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NONLINEAR ANALYSES OF COMPOSITE AEROSPACE STRUCTURES IN SONIC FATIGUE

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By

Chuh Mei, Principal Investigator

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TABLE OF CONTENTS

Page

1. ACOUSTICS-STRUCTURE INTERACTIONS USING BOUNDARY/FINITE ELEMENT METHODS 1

2. RESPONSE OF THERMALLY BUCKLED COMPOSITE PANELS EXCITED BY RANDOM NOISE 3

3. NUMERICAL SIMULATION OF NONLINEAR RESPONSE OF COMPOSITE PLATES UNDER COMBINED ACOUSTIC AND THERMAL LOADS 4

4. SONIC FATIGUE DESIGN AND PASSIVE CONTROL OF RANDOM RESPONSE OF SHAPE MEMORY ALLOY HYBRID COMPOSITES 8

5. REFERENCES 13

6. PRESENTATIONS AND PROCEEDING PAPERS 16

NONLINEAR ANALYSIS OF COMPOSITE AEROSPACE STRUCTURES IN SONIC FATIGUE

By
Chuh Mei*

This report summarizes the semiannual research progress, accomplishments and future plans performed under the NASA Langley Research Center Grant No. NAG-1-1358, entitled, "Nonlinear Analysis of Composite Aerospace Structures in Sonic Fatigue," for the period December 16, 1992 to June 15, 1993. The primary research effort of this project is the development of analytical methods for the prediction of nonlinear random response of composite aerospace structures subjected to combined acoustic and thermal loads. The progress, accomplishments and future plates on four sonic fatigue research topics are described. The sonic fatigue design and passive control of random response of shape memory alloy hybrid composites presented in section 4, which is suited especially for HSCT, is a new initiative.

1. ACOUSTICS-STRUCTURE INTERACTIONS USING BOUNDARY/FINITE ELEMENT METHODS

Over the past ten years, there has been increasing attempts to apply the boundary element method (BEM) to acoustic problems. BEM has some advantages and disadvantages of its use for certain types of problems. Some disadvantages include: low degree of freedom elements, no nonlinear analysis, and full populated matrices. Some advantages are: simple element models, low computation time, and excellent model of infinite domains.

Knowing the advantages and disadvantages of the finite element method (FEM), the research is focuses on coupling FEM and BEM for structural-acoustic interaction problems. Thus, we wanted to take advantage of the good aspects of each method. It is decided to first study the interaction between the sound pressure inside a three-dimensional duct and a vibrating plate. The duct was considered to have all rigid walls and the governing equations of BEM and FEM were coupled to analyze the problem. The acoustics in the duct using BEM is governed by

$$[A]p = j\rho\omega[B]v \tag{1}$$

where p is the acoustic pressure and v is the acoustic velocity. The constants ρ and ω are the fluid density and forcing frequency, respectively. The governing equation of a harmonic motion of a structure for the FEM analysis is given as

$$[H]q = [\Phi]^T([K] + j\omega[C] - \omega^2[M])[\Phi]q = [\Phi]^T(f_e + f_p) \tag{2}$$

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where $[K]$, $[C]$, $[M]$, $[H]$ and $[\Phi]$ are the system stiffness, damping, mass, modal response, and modal matrices, respectively. The variables, q and f_p, are the modal displacement vector and the modal force vector related to the acoustic pressure inside the duct, f_e is the external force vector. Equations (1) and (2) can be coupled and expressed as

$$\begin{bmatrix} [A] & -\omega^2\rho[B][\Phi] \\ -[\Phi]^T[S] & [H] \end{bmatrix} \begin{Bmatrix} p \\ q \end{Bmatrix} = \begin{Bmatrix} 0 \\ [\Phi]^T f_e \end{Bmatrix} \tag{3}$$

to solve for the pressure and modal displacements anywhere on the plate or in the duct. The matrix $[S]$ is a diagonal matrix that represents the area of each element. Figure 1 shows the coupled duct and plate system. A modal summation expression for the transmission loss at the end of the duct was calculated by Bokil [1] using a classic analytical continuum solution. Experimental results are also shown, which were obtained by Guy and Bhattacharya [2]. The coupled BEM/FEM results are compared for various element types in Figure 2. The coupled BEM/FEM approach gives very good approximations.

While learning to couple BEM/FEM, research has also continued in the field of analyzing various 3–D duct systems using the boundary element method. BEM results were compared with exact solutions for some regular type ducts and the comparisons were very good. To show the versatility of BEM, BEM is applied to two types of irregular duct systems, shown in Figures 3 and 4. The duct system in Figure 3 was analyzed using finite elements by Munjal [3], and 3 element cases using the BEM were compared. In Figure 4, a four-inlet/four-outlet duct was created and two BEM element cases were run to calculated the transmission loss at the end of the duct. The duct results were presented by Carl Pates III at the 1993 National Conference on Noise Control Engineering, Williamsburg, Virginia on May 2–5. The title of the paper is "Boundary Element Analysis of the Acoustic Field in Three-Dimensional Regular and Irregular Ducts." We expect to present a paper on the coupling of BEM/FEM at the upcoming 1994 AIAA/ASME/ASCE/AHS/ASC 35th Structures, Structural Dynamics and Materials Conference. We also expect to send a paper to the Journal of Acoustical Society of America in the same area of research.

Future research work will include applying composite materials in the plate, and advancing to nonlinear structural analysis of the plate. Carl S. Pates III is expected to complete his Ph.D. degree in May 1994.

2. RESPONSE OF THERMALLY BUCKLED COMPOSITE PANELS EXCITED BY RANDOM NOISE

This investigation can be broken into two parts. There is analytical as well as experimental progress to be discussed.

2.1 Analytical Investigation

The analytical work has centered on the nonlinear vibrations of beams and more recently plates. Initially, work has been centered on solving the nonlinear governing equation for a beam buckled by an end displacement. A relationship was determined to relate the axial and transverse displacements along with the forces produced in the beam. This was done using a Galerkin single mode solution. Results of this investigation were then compared with those by Yamaki [4] and experimental data performed at ODU. Results from the three were found to be in good agreement and thus work on the nonlinear vibration problem was conducted. This was done by employing the harmonic balance method to the nonlinear equations of motion and results were found to be in good agreement with those of Tseng and Dugundji [5]. These results were also used to compare with a finite element analysis conducted by Peter O'Donoghue [6].

Work has also been initiated to analyze the response of a composite panel that has been thermally buckled. The work being conducted analyzes the previously mentioned plate acted on by a harmonic base excitation. This work will hopefully be completed this summer and be extended to include random excitations. It is also attempted in this analysis to include the possibility of snap-through motions. Experimental work which will be discussed later will also be used to compare with analytic predictions. In addition, an attempt will be made to extend the work of Peter O'Donoghue to include beams acted on by random noise and to define regions of snap-through for such a dynamic condition.

2.2 Experimental Work

Experimental work conducted over the past year has concentrated primarily on preparations for experiments to be conducted this summer by Ray Istenes at Wright Laboratories in Dayton, Ohio. A study was first conducted to determine adequate plate sizes to be tested in the summer. This analysis was conducted using computer codes developed by Chuck Gray [7]. This study was concerned primarily with ensuring that buckling temperatures, frequency ranges, and buckled mode shapes could be measured and fall within required limitations of the 12,000-lb shaker at WL, in order that meaningful data is taken. Work was then conducted for clamping fixture designs. Two fixtures were designed and manufactured for each of two plate sizes to be tested this summer. The test specimens are of dimensions 6×12 in. and 7×9 in. with two laminations

$[0/\pm45/90]_s$ and $[0/90/0/90]_s$. Insulating materials were also considered to insulate the clamping fixture from the composite panel and the lamp which will be used for heating the panels. The reason for this study is that the composite panels have a lower coefficient of thermal expansion than the support steel fixture. Consequently, if insulation is not present the panels may not buckle when a constant temperature distribution is present on the panel surface.

Time was also spent to investigate options available for instrumenting the composite panels. It was determined that strain gages would be the best choice since they will not have the effect of mass loading the structure and can be used to get both bending and axial information. A laser vibrometer may also be used to obtain information on points of particular interest on the panels.

Testing of the panels will make up the bulk of the progress experimentally over the next several months. During this summer some of the objectives to be accomplished are:

(1) Measure surface temperature distribution of composite panels as well as the temperature gradient through the panel thickness;

(2) Measure linear natural frequencies of composite panels;

(3) Measure buckled mode shape and compare with analytic predictions of Chuck Gray [7];

(4) Conduct dynamic vibrations tests with harmonic and random inputs;

(5) Measure the cofficients of thermal expansion of the composite material that will be used in the above investigation (this will be used to fulfill my research skill requirement);

(6) Conduct experimental investigation to determine the regions of snap-through for an aluminum beam under harmonic excitation (to be presented with results obtained by Peter O'Donoghue [6] at the 35th Structures, Structural Dynamics and Materials Conference).

3. NUMERICAL SIMULATION OF NONLINEAR RESPONSE OF COMPOSITE PLATES UNDER COMBINED ACOUSTIC AND THERMAL LOADS

This problem does a time domain study of the response of the composite panel subject to simultaneous thermal and acoustic loads. The characteristics of the problem, in particular, the high decibel sound pressure level, the thermal gradient across thickness which induces buckling, along with the geometrical nonlinearities in the behavior of a thin flat panel, call for analysis under large deformation assumptions. Accordingly von Karman strain-displacement relations are used. Other complexities accommodated in the finite element formulated include:

(1) transverse shear strains;

(2) initial imperfection in flatness of the panel;

(3) initial stress distribution;

(4) arbitrarily laminated composite panel with a completely anisotropic property;

(5) improvement in modeling of pressure distribution from one that is normal to one that is inclined at an arbitrary angle

(6) general temperature distribution $\Delta T(x, y, z)$.

The laminate is still restricted to small strains and linear elasticity characteristics. Based on this problem definition, the equation of motion had been derived and is expressed in the matrix form as

$$\begin{aligned}
&\begin{bmatrix} [m]_b & 0 \\ 0 & [m]_m \end{bmatrix} \begin{Bmatrix} \ddot{w}_b \\ \ddot{w}_m \end{Bmatrix} \\
&+ \left(\begin{bmatrix} [k]_b & [k]_{bm} \\ [k]_{mb} & [k]_m \end{bmatrix} + \begin{bmatrix} [k_o]_b & [k_o]_{bm} \\ [k_o]_{mb} & 0 \end{bmatrix} + \begin{bmatrix} [k_{No}] - [k_{N\Delta T}] & 0 \\ 0 & 0 \end{bmatrix} \right. \\
&+ \begin{bmatrix} [k1_{Nm}]_b + [k1_{Nb}]_b & [k1]_{bm} \\ [k1]_{mb} & 0 \end{bmatrix} + \begin{bmatrix} [k1_o] & 0 \\ 0 & 0 \end{bmatrix} \\
&\left. + \begin{bmatrix} [k2] & 0 \\ 0 & 0 \end{bmatrix} \right) \begin{Bmatrix} w_b \\ w_m \end{Bmatrix} = \begin{Bmatrix} p(t) \\ 0 \end{Bmatrix} + \begin{Bmatrix} \{p_o\}_b \\ \{p_o\}_m \end{Bmatrix} + \begin{Bmatrix} \{p_{\Delta T}\}_b \\ \{p_{\Delta T}\}_m \end{Bmatrix}
\end{aligned} \tag{4}$$

or

$$\begin{aligned}
[m]\{\ddot{w}\} + ([k] + [k_o] + [k_{NO}] - [k_{N\Delta T}] \\
+ [k1] + [k1_o] + [k2])\{w\} \\
= \{p(t)\} + \{p_o\} + \{p_{\Delta T}\}
\end{aligned} \tag{5}$$

where $[m], [k], \{p\}$ and $\{w\}$ denote the element mass, linear stiffness matrices, load and displacement vectors, respectively; and $[k1]$ and $[k2]$ denote the first and second order nonlinear stiffness matrices, respectively. The subscripts b and m denote the bending and membrane components, respectively. The subscripts $o, NO, N\Delta T, Nm$ and Nb denote the corresponding stiffness matrix due to initial imperfection $w_o(x, y)$, initial stress resultant $\{N_o\}$, thermal stress resultant $\{N_{\Delta T}\}$, inplane stress resultant component $\{N_m\}(= [A]\{\epsilon_m^o\})$ and bending stress resultant component $\{N_b\}(= [B]\{\kappa\})$, respectively.

The element implemented in this study is the nine-node, C^1 continuous, quadrilateral isoparametric element [8]. This element has total 45 degrees of freedom, 5 degrees of freedom per node.

After assembling the individual finite elements for the entire panel and applying the kinematic boundary conditions, the system equations of motion become

$$\begin{aligned}
[M]\left\{\ddot{W}\right\} + ([K] + [K_o] + [K_{No}] - [K_{N\Delta T}] \\
+ [K1] + [K1_o] + [K2])\{W\} \\
= \{P(t)\} + \{P_o\} + \{P_{\Delta T}\}
\end{aligned} \tag{6}$$

Certain significant modifications have been made to the problem formulation. The original finite element model (equation 6) was in the system's global nodal degrees of freedom, representing physical values of displacements and rotations. It has, since then, been derived in terms of modal coordinates.

As a first-step, this involves coming up with a modal transformation matrix which is a matrix of eigenvectors obtained upon solving a linear free vibration eigenvalue problem for the given panel configuration, i.e.,

$$\omega_r^2[M]\{\phi_r\} = [K]\{\phi_r\} \tag{7}$$

gives

$$\{W\} = \sum_{r=1}^{n} q_r\{\phi_r\} = [\Phi]\{q\} \tag{8}$$

The eigenvectors $\{\phi_r\}$ are the mode shapes corresponding to the different natural frequencies of vibration ω_r of the panel.

A linear combination of these mode shapes (equation 8) defines the vector of unknown nodal displacements and rotations. The time-dependent terms, q_r, that form the linear combination of the mode shapes are now the new unknown in the system of equations. These are the generalized modal coordinates.

However, since the response of a structure when excited is largely in the neighborhood of the lowest few natural frequencies, it does suffice to use a truncated transformation matrix with just the first few (say n) mode shape vectors or eigenvectors, and the corresponding number of modal coordinates. In doing so, we gain a huge advantage by reducing the size of the finite element model from hundreds of degrees of freedom (in the physical coordinates $\{W\}$) to just a handful (n, about ten), in the modal coordinates $\{q\}$. This, in fact, was the single most important purpose of switching the solution procedure from a system of coupled nonlinear equations in physical degrees of freedom to one that is still coupled and nonlinear, but of a drastically reduced size in modal degrees of freedom.

The formulation of the problem in terms of modal coordinates, necessitates transformation of the individual matrices in the system of equations, namely, the mass matrix, the linear and nonlinear stiffness matrices and the thermal and initial stress stiffness matrices, to their equivalent 'modal' matrices. So is the case with the various load vectors which form their modal

counterparts. For example the nonlinear stiffness matrices $[K1]$ and $[K2]$ can be expressed in terms of the modal coordinates as

$$[K1] = \sum_{r=1}^{n} q_r [K1]^{(r)} \tag{9}$$

and

$$[K2] = \sum_{r=1}^{n} \sum_{s=1}^{n} q_r q_s [K2]^{(rs)} \tag{10}$$

where the nonlinear modal stiffness matrices $[K1]^{(r)}$ and $[K2]^{(rs)}$ are evaluated with the corresponding element components from modes $\{\phi_r\}$ and $\{\phi_s\}$ as

$$[K1]^{(r)} = \sum_{\substack{all \\ elements}} \begin{bmatrix} [k1_{Nm}]_b^{(r)} + [k1_{Nb}]_b^{(r)} & [k1]_{bm}^{(r)} \\ [k1]_{mb}^{(r)} & 0 \end{bmatrix} \tag{11}$$

and

$$[K2]^{(rs)} = \sum_{\substack{all \\ elements}} \begin{bmatrix} [k2]^{(rs)} & 0 \\ 0 & 0 \end{bmatrix} \tag{12}$$

The system equations of motion, equation (6), are thus transformed to the forced Duffing equations in truncated modal coordinates as

$$[\bar{M}]\{\ddot{q}\} + ([\bar{K}] + [K_q] + [K_{qq}])\{q\} = \{F\} \tag{13}$$

where the modal mass $[\bar{M}]$ and linear modal stiffness $[\bar{K}]$ are diagonal as

$$[\bar{M}] = [\Phi]^T [M][\Phi] \tag{14}$$

and

$$[\bar{K}] = [\Phi]^T ([K] + [K_o] + [K_{No}] - [K_{N\Delta T}])[\Phi] \tag{15}$$

The nonlinear modal stiffness matrices $[K_q]$ and $[K_{qq}]$ are linearly and quadratically in $\{q\}$ as

$$[K_q] = [\Phi]^T \left\{ \sum_{r=1}^{n} q_r \left([K1]^{(r)} + [K1_o]^{(r)} \right) \right\} [\Phi] \tag{16}$$

and

$$[K_{qq}] = [\Phi]^T \left(\sum_{r=1}^{n} \sum_{s=1}^{n} q_r q_s [K2]^{(rs)} \right) [\Phi] \tag{17}$$

and the modal force vector is

$$\{F\} = [\Phi]^T (\{P(t)\} + \{P_o\} + \{P_{\Delta T}\}) \tag{18}$$

For isotropic or symmetrically laminated composite ($[B] = 0$) panels, the bending, $\{\phi_r\}_b$, and membrane, $\{\phi_r\}_m$, mode shapes are uncoupled. In this case, the system modal equations of motion have the form

$$[\bar{M}]\{\ddot{q}\} + ([\bar{K}] + [K_{qq}])\{q\} = \{F\} \tag{19}$$

The expressions for $[\bar{M}]$, $[\bar{K}]$ and $[K_{qq}]$ modal matrices in Eq. (19) are derived in reference [9].

Equation (13) or (19) now results in a system of nonlinear ordinary differential equations in the unknown modal coordinates vector. These equations are then broken down to a numerical integration scheme following the single-step algorithm of Zienkiewicz et al. [10], coupled with the Wilson-θ method [11]. Therefore, in their final form the equations of motion are a system of nonlinear algebraic equations at any given time-step, to be solved iteratively using the Newton-Raphson technique.

Thus, at the present stage, the program has been updated to accommodate these modifications in the formulation. Meanwhile, the Monte Carlo simulation technique for generating a spatially uniform random Gaussian pressure load has been completed. The first set of solutions for a flat panel (no initial imperfection) under the above described pressure distribution with no thermal loads, is being worked on presently. The next step will be to arrive at a solution for the combined thermal-acoustic loading problem. Jayashree Moorthy is expected to complete her Ph.D. degree in August 1994.

4. SONIC FATIGUE DESIGN AND PASSIVE CONTROL OF RANDOM RESPONSE OF SHAPE MEMORY ALLOY HYBRID COMPOSITES

Shape memory alloys (SMA) in the low temperature martensitic condition, when plastically deformed and the external stress removed, will regain its original (memory) shape when heated. Strains of typically six to eight percent can be completely recovered by heating the nickel-titanium alloys (Nitinol) above its phase transformation temperature (the austenite finish temperature A_f). This characteristic transformation temperature can be altered by changing the composition of the alloy. In addition, the Young's modulus increases three to four times and the yielding strength increases approximately ten times when Nitinol is heated causing the material transformation from the martensitic phase to the austenite phase [12, 13]. Therefore, SMA will be ideal for sonic fatigue application for HSCT such that the SMA fibers are embedded in the conventional composite panels (such as graphite-epoxy laminates). The aerodynamic heating will be the heat source and heat the skin panels above the transformation temperature A_f. The overall stiffness of the SMA hybrid composite panel will be increased due to: (1) the increase of the Young's modulus of the SMA fibers and (2) the internal tensile inplane forces induced in the panel

from the recovery of initial strains of the SMA fibers. The root-mean-square (RMS) maximum deflection and RMS maximum strain will be thus greatly reduced. This has been demonstrated from the preliminary results of an investigation of passive control of random response of SMA fiber-reinforced composite plates [14] using the classical analytical continuum method.

4.1 Mathematical Formulation

A limited amount of investigations on structural response subjected to intense acoustic and thermal loads exists in the literature. Seide and Adami [15] were the first who studied large deflection random response of a thermally buckled simply supported beam. The well-known classic Woinowsky-Krieger large amplitude beam vibration equation is used. The Galerkin's method and time domain numerical simulation are then applied to obtain random response. Most recently, the Galerkin/numerical simulation approach was applied to simply supported metal and orthotropic composite rectangular plates by Vaicaitis [16, 17]. The classic von Karman large deflection plate equations including temperature and orthotropic property effects are employed. Lee [18] further extended to nonuniform temperature distributions. He studied isotropic rectangular plates with either simply supported or clamped edges. The Galerkin/equivalent linearization method was used. The classic continuum approaches thus have been limited to simple beam [15], and isotropic or orthotropic rectangular plates [16–18].

For over three decades, the finite element methods have been the predominant approach for structural mechanics of complex structural geometry. However, there are only few studies where nonlinear random response of structures subjected to combined acoustic and thermal loads are involved. Locke and Mei [19, 20] extended the finite element method the first time to structures under combined thermal and acoustic loads. The thermal load considered is a steady-state temperature distribution $\Delta T(x, y)$. The linear vibration mode shapes of the thermally buckled structure are used to reduce the order of the system equations of motion to a set of nonlinear modal equations of a much smaller order. The equivalent linearization technique is then employed to iteratively obtain the RMS responses. Excellent agreement between the finite element and the Galerkin/numerical simulation [15] results is obtained. Chen and Mei [21] recently refined further the finite element formulation for nonlinear random response of structures by considering that the acoustic pressure and thermal load are applied simultaneously. It is found that there are significant differences between the random response of acoustic and thermal loads applied sequentially [15, 19, 20] and simultaneously [21] at high temperatures.

Shape memory alloys have been applied as actuators for active control of buckling of beams [22] and shape control of beams [23]. It is also being studied in using in active vibration control of beams [24, 25] and large space structures [26]. Active vibration control of flexible

linkage mechanisms using SMA fiber-reinforced composites has been investigated by Venkatesh et al. [27]. Acoustic transmission and radiation control by use of the SMA hybrid composite was presented by Rogers and Fuller [28, 29]. In all those investigations [22–29] the SMA fibers are heated by applying an electrical current with control devices. However, the heat source for the present sonic fatigue design of SMA hybrid composite panels is from the aerodynamic heat, therefore, no control device is needed. In the previous studies of the application of SMA [12, 13, 28, 29], the thermal effects of the composite materials were neglected, thus a SMA hybrid composite panel will have only the recovery tensile forces included. Following their formulation there will be no thermal buckling and postbuckling possible for SMA composite panels. This is not reasonable in reality. In this study, the thermal effects of the composite matrix (such as graphite-epoxy) are considered and the finite element formulation for acoustic fatigue design of SMA fiber-reinforced hybrid composite laminated plates is developed.

4.2 Properties of a SMA Hybrid Composite Lamina

The one-dimensional stress-strain relation of SMA fiber can be described as [12, 13]

$$\sigma_s = E_s^* \epsilon + \sigma_r^* \tag{20}$$

where the Young's modulus E_s^* and the recovery stress σ_r^* are both temperature dependent. The superscript (*) indicates that the corresponding property is temperature dependent. The nonlinear recovery stress σ_r^* is related to the initial recovery strain ϵ_r from a semi-empirical expression [30]

$$\sigma_r^* = 5.34 \times 10^6 \epsilon_r^3 - 4.89 \times 10^5 \epsilon_r^2 + 1.68 \times 10^4 \epsilon_r \tag{21}$$

The one-dimensional stress-strain relation of graphit-epoxy matrix with temperature effects can be written as

$$\sigma_m = E_m(\epsilon - \alpha \Delta T) \tag{22}$$

Using an engineering approach, the stress-strain relation of a thin lamina having graphite-epoxy matrix and SMA fibers can be derived as

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11}^* & Q_{12}^* & 0 \\ Q_{12}^* & Q_{22}^* & 0 \\ 0 & 0 & Q_{66}^* \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} + \begin{Bmatrix} \sigma_r^* v_s \\ 0 \\ 0 \end{Bmatrix} - \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} v_m \Delta T$$

$$= [Q^*] \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} + \begin{Bmatrix} \sigma_r^* v_s \\ 0 \\ 0 \end{Bmatrix} - [Q]_m \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} v_m \Delta T \tag{23}$$

where $[Q]_m$ is the reduced stiffness matrix of a conventional graphite-epoxy lamina, v_m and v_s are the volume fractions of graphite-epoxy matrix and SMA fibers and $v_m + v_s = 1$. The reduced stiffness matrix of the SMA hybrid composite lamina $[Q^*]$ is evaluated from E_1^*, E_2^*, ν_{12}^* and G_{12}^* as [13]

$$E_1^* = E_{1m} v_m + E_s^* v_s$$
$$E_2^* = E_{2m} E_s^* / (E_{2m} v_s + E_s^* v_m)$$
$$\nu_{12}^* = \nu_m v_m + \nu_s^* v_s$$

and

$$G_{12}^* = G_{12m} G_s^* / (G_{12m} v_s + G_s^* v_m) \tag{24}$$

For a general k-th layer with an orientation angle θ, the stress-strain relation is thus

$$\{\sigma\}_k = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = [\bar{Q}^*]_k \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{Bmatrix} \sigma_x^* \\ \sigma_y^* \\ \tau_{xy}^* \end{Bmatrix}_k - \left([\bar{Q}]_m \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} v_m \right)_k \Delta T \tag{25}$$

The resultant force and moment vectors of SMA fiber-reinforced hybrid composite plate are defined as

$$\{N, M\} = \int_{-h/2}^{h/2} \{\sigma\}_k (1, z) dz$$

or

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A^* & B^* \\ B^* & D^* \end{bmatrix} \begin{Bmatrix} \epsilon^o \\ \kappa \end{Bmatrix} + \begin{Bmatrix} N^* \\ M^* \end{Bmatrix} - \begin{Bmatrix} N_{\Delta T} \\ M_{\Delta T} \end{Bmatrix} \tag{26}$$

where the inplane strain and curvature vectors are defined from the von Karman strain-displacement relations as

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \{\epsilon^o\} + z\{\kappa\}$$
$$= \begin{Bmatrix} u_{,x} \\ v_{,y} \\ u_{,y} + v_{,x} \end{Bmatrix} + \frac{1}{2} \begin{Bmatrix} w_{,x}^2 \\ w_{,y}^2 \\ 2 w_{,x} w_{,y} \end{Bmatrix} - z \begin{Bmatrix} w_{,xx} \\ w_{,yy} \\ 2 w_{,xy} \end{Bmatrix} \tag{27}$$

4.3 Finite Element Equations of Motion

Application of the principle of virtual work, the governing equations of motion can be derived for a SMA fiber-reinforced hybrid composite panel subjected to a combined thermal and acoustic loads as

$$\begin{bmatrix} M_b & 0 \\ 0 & M_m \end{bmatrix} \begin{Bmatrix} \ddot{W}_b \\ \ddot{W}_m \end{Bmatrix} + \left(\begin{bmatrix} K_b^* & K_B^* \\ K_B^{T*} & K_m^* \end{bmatrix} + \begin{bmatrix} K_\sigma^* & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} K_{N\Delta T} & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} N1_{Bm}^* & N1_{bm}^* \\ N1_{mb}^* & 0 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} N2_b^* & 0 \\ 0 & 0 \end{bmatrix} \right) \begin{Bmatrix} W_b \\ W_m \end{Bmatrix} = \begin{Bmatrix} P_b(t) \\ 0 \end{Bmatrix} + \begin{Bmatrix} P_{b\Delta T} \\ P_{m\Delta T} \end{Bmatrix} + \begin{Bmatrix} P_b^* \\ P_m^* \end{Bmatrix} \tag{28}$$

or

$$[M]\left\{\ddot{W}\right\} + ([K^*] + [K_\sigma^*] - [K_{N\Delta T}] + \frac{1}{2}[N1^*] + \frac{1}{3}[N2^*])\{W\} = \{P(t)\} + \{P_{\Delta T}\} + \{P^*\} \tag{29}$$

where $[M]$ and $[K^*]$ are the mass and linear stiffness matrices; $[K_\sigma^*]$ and $[K_{N\Delta T}]$ are the geometric stiffness matrices due to the recovery stress σ_r^* and thermal inplane force vector $\{N_{\Delta T}\}$, respectively; $[N1^*]$ and $[N2^*]$ are the first and second order nonlinear stiffness matrices which depend linearly and quadratically upon displacement $\{W\}$, respectively; $\{P(t)\}$ is the random acoustic excitation load vector, $\{P_{\Delta T}\}$ is the thermal load vector, and $\{P^*\}$ is the SMA recovery force vector. The solution procedures developed in references 19 and 21 are to be employed to determine:

(1) the critical buckling temperature, T_{cr};

(2) the thermal postbuckling behavior when the temperature is higher than T_{cr};

(3) the RMS maximum deflection and strain at intense acoustic load and elevated temperature;

(4) the fatigue life; and

(5) parameter study on random response and fatigue life with the initial recovery strain of SMA, ϵ_r, varying from 2% to 4% and the volume fraction of SMA fibers, v_s, varying from 10% to 20%.

5. REFERENCES

1. Bokil, Vijay, "Modal Analysis of Gyroscopically Coupled Sound-Structure Interaction Problems," Master's Thesis, Department of Mechanical Engineering and Mechanics, Old Dominion University, May 1992.

2. Guy, R. W., Bhattacharya, M. C., "The Transmission of Sound Through a Cavity-Backed Finite Plate," Journal of Sound and Vibration, 1973, Vol. 27–2, pp. 207–223.

3. Munjal, M. L., Acoustics of Ducts and Mufflers, Wiley and Sons, 1987, pp. 274–282.

4. Yamaki, N. and Mori, A., "Nonlinear Vibrations of a Clamped Beam with Initial Deflection and Initial Axial Displacement, Part 1: Theory, "Journal of Sound and Vibrations, Vol. 71, No. 3, 1980, pp. 333–346.

5. Tseng, W. Y. and Dugundji, J., "Nonlinear Vibrations of a Beam under Harmonic Excitation," Journal of Applied Mechanics, Vol. 38, 1971, pp. 467–476.

6. O'Donoghue, P. J., "An Analysis of the Response of a Buckled Beam with Fixed Ends to an Harmonic Base Excitation Using the Finite Element Method," Master's Thesis, Department of Mechanical Engineering and Mechanics, Old Dominion University, May 1993.

7. Gray, C. C., "Finite Element Analysis of Thermal Postbuckling and Vibrations of Thermally Buckled Composite Plates," Master's Thesis, Department of Mechanical Engineering and Mechanics, Old Dominion University, May 1991.

8. Ergatoudis, I., Irons, B. M., Zienkiewicz, O. C., "Curved, Isoparametric Quadrilateral Elements for Finite-Element Analysis," International Journal of Solids and Structures, Vol. 4, 1968, pp. 31–42.

9. Lai, Z., Zhou, R. C., Xue, D. Y., Huang, J. K. and Mei, C., "Suppression of Nonlinear Panel, Flutter at Elevated Temperature with Piezoelectric Actuators," Proceedings of the 34th Structures, Structural Dynamics and Materials Conference, LaJolla, CA, April 19–22, 1993, pp. 3466–3474.

10. Zienkiewicz, O. C., Wood, W. L., Hine, N. W. and Taylor, R. L., "A Unified Set of Single-Step Algorithms, Part I: General Formulation and Applications," International Journal for Numerical Methods in Engineering, Vol. 20, 1984, pp. 1529–1552.

11. Wilson, E. L., Farhoomand, I., Bathe, K. J., "Nonlinear Dynamics Analysis of Complex Structures," Earthquake Engrg. Structural Dynamics 1, 1973, 283–291.

12. Rogers, C. A., Liang, C. and Jia, J., "Behavior of Shape Memory Alloy Reinforced Composite Plates — Part 1: Model Formulation and Control Concepts," Proceedings of

the 30th Structures, Structural Dynamics and Materials Conference, Mobile, AL, April 3–5, 1989, pp. 2011–2017.

13. Jia, J. and Rogers, C. A., "Formulation of a Mechanical Model of Composites with Embedded SMA Actuators," Proceedings of the 8th Biennial Conference on Failure Prevention and Reliability, Montreal, Canada, September 17–21, 1989, pp. 203–210.

14. Pates III, C. S., Zhong, Z. W. and Mei, C., "Passive Control of Random Response of Shape Memory Alloy Fiber-Reinforced Composite Plates," Submitted for Presentation at the 5th International Conference on Recent Advances in Structural Dynamics, Institute of Sound and Vibration, University of Southampton, July 18–21, 1994.

15. Seide, P. and Adami, C., "Dynamic Stability of Beams in a Combined Thermal-Acoustic Environment," AFFWAL-TR-83-3027, Wright-Patterson AFB, October 1983.

16. Vaicaitis, R. and Arnold, R. R., "Nonlinear Response and Fatigue of Surface Panels by the Time Domain Monte Carlo Approach," WRDC-TR-90-3081, Wright-Patterson AFB, May 1992.

17. Vaicaitis, R. and Kavallieratos, P. A., "Nonlinear Response of Composite Panels to Random Excitation," Proceedings of the 34th Structures, Structural Dynamics and Materials Conference, LaJolla, CA, April 19–22, 1993, pp. 1041–1049.

18. Lee, J., "Large-Amplitude Plate Vibration in an Elevated Thermal Environment," WL-TR-92-3049, Wright-Patterson AFB, June 1992.

19. Locke, J. E., "A Finite Element Formulation for the Large Deflection Random Response of Thermally Buckled Structures," Ph.D. Dissertation, Old Dominion University, Norfolk, VA, July 1988.

20. Locke, J. E. and Mei, C. "A Finite Element Formulation for the Large Deflection Random Response of Thermally Buckled Beams," AIAA Journal, Vol. 28, 1990, pp. 2125–2131.

21. Chen, R. and Mei, C., "Finite Element Nonlinear Random Response of Beams to Acoustic and Thermal Loads Applied Simultaneously," Proceedings of the 34th Structures, Structural Dynamics and Materials Conference, LaJolla, CA, April 19–22, 1993, pp. 1050–1057.

22. Baz, A. and Tampe, L., "Active Control of Buckling of Flexible Beams," Proceedings of the 8th Biennial Conference on Failure Prevention and Reliability, Montreal, Canada, September 17–21, 1989, pp. 211–218.

23. Chaudhry, Z. and Rogers, C. A., "Bending and Shape Control of Beams Using SMA Actuators," Journal of Intelligent Systems and Structures, Vol. 2, 1991, pp. 581–602.

24. Baz, A., Iman, K. and McCoy, J., "Active Vibration Control of Flexible Beams Using Shape Memory Actuators," Journal of Sound and Vibration, Vol. 140, 1990, pp. 437–456.

25. Srinivasan, A. V., Cutts, D. G. and Schetky, L. M., "Thermal and Mechanical Consideration in Using Shape Memory Alloys to Control Vibrations in Flexible Structures," Metallurgical Transactions A, Vol. 22A, 1991, pp. 623–627.

26. Maclean, B. J., Patterson, G. J. and Misra, M. S., "Modeling of a Shape Memory Integrated Actuator for Vibration Control of Large Space Structures," Journal of Intelligent Materials Systems and Structures, Vol. 2, 1991, pp. 72–94.

27. Venkatesh, A., Hilborn, J., Bidaux, J. E. and Gotthardt, R., "Active Vibration Control of Flexible Linkage Mechanisms Using Shape Memory Alloy Fiber-Reinforced Composites," The 1st European Conference on Smart Structures and Materials, Glasgow, UK, May 12–14, 1992, pp. 185–188.

28. Liang, C., Rogers, C. A. and Fuller, C. R., "Acoustic Transmission and Radiation Analysis of Adaptive Shape Memory Alloy Reinforced Laminated Plates," Journal of Sound and Vibration, Vol. 145, 1991, pp. 23–41.

29. Anders, W. S., Rogers, C. A. and Fuller, C. R., "Vibration and Low Frequency Acoustic Analysis of Piecewise-Activated Adaptive Composite Panels," Journal of Composite Materials, Vol. 26, 1992, pp. 103–120.

30. Cross, W. B., Kariotis, A. H. and Stimler, F. J., "Nitinol characterization Study," NASA CR-1433, 1969.

6. PRESENTATIONS AND PROCEEDING PAPERS

1. Chiang, C. K., Xue, D. Y. and Mei, C., "Nonlinear Vibrations of Thin Arbitrarily Laminated Composite Plates Subjected to Harmonic Excitations using DKT Elements," Proceedings of the 34th Structures, Structural Dynamics and Materials Conference, LaJolla, CA, April 1993, pp. 245–254.

2. Chen, R. and Mei, C., "Finite Element Nonlinear Random Response of Beams to Acoustic and Thermal Loads Applied Simultaneously," Proceedings of the 34th Structures, Structural Dynamics and Materials Conference, LaJolla, CA, April 1993, pp. 1050–1060.

3. Pates III, C. S., "Boundary Element Analysis of the Acoustic Field in Three-Dimensional Regular and Irregular Ducts," Proceedings of the NOISE-CON 93, Williamsburg, VA, May 1993, pp. 111–116.

Coupled Duct/Plate System

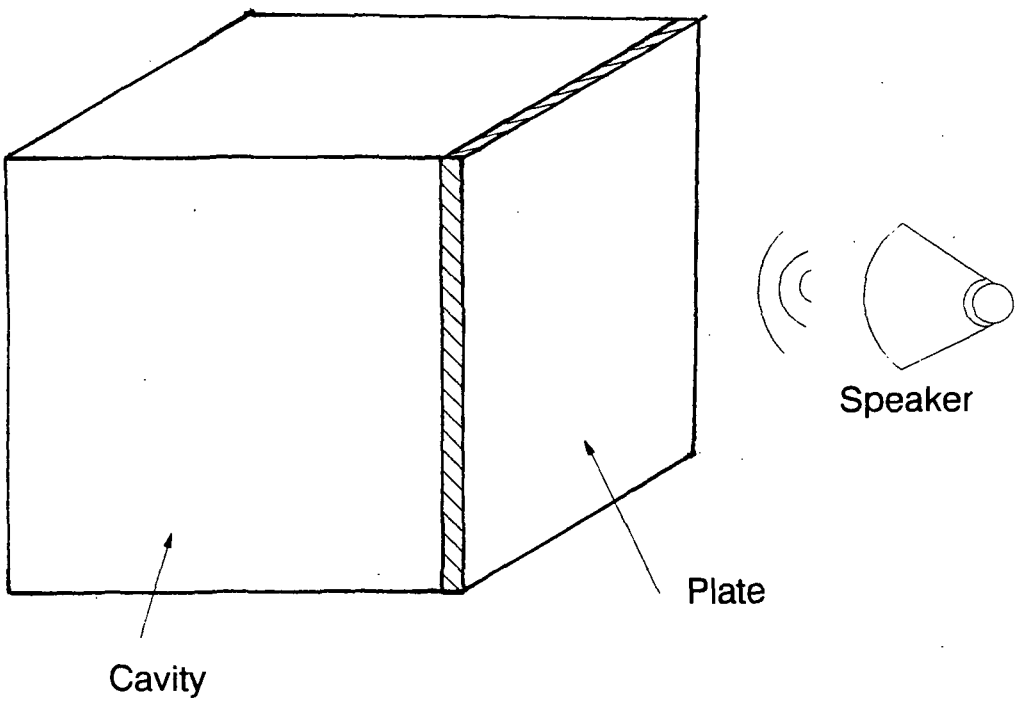

Figure 1

TRANSMISSION LOSS FOR COUPLED ACOUSTIC STRUCTURE PROBLEM

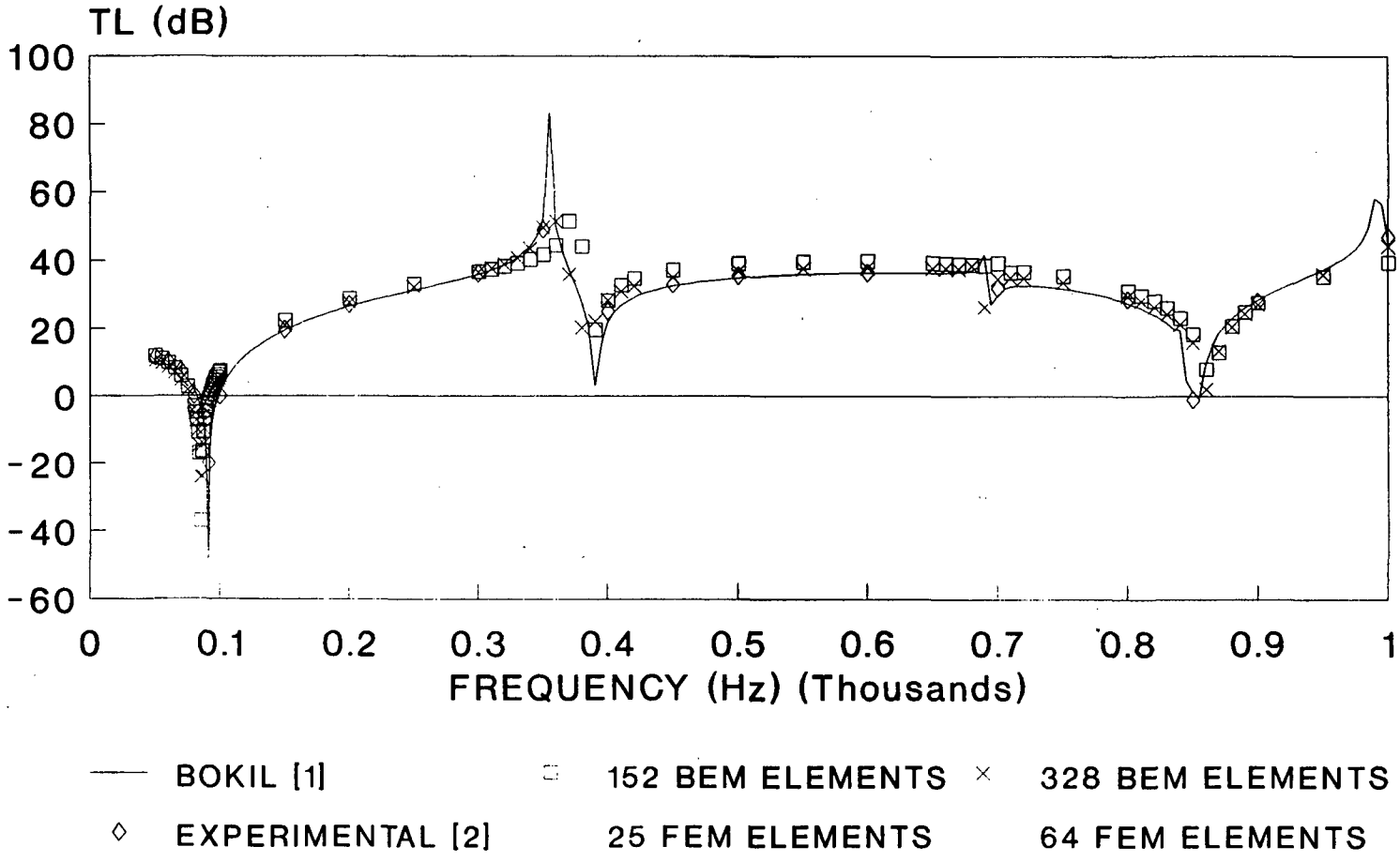

Figure 2

FEM and BEM Transmission Loss of Offset Inlet and Outlet Duct

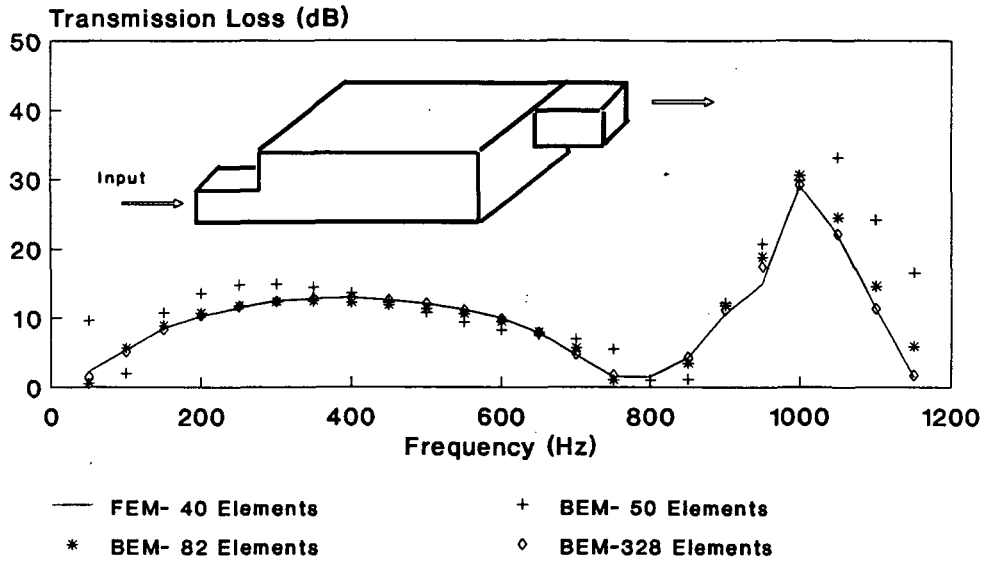

Figure 3

BEM Transmission Loss of Four Inlet and Four Outlet Duct

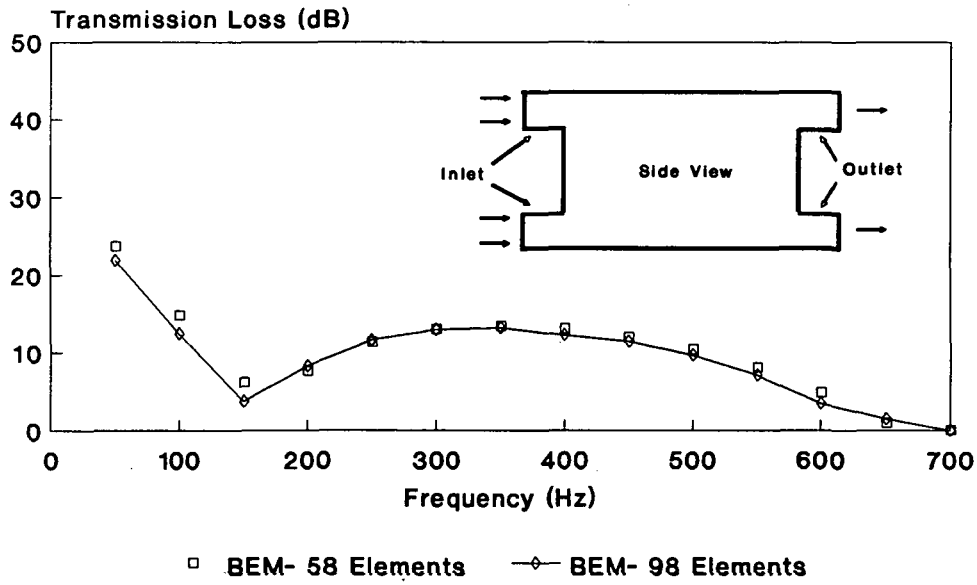

Figure 4