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UNCERTAINTY REASONING IN EXPERT SYSTEMS

Final Report

ABSTRACT

Intelligent control is a very successful way to transform the expert's knowledge of the type "if the velocity is big and the distance from the object is small, hit the brakes and decelerate as fast as possible" into an actual control. To apply this transformation, one must choose an appropriate methods for reasoning with uncertainty, i.e., one must:

1) choose the representation for words like "small", "big";
2) choose operations corresponding to "and" and "or";
3) choose a method that transforms the resulting uncertain control recommendations into a precise control strategy.

The wrong choice can drastically affect the quality of the resulting control, so the problem of choosing the right procedure is very important. From a mathematical viewpoint these choice problems correspond to non-linear optimization and are therefore extremely difficult.

In this project, we develop a new mathematical formalism (based on group theory) that allows us to solve the problem of optimal choice and thus:

1) explain why the existing choices are really the best (in some situations);
2) explain a rather mysterious fact that *fuzzy control* (i.e., control based on the experts' knowledge) is often better than the control by these same experts;
3) give choice recommendations for the cases when traditional choices do not work.

Perspectives of space applications will be also discussed.

Keywords. *Uncertainty, fuzzy control, optimization, stability, smoothness, space applications.*

CONTENTS

1. FORMULATION OF THE PROBLEM

Uncertainty reasoning is vitally important for space exploration. The high cost and importance of space missions lead to the necessity for automated systems that help human operators in unexpected situations.

In addition, the distance between the Earth mission control centers and the spacecraft demands that some intelligent software also be present on board the mission in order to make quick decisions in emergency situations.

One of the main problems in solving such control problems (and therefore in creating the corresponding software) is that we have to devise solutions on the basis of an *uncertain* knowledge of the situation. Thus it is necessary to have methods which allow representation of this uncertain knowledge and which allow automated conclusions based upon such uncertain knowledge.

Two main types of uncertain reasoning. Several techniques have been developed for expressing uncertainty in intelligent systems (see, e.g., a survey [S88]). In the majority of these techniques, uncertainty of an expert statement A is represented by a number from 0 to 1: 1 means that we are absolutely sure that this statement is true; 0 means that we are absolutely sure that A is false, and numbers from 0 to 1 represent intermediate degrees of belief. These methods can be divided into two big groups:

- methods in which these numbers are interpreted and processed as *probabilities* (e.g., *probabilistic logic*, *Dempster-Shafer formalism*), and
- methods that use *non-probabilistic* processing techniques (*certainty values*, *fuzzy* techniques).

For both types of methods, uncertainty of expert statements leads to uncertainty in the final decisions.

Probabilistic methods. If we use probabilistic methods to process initial uncertainties, then (in the majority of cases) the formulas for the resulting uncertainties are known from probability theory. The *main problem* with these methods is that often the existing algorithms are too slow, and therefore, *faster algorithms are required* if we want to apply these methods to real-time control problems.

Non-probabilistic techniques. For non-probabilistic techniques, there is *another important problem:* since we are not using probabilistic formulas, *what formulas to use?* In particular, we must make the following choices:

1) First, we must choose a method to represent the initial expert statements. For each term like "small", "medium", etc, that an expert uses to express his knowledge, and for every possible value x of the corresponding variable, we must express his degree of belief that this x is small by a number $\mu(x)$ from an interval [0,1].
2) Second, we must choose a method to combine the resulting degrees of belief that corresponds to "and" and "or", e.g., what is our degree of belief that x is "small" *and* velocity $\dot{x}$ is "medium"?

Using these two stages, we are able, for each possible value u of control, to compute the resulting degree of belief $\mu_C(u)$ that u is appropriate for this control situation.

3) Third, we must transform this uncertain knowledge about u into a single value $\bar{u}$; this procedure is called *defuzzification.*

Methods for using uncertain knowledge in control are highly successful. Practically all of the proposed methods of intelligent control have been experimentally tested and proved to be appropriate for some real-life situations (see, e.g., surveys [S85], [L90], [B91], [K92]). Experiments performed at Johnson Space Center on the Shuttle and rover simulators, showed that these methods really lead to high quality control of space missions and planet rovers [L88], [L89], [LJ90].

The choice of uncertainty representation is vitally important for an expert system. This importance is demonstrated by the history of the first efficient expert system - MYCIN [S76]. The operations for combining degrees of belief that are implemented in this system are very complicated. The reason is that while creating MYCIN the authors tried to choose an uncertainty representation that would make the percentage of correct diagnoses higher. They experimented with several formulas, spent a lot of time and money and came to the ultimate formulas that are now implemented. This stage of creating the expert system turned out to be time-consuming but necessary, because all other choices essentially decreased the efficiency of this system - and sometimes even made it useless.

Unfortunately the result of this experiment turned out to be essentially dependent on the concrete domain: the same formulas that are extremely efficient in MYCIN fail in application to other domains. So the problem of choosing the appropriate uncertainty representation technique is really vitally important, because the wrong choice can lead to complete failure. Further, the proper choice can lead to an essential increase in the efficiency of the system - an increase that can otherwise be obtained only by additional expenditures on hardware.

Trial-and-error choice is impossible for a space mission. Usually the choice of an appropriate technique is made on a trial-and-error basis, but this is impossible for a billion-dollar project. So, *we need theoretical methods for this choice.*

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2. MAIN OBJECTIVES OF THIS RESEARCH PROJECT

The eventual goal of this research is *to develop methods for choosing the appropriate representation of uncertainty* (either from one of the already existing formalisms or by developing a new one).

In order to achieve this goal, it is necessary to accomplish the following:

- To give a *survey* of the *existing* uncertainty reasoning *techniques.*
- To *describe characteristics of different* uncertainty reasoning *techniques* that are maximally relevant to our engineering problems.
- To describe algorithms that *estimate the values of the chosen characteristics* for different uncertainty reasoning techniques.
- To *formulate the problem of finding the best technique* as a mathematical problem, and
- To solve this optimization problem, i.e., to *find* uncertainty reasoning *techniques that are optimal* with respect to different optimality criteria.

3. MAIN RESULTS

The main results of this research project are published in the conference proceedings paper [26], and in other papers published under this grant. With R. Lea from Johnson Space Center, we are currently working on a book that would incorporate all these results. These results include the following:

A survey of different uncertainty reasoning techniques is given in [26]. In this survey, we not only give the list of all possible techniques, but give a theoretical explanation of why exactly these techniques turned out to be workable.

Characteristics of uncertainty reasoning techniques. For every technique, we must:

1) first, elicit the knowledge from the experts,
2) then, process this knowledge using the corresponding techniques,
3) and finally, apply the resulting control to a spacecraft.

For each stage, we can choose a natural criterion that makes this particular stage most successful:

1) The time and effort needed to solicit the knowledge from an expert, can be described by the average *number of* binary ("yes–no") *questions* that we need to ask an expert. This number, in its turn, is related to the accuracy with which we need to know the experts' degrees of belief: if the resulting control is *sensitive* to this accuracy, then we must determine the degree of belief with better precision, and thus, ask more questions.

2) The time spent on the processing stage is *computation time* of an algorithm.

3) The quality of the resulting control can be characterized by the criteria that are traditionally used in control theory: *stability* and *smoothness.*

So, we have five main criteria for choosing uncertainty reasoning techniques:

- *entropy*, i.e., the average number of binary questions required to complete the knowledge;
- *robustness*, i.e., sensitivity of the resulting control relative to the inaccuracies in the initial degrees of belief;
- *computational complexity*, i.e., the time required for the programs to run;
- *stability* of the resulting control strategies;
- *smoothness* of the resulting trajectories.

With the exception of computational complexity (that is a well-defined notion), for all other criteria we could not use the existing criteria, and therefore, had to provide appropriate mathematical definitions. For example, in traditional control theory, stability is usually understood as the following condition: after a small fluctuation, when time $t \to \infty$, the trajectory eventually returns to its original position. However, if this time t exceeds the time of the space mission, then this theoretical "stability" is of no use. So, instead of using the existing theoretical criteria, we give a new definition that formalized the engineering practice rather than the existing theory.

In particular, as a source of the notion of stability, we considered *spacecraft orientation* problems, and as a source of smoothness, *spacecraft docking* problems (see [26] for details).

For *entropy*, these new definitions are given in [2], [17], [33]; for *robustness*, in [31], [32], [TR3], [A14]; for *stability* and *smoothness*, in [21], [26].

In real-life situations, we must use a *combined* criterion to find a reasonable trade-off between all five main criteria.

Algorithms that estimate the values of these characteristics:

- For *entropy*, such algorithms are presented in [2], [6], [17], [33]. In particular, [2] and [17] cover the Dempster-Shafer and probabilistic approaches, and [6] covers the special case of *non-transitive* preferences.
- For *robustness*, methods are presented in [31] and [32].
- *Computational complexity* is not a very serious problem for non-probabilistic methods, because we can always choose a technique that is reasonably efficient. For *probabilistic methods*, however, the situation is radically different: here, the formulas are given, and we cannot change them at will. Traditional algorithms of Dempster-Shafer approach require too long computations, and this is one of the main reasons why these methods are not universally used. In [1], [11], [A10], we describe an alternative computational algorithm that enables us to use polynomial-time (i.e., feasible) algorithms.
- For *stability* and *smoothness*, methods are given in [21] and [26].

Formulation of the problem of finding the best technqiue as a mathematical problem. As we have already mentioned, in real-life situations, it is reasonable to use not only the five main criteria, but their combinations as well. What exactly combination to use depends on a specific problem. To describe a general case, we developed a general optimization formalism [19], [25], [26] (see also [20], [22], [23], [24]).

This formalism is based on the so-called *group-theoretic* (*symmetry*) approach that has been so successful in modern theoretical physics.

The main idea of applying this approach to non-probabilistic uncertainty is as follows. In probabilistic case, the value $t(A)$ that is assigned as a truth value to a statement A has a very precise meaning, e.g., it describes the ratio of cases in which an expert considers A to be true. In non-probabilistic case, an expert describes his uncertainty in terms of words of natural languages ("probably", "maybe", etc), and how to represent these words by numbers is not really that important.

Therefore, if we use a reasonable criterion for choosing a technique, it is natural to expect that the relative quality of different techniques (with respect to this criterion) should not depend on what exactly mapping from words to numbers we use.

In other words, we have here a family of natural *transformations* (that transform uncertainty values obtained by using one mapping into values obtained by another mapping), and the ordering between the techniques that corresponds to optimality criteria must be *invariant* w.r.t. these transformations.

Optimal choice of techniques. Group-theoretic approach not only allows us to formulate the family of reasonable criteria, but also to find the techniques that are optimal with respect to these criteria.

As a criterion, we can take many different combinations of main criteria. Since there are infinitely many possible combinations, it is impossible to describe a technique that is optimal for each of these combinations.

So, in this project, we do the following:

- for the *main* criteria, we solve the optimization problem precisely, and find techniques that are optimal with respect to these criteria;
- for *combined* criteria, we describe a family of techniques that are optimal under different combination criteria; then, when a criteria is given, to find a technique that is optimal with respect to this criteria. it is sufficient to test only techniques from this family.

The description of all techniques that can be optimal under reasonable optimality criteria is given in [26].

This family includes all the techniques that have been empirically shown to be good, and also other techniques that are worth trying.

For *main* criteria, the optimal techniques are described in the following papers:

- for *entropy*, in [33]; in particular, we get $\min(a, b)$ for "and", and $\min(a + b, 1)$ for "or";
- for *robustness*, in [31] and [32]; in particular, we get $\min(a, b)$ for "and", $\max(a, b)$ for "or", piece-wise linear *membership functions* $\mu(x)$, and standard (center-of-mass) defuzzification;
- for *computational complexity*, min and max are evidently the simplest;
- for *stability*, in [26]; in particular, we get $\min(a, b)$ for "and", and $\min(a + b, 1)$ for "or";
- for *smoothness*, in [26]; in particular, we get ab for "and", and $\max(a, b)$ for "or".

4. ADDITIONAL RESULTS

In this research, we also applied group-theoretic and similar techniques to solve additional related problems.

Is expert knowledge really necessary? Many researchers, especially in traditional control community, still doubt that expert knowledge is necessary. Why not make more experiments and determine the properties of the system?

This is rather a fundamental and theoretical question than a practical one. However, we thought that it would be nice to have an answer to this challenge.

To provide such an answer, we analyzed the most general physical systems.

- One argument in favor of expert knowledge is that for some physical systems, there is simply no way to find their description based on the experimental data only. In particular, such an argument was provided for background microwave radiation: supposedly, there is no way to provably confirm whether it is of cosmological origin or not. We analyzed this example in [9], and proved that this in principle, if we have sufficiently accurate experimental data, it *will be* possible to distinguish between the cosmological and other models of a 3K radiation. (Another example of *cognizability* of the physical world is given in [10].)
- However, from the viewpoint of *computation time* that is necessary to make predictions based on the experimental data, we showed that in the general case, this computation time grows *exponentially* with the size of data, and therefore, this problem is not feasible [8], [28]. This means that to be able *to design feasible algorithms we do need expert knowledge* in addition to experimental data.
- Another case when expert knowledge is extremely helpful is the case of the so-called *inverse problems.* This is a generic mathematical term for the problems in which we reconstruct the parameters of the system from the noisy measurement data. Usually, such problems are *ill-posed*, i.e., small errors in the measured data can lead to large errors in the parameters. In [27] and [16], we prove that with the expert knowledge added, such problems not only become well-posed, but also that we can apply reasonably fast algorithms to solve them. Unlike the above two fundamental theoretical results, this is also a *practical result.*

If several different techniques are already used to represent the expert knowledge, what is the best way to combine them? Our main result was aimed at the case when we start "from scratch", i.e., when we first choose the uncertainty representation technique, and then apply this technique to elicit knowledge from the experts and process it. In many cases, however, when we start the problem, we already have some expert knowledge, and this knowledge is already represented by using non-optimal technique. So, in order to apply the better technique, we must first translate this knowledge from one representation into another.

For *non-probabilistic* uncertainty reasoning techniques, in [20], we applied the group-theoretic approach to describe the best "interface" translation algorithms. For translation between *probabilistic* and *non-probabilistic* methods, a translation is proposed in [30].

How to combine intelligent control with more traditional control techniques. Traditional control is best developed for *linear* systems. So, if we have a non-linear system, it is reasonable to apply some non–linear re-scaling of its variables so that after this re-scaling the system will be either linear, or closer to being linear.

Since this re-scaling works for traditional control, it sounds reasonable to apply it also to the case of *intelligent control*, i.e., control based on the expert knowledge. This idea was used in [34], [A16], and shown to be reasonably efficient. Moreover, we apply group-theoretic methodology to find the *optimal* re-scaling.

How to make expert systems smarter? In the majority of applications of an expert system to control, we just translate the expert's knowledge into an actual control strategy. The resulting control is sometimes worse than the control by a human operator, because an operator can not only apply his rules, but he can also combine them into more complicated ones (i.e., in other words, he can make *logical conclusions.*

In [18], [A3], [A11], we show how this can done automatically in a general case. The idea developed in these papers comes from the analysis of chemical systems [A7].

For important problems of *pattern recognition* and *cluster analysis*, such algorithms are presented in [TR1] and [TR2].

How to make an expert system learn? In the above formulation, we analyzed the problem of how to translate the expert knowledge into the actual control strategy.

The resulting control strategy is not perfect: first, the expert was not perfect; second, the translation might not grasp some nuances of his knowledge. So, it is desirable to make the resulting automated system *learn.* How?

Several techniques are known that make intelligent systems learn, among them *analytical* techniques (i.e., crudely speaking, numerical computations methods), *neural networks*, *genetic algorithms*, etc. All these techniques have been successfully applied to tune intelligent control systems.

But here, we encounter the same problem of choice: there are several different neural network techniques, and in some cases, some of them work fine, and some fail to improve the quality of the system. *How to choose?*

To solve this problem, we applied a similar group-theoretic methodology, and arrived at the following results:

- for the simplest case when we are changing the degree of belief of a single statement, the optimal technique is presented in [23];
- for *neural networks*, the optimal techniques are presented in [22]; namely, in [22], we describe the optimal choice of a non-linear basic element (*neuron*);

- for *genetic algorithms*, the optimal techniques are presented in [24]; namely, we describe the optimal choice of *re-scaling.*

Two additional results about learning are presented in [29], [7], [A8], and [A13]:

- in [29], we describe *how to combine* different learning techniques (namely, neural and analytical);
- in [7] (see also [A8]), we show that neural networks are (in principle) capable to learn anything (if we use an appropriate learning algorithm). In [7], we prove that such a learning procedure is possible, but provide no example. Such an example is given in [A13] (*warning:* this algorithm is not practically efficient; the main reason for providing it was to prove that in principle, such an algorithm is possible).

Taking into consideration the uncertainty of the expert knowledge and of the measurement results. As a result of applying uncertainty reasoning techniques, we design a system that transforms the measurement results into the actual control. The resulting values of control are not precise for two reasons:

- first, experts are not absolutely confident in the rules that they use;
- measurement results are not absolutely precise, because every real measuring device is not perfect.

It is therefore important to take *both* uncertainties into consideration when designing an intelligent control system. This is partially done.

In particular, when we analyzed *sensitivity* [31], [A14], we actually considered sensitivity with respect to both types of uncertainty.

For other criteria, we have just started such analysis (see [5], [12–15], [A1–A2], [A4], [A9], [A12], [A17]).

By-product of this research: group-theoretical approach leads to new algorithms and results. The same ideas of optimization under uncertainty have also been successfully applied to other problems:

- In [3], we describe an algorithm for *computer graphics* that is better than the existing ones. This algorithm solves the problem that is very important for space applications, with its 3D areas: the problem of rotating an image around an arbitrary axis.
- In [4], we describe an algorithm for image processing from radar measurements. Here, group-theoretic approach is used to justify the choice of an entropy-like function that we optimize to get the best quality of a reconstructed image.
- Other results (not yet published) are contained in the theses defended under this grant.

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Technical Reports

TR1. Vladik Kreinovich. "Fast parallel algorithms that compute transitive closure of a fuzzy relation", *Center for Fuzzy Logic and Intelligent Systems Research*, Texas A&M University, Technical Report CFL-93-001, January 1993, 8 pp.

TR2. Vladik Kreinovich. "Strongly transitive fuzzy relations: a more adequate way to describe similarity", *Center for Fuzzy Logic and Intelligent Systems Research*, Texas A&M University, Technical Report CFL-93-002, January 1993, 10 pp.

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A1. Mark Baker, Diane Doser, Vladik Kreinovich, Vitaly Kozlenko. "Why linear regression methods work so well for non- linear problems?". *Abstracts for a Workshop on Interval Methods in International Conference on Interval Computations*, Lafayette, LA, February 25–March 1, 1993, p. 2.

A2. Carlos Ferregut, Soheil Nazarian, Krishnamohan Vennalaganti*, Ching-Chuan Chang*, Vladik Kreinovich. "Fast interval error estimates for the results of nonlinear least squares method: application to pavement engineering". *Abstracts for a Workshop on Interval Methods in International Conference on Interval Computations*, Lafayette, LA, February 25–March 1, 1993, p. 14.

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A4. Olga Kosheleva*, Andrew Bernat, Andrei Finkelstein, Vladik Kreinovich. "Interval estimates for closure-phase and closure amplitude imaging in radio astronomy". *Abstracts for a Workshop on Interval Methods in International Conference on Interval Computations*, Lafayette, LA, February 25–March 1, 1993, p. 18.

A5. O. M. Kosheleva* and V. Ya. Kreinovich. "What to do if there exist no von Neumann-Morgenstern solution" *Abstracts of Papers Presented to the American Mathematical Society*, 1990, Vol. 11, No. 5 (October 1990), p. 475.

A6. V. Ya. Kreinovich "Optimal non-binary coding for analog-to-digital converters". *Abstracts of Papers Presented to the American Mathematical Society*, 1990, Vol. 11, No. 5 (October 1990), p. 475.

A7. V. Ya. Kreinovich. "All kinds of behavior are possible in chemical kinetics." *Abstracts of the Southwestern Conference on Theoretical Chemistry*, The University of Texas at El Paso, November 1990, p. 21.

A8. V. Kreinovich. Letter to the Editor, *Neural Networks*, 1993, Vol. 6, No. 1, pp. 3–4.

A9. Vladik Kreinovich. "Interval, mean value, standard deviation, what else? Group-theoretic approach to describing uncertainty". *Abstracts for a Workshop on Interval Methods in International Conference on Interval Computations*, Lafayette, LA, February 25–March 1, 1993, p. 19.

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A12. Joe Lorkowski*, Vladik Kreinovich. "If we measure a number we get an interval. What if we measure a function or an operator?". *Abstracts for a Workshop on Interval Methods in International Conference on Interval Computations*, Lafayette, LA, February 25–March 1, 1993, p. 21.

A13. Ray Mines, Mutsumi Nakamura*, Vladik Kreinovich. "Constructive proof of Kolmogorov's theorem, neural networks and intervals". *Abstracts for a Workshop on Interval Methods in International Conference on Interval Computations*, Lafayette, LA, February 25–March 1, 1993, p. 24–25.

A14. Hung T. Nguyen, Vladik Kreinovich, Bob Lea, Dana Tolbert*. "How to control if even experts are not sure: minimizing intervals of uncertainty". *Abstracts for a Workshop on Interval Methods in International Conference on Interval Computations*, Lafayette, LA, February 25–March 1, 1993, p. 27.

A15. J. H. Perluissi and V. Ya. Kreinovich. "Computational complexity problems in measuring strong currents. 1." *Abstracts of Papers Presented to the American Mathematical Society*, 1990, Vol. 11, No. 5 (October 1990), p. 475.

A16. H. VanLangingham, A. Tsoukkas*, V. Kreinovich, and C. Quintana*. "Nonlinear rescaling of control values simplifies fuzzy control". *Abstracts of Papers. Third International Workshop on Neural Networks and Fuzzy Logic '92*, NASA Johnson Space Center, Houston, TX, June 1–3, 1992, p. 40.

A17. Karen Villaverde*, Vladik Kreinovich. "Linear-time algorithms that locate local maxima and mimima of a function from approximate measurement results". *Abstracts for a Workshop on Interval Methods in International Conference on Interval Computations*, Lafayette, LA, February 25–March 1, 1993, p. 35.

APPENDIX 1. LIST OF GRADUATE THESES AND PROJECTS DEFENDED UNDER THE GRANT

1. Jaswinder Singh Chadha. "An automated system for inspection of solder joints using machine vision", *Master Thesis*, Department of Mechanical and Industrial Engineering, University of Texas at El Paso (co-supervised by W. C. Johnson), December 1991.

2. Chi-Cheng Chang."Comparison of different fuzzy control algorithms based on computer simulation of car back-parking problem", *Master Project*, Computer Science Department, University of Texas at El Paso, June 1992.

3. Ching-Chuan Chang. "Fast algorithm that estimates the precision of indirect measurements", *Master Project*, Computer Science Department, University of Texas at El Paso, November 1992.

4. Luis Olac Fuentes. "Applying uncertainty formalisms to well-defined problems: experimental and theoretical foundations", *Master Thesis*, Computer Science Department, University of Texas at El Paso, July 1991.

5. Sunil Kamat. "Efficient spare allocation for restructrable VLSI RAM chips." *Master Thesis*, Department of Electrical Engineering, University of Texas at El Paso, December 1992.

6. Liz Kamoroff. "How to extract knowledge from an expert so that his effort is minimal", *Master Project*, Computer Science Department, University of Texas at El Paso, October 1992.

7. Sukanya Krishnamurthy. "Robust statistical methods in computer vision. *Master Thesis*, Department of Computer Science, University of Texas at El Paso, November 1992.

8. Nilesh Nabar. "Methodology that combines neural and analytical models and its application to composite materials, *Master Thesis*, Department of Electrical Engineering, University of Texas at El Paso, December 1991.

9. Prakash Narasimhamurthy. "Application of intelligent control to congestion in computer networks". *Master Thesis*, Department of Electrical Engineering, University of Texas at El Paso, December 1992.

10. Nitin Nilkanth Okade. "A real time algorithm for fractal analysis and its application to an early detection of epileptic seizures." *Master Thesis*, Department of Electrical Engineering, University of Texas at El Paso, January 1993.

11. Monu Pradhan-Advani. "Catastrophe theory and congestion in computer networks", *Master Project*, Computer Science Department, University of Texas at El Paso, March 1992.

12. Dharmendran Rajendran. "Application of discrete optimization techniques to the diagnostics of industrial systems", *Master Thesis*, Department of Mechanical and Industrial Engineering, University of Texas at El Paso, August 1991.
13. Ketan Shah. "A fractionally linear method to decrease window size in the case of congestion in computer networks", *Master Thesis*, Department of Electrical Engineering, University of Texas at El Paso, November 1992.