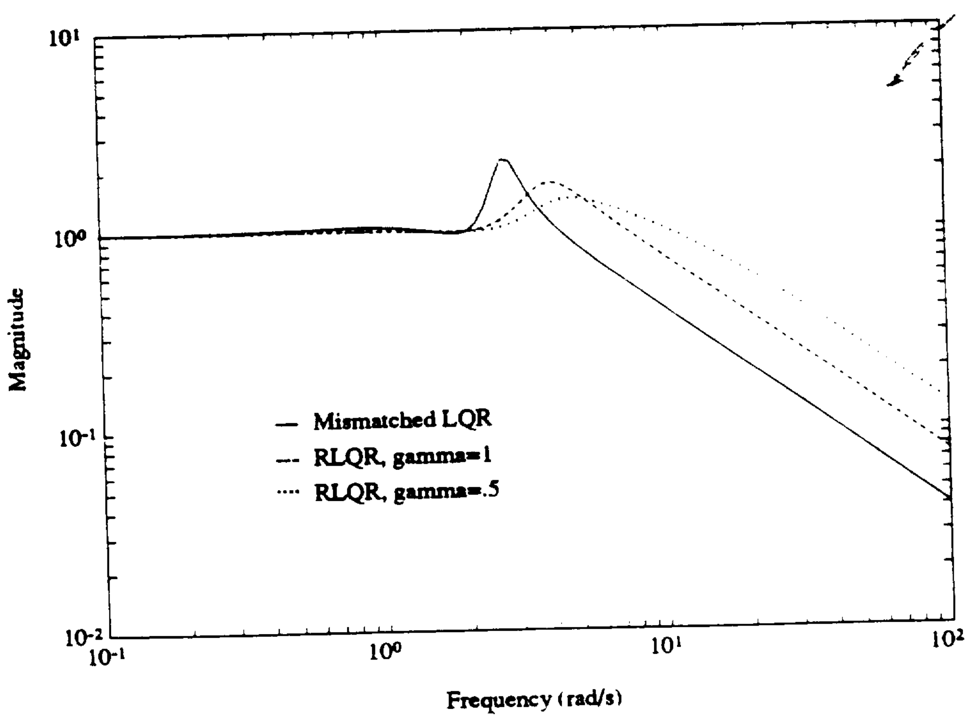


RLQR DESIGN


k=.5
k=.875
k=1.25
k=1.625
k=2
Disturbance
0.02
0.01
0
-0.01
-0.02
2
1
-1
-2
y
d
0
5
10
15
20
Time

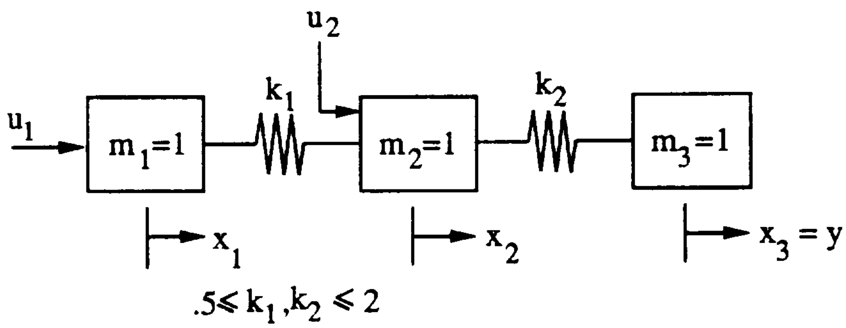


Magnitude
Frequency (rad/s)
Mismatched LQR
RLQR, gamma=1
RLQR, gamma=.5

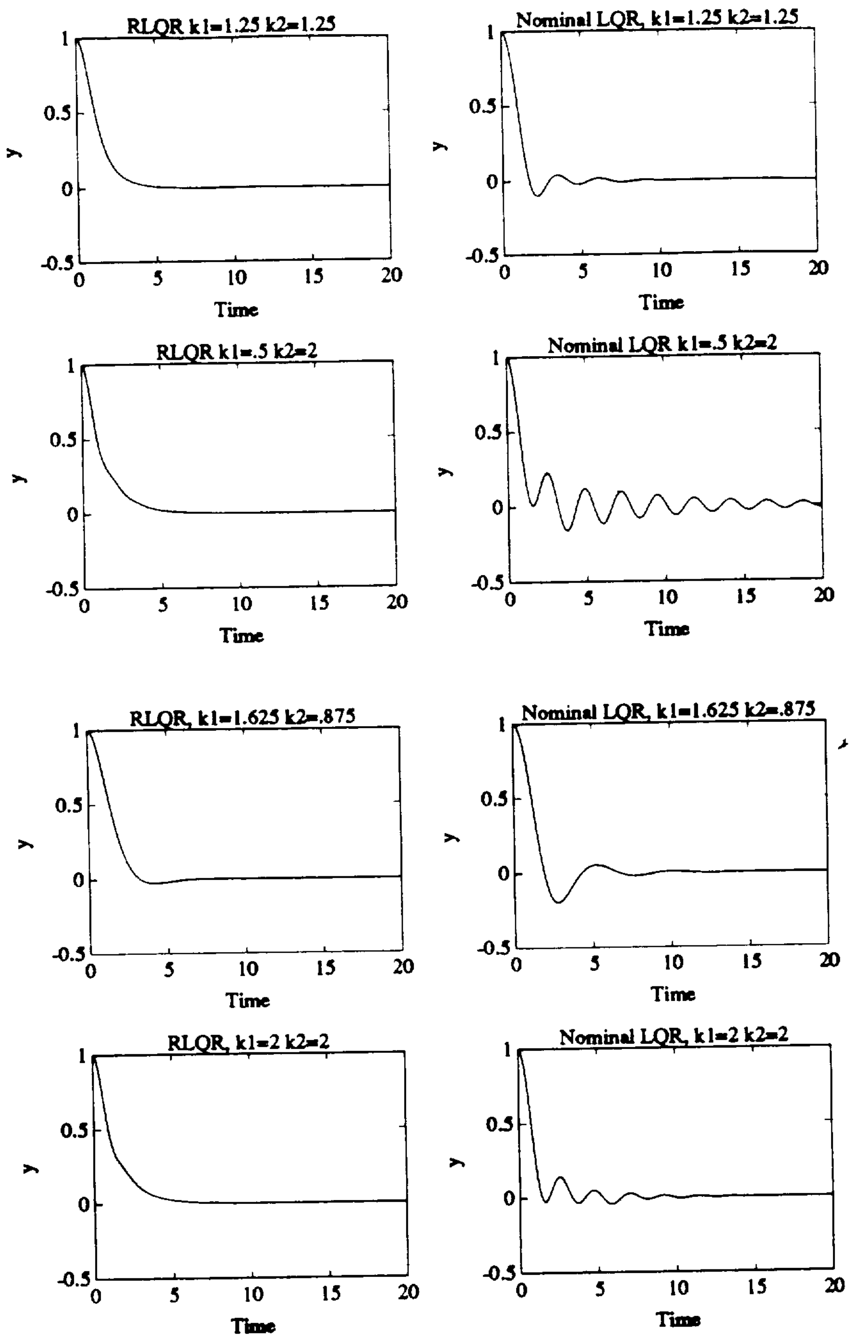


Magnitude
Frequency (rad/s)
Mismatched LQR
RLQR, gamma=1
RLQR, gamma=.5

THREE-MASSES, TWO UNCERTAIN SPRINGS

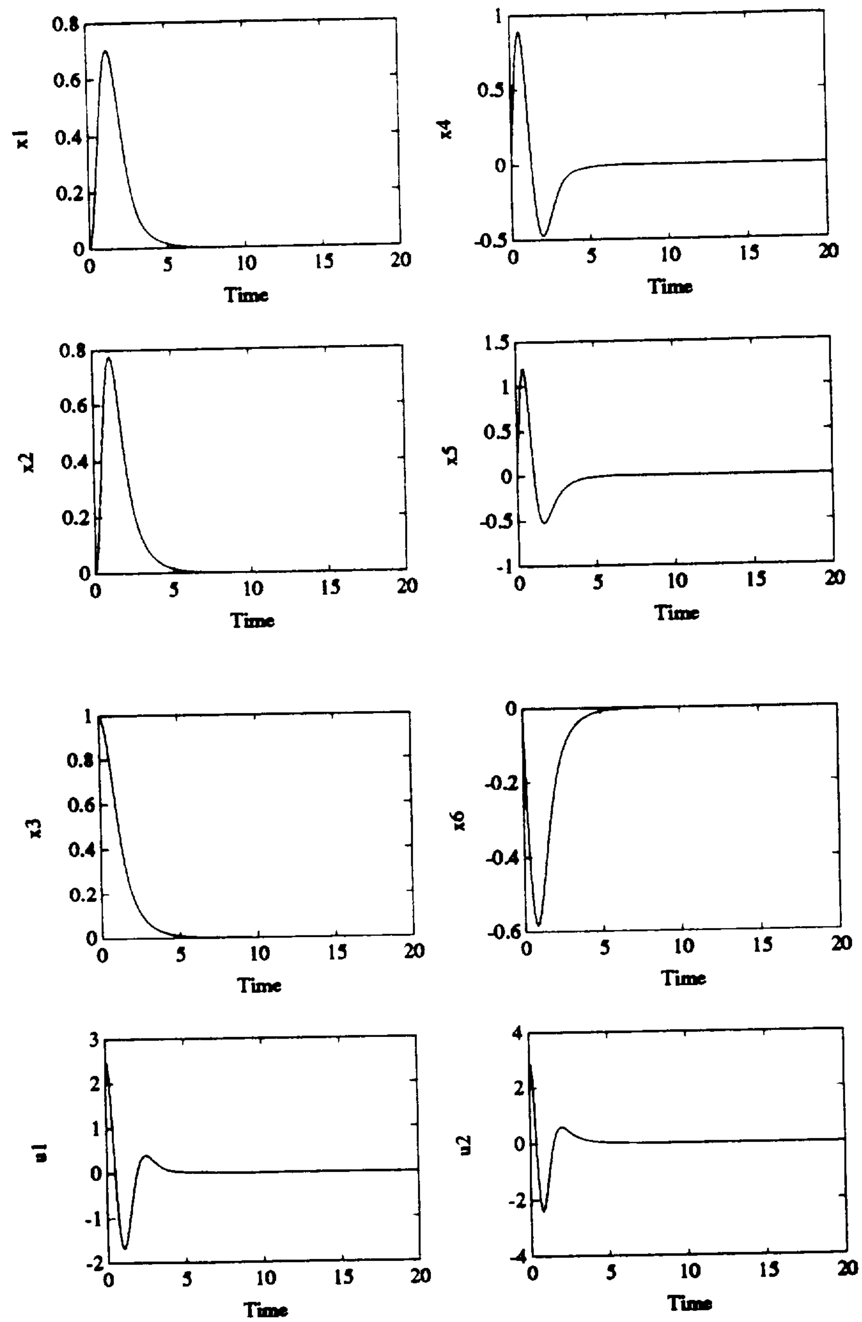


PERFORMANCE COMPARISONS:
RLQR (LEFT) VS MISMATCHED LQR (RIGHT)



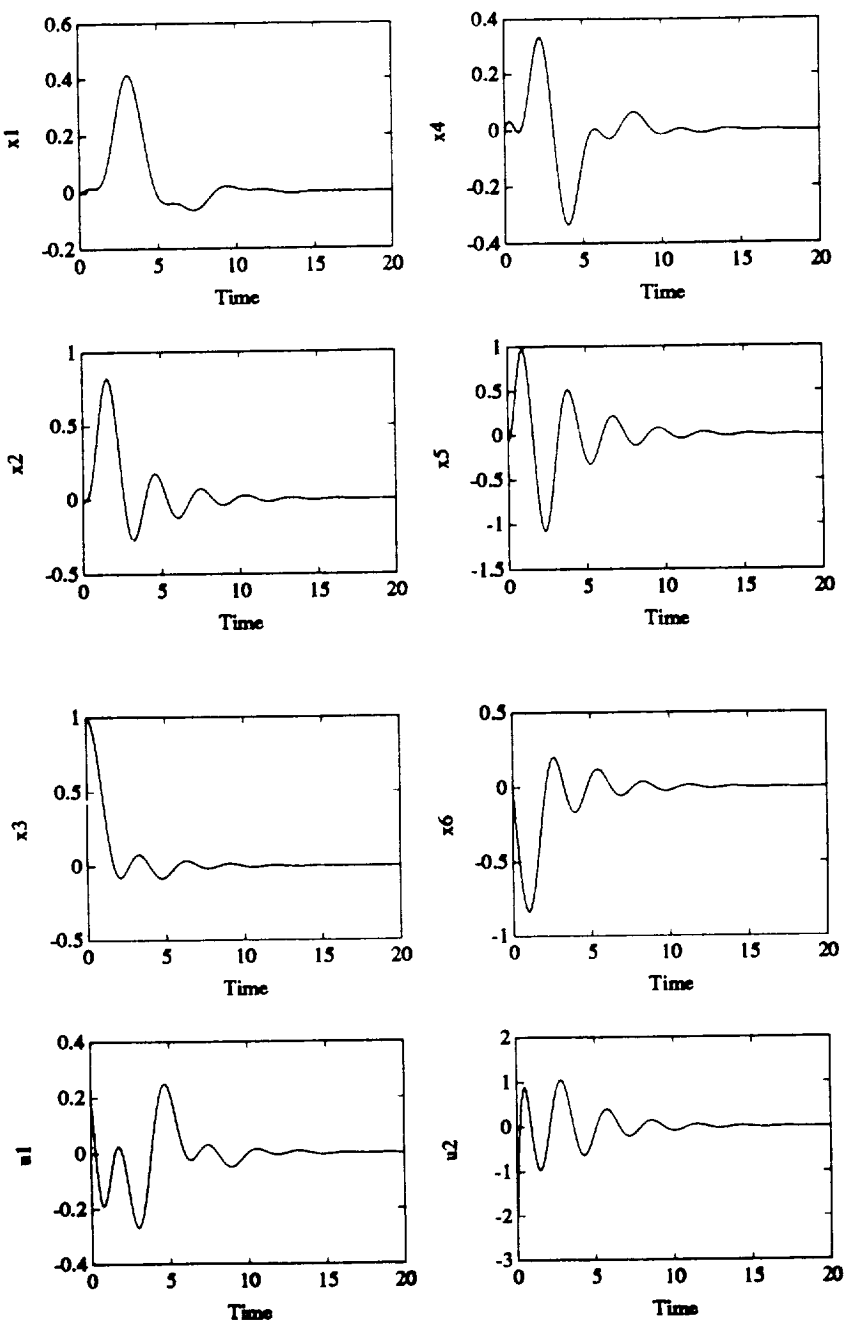
RLQR TRANSIENTS: 2-SPRING SYSTEM

$K_1 = .5$, $K_2 = 1.25$



MISMATCHED LQR TRANSIENTS: 2-SPRING SYSTEM

$K_1 = .5$, $K_2 = 1.25$



CONCLUSIONS

- RLQR design is a full state method
- Guarantees stability as well as some robustness
- Interesting energy interpretations
- Understanding underlying fundamentals will help us when we extend to output feedback