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# NASA Contractor Report 4515

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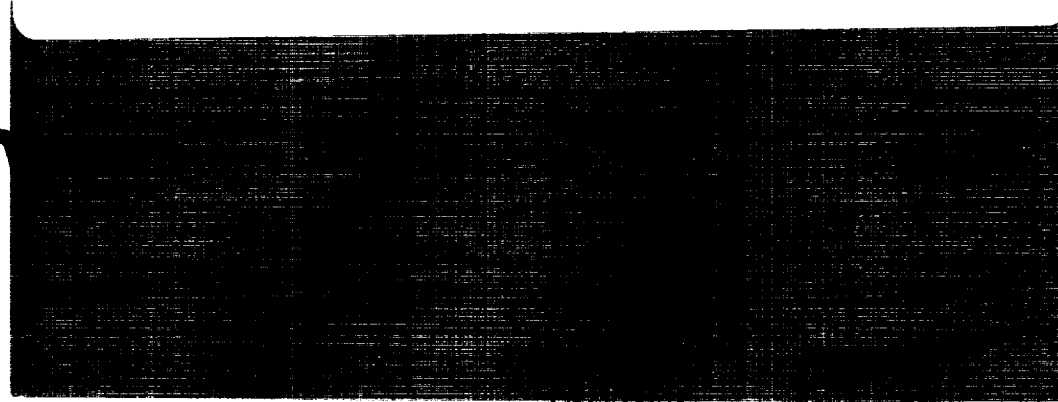
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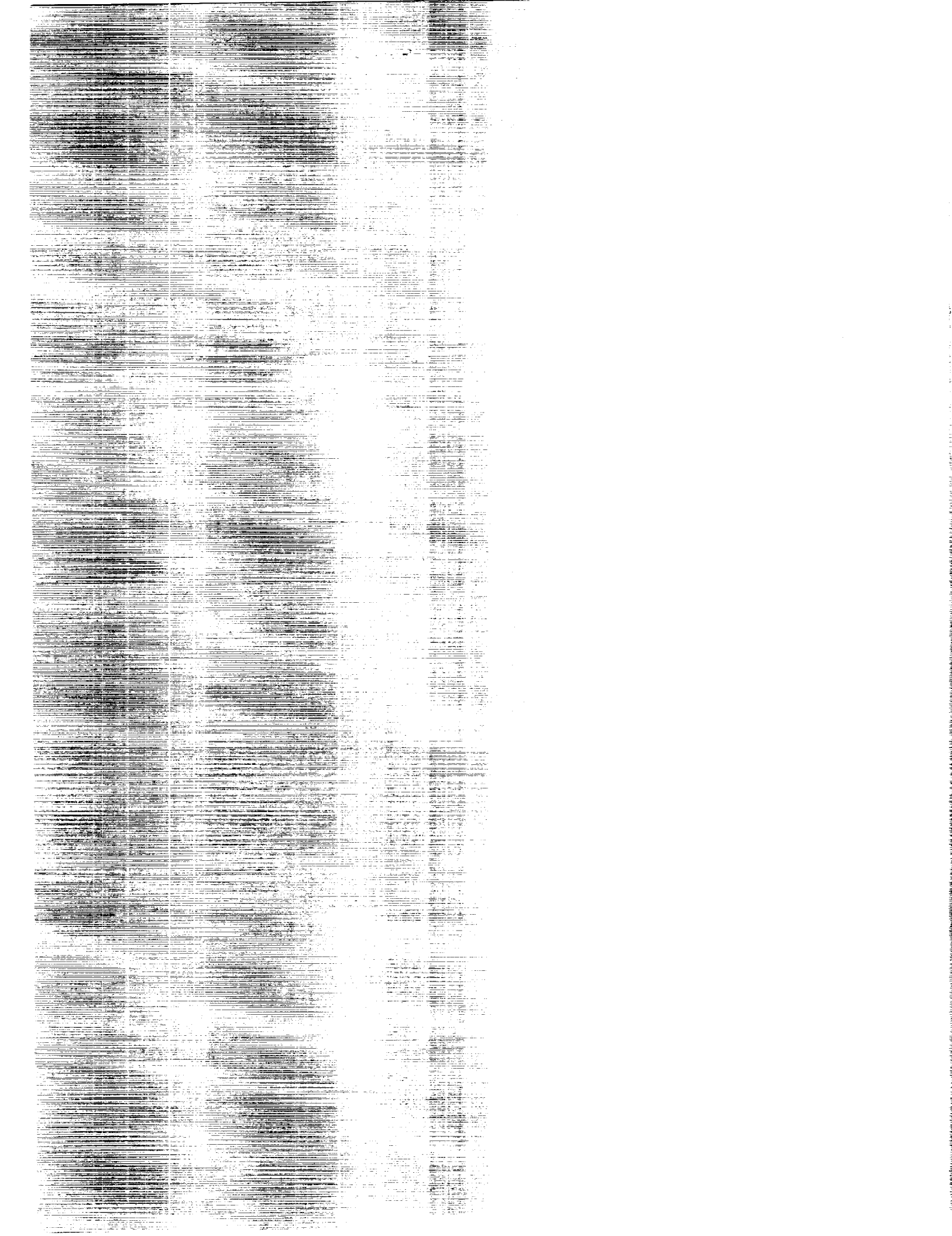
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## Summary

A second-order model for the velocity field and a two-equation model for the temperature field are used to calculate supersonic boundary layers assuming negligible real gas effects. The modeled equations are formulated on the basis of an incompressible assumption and then extended to supersonic flows by invoking Morkovin's hypothesis, which proposes that compressibility effects are completely accounted for by mean density variations alone. In order to calculate the near-wall flow accurately, correcting functions are proposed to render the modeled equations asymptotically consistent with the behavior of the exact equations near a wall and, at the same time, display the proper dependence on the molecular Prandtl number. Thus formulated, the near-wall second-order turbulence model for heat transfer is applicable to supersonic flows with different Prandtl numbers. The model is validated against supersonic flows with free-stream Mach numbers as high as 10 and wall temperature ratios as low as 0.3. Among the flow cases considered, the momentum thickness Reynolds number varies from  $\sim 4,000$  to  $\sim 21,000$ . Good correlation with measurements of mean velocity and temperature is obtained. Discernible improvements in the law-of-the-wall are observed, especially in the range where the log-law applies.

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## 1. Introduction

Until recently, it was not possible to calculate supersonic flat plate turbulent boundary layers accurately when the free-stream Mach numbers are higher than 5 (Bradshaw et al. 1991). The reason can be traced to an incorrect estimate of the near-wall flow using wall functions (Zhang et al. 1992). With the advent of supercomputers and numerical techniques, it was possible to numerically simulate simple turbulent flows with and without heat transfer (Moser and Moin 1987; Kim et al. 1987; Mansour et al. 1988; Spalart 1988; Kim and Moin 1989; Kasagi et al. 1991). Consequently, asymptotically correct near-wall two-equation models for the velocity and temperature fields have been proposed (So et al. 1991b; Nagano and Kim 1988; Nagano et al. 1991; Sommer et al. 1992). These models were based on conventional high-Reynolds-number models with near-wall correcting functions that were derived to satisfy the asymptotic behavior of the exact equations. As such, the models were formulated for fluids with Prandtl number,  $Pr \approx 1$  (So and Sommer 1993). These models have been applied to calculate a wide variety of incompressible flows with and without heat transfer, including direct simulation data, and good agreement was obtained for all flow cases tested. Extensions to second-order models have been proposed and validated (Lai and So 1990a,b; So et al. 1991a). Again, the high-Reynolds-number second-order models were found to give good results when they were modified to yield asymptotically correct near-wall behavior. The modifications were in the form of near-wall correcting functions added to the Reynolds-stress and dissipation-rate equations.

These successes, therefore, justify the extension of the near-wall correcting functions to supersonic flows. The extension was first carried out with two-equation models (Zhang et al. 1992) assuming the validity of Morkovin's hypothesis (1962) and the results indicated that, with an asymptotically consistent near-wall correction, the models were able to mimic supersonic flows up to a free-stream Mach number of 10 with an adiabatic wall. In this first attempt, real gas effects were neglected and the turbulent Prandtl number,  $Pr_t$ , was assumed to be 0.9. On the other hand, the calculations of supersonic flows with cooled wall boundary condition and high Mach numbers were not as satisfactory. Strictly speaking, there is no dynamic similarity between momentum and

heat transport, even in incompressible flows (Antonia and Kim 1991). Therefore, the constant  $Pr_t$  assumption needs to be relaxed. An attempt for supersonic flows has been carried out (Sommer et al. 1993) and the variable  $Pr_t$  model used was a modification of the two-equation incompressible model proposed by Sommer et al. (1992). Therefore, the proposed model was applicable to flows with  $Pr \approx 1$  only. The variable  $Pr_t$  calculations assuming negligible real gas effects were in good agreement with measurements (Fernholz and Finley 1977; Kussoy and Horstman 1991) and gave significant improvement over those obtained assuming constant  $Pr_t$ . The same methodology has been applied to modify second-order models for supersonic flows (Zhang et al. 1993) and these calculations with constant  $Pr_t$  were found to give improvements over those reported by Zhang et al. (1992), especially the calculations of high Mach number flows with low wall temperature ratios. This means that, for the first time, a model to treat complex compressible flows is available. However, the second-order model is still limited by the constant  $Pr_t$  assumption and by the fact that the model is only applicable to fluids with  $Pr \approx 1$ .

Most engineering flows of importance involve fluids whose Prandtl numbers vary with temperature and the range of variation could be large. Furthermore, some fluids have a  $Pr$  that is vastly different from 1. In view of this, an appropriate variable  $Pr_t$  model for flows with heat transfer would be one that could handle a wide variety of  $Pr$  in addition to being able to account for variable  $Pr$  effect. This means that a more general incompressible heat transfer model to those proposed by Nagano and Kim (1988), Nagano et al. (1991) and Sommer et al. (1992) has to be formulated and validated before its extension to supersonic flows. The task has been attempted by So and Sommer (1993). Their approach is based on the proposal of Sommer et al. (1992). In addition to requiring the modeled equations to satisfy the asymptotic behavior of their exact counterparts, they also try to model the correcting functions so that the parametric dependence on  $Pr$  of the near-wall flow is properly accounted for. The result is a variable  $Pr_t$  model that could correctly predict incompressible heat transfer with  $Pr$  that varies from a low of  $10^{-2}$  to a high of  $10^3$ . The model is applicable to flows with constant wall heat flux as well as constant wall

temperature boundary conditions and the predictions are in good agreement with such diverse data as those given by Kader (1981), Kim and Moin (1989) and Kasagi et al. (1991).

The present objective is to formulate a second-order variable  $Pr_t$  model for supersonic flows that is valid for a wide range of  $Pr$ . This is accomplished by relaxing the constant  $Pr_t$  assumption made in the second-order model of Zhang et al (1993) and by extending the heat transfer model of So and Sommer (1993) to supersonic flows. In the present work, Morkovin's hypothesis (1962) is again invoked and the approach taken is similar to that outlined in Sommer et al. (1992). Consequently, the second-order modeled equations of Zhang (1993) for the velocity field and the two-equation model of So and Sommer (1993) for the temperature field are extended to supersonic flows. An established boundary-layer code (Anderson and Lewis 1971) is modified to solve the set of governing equations and the calculations of compressible boundary layers with adiabatic and cooled wall boundary conditions over a wide range of Mach numbers are compared with measurements (Fernholz and Finley 1977; Kussoy and Horstman 1991) and the constant  $Pr_t$  calculations of Zhang et al. (1993).

## 2. Mathematical Formulation

The supersonic flow of an ideal gas with real gas effects, bulk viscosity and body forces neglected is considered. A density-weighted average is used to decompose the fluctuating quantities, besides pressure and density, into a mass-weighted mean part and a mass-weighted fluctuating part. On the other hand, the pressure and density are decomposed using Reynolds average, which results in a time-averaged mean part and a time-averaged fluctuating part. For any variable  $F$ , the mass-weighted mean is denoted by  $\tilde{F}$ , the mass-weighted fluctuating part by  $f$ , the time-averaged mean by  $\bar{F}$  and the time-averaged fluctuating part by  $f'$ . The fluid density is taken to be  $\rho$ , the dynamic viscosity  $\mu$ , the thermal conductivity  $k$ , the specific heat at constant pressure  $C_p$ , and the gas constant is denoted by  $R$ . In terms of these variables and the pressure  $p$ , the temperature  $\Theta$ , and the  $i$ th component of the velocity  $u_i$ , the mean equations of motions for compressible turbulence can be written as:

$$\frac{\partial \bar{\rho}}{\partial t} + (\bar{\rho} \tilde{U}_i)_{,i} = 0 \quad , \quad (1)$$

$$\frac{\partial \bar{\rho} \tilde{U}_i}{\partial t} + (\bar{\rho} \tilde{U}_i \tilde{U}_j)_{,j} = -\bar{P}_{,i} - \frac{2}{3}(\bar{\mu} \tilde{U}_{j,j})_{,i} + [\bar{\mu}(\tilde{U}_{ij} + \tilde{U}_{j,i})]_{,j} - (\bar{\rho} \tau_{ij})_{,j} \quad , \quad (2)$$

$$\begin{aligned} \frac{\partial \bar{\rho} \bar{C}_p \tilde{\Theta}}{\partial t} + (\bar{\rho} \tilde{U}_i \bar{C}_p \tilde{\Theta})_{,i} &= \frac{\partial \bar{P}}{\partial t} - \tilde{U}_i \bar{P}_{,i} + \bar{u}_i \bar{P}_{,i} + \overline{u_i p_{,i}} + \bar{\sigma}_{ij} \tilde{U}_{i,j} \\ &+ \bar{\sigma}_{ij} \bar{u}_{i,j} + \bar{\rho} \varepsilon - (\bar{\rho} \bar{C}_p Q_i)_{,i} + (\bar{k} \tilde{\Theta}_{,i})_{,i} \quad , \end{aligned} \quad (3)$$

$$\bar{P} = \bar{\rho} R \tilde{\Theta} \quad , \quad (4)$$

where  $(\cdot)_{,i}$  denotes a gradient with respect to the spatial coordinate  $x_i$ , the Einstein summation convention applies to repeated indices, and the Reynolds stress tensor, the Reynolds heat flux vector, the turbulent dissipation rate are defined as  $\tau_{ij} = \overline{u_i u_j}$ ,  $Q_i = \overline{u_i \theta}$ ,  $\bar{\rho} \varepsilon = \overline{\sigma_{ij} u_{ij}}$ , respectively. The mean viscous stress tensor is given by:

$$\bar{\sigma}_{ij} = -\frac{2}{3} \bar{\mu} \tilde{U}_{k,k} \delta_{ij} + \bar{\mu} (\tilde{U}_{ij} + \tilde{U}_{j,i}) \quad . \quad (5)$$

When deriving these equations, additional assumptions are made regarding the neglect of turbulent fluctuations of dynamic viscosity, thermal conductivity and specific heat. Also, according to Speziale and Sarkar (1991), the velocity-pressure gradient correlation term  $\overline{u_i' p_{,i}'}$  can be written in the equivalent form as

$$\overline{u_i' p_{,i}'} = -\left(\overline{\rho} R \widetilde{\Theta} \bar{u}_i\right)_{,i} + \left(\overline{\rho} R \widetilde{u_i \theta}\right)_{,i} - \overline{p' u_{i,i}'} . \quad (6)$$

From these equations, it can be seen that, to achieve closure, models are required for the Reynolds stress tensor  $\tau_{ij}$ , the Reynolds heat flux vector  $Q_i$ , the pressure dilatation correlation  $\overline{p' u_{i,i}'}$ , the turbulent dissipation rate  $\varepsilon$  and the mass flux vector  $\bar{u}_i$ . In the following, appropriate near-wall models are proposed for  $\tau_{ij}$ ,  $Q_i$  and  $\varepsilon$ , while Morkovin's hypothesis (1962) is invoked to justify the neglect of  $\overline{p' u_{i,i}'}$  and  $\bar{u}_i$  in the modeling of supersonic turbulent flows. The models for  $\tau_{ij}$ , and  $\varepsilon$  are presented first and this is followed by a discussion of the model for  $Q_i$ .

### 3. Second-Order Model for the Velocity Field

The modeling of the Reynolds stress tensor is provided by the Reynolds-stress transport equation which is closed by postulating models for the terms representing turbulent diffusion, viscous dissipation and velocity-pressure gradient correlation in the exact equation. Incompressible models for these terms are proposed. Usually the models are formulated for high Reynolds-number flows, therefore, they are not suitable for near-wall flow calculations. Furthermore, the modeled equation is not valid at the wall. Consequently, some kind of wall functions have to be invoked to connect the modeled equation to the wall so that the wall boundary conditions for the Reynolds stresses can be satisfied. This approach is not satisfactory because it is too restrictive in the sense that the wall functions proposed are very much flow-type dependent and thus render the Reynolds-stress model less general compared to other models that are not as sophisticated or are of lower order. Various remedies have been proposed. However, the most promising approach is to modify the modeled equations so that they are valid for near-wall and/or low-Reynolds-number flows.

The approach taken by Zhang (1993) is to derive near-wall correcting functions for the Reynolds-stress transport equation and the conventional dissipation-rate equation. Certain constraints are imposed and these include the requirements that the wall boundary conditions for the Reynolds stresses and the dissipation rate have to be satisfied exactly and that, to the lowest order of the wall normal coordinate  $x_2$ , the near-wall behavior of the modeled equations is consistent with that of the exact equations. The correcting functions derived by Zhang (1993) give asymptotically correct matching up to order  $x_2$ . Of course, more accurate correcting functions can be derived; however, these fairly simple near-wall models are found to give results that are in good agreement with measurements covering a wide range of flow Reynolds numbers (Zhang 1993). These successes prompt the extension of the incompressible modeled equations to supersonic flows by invoking Morkovin's hypothesis (1962) and the results are extremely encouraging (Zhang et al. 1993). In view of this, Zhang et al.'s (1993) modeled Reynolds-stress equations are adopted in the present work.

The compressible Reynolds-stress equation written in the same form as its incompressible counterpart is given by:

$$\begin{aligned} \frac{\partial}{\partial t}(\bar{\rho}\tau_{ij}) + (\tilde{U}_k\bar{\rho}\tau_{ij})_{,k} = & \left(\overline{u'_i\sigma'_{jk}} + \overline{u'_j\sigma'_{ki}}\right)_{,k} + C_{ijk,k} + (-\bar{\rho}\tau_{ik}\tilde{U}_{j,k} - \bar{\rho}\tau_{jk}\tilde{U}_{i,k}) \\ & + \Pi_{ij} - \varepsilon_{ij} + \left(\frac{2}{3}\overline{p'u'_{k,k}}\delta_{ij}\right) - (\bar{u}_i\bar{p}_{,j} + \bar{u}_j\bar{p}_{,i}) + (\bar{u}_i\bar{\sigma}_{jk,k} + \bar{u}_j\bar{\sigma}_{ik,k}) \quad , \quad (7) \end{aligned}$$

where  $C_{ijk,k} = -\left(\overline{\rho u'_i u'_j u'_k} + \frac{2}{3}\overline{p'u'_{k,k}}\delta_{ij}\right)_{,k}$  ,

$$\Pi_{ij} = -\left(\overline{u'_i p'_{,j}} + \overline{u'_j p'_{,i}}\right) + \frac{2}{3}\overline{u'_k p'_{,k}}\delta_{ij} \quad ,$$

$$\varepsilon_{ij} = \overline{\sigma'_{ik}\mu'_{j,k}} + \overline{\sigma'_{jk}\mu'_{i,k}}$$

are the turbulent diffusion of  $\tau_{ij}$ , the velocity-pressure gradient correlation and the viscous dissipation rate of  $\tau_{ij}$ , respectively. The last three bracketed terms in (7) arise as a result of compressibility and are identically zero for incompressible flows. Therefore, if Morkovin's hypothesis (1962) is invoked, the last three bracketed terms in (7) can be neglected and the turbulent diffusion, viscous dissipation and velocity-pressure gradient correlation terms can be modeled as in constant-density flows. Consistent with this assumption, the term  $\overline{p'u'_{i,i}}$  in (6) is also neglected. Finally, the viscous diffusion term  $\left(\overline{u'_i\sigma'_{jk}} + \overline{u'_j\sigma'_{ik}}\right)_{,k}$  is approximated by  $(\bar{\mu}\tau_{ij,k})_{,k}$ . Thus simplified, (7) can be closed by adopting the near-wall models proposed by Zhang (1993). Without derivation, these models are given as:

$$C_{ijk,k} = \left[ C_{\mathcal{P}}\bar{\rho}\frac{K}{\varepsilon}(\tau_{ii}\tau_{ij,l} + \tau_{jl}\tau_{ki,l} + \tau_{il}\tau_{jk,l}) \right]_{,k} \quad , \quad (8)$$

$$\Pi_{ij} = \Phi_{ij} + \Phi_{ij}^R + \Phi_{ij}^W \quad , \quad (9)$$

$$\varepsilon_{ij} = \frac{2}{3}\bar{\rho}\varepsilon\delta_{ij} + \varepsilon_{ij}^W \quad . \quad (10)$$

Here,  $\Phi_{ij}$  is given by the high-Reynolds-number model of Launder et al. (1975),  $\Phi_{ij}^R$  is the "pressure echo" term, and  $\Phi_{ij}^W$  and  $\varepsilon_{ij}^W$  are near-wall corrections. Zhang (1993) have shown that, to order  $x_2^2$ , the near-wall correction terms are not affected by the presence of the "pressure echo"

term. Therefore, the proposal for  $\Phi_{ij}^w$  is applicable irrespective of whether the "pressure echo" term is included in the modeling of  $\Pi_{ij}$  or not. The models for these different terms can now be generalized for compressible flows as:

$$\Phi_{ij} = -C_1 \bar{\rho} \frac{\epsilon}{K} \left( \tau_{ij} - \frac{2}{3} K \delta_{ij} \right) - \alpha_1 \left( P_{ij} - \frac{2}{3} \tilde{P} \delta_{ij} \right) - \beta_1 \left( D_{ij} - \frac{2}{3} \tilde{P} \delta_{ij} \right) - 2\gamma_1 K S_{ij} \quad , \quad (11)$$

$$\Phi_{ij}^R = 2C_w \bar{\rho} K S_{ij} (K^{3/2}/\epsilon x_2) \quad , \quad (12)$$

$$\Phi_{ij}^w = f_{w1} \left[ C_1 \bar{\rho} \frac{\epsilon}{K} \left( \tau_{ij} - \frac{2}{3} K \delta_{ij} \right) - \bar{\rho} \frac{\epsilon}{K} (\tau_{ik} n_k n_j + \tau_{jk} n_k n_i) + \alpha^* \left( P_{ij} - \frac{2}{3} \tilde{P} \delta_{ij} \right) \right] \quad , \quad (13)$$

$$\epsilon_{ij}^w = f_{w1} \left[ -\frac{2}{3} \bar{\rho} \epsilon \delta_{ij} + \bar{\rho} \frac{\epsilon}{K} \frac{(\tau_{ij} + \tau_{ik} n_k n_j + \tau_{jk} n_k n_i + n_i n_j \tau_{kl} n_k n_l)}{(1 + 3\tau_{kl} n_k n_l / 2K)} \right] \quad , \quad (14)$$

where  $P_{ij} = -(\bar{\rho} \tau_{ik} \tilde{U}_{j,k} + \bar{\rho} \tau_{jk} \tilde{U}_{i,k})$  represents turbulence production,  $K = \tau_{ij}/2$  is the turbulent kinetic energy,  $\tilde{P} = P_{ii}/2$ ,  $S_{ij} = \frac{1}{2}(\tilde{U}_{i,j} + \tilde{U}_{j,i})$  and  $D_{ij} = -(\bar{\rho} \tau_{ik} \tilde{U}_{k,j} + \bar{\rho} \tau_{jk} \tilde{U}_{k,i})$ . The damping function is defined as  $f_{w1} = \exp[-(Re_t/150)^2]$  while the turbulent Reynolds number is given by  $Re_t = K^2/\bar{\nu}\epsilon$ . Unit normal to the wall is denoted by  $n_i$  and the  $C$ 's are model constants whose values are chosen to be the same as those given by Zhang (1993). Furthermore, the model constants  $\alpha_1$ ,  $\beta_1$  and  $\gamma_1$ , are related to a single constant  $C_2$  and the relations are as given by Launder et al. (1975). For ease of reference later on, the second-order model with the term  $\Phi_{ij}^R$  included is referred to as LRR/WR, while the model with the term  $\Phi_{ij}^R$  excluded is denoted as LRR.

The dissipation rate  $\epsilon$  is decomposed into a solenoidal part and a compressible part so that  $\epsilon = \epsilon_s + \epsilon_c$ . These latter terms are defined as:  $\bar{\rho} \epsilon_s = \overline{\mu \omega_i' \omega_i'}$ ,  $\bar{\rho} \epsilon_c = \frac{4}{3} \overline{\mu (u_{i,i}')^2}$  and  $\omega_i'$  is the fluctuating vorticity. The solenoidal dissipation rate is associated with the energy cascade, therefore, it approaches its incompressible limit correctly. Consistent with Morkovin's hypothesis (1962), the compressible part of  $\epsilon$  is neglected in the present formulation and  $\epsilon$  is taken to be given by  $\epsilon_s$  alone. A modeled dissipation-rate equation similar to its incompressible counterpart can be written for compressible flows as:



$$\frac{\partial \bar{\rho} \varepsilon}{\partial t} + (\bar{\rho} \varepsilon \tilde{U}_k)_{,k} = (\bar{\mu} \varepsilon_{,k})_{,k} + \left( C_{\varepsilon} \bar{\rho} \frac{K}{\varepsilon} \tau_{ki} \varepsilon_{,i} \right)_{,k} + C_{\varepsilon 1} \frac{\varepsilon}{K} \tilde{P} - C_{\varepsilon 2} \frac{\bar{\rho} \varepsilon \tilde{\varepsilon}}{K} + \xi \quad (15)$$

The near-wall correcting function  $\xi$  of Zhang (1993) can be generalized for compressible flows to give

$$\xi = f_{w2} \bar{\rho} \left( -2 \frac{\varepsilon \tilde{\varepsilon}}{K} + 1.5 \frac{\tilde{\varepsilon}^2}{K} - 1.5 C_{\varepsilon 1} \frac{\varepsilon}{K} \tilde{P} \right) \quad (16)$$

In (16),  $\tilde{\varepsilon}$  and  $\bar{\varepsilon}$  are defined by  $\bar{\rho} \tilde{\varepsilon} = \bar{\rho} \varepsilon - 2\bar{\mu} (\partial \sqrt{K} / \partial x_2)^2$  and  $\bar{\rho} \bar{\varepsilon} = \bar{\rho} \varepsilon - 2\bar{\mu} K / x_2^2$ , respectively, and the damping function is given by  $f_{w2} = \exp[-(Re_t / 40)^2]$ . Again, the  $C$ 's are model constants and their values specified by Zhang (1993) are adopted.

For the sake of completeness, the model constants used are specified as  $C_1 = 1.5$ ,  $C_2 = 0.4$ ,  $C_{\varepsilon 1} = 1.5$ ,  $C_{\varepsilon 2} = 1.83$ ,  $C_s = 0.11$ ,  $C_\varepsilon = 0.1$ ,  $\alpha^* = 0.45$  and  $\alpha_1 = (8 + C_2)/11$ ,  $\beta_1 = (8C_2 - 2)/11$  and  $\gamma_1 = (30C_2 - 2)/55$ . The constant  $C_w$  is introduced as a result of "pressure echo" modeling in the velocity-pressure gradient correlation. For compressible flow calculations, a generally valid relation is given by Zhang et al. (1993) as:  $C_w = (C_w)_{in} - (5.8 \times 10^{-4})M_\infty$  for  $M_\infty > 2.5$ , where  $M_\infty$  is the free stream Mach number. For  $M_\infty \leq 2.5$ ,  $C_w = (C_w)_{in}$ , where  $(C_w)_{in}$  is given by:  $(C_w)_{in} = 4.14 \times 10^{-3} + 3 \times 10^{-3}(\log Re_\theta)$  for  $Re_\theta \leq 5.500$  and  $(C_w)_{in} = 0.0153$  for  $Re_\theta > 5,500$ . Here,  $Re_\theta$  is the momentum thickness Reynolds number. Finally, the boundary conditions for the mean and turbulent velocity field are given by:

$$\tilde{U} = \tilde{V} = K = \tau_{ij} = 0; \quad \varepsilon = 2 \bar{v}_w (\partial \sqrt{K} / \partial y)^2 \quad (17)$$

#### 4. Two-Equation Model for the Temperature Field

In the previous section, a near-wall second-order model is outlined for the velocity field. Since it is advisable to calculate turbulent heat transfer using a turbulence model of equal or lesser order, a two-equation model for the temperature field would be most appropriate for the present work. Therefore, gradient transport is assumed and the  $i$ th component of the turbulent heat flux is given by  $-Q_i = -\widetilde{u_i\theta} = \alpha_t \left( \partial \widetilde{\theta} / \partial x_i \right)$ , where  $\alpha_t$  is the eddy thermal diffusivity. Dimensionally,  $\alpha_t$  is the product of a velocity scale and a length scale. A characteristic velocity scale for turbulent flow is  $K^{1/2}$ . If the interactions between momentum and heat transport are to be modeled properly, an appropriate length scale would be one given by a combination of  $K^{1/2}$  and the time scales of both the thermal and velocity fields. The time scale characteristic of the thermal field can be evaluated from the temperature variance  $\widetilde{\theta^2}$  and its dissipation rate  $\varepsilon_\theta$ , while the time scale for the velocity field is given by  $K$  and its dissipation rate  $\varepsilon$ . In view of this, the simplest proposal for  $\alpha_t$  will be:

$$\alpha_t = C_\lambda f_\lambda K \left[ K \widetilde{\theta^2} / \varepsilon \varepsilon_\theta \right]^{1/2}, \quad (18)$$

where the velocity scale  $K^{1/2}$  is multiplied by the combined time scale  $\left[ K \widetilde{\theta^2} / \varepsilon \varepsilon_\theta \right]^{1/2}$  to give an appropriate length scale for the definition of  $\alpha_t$ ,  $C_\lambda$  is a model constant and  $f_\lambda$  is a damping function to be defined. A constant value of  $C_\lambda = 0.11$  has been put forward by Nagano and Kim (1988) and adopted by Sommer et al. (1992, 1993). Furthermore,  $C_\lambda = 0.10$  has been assumed by Nagano et al. (1991). In all these calculations, the 0.1 value is found to give good results for the flow cases tested. Therefore, it is prudent to assume a value not too different from 0.1. For the present,  $C_\lambda = 0.095$  is found to give the best results. As for  $f_\lambda$ , it has to be parametric in  $Pr$ , otherwise, the model cannot be applied to calculate heat transfer in fluids with vastly different  $Pr$ . In the following, a near-wall model for  $\alpha_t$  is first discussed, then the appropriate expression for  $f_\lambda$  is presented.

Since  $K$  and  $\varepsilon$  are defined by the solution of the modeled equations outlined in the previous section,  $\alpha_t$  can be determined by solving two equations governing the transport of  $\widetilde{\theta^2}$  and  $\varepsilon_\theta$ . For

incompressible flows, various modeled equations for  $\widetilde{\theta}^2$  and  $\varepsilon_\theta$  have been proposed (Launder 1976). These equations are formulated for high-Reynolds-number flows, therefore, they cannot be used consistently with other near-wall models. Several near-wall two-equation models have been put forward by Nagano and Kim (1988), Nagano et al. (1991), Sommer et al. (1992, 1993) and So and Sommer (1993). One of these models is asymptotically incorrect and gives a zero  $\varepsilon_\theta$  at the wall (Nagano and Kim 1988); therefore, it is not consistent with the behavior of the near-wall turbulence model described above. The other models give asymptotically correct results near a wall and are appropriate for the present application. With the exception of So and Sommer (1993), most of the models formulated to-date are, strictly speaking, valid for fluids with  $Pr \approx 1$ . If the compressible turbulence model for heat transfer is to be general enough for fluids with vastly different  $Pr$ , then the appropriate incompressible model to be adopted for extension to compressible flow is that proposed by So and Sommer (1993). Therefore, the present approach adopts that model as a base and proceeds to generalize it for supersonic turbulent flows.

Again, Morkovin's hypothesis (1962) is invoked in order to extend the incompressible model to compressible flows. Since the incompressible modeled equations for  $\widetilde{\theta}^2$  and  $\varepsilon_\theta$  have been given by So and Sommer (1993) and their extensions to compressible flows are straight forward, these equations can be written down without derivation as:

$$\bar{\rho} \frac{\partial \widetilde{\theta}^2}{\partial t} + \bar{\rho} \widetilde{U}_k \frac{\partial \widetilde{\theta}^2}{\partial x_k} = \frac{\partial}{\partial x_k} \left( \bar{\rho} \bar{\alpha} \frac{\partial \widetilde{\theta}^2}{\partial x_k} \right) + \frac{\partial}{\partial x_k} \left( C_{\theta^2} \bar{\rho} \tau_{kj} \frac{K}{\varepsilon} \frac{\partial \widetilde{\theta}^2}{\partial x_j} \right) - 2\bar{\rho} Q_k \frac{\partial \widetilde{\theta}}{\partial x_k} - 2\bar{\rho} \varepsilon_\theta, \quad (19)$$

$$\begin{aligned} \bar{\rho} \frac{\partial \varepsilon_\theta}{\partial t} + \bar{\rho} \widetilde{U}_k \frac{\partial \varepsilon_\theta}{\partial x_k} &= \frac{\partial}{\partial x_k} \left( \bar{\rho} \bar{\alpha} \frac{\partial \varepsilon_\theta}{\partial x_k} \right) + \frac{\partial}{\partial x_k} \left( C_{\varepsilon\theta} \bar{\rho} \tau_{kj} \frac{K}{\varepsilon} \frac{\partial \varepsilon_\theta}{\partial x_j} \right) + C_{d1} \frac{\varepsilon_\theta}{\theta^2} P_\theta + C_{d2} \frac{\varepsilon}{K} P_\theta \\ &+ C_{d3} \frac{\varepsilon_\theta}{K} P_w - C_{d4} \frac{\widetilde{\varepsilon}_\theta}{\theta^2} \bar{\rho} \varepsilon_\theta - C_{d5} \frac{\widetilde{\varepsilon}_\theta}{K} \bar{\rho} \varepsilon_\theta + \xi_{\varepsilon\theta}, \end{aligned} \quad (20)$$

where the effects of both thermal ( $\widetilde{\theta}^2 / \varepsilon_\theta$ ) and velocity ( $K/\varepsilon$ ) time scale on the dissipation of  $\widetilde{\theta}^2$  are modeled into the  $\varepsilon_\theta$  equation. In these equations,  $\bar{\alpha}$  is the mean thermal diffusivity  $P_\theta = -\bar{\rho} Q_k (\partial \widetilde{\theta} / \partial x_k)$  is turbulence production due to mean temperature gradients,  $\xi_{\varepsilon\theta}$  is a near-

wall correcting function,  $\tilde{\varepsilon}_\theta = \varepsilon_\theta - \bar{\alpha} \left( \partial \sqrt{\theta^2} / \partial x_2 \right)^2$  and the  $C$ 's are model constants to be defined later.

If the proposed model is to approach the high-Reynolds-number limit correctly, the model constants in (19) and (20) cannot differ from conventional values adopted by other researchers. A generally acceptable set of constants for the  $C$ 's are given by:  $C_\theta^2 = 0.11$ ,  $C_{\varepsilon\theta} = 0.11$ ,  $C_{d1} = 1.8$ ,  $C_{d2} = 0$ ,  $C_{d3} = 0.72$ ,  $C_{d4} = 2.2$  and  $C_{d5} = 0.8$ . In other words, the near-wall correcting function  $\xi_{\varepsilon\theta}$  has to be determined so that (19) and (20) would approach their high-Reynolds-number counterparts correctly, i.e.  $\xi_{\varepsilon\theta}$  would asymptote to zero away from a wall. When these constraints are used to derived  $\xi_{\varepsilon\theta}$  correct to order  $x_2$  near a wall, the following expression is obtained:

$$\xi_{\varepsilon\theta} = f_{w,\varepsilon\theta} \bar{\rho} \left[ (C_{d4} - 4) \frac{\varepsilon_\theta}{\theta^2} \tilde{\varepsilon}_\theta + C_{d5} \frac{\tilde{\varepsilon}_\theta}{K} \varepsilon_\theta - \frac{\varepsilon_\theta^{*2}}{\theta^2} + (2 - C_{d1} - C_{d2} Pr) \frac{\varepsilon_\theta}{\theta^2} P_\theta^* \right] , \quad (21)$$

where  $P_\theta^*$  is the mean production term in  $P_\theta$  due to  $\partial \tilde{\Theta} / \partial x_1$  alone and  $\varepsilon_\theta^* = \varepsilon_\theta - \bar{\alpha} \tilde{\theta}^2 / x_2^2$ . The presence of  $P_\theta^*$  is a consequence of the constant wall heat flux boundary condition where  $\partial \tilde{\Theta} / \partial x_1$  is finite. Therefore, the near-wall correcting function is valid for all thermal wall boundary conditions. A damping function  $f_{w,\varepsilon\theta} = \exp[-(Re_t/80)^2]$  is introduced to ensure that the contribution of  $\xi_{\varepsilon\theta}$  would vanish away from the wall. Thus formulated, the model has no new constants compared to its high-Reynolds-number counterpart.

With the exception of Nagano and Kim (1988), the various damping functions proposed for  $f_\lambda$  satisfy the requirement that  $\alpha_t$  behave like  $x_2^3$  as a wall is approached. This is consistent with the exact near-wall behavior of the normal heat flux. If, in addition, the model is to work well with flows having different Pr, then  $f_\lambda$  has to be parametric in Pr. When these requirements are used to deduce an expression for  $f_\lambda$ , the following is obtained (So and Sommer 1993):

$$f_\lambda = C_{\lambda 1} (1 - f_{\lambda 1}) / Re_t^{1/4} + f_{\lambda 1} , \quad (22)$$

where  $C_{\lambda I}$  is a model constant taken to be parametric in  $Pr$ , the damping function  $f_{\lambda I}$  is defined by  $f_{\lambda I} = [1 - \exp(-x_2^+ / A^+)]^2$ ,  $x_2^+ = x_2 u_\tau / \bar{v}$  is a normal coordinate made dimensionless by the local length scale  $\bar{v} / u_\tau$  and the model constant  $A^+$  is also assumed to be parametric in  $Pr$ . The friction velocity  $u_\tau$  is defined as  $(\tau_w / \bar{\rho})^{1/2}$ . Typically, for flows with  $Pr \approx 1$ , the values for  $C_{\lambda I}$  and  $A^+$  are 0.1 and 40, respectively. Their variations with  $Pr$  have been determined by So and Sommer (1993) and are given by:  $A^+ = 10/Pr$  for  $Pr < 0.25$  and  $A^+ = 39/Pr^{1/6}$  for  $Pr \geq 0.25$ ;  $C_{\lambda I} = 0.4/Pr^{1/4}$  for  $Pr < 0.1$  and  $C_{\lambda I} = 0.07/Pr$  for  $Pr \geq 0.1$ .

Finally, the wall boundary conditions for the temperature field and  $\tilde{\theta}^2$  and  $\epsilon_\theta$  can be stated as follows. At the wall, the thermal boundary conditions can either be adiabatic or constant temperature, while  $\tilde{\theta}^2 = 0$  and  $(\epsilon_\theta)_w = \bar{\alpha} \left( \partial \sqrt{\tilde{\theta}^2} / \partial x_2 \right)^2$  are appropriate for both thermal wall boundary conditions.

## 5. Results and Discussion

The ability of the near-wall models to calculate heat transfer in fluids with widely different  $Pr$  is illustrated first. In these calculations, fully-developed pipe and channel flows with adiabatic and constant wall temperature boundary conditions are considered. Therefore, the governing equations outlined above can be reduced to ordinary differential equations and solved fairly easily by some standard numerical techniques, such as Newton iteration or relaxation methods. Both high and low Reynolds-number flows are calculated and compared with measurements and direct numerical simulation (DNS) data. Some sample comparisons are shown here. These are the comparisons with the empirical temperature log-law of Kader (1981) and with the DNS data of Kim and Moin (1989).

In Kader's (1981) study, an empirical temperature log-law is proposed after careful analysis of numerous temperature measurements in pipe and channel flows with widely different  $Pr$ . The resultant log-law is found to correlate well with measurements over a broad range of Reynolds number and  $Pr$ . Since a more careful comparison with this empirical log-law has been given by So and Sommer (1993), only a representative comparison in the  $Pr$  range  $10 < Pr < 100$  is shown in Fig. 1. In this figure,  $\Theta^+ = \bar{\Theta} / \Theta_\tau$ ,  $y^+ = y u_\tau / \nu$ ,  $y$  is the normal coordinate to the wall and  $\Theta_\tau$  is the friction temperature. The model calculations are in very good agreement with Kader's temperature log-law. Such good agreement extends to  $Pr$  as low as 0.025 and as high as  $10^3$ . A sample comparison with low-Reynolds-number channel flows is carried out with the constant wall temperature DNS data of Kim and Moin (1989). The Reynolds number based on  $u_\tau$  is 180 and three different values of  $Pr$  are investigated. These are:  $Pr = 0.1, 0.71$  and  $2$ . The velocity field comparisons have been given by So and Sommer (1993) and the results are in good agreement with DNS data. Here, only the comparisons with  $\Theta^+$  and  $\theta_{rms} = (\overline{\theta^2})^{1/2} / \Theta_\tau$ , the root-mean square temperature variance, are shown in Figs. 2 and 3, respectively. It can be seen that the calculated mean temperature profiles agree well with DNS data and their dependence on  $Pr$  is modeled correctly (Fig. 2). A slight discrepancy exists in the prediction of the maximum  $\theta_{rms}$  and

this is most notable for the case where  $Pr = 2$  (Fig. 3). In general, the heat transfer model gives good predictions of incompressible flows with widely different  $Pr$ .

The compressible flow calculations are compared with the measurements of Fernholz and Finley (1977) and Kussoy and Horstman (1991) and the constant  $Pr_t$  model of Zhang et al. (1993). This way, the validity of the constant  $Pr_t$  assumption for compressible flows can be examined. Measurements obtained under both adiabatic and cooled wall boundary conditions are considered. In this preliminary attempt, all calculated cases are limited to flat plate boundary layers only, while attempts to calculate complex supersonic flows will be discussed in a subsequent report. Two adiabatic cases are chosen from Fernholz and Finley (1977). The free-stream Mach numbers of these two cases are  $M_\infty = 2.244$  and  $10.31$ , while the corresponding momentum thickness Reynolds number,  $R_\theta$ , are  $20,797$  and  $15,074$ , respectively. In addition to mean velocity, wall friction is also reported. Therefore, the comparisons are made with the mean velocity profiles in semi-log plots of  $U^+$  versus  $\ln y_w^+$ , where  $U^+ = \tilde{U}/u_\tau$  and  $y_w^+ = y u_\tau / \bar{v}_w$ . Calculations are made of the skin friction coefficient,  $C_f = 2\tau_w / \bar{\rho}_\infty U_\infty^2$  and the heat transfer coefficient,  $C_h = q_w / \bar{\rho}_\infty U_\infty \bar{C}_p (\Theta_w - \Theta_{aw})$ , where  $q_w$  is the wall heat flux. Furthermore, comparisons are made with the mean temperature profiles in the form of  $\tilde{\Theta} / \Theta_\infty$  versus  $y / \delta$ , where  $\Theta_\infty$  is the freestream temperature and  $\delta$  is the measured boundary-layer thickness. However, these are not independent comparisons because the temperature profiles are inferred from the measured velocity profiles by assuming constant total enthalpy and pressure across the boundary layers. In order to verify the present heat transfer model, comparisons with supersonic cooled wall boundary layers are carried out. Only one case is presented and this has a wall temperature ratio  $\Theta_w / \Theta_{aw} = 0.3$ . The data is extracted from Kussoy and Horstman (1991). The corresponding  $M_\infty$  and  $R_\theta$  are  $3,939$  and  $4,600$ , respectively. Since the measurements of velocity and temperature are obtained independently, the performance of the compressible heat transfer model can be evaluated. Again, the skin friction coefficient and the heat transfer coefficient are calculated and compared with measurements.

Comparisons with the cases where an adiabatic wall boundary condition is specified are presented first. These results are shown in Figs. 4 - 7. Only the mean velocity (Figs. 4 and 6) and mean temperature (Figs. 5 and 7) profiles are compared. The measured velocity profiles can be correlated by the log-law (Zhang et al. 1992a)

$$U^+ = \frac{1}{\kappa} \ln y^+ + B \quad (23)$$

where  $\kappa$  is the von Karman constant. At low Mach numbers, the log-law with  $\kappa = 0.41$  and  $B = 4.3$  correlates well with data and model calculations (see log-law plotted in Fig. 4). As Mach number increases,  $B$  decreases with Mach numbers and at  $M_\infty = 10.31$ , the  $B$  value determined from the present model is 3.35 while that from the constant  $Pr_t$  model is 3.8. Both models give the same von Karman constant, i.e.  $\kappa = 0.41$ . However, the present model yields a longer range log-law compared to the constant  $Pr_t$  model and is more consistent with measurements (see the log-laws plotted in Fig. 6). Therefore, the  $B$  value thus determined is more reliable (Fig. 6). As for the temperature comparison (Figs. 5 and 7), again, there is little difference between the two model predictions at low Mach numbers. At  $M_\infty = 10.31$ , there is a slight discrepancy between the present model and that of the constant  $Pr_t$  model in the region bounded by  $0.1 < y/\delta < 0.5$ . It seems that the present model over-predicts the mean temperature in this region. Since the mean temperature data are inferred from the mean velocity measurements and the assumption of constant total enthalpy, it follows that a high measured velocity would lead to a low temperature estimation. The measured velocities seem to be high compared to the log-law in this region, therefore, the inferred temperatures are low. Measurements of  $C_f \times 10^{-3}$  for the  $M_\infty = 2.244$  and 10.31 cases are 1.62 and 0.24, respectively. The corresponding values determined from the constant  $Pr_t$  model of Zhang et al. (1993) are 1.69 and 0.24, while the present model gives 1.71 and 0.24, respectively. As far as the prediction of  $C_f$  is concerned, both models are quite good. In other words, the integral boundary-layer parameters are not as much affected by the model used to calculate the flow.



The cooled wall results are compared in Figs. 8 and 9. It has been demonstrated by Zhang et al. (1992) that the von Karman constant changes as the wall temperature ratio decreases. Furthermore,  $B$  also varies as  $\Theta_w/\Theta_{aw}$  decreases. The same behavior is predicted by the constant  $Pr_t$  model and the present model. When  $\Theta_w/\Theta_{aw}$  decreases to 0.3, the  $\kappa$  and  $B$  values determined from the model calculations are: 0.29 and 2.87 from the present model and 0.29 and 3.74 from the constant  $Pr_t$  model (see the log-laws plotted in Fig. 8), respectively. It is difficult to say which set of values agrees better with measurements. However, as before, the present model yields a longer range of log-law compared to the constant  $Pr_t$  model and is consistent with the measurements. In view of this, it can be said that the set of values determined from the present model is more reliable. There is little difference in the predicted mean temperature at low values of  $\Theta_w/\Theta_{aw}$ . As  $\Theta_w/\Theta_{aw}$  decreases to 0.3, a slight discrepancy exists between the model predictions in the region bounded by  $0.1 < y/\delta < 0.5$  (Fig. 9), which is the same as observed in the  $M_\infty = 10.31$  case with an adiabatic wall boundary condition. This time, the mean temperatures are measured independently and they seem to agree better with the predictions of the constant  $Pr_t$  model. The measured  $C_f \times 10^{-3}$  and  $C_h \times 10^{-3}$  for the  $M_\infty = 8.18$  case are 0.98 and 0.53, respectively, while the corresponding calculated values are 0.95 and 0.57 from the constant  $Pr_t$  model and 0.99 and 0.60 from the present model. It can be seen that there is an improvement in the prediction of  $C_f$  but a deterioration in the calculation of  $C_h$  when the present model is used to simulate the temperature field. However, according to Kussoy and Horstman (1991), the measurement of  $C_h$  is not as accurate as that of  $C_f$ . Therefore, it might turn out that there is no deterioration in the prediction of  $C_h$  after all.

Finally, some sample plots of the temperature variance  $\widetilde{\theta}^2$  and turbulent Prandtl number  $Pr_t$  are shown in Figs. 10 and 11, respectively. The root mean square temperature variance normalized by  $(\Theta_w - \Theta_\infty)$  is shown in Fig. 10. It can be seen that, as  $M_\infty$  increases, the maximum  $\widetilde{\theta}^2$  increases. The same is true when  $\Theta_w/\Theta_{aw}$  decreases; however, the increase is substantially larger than that due to Mach number enhancement. Therefore, these results indicate that temperature fluctuations are promoted by compressibility and most significantly by wall cooling.

The calculated  $Pr_t$  is not constant across the boundary layers (Fig. 11). For an adiabatic wall boundary condition, the calculated  $Pr_t$  reaches a maximum of about 2 at the wall and decreases rapidly to about 1.5 in the region  $4 < y^+ < 8$ . Thereafter,  $Pr_t$  continues to decrease towards the edge of the boundary layer. Essentially the same trend is followed by the  $M_\infty = 2.244$  case, except that the level is lower. On the other hand,  $Pr_t$  has a value of about 1.1 at the wall for the cooled wall case. Its value decreases to a minimum at  $y^+ = 6$  and then rises to a maximum of about 1.7 at  $y^+ = 20$ . At the edge of the boundary layer, the value of  $Pr_t$  is about 1.0, which is substantially higher than the values attained in the adiabatic wall cases. The constant  $Pr_t$  model calculations show that, irrespective of the fact that  $Pr_t$  varies significantly across the boundary layers, a constant value of 0.9 yields mean flow results that are as good as the present model. As for the turbulence statistics, no reliable data is available for comparisons. Therefore, the merit of the present model versus that of constant  $Pr_t$  cannot be commented on in this work.

## 6. Conclusions

A two-equation turbulence model for the temperature field is proposed for supersonic flows. The model equations are derived directly from their incompressible counterparts by invoking Morkovin's hypothesis, where it is postulated that compressibility effects could be accounted for by the variations of mean density alone. An eddy thermal diffusivity is assumed and it is determined from the temperature variance and its dissipation rate, whose transport equations are modeled and solved in the present approach. The eddy thermal diffusivity is taken to be the product of a turbulence velocity scale and a length scale. It is further assumed that the turbulence velocity scale and an appropriately defined time scale can be used to define the length scale. Both thermal and velocity time scales are used to determine the appropriate time scale. This is necessary because the interactions of the velocity and temperature fields have to be accounted for properly. The present approach invokes a second-order compressible turbulence model for the velocity field. The model equations are applied to study supersonic flows with freestream Mach numbers and wall temperature ratios that vary from 2.244 to 10.31 and 0.3 to 1.0, respectively. In calculating these supersonic flows, real gas effects are neglected. The calculated results are compared with measurements covering the same range of Mach numbers and wall temperature ratios. A similar model assuming the turbulent Prandtl number to be 0.9 is used to calculate the test cases and the results compared with the variable  $Pr_t$  model calculations and measurements.

The two model calculations yield results that are in good agreement with measurements. One possible difference is in the prediction of the range of the log-law. The present model predicts a longer range for all test cases examined compared to the constant  $Pr_t$  model. In the test cases studied, the longer log-law range seems to be more consistent with measurements. Thus compared, the constant  $Pr_t$  assumption is found to be valid for the range of Mach numbers and wall temperature ratios investigated. On the other hand, compared to the constant  $Pr_t$  calculations, the present model over-predicts slightly the mean temperature in the range  $0 < y/\delta < 0.5$  at high Mach numbers and low wall temperature ratios. Consequently, the relative merits of the present

model and the constant  $Pr_t$  model have to be further analysed by comparing the calculations with accurately measured turbulence statistics, which are presently lacking.

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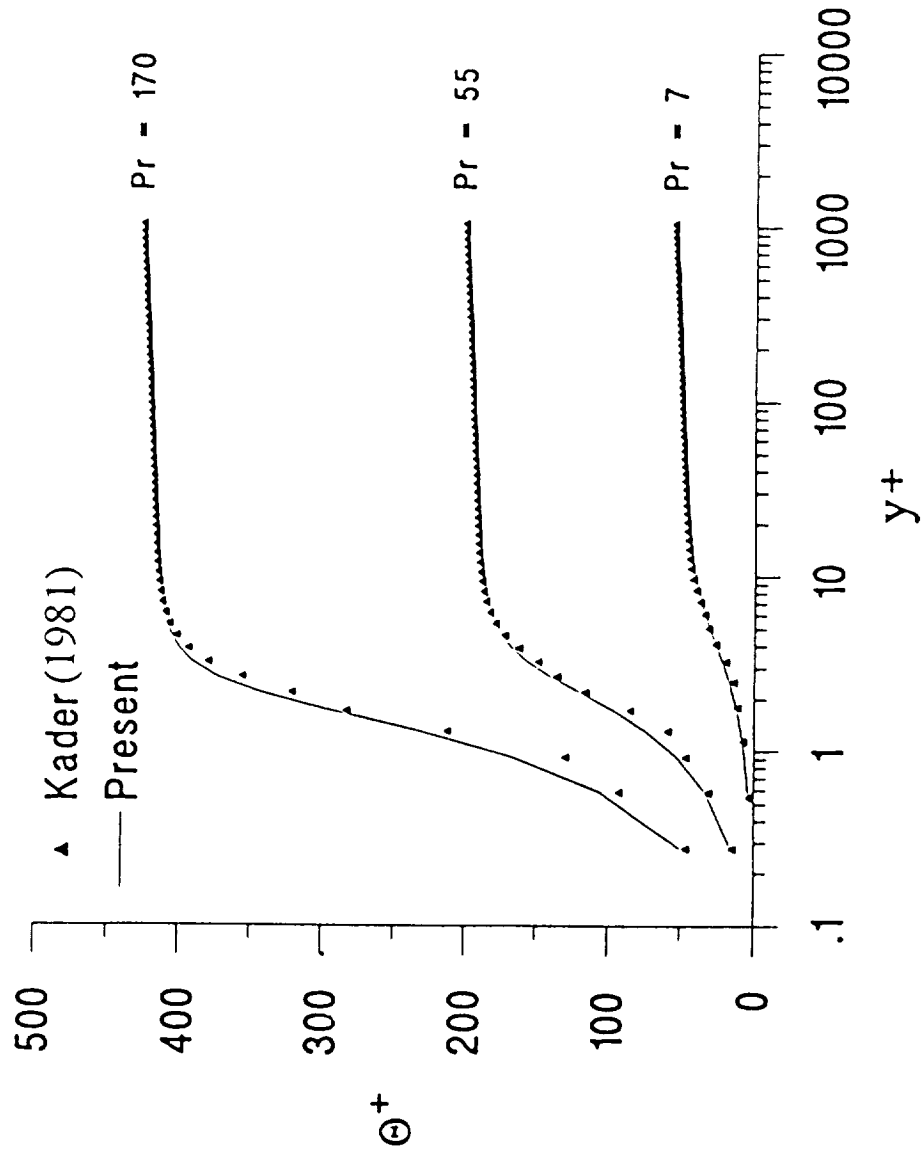


Figure 1. Comparison of calculated and measured  $\Theta^+$  for incompressible flows with different  $Pr$ .

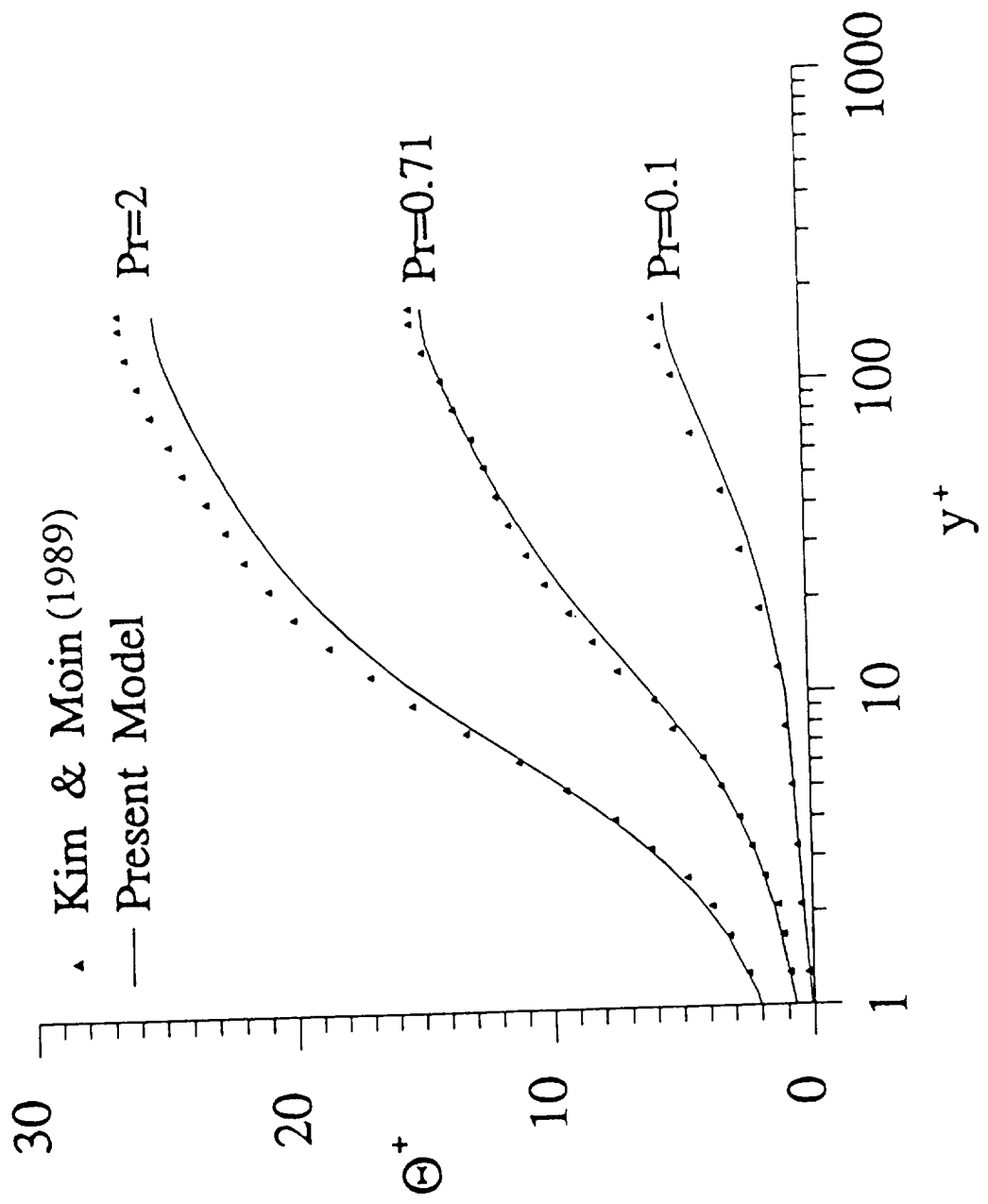


Figure 2. Comparison of calculated  $\Theta^+$  with direct simulation data for three values of  $Pr$ .



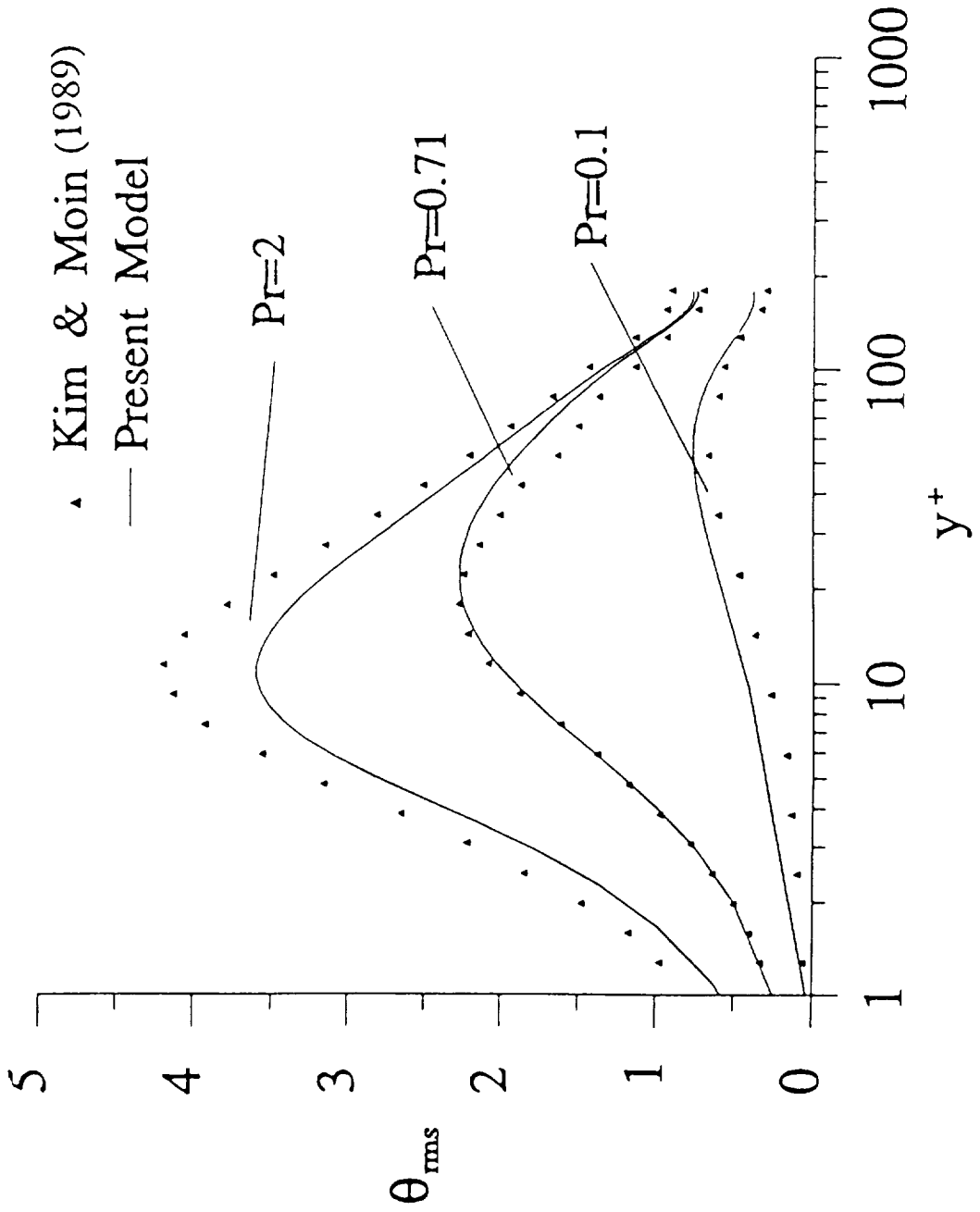


Figure 3. Comparison of calculated  $\theta_{rms}$  with direct simulation data for three values of Pr.

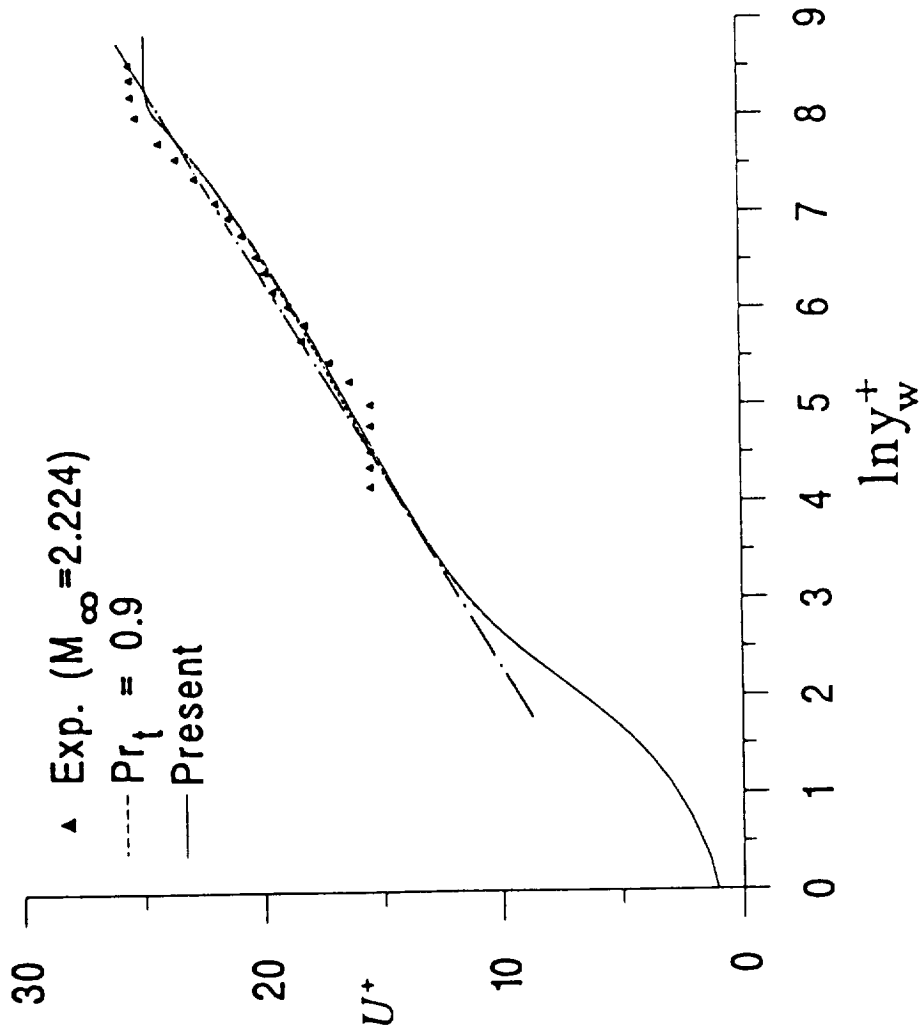


Figure 4. Comparison of calculated and measured  $U^+$  for the  $M_\infty = 2.244$  case.

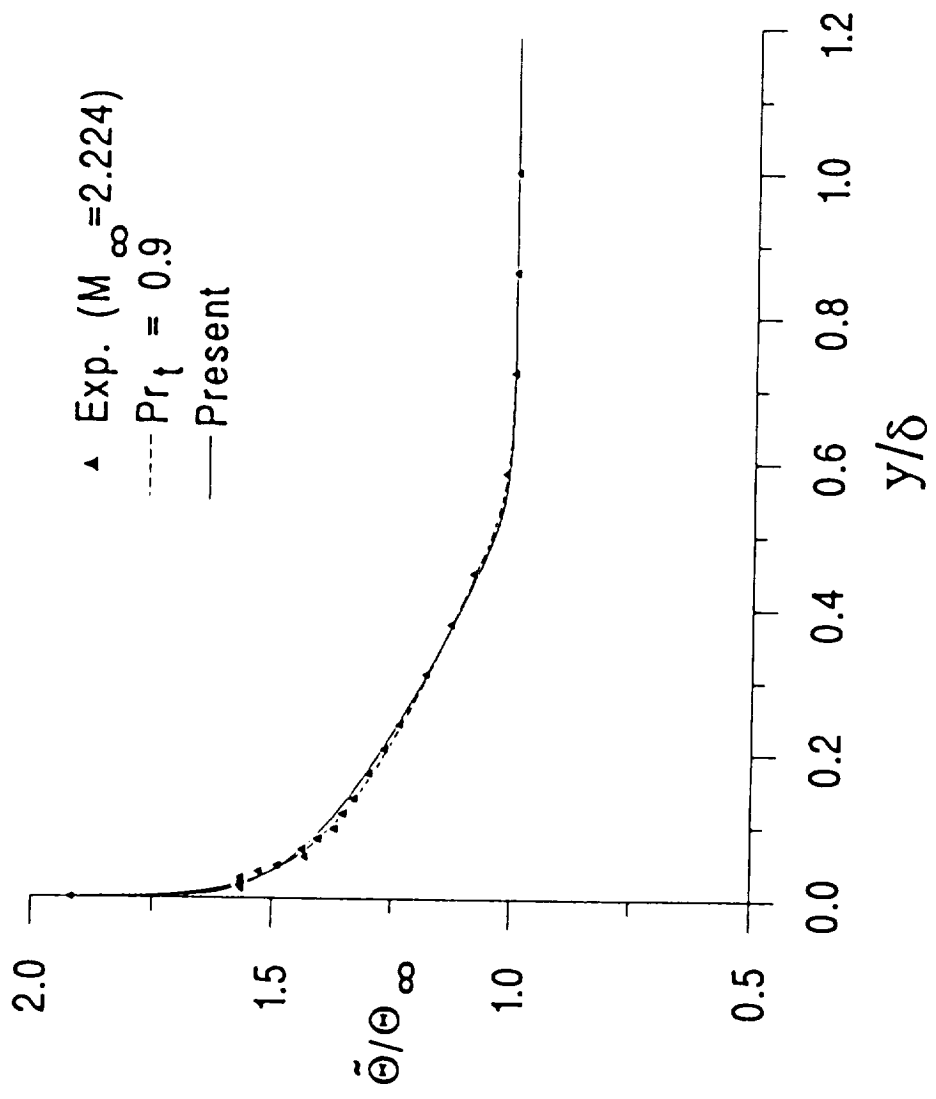


Figure 5. Comparison of calculated and measured  $\tilde{\Theta}/\Theta_{\infty}$  for the  $M_{\infty} = 2.244$  case.

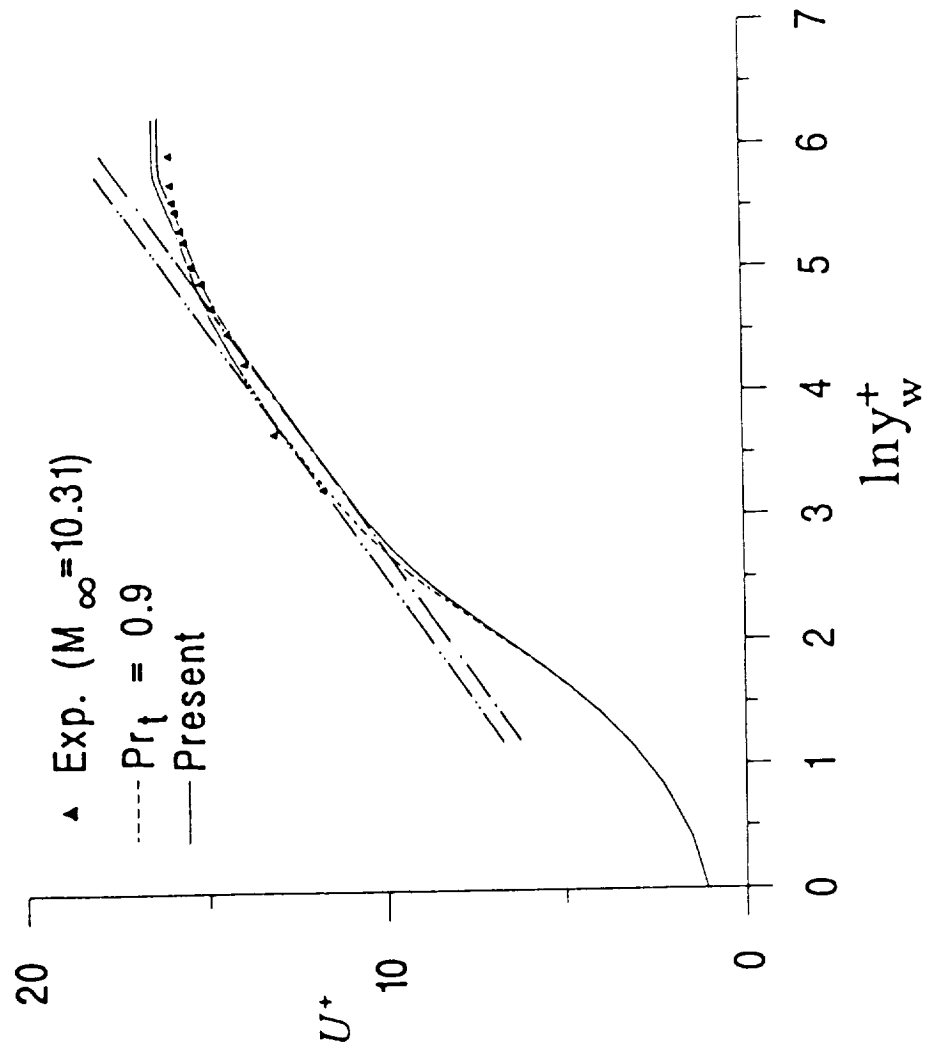


Figure 6. Comparison of calculated and measured  $U^+$  for the  $M_\infty = 10.31$  case.

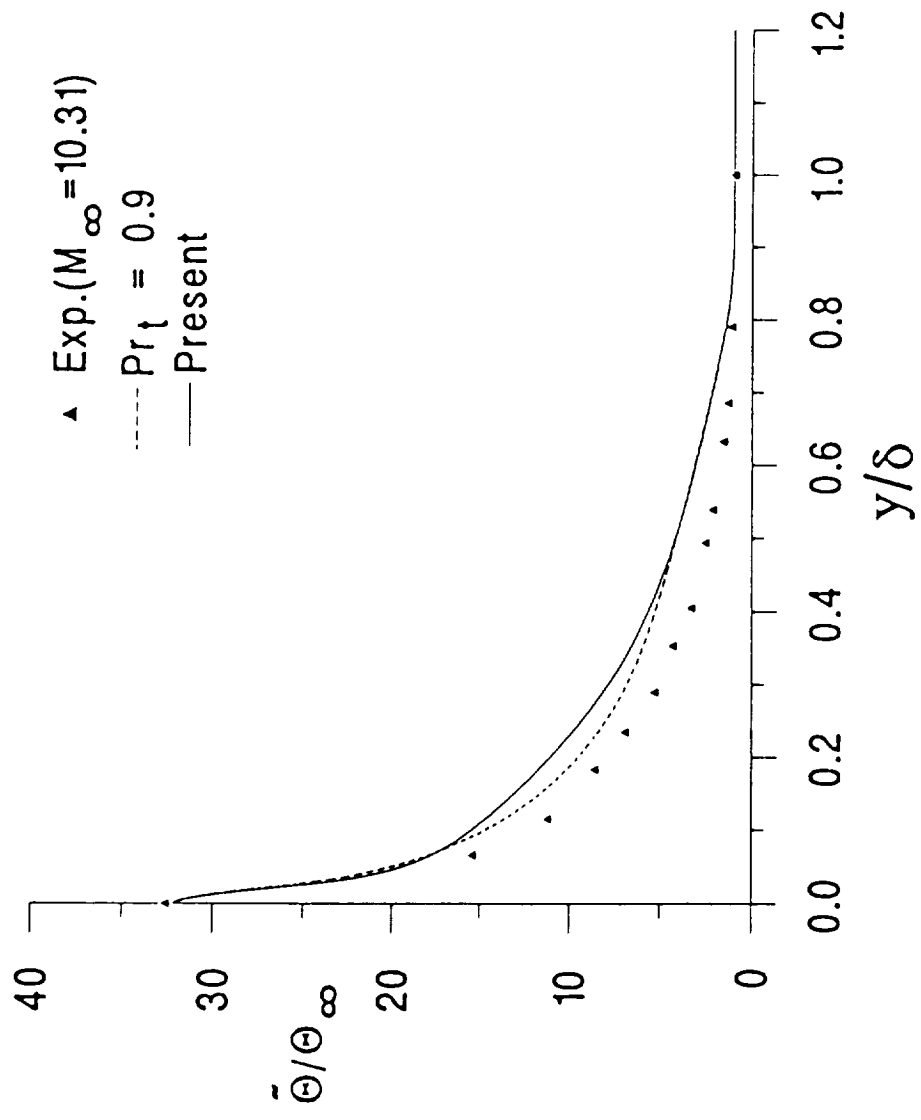


Figure 7. Comparison of calculated and measured  $\tilde{\Theta}/\Theta_\infty$  for the  $M_\infty = 10.31$  case.

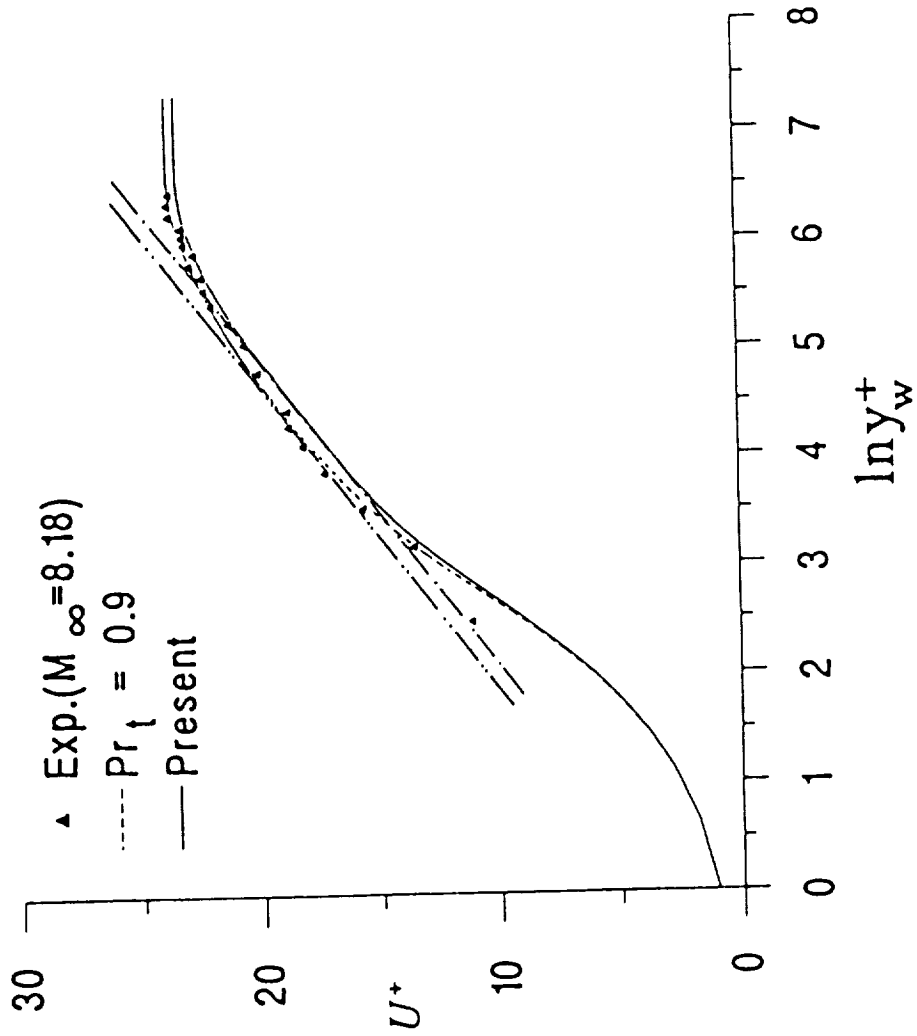


Figure 8. Comparison of calculated and measured  $U^+$  for the  $M_\infty = 8.18$  case.

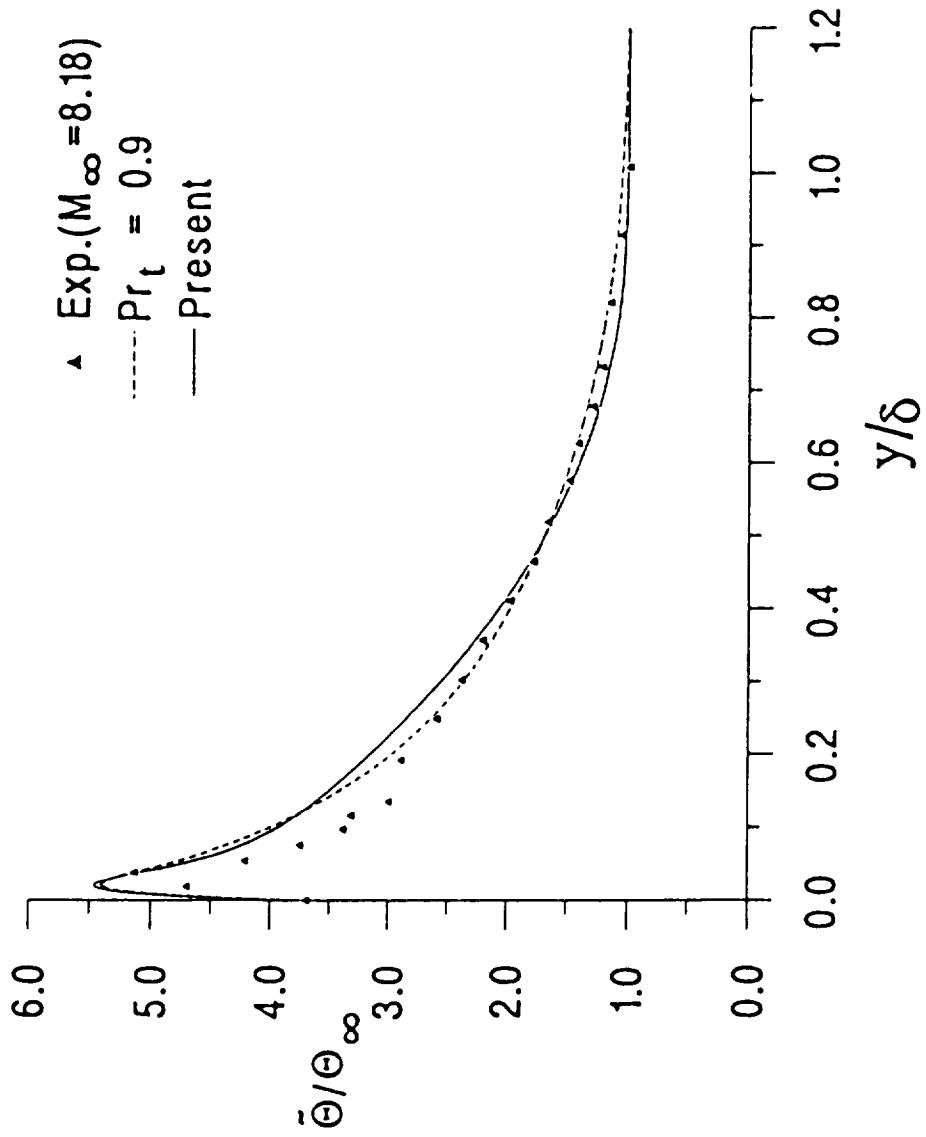


Figure 9. Comparison of calculated and measured  $\bar{\Theta}/\Theta_\infty$  for the  $M_\infty = 8.18$  case.

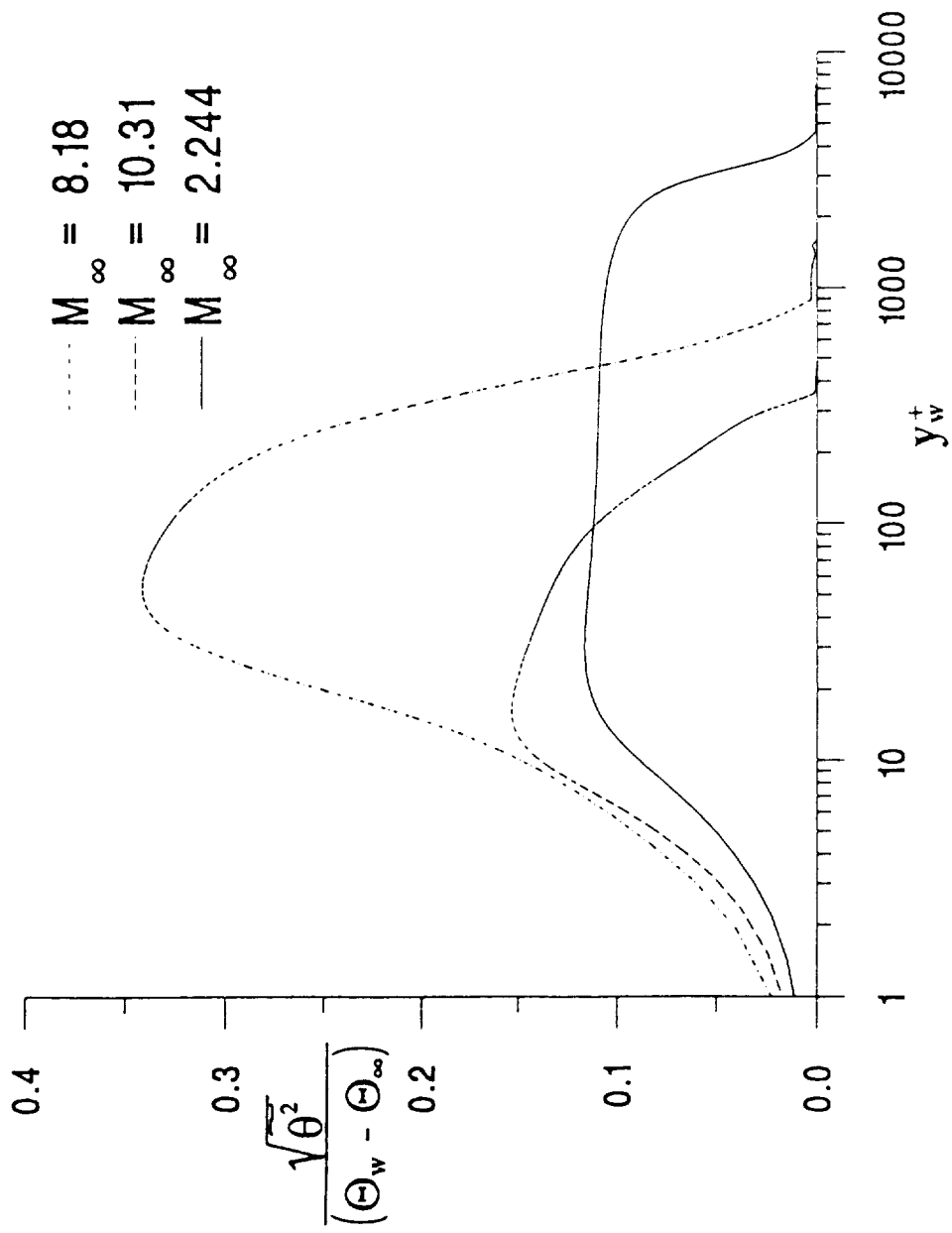


Figure 10. Distributions of  $\sqrt{\theta^2} / (\Theta_w - \Theta_\infty)$  across the boundary layers for both adiabatic and cooled wall boundary conditions.



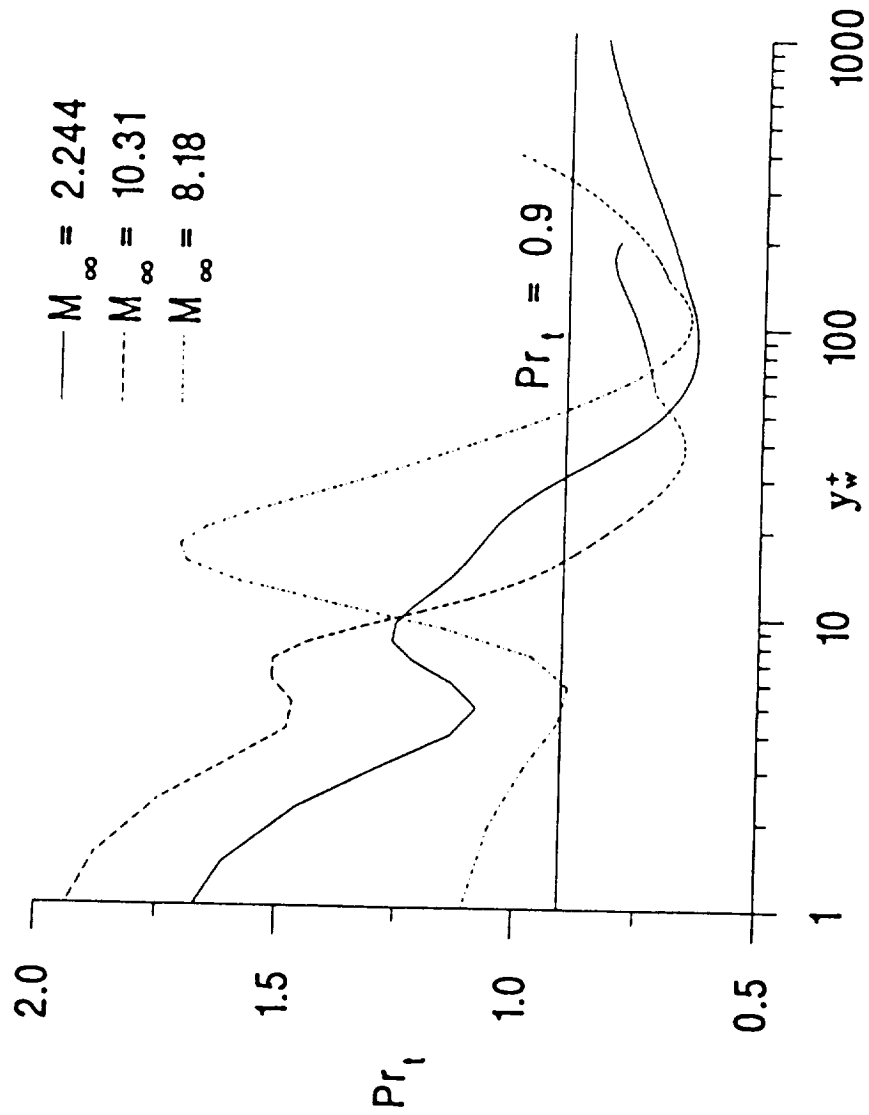


Figure 11. Distributions of  $Pr_t$  across the boundary layers for both adiabatic and cooled wall boundary conditions.

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