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ATTITUDE STABILIZATION OF A RIGID SPACECRAFT USING GAS JET ACTUATORS OPERATING IN A FAILURE MODE*

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Abstract

We consider the attitude stabilization of a rigid spacecraft using control torques supplied by gas jet actuators about only two of its principal axes. First, the case where the uncontrolled principal axis of the spacecraft is not an axis of symmetry is considered. In this case, the complete spacecraft dynamics are small time locally controllable. However, the spacecraft cannot be asymptotically stabilized to an equilibrium attitude using time-invariant continuous feedback. A discontinuous stabilizing feedback control strategy is constructed which stabilizes the spacecraft to an equilibrium attitude. Next, the case where the uncontrolled principal axis of the spacecraft is an axis of symmetry is considered. In this case, the complete spacecraft dynamics are not even accessible. However, the spacecraft dynamics are strongly accessible and small time locally controllable in a reduced sense. The reduced spacecraft dynamics cannot be asymptotically stabilized to an equilibrium attitude using time-invariant continuous feedback, but again a discontinuous stabilizing feedback control strategy is constructed. In both cases, the discontinuous feedback controllers are constructed by switching between one of several feedback functions.

1. Introduction

We consider the attitude stabilization of a rigid spacecraft using control torques supplied by gas jet actuators about only two of its principal axes. A rigid spacecraft in general is controlled by three independent actuators about its principal axes. The situation considered here may arise due to the failure of one of the actuators. The linearization of the complete spacecraft dynamic equations at any equilibrium attitude has an uncontrollable eigenvalue at the origin. Consequently, controllability and stabilizability properties of the spacecraft cannot be inferred using classical linearization ideas and requires inherently nonlinear analysis. Moreover, a linear feedback control law cannot be used to asymptotically stabilize the spacecraft to an equilibrium attitude. An analysis of the controllability properties of a spacecraft with two independent control torques is made in [7]. In [7] it is shown that a necessary and sufficient condition for complete controllability of a spacecraft with control torques supplied by gas jet actuators about only two of its principal axes is that the uncontrolled principal axis must not be an axis of symmetry of the spacecraft. In [6], it is shown that a rigid spacecraft controlled by two pairs of gas jet actuators about its principal axes cannot be asymptotically stabilized to an equilibrium attitude using a time-invariant continuously differentiable, i.e., C^1, feedback control law. Moreover, using some of the theoretical results in [9] and [12], it also follows that there does not exist any time-invariant continuous feedback control law which asymptotically stabilizes the spacecraft to an equilibrium attitude. However a smooth C^1 feedback control law is derived in [6] which locally asymptotically stabilizes the spacecraft to a circular attractor, rather than an isolated equilibrium.

We first consider the case where the uncontrolled principal axis of the spacecraft is not an axis of symmetry. In this case, the complete spacecraft dynamics are small time locally controllable at any equilibrium attitude. However, as stated earlier, the spacecraft cannot be asymptotically stabilized to any equilibrium attitude using a time-invariant continuous feedback control law. Using local controllability results, an algorithm which locally asymptotically stabilizes the spacecraft to an isolated equilibrium is proposed in [7]. That algorithm is extremely complicated and is based on Lie algebraic methods in [8]. The algorithm yields a piecewise constant discontinuous control. Although very complicated, the algorithm is the only one proposed in the literature thus far which locally asymptotically stabilizes the spacecraft attitude to an equilibrium. In this paper a new discontinuous stabilizing feedback control strategy is constructed which stabilizes the spacecraft to an equilibrium attitude. The control strategy is simple and is based on physical considerations of the problem.

We next consider the case where the uncontrolled principal axis of the spacecraft is an axis of symmetry. In this case, the complete spacecraft dynamics are not even accessible. Under some rather weak assumptions, the spacecraft dynamic equations are strongly accessible and small time locally controllable at any equilibrium attitude in a reduced sense. The reduced spacecraft dynamics cannot be asymptotically stabilized to an equilibrium attitude using time-invariant continuous feedback. Nevertheless, a discontinuous feedback control strategy is constructed which achieves attitude stabilization of the spacecraft.

2. Kinematic and Dynamic Equations

The orientation of a rigid spacecraft can be specified using various parametrizations of the special orthogonal group $SO(3)$. Here we use the following Euler angle convention. Consider an inertial $X_1 X_2 X_3$ coordinate frame; let $x_1 x_2 x_3$ be a coordinate frame aligned with the principal axes of the spacecraft with origin at the center of mass of the spacecraft. If the two frames are initially coincident, a series of three rotations about the body axes, performed in the proper sequence, is sufficient to allow the spacecraft to reach any orientation. The three rotations are [14]:

- a positive rotation of frame $X_1 X_2 X_3$ by an angle ψ about the X_3 axis; let $x_1' x_2' x_3'$ denote the resulting coordinate frame;
- a positive rotation of frame $x_1' x_2' x_3'$ by an angle θ about the x_2' axis; let $x_1'' x_2'' x_3''$ denote the resulting frame;
- a positive rotation of frame $x_1'' x_2'' x_3''$ by an angle ϕ about the x_1'' axis; let $x_1 x_2 x_3$ denote the final coordinate frame.

A rotation matrix $R(\psi,\theta,\phi)$ relates components of a vector in the inertial frame to components of the same vector in the body frame [14]. We assume that the Euler angles are limited to the ranges

$$-\pi < \psi, \phi < \pi, \ -\pi/2 < \theta < \pi/2 . \tag{2.1}$$

Suppose ω_1, ω_2, ω_3 are the principal axis components of the absolute angular velocity vector ω of the spacecraft. Then expressions for ω_1, ω_2, ω_3 are given by

$$\omega_1 = \dot{\phi} - \dot{\psi}\sin\theta , \tag{2.2}$$

$$\omega_2 = \dot{\theta}\cos\phi + \dot{\psi}\cos\theta \sin\phi , \tag{2.3}$$

$$\omega_3 = -\dot{\theta}\sin\phi + \dot{\psi}\cos\theta \cos\phi . \tag{2.4}$$

Since these equations are invertible, we can solve for $\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$ in terms of ω_1, ω_2, ω_3 obtaining

$$\dot{\phi} = \omega_1 + \omega_2 \sin\phi \tan\theta + \omega_3 \cos\phi \tan\theta , \tag{2.5}$$

$$\dot{\theta} = \omega_2 \cos\phi - \omega_3 \sin\phi , \tag{2.6}$$

$$\dot{\psi} = \omega_2 \sin\phi \sec\theta + \omega_3 \cos\phi \sec\theta . \tag{2.7}$$

Next we consider the dynamic equations which describe the evolution of the angular velocity components of the spacecraft. Let $J = \mathrm{diag}(J_1, J_2, J_3)$, $J_i > 0$, $i = 1, 2, 3$, be the inertia matrix of the spacecraft in a coordinate frame defined by its principal axes. Let H be the angular momentum vector of the spacecraft relative to the inertial frame. Then we have

$$J\omega = R(\psi,\theta,\phi)H . \tag{2.8}$$

Differentiating (2.8) we obtain

$$J\dot{\omega} = S(\omega)R(\psi,\theta,\phi)H + R(\psi,\theta,\phi)\dot{H} , \tag{2.9}$$

where

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$$S(\omega) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix} . \quad (2.10)$$

We assume that the control torques u'_1 and u'_2 are applied about axes represented by unit vectors b_1 and b_2 respectively. This implies that

$$R(\psi,\theta,\phi)\dot{H} = b_1 u'_1 + b_2 u'_2 . \quad (2.11)$$

Without loss of generality, we assume that $b_1 = (1, 0, 0)^T$ and $b_2 = (0, 1, 0)^T$. Thus the equations describing the evolution of the angular velocity of the spacecraft are given by

$$J_1\dot{\omega}_1 = (J_2 - J_3)\omega_2\omega_3 + u'_1 , \quad (2.12)$$

$$J_2\dot{\omega}_2 = (J_3 - J_1)\omega_3\omega_1 + u'_2 , \quad (2.13)$$

$$J_3\dot{\omega}_3 = (J_1 - J_2)\omega_1\omega_2 . \quad (2.14)$$

3. Controllability and Stabilizability Properties of Complete Spacecraft Dynamics

As background for our subsequent development, we consider the controllability and stabilizability properties for the complete dynamics of the spacecraft with control torques only about two principal axes. Define

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{u'_1}{J_1} \\ \frac{u'_2}{J_2} \end{bmatrix} .$$

From Section 2 the state equations can be rewritten as

$$\dot{\omega}_1 = a_1\omega_2\omega_3 + u_1 , \quad (3.1)$$

$$\dot{\omega}_2 = a_2\omega_1\omega_3 + u_2 , \quad (3.2)$$

$$\dot{\omega}_3 = a_3\omega_1\omega_2 , \quad (3.3)$$

$$\dot{\phi} = \omega_1 + \omega_2\sin\phi \tan\theta + \omega_3\cos\phi \tan\theta , \quad (3.4)$$

$$\dot{\theta} = \omega_2\cos\phi - \omega_3\sin\phi , \quad (3.5)$$

$$\dot{\psi} = \omega_2\sin\phi \sec\theta + \omega_3\cos\phi \sec\theta , \quad (3.6)$$

where

$$a_1 = \frac{J_2 - J_3}{J_1} , \; a_2 = \frac{J_3 - J_1}{J_2} , \; a_3 = \frac{J_1 - J_2}{J_3} .$$

It is easily verified that the linearization of the equations about an equilibrium has an uncontrollable eigenvalue at the origin. This implies that an inherently nonlinear analysis is necessary in order to characterize the controllability and stabilizability properties of the complete spacecraft dynamics. Moreover, a linear feedback control law cannot be used to asymptotically stabilize the spacecraft to an equilibrium attitude.

We now present fundamental results on the controllability and stabilizability properties of the complete spacecraft dynamics described by equations (3.1)-(3.6). The reader in referred to [13] for additional details.

Theorem 3.1: The complete spacecraft dynamics described by state equations (3.1)-(3.6) are strongly accessible if and only if $J_1 \neq J_2$, i.e., the uncontrolled principal axis is not an axis of symmetry.

Theorem 3.2: The complete spacecraft dynamics described by state equations (3.1)-(3.6) are small time locally controllable at any equilibrium if and only if $J_1 \neq J_2$.

Theorem 3.3: The complete spacecraft dynamics described by state equations (3.1)-(3.6) cannot be locally asymptotically stabilized to an equilibrium by any time-invariant continuous state feedback control law.

Theorem 3.3 holds if $J_1 \neq J_2$ and also if $J_1 = J_2$. A weaker version (with "continuous" replaced by "C^1") was proved in [6]. However, Theorem 3.3 follows from [6] using results in [9] and [12]. This negative result also implies that feedback control approaches based on linearization, Lyapunov methods, center manifold theory, or zero dynamics cannot be used to asymptotically stabilize the spacecraft to an equilibrium attitude.

Although the full set of equations (3.1)-(3.6) cannot be asymptotically stabilized to an equilibrium via continuous feedback, one may still wish to design a smooth control law which stabilizes at least a particular subset of state variables. Consider the state equations for ω_1, ω_2, ω_3, ϕ and θ given by equations (3.1)-(3.5). These equations are invariant with respect to the Euler angle ψ. Asymptotic stabilization of this subset of the original equations corresponds to stabilization of the motion of the spacecraft about an attractor, which is not an isolated equilibrium. A result from [6] shows that the closed loop trajectories can be asymptotically stabilized to the manifold

$$\Omega = \{(\omega_1,\omega_2,\omega_3,\phi,\theta,\psi) : \omega_1 = \omega_2 = \omega_3 = \phi = \theta = 0\} \quad (3.8)$$

using smooth C^1 feedback.

We mention that although the complete spacecraft dynamics described by equations (3.1)-(3.6) cannot be asymptotically stabilized to an equilibrium by continuous feedback, an algorithm generating a piecewise constant discontinuous control has been developed in [7] which locally asymptotically stabilizes the complete spacecraft dynamics to an equilibrium. The algorithm requires that $J_1 \neq J_2$, i.e., the uncontrolled principal axis must not be an axis of symmetry. The algorithm is based on Lie algebraic methods in [8]. The algorithm is extremely complicated and is not an easily implementable control strategy. However, stabilization of the complete spacecraft dynamic equations (3.1)-(3.6) is an inherently difficult problem and the algorithm in [7] is the only control strategy proposed in the literature thus far.

4. Attitude Stabilization of a Non-Axially Symmetric Spacecraft with Two Control Torques

In this section, we consider the equations (3.1)-(3.6) describing the motion of a spacecraft controlled by input torques only about two of its principal axes. It is assumed that the uncontrolled principal axis is not an axis of symmetry of the spacecraft; i.e., $J_1 \neq J_2$. As a consequence of the negative result of Theorem 3.3, we restrict our study to the class of discontinuous feedback controllers in order to asymptotically stabilize the complete spacecraft dynamics. However, as shown in the previous section, the complete spacecraft dynamics are small time locally controllable at any equilibrium attitude. This suggests that a piecewise analytic feedback control law can be constructed which asymptotically stabilizes the complete spacecraft dynamics to an equilibrium attitude. Here we present a particular discontinuous feedback strategy, which is obtained by requiring that the spacecraft undergo a sequence of specified maneuvers. Without loss of generality, we assume that the equilibrium attitude to be stabilized is the origin. We first present a physical interpretation of the sequence of maneuvers that transfers any initial state to the origin.

Maneuvers 1-3. Transfer the initial state of the spacecraft to an equilibrium state in finite time; i.e., bring the spacecraft to rest.

There are control laws based on center manifold theory [1] and zero dynamics theory [6] which accomplish this in an asymptotic sense. Here we use a sequence of three maneuvers, and corresponding feedback control laws, which bring the spacecraft to rest in finite time.

Maneuver 4. Transfer the resulting state to an equilibrium state where $\phi = 0$ in finite time; i.e., so that the spacecraft is at rest with $\phi = 0$. This maneuver is accomplished using the control torque u_1 only.

Maneuver 5. Transfer the resulting state to an equilibrium state where $\phi = 0$, $\theta = 0$ in finite time; i.e., so that the spacecraft is at rest with $\phi = 0$, $\theta = 0$. This maneuver is accomplished using the control torque u_2 only.

In order to complete specification of the sequence of maneuvers, the Euler angle ψ must be brought to zero. This cannot be accomplished directly since a control torque cannot be applied about the third principal axis of the spacecraft. However, the resulting state can be transferred to the origin indirectly using three maneuvers. The three maneuvers correspond to three consecutive rotations about the two controlled principal axes of the spacecraft, the first and the third being around the first principal axis. This produces a net change in the orientation of the spacecraft so that the state of the spacecraft is transferred to the origin in finite time. The three maneuvers are described as follows.

Maneuver 6. Transfer the resulting state to an equilibrium state where $\phi = 0.5\pi$, $\theta = 0$ in finite time; i.e., so that the spacecraft is at rest with $\phi = 0.5\pi$, $\theta = 0$. This maneuver is accomplished using the control torque u_1 only.

Maneuver 7. Transfer the resulting state to the equilibrium state $(\omega_1,\omega_2,\omega_3,\phi,\theta,\psi) = (0,0,0,0.5\pi,0,0)$ in finite time. This maneuver is accomplished using the control torque u_2 only.

Maneuver 8. Transfer the equilibrium state $(\omega_1,\omega_2,\omega_3,\phi,\theta,\psi) = (0,0,0,0.5\pi,0,0)$ to the equilibrium state $(0,0,0,0,0,0)$ in finite time. This maneuver is accomplished using the control torque u_1 only.

Note that, excluding the first three maneuvers where the spacecraft is brought to rest, all subsequent maneuvers are such that the angular velocity component ω_3 is maintained identically zero. This is accomplished by carrying out maneuvers which require use of only a single control torque at a time.

It is convenient to introduce some notation. Throughout, assume $k > 0$, and define

$$G(x_1, x_2) = \begin{cases} k & \text{if } \{x_1 + \frac{x_2|x_2|}{2k} > 0\} \text{ or} \\ & \{x_1 + \frac{x_2|x_2|}{2k} = 0 \text{ and } x_2 > 0\} \\ -k & \text{if } \{x_1 + \frac{x_2|x_2|}{2k} < 0\} \text{ or} \\ & \{x_1 + \frac{x_2|x_2|}{2k} = 0 \text{ and } x_2 < 0\} \\ 0 & \text{if } \{x_1 = 0 \text{ and } x_2 = 0\} \end{cases}$$

We use the well-known property that the feedback control

$$u = -G(x_1 - \bar{x}_1, x_2)$$

for the system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u$$

transfers any initial state to the final state $(\bar{x}_1, 0)$ in a finite time. We also use the standard notation that

$$\text{sign}(x_1) = \begin{cases} 1 & \text{if } x_1 > 0 \\ -1 & \text{if } x_1 < 0 \\ 0 & \text{if } x_1 = 0 \end{cases}.$$

Our mathematical construction of a control strategy which transfers an arbitrary initial state of the spacecraft to the origin is based on a sequence of equilibrium subsets and a sequence of control functions which transfers a state in one subset to another. Consider the following equilibrium subsets

$$M_1 = \{(\omega_1,\omega_2,\omega_3,\phi,\theta,\psi) = (0,0,0,\phi,\theta,\psi),\ \phi,\theta,\psi \text{ arbitrary}\},$$
$$M_2 = \{(\omega_1,\omega_2,\omega_3,\phi,\theta,\psi) = (0,0,0,0,\theta,\psi),\ \theta,\psi \text{ arbitrary}\},$$
$$M_3 = \{(\omega_1,\omega_2,\omega_3,\phi,\theta,\psi) = (0,0,0,0,0,\psi),\ \psi \text{ arbitrary}\},$$
$$M_4 = \{(\omega_1,\omega_2,\omega_3,\phi,\theta,\psi) = (0,0,0,0.5\pi,0,\psi),\ \psi \text{ arbitrary}\}.$$

We now present the feedback control laws that accomplish the sequential maneuvers described above; for each case we show that a desired terminal state which defines the maneuver is reached.

Transferring any initial state to a state in M_1

In order to transfer the arbitrary initial state to a final state which satisfies $\omega_1 = \omega_2 = \omega_3 = 0$ three sequential maneuvers are required. The first maneuver results in $\omega_1 = \omega_2 = 0$ while $\omega_3 \neq 0$ in general; the second maneuver results in $\omega_1 = \omega_1^*$ and $\omega_2 = \omega_2^*$, where ω_1^*, ω_2^* are chosen to guarantee that at the end of the third maneuver $\omega_1 = \omega_2 = \omega_3 = 0$. These three maneuvers are described in detail as follows.

Maneuver 1. Let $(\omega_1^0,\omega_2^0,\omega_3^0,\phi^0,\theta^0,\psi^0)$ denote an initial state for the complete spacecraft dynamics described by equations (3.1)-(3.6). Define

$$v_1 = a_1\omega_2\omega_3 + u_1,$$
$$v_2 = a_2\omega_3\omega_1 + u_2.$$

Equations (3.1)-(3.3) can now be rewritten as

$$\dot{\omega}_1 = v_1, \tag{4.1}$$
$$\dot{\omega}_2 = v_2, \tag{4.2}$$
$$\dot{\omega}_3 = a_3\omega_1\omega_2. \tag{4.3}$$

Apply the feedback control functions

$$v_1 = -k\,\text{sign}\omega_1,$$
$$v_2 = -k\,\text{sign}\omega_2.$$

It is easy to see that after a finite time given by $\max(\frac{|\omega_1^0|}{k}, \frac{|\omega_2^0|}{k})$, $\omega_1 = \omega_2 = 0$; at this instant let $\omega_3 = \bar{\omega}_3$ where the constant value $\bar{\omega}_3$ can be evaluated.

Maneuver 2. Apply the feedback control functions

$$v_1 = -k\,\text{sign}(\omega_1 - \omega_1^*),$$
$$v_2 = -k\,\text{sign}(\omega_2 - \omega_2^*),$$

where

$$\omega_1^* = \left[\frac{3k|\bar{\omega}_3|}{2|a_3|}\right]^{\frac{1}{3}}, \quad \omega_2^* = -\omega_1^*\,\text{sign}\bar{\omega}_3\,\text{sign}a_3.$$

It is again easy to see that after a finite time given by $\frac{\omega_1^*}{k}$, $\omega_1 = \omega_1^*$, $\omega_2 = \omega_2^*$, and in addition it can be shown that $\omega_3 = \frac{\bar{\omega}_3}{2}$.

Maneuver 3. Apply the feedback control functions

$$v_1 = -k\,\text{sign}\omega_1,$$
$$v_2 = -k\,\text{sign}\omega_2.$$

It can be seen that after a finite time given by $\frac{\omega_1^*}{k}$, $\omega_1 = 0$, $\omega_2 = 0$ and it can be shown that $\omega_3 = 0$.

Consequently, the resulting state after these three sequential maneuvers is $(0,0,0,\phi^1,\theta^1,\psi^1) \in M_1$ for some ϕ^1, θ^1, ψ^1.

Transferring a state in M_1 to a state in M_2 (Maneuver 4)

Let $(0,0,0,\phi^1,\theta^1,\psi^1) \in M_1$ denote a state of the spacecraft. Apply the feedback control functions

$$u_1 = -G(\phi, \omega_1),$$
$$u_2 = 0.$$

It follows that

$$\omega_2 = 0, \ \omega_3 = 0,$$
$$\theta = \theta^1, \ \psi = \psi^1,$$

satisfy equations (3.2), (3.3), (3.5), (3.6) while equations (3.1), (3.4) become

$$\dot{\omega}_1 = -G(\phi, \omega_1),$$
$$\dot{\phi} = \omega_1.$$

Consequently, after a finite time $\omega_1 = 0$, $\phi = 0$; and thus a state $(0,0,0,\phi^1,\theta^1,\psi^1) \in M_1$ is transferred to the state $(0,0,0,0,\theta^1,\psi^1) \in M_2$ in finite time.

Transferring a state in M_2 to a state in M_3 (Maneuver 5)

Let $(0,0,0,0,\theta^1,\psi^1) \in M_2$ denote a state of the spacecraft. Apply the feedback control functions

$$u_1 = 0,$$
$$u_2 = -G(\theta, \omega_2).$$

It follows that

$$\omega_1 = 0, \ \omega_3 = 0,$$
$$\phi = 0, \psi = \psi^1,$$

satisfy equations (3.1), (3.3), (3.4), (3.6) while equations (3.2), (3.5) become

$$\dot{\omega}_2 = -G(\theta, \omega_2),$$
$$\dot{\theta} = \omega_2.$$

Consequently, after a finite time $\omega_2 = 0$, $\theta = 0$; and thus a state $(0,0,0,0,\theta^1,\psi^1) \in M_2$ is transferred to the state $(0,0,0,0,0,\psi^1) \in M_3$ in finite time.

Transferring a state in M_3 to a state in M_4 (Maneuver 6)

Let $(0,0,0,0,0,\psi^1) \in M_3$ denote a state of the spacecraft. Apply the feedback control functions

$$u_1 = -G(\phi - 0.5\pi, \omega_1),$$
$$u_2 = 0.$$

It follows that

$$\omega_2 = 0, \omega_3 = 0,$$
$$\theta = 0, \psi = \psi^1,$$

satisfy equations (3.2), (3.3), (3.5), (3.6) while equations (3.1), (3.4) become

$$\dot\omega_1 = -G(\phi - 0.5\pi, \omega_1),$$
$$\dot\phi = \omega_1.$$

Consequently, after a finite time $\omega_1 = 0$, $\phi = 0.5\pi$; and thus a state $(0,0,0,0,0,\psi^1) \in M_3$ is transferred to the state $(0,0,0,0.5\pi,0,\psi^1) \in M_4$ in finite time.

Transferring a state in M_4 to $(0,0,0,0.5\pi,0,0)$ (Maneuver 7)

Let $(0,0,0,0.5\pi,0,\psi^1) \in M_4$ denote a state of the spacecraft. Apply the feedback control functions

$$u_1 = 0,$$
$$u_2 = -G(\psi, \omega_2).$$

It follows that

$$\omega_1 = 0, \omega_3 = 0,$$
$$\phi = 0.5\pi, \theta = 0,$$

satisfy equations (3.1), (3.3), (3.4), (3.5) while equations (3.2), (3.6) become

$$\dot\omega_2 = -G(\psi, \omega_2),$$
$$\dot\psi = \omega_2.$$

Consequently, after a finite time $\omega_2 = 0$, $\psi = 0$; and thus a state $(0,0,0,0.5\pi,0,\psi^1) \in M_4$ is transferred to the state $(0,0,0,0.5\pi,0,0)$ in finite time.

Transferring $(0,0,0,0.5\pi,0,0)^T$ to $(0,0,0,0,0,0)$ (Maneuver 8)

Let $(0,0,0,0.5\pi,0,0)$ denote the state of the spacecraft. Apply the feedback control functions

$$u_1 = -G(\phi, \omega_1),$$
$$u_2 = 0.$$

It follows that

$$\omega_2 = 0, \omega_3 = 0,$$
$$\theta = 0, \psi = 0,$$

satisfy equations (3.2), (3.3), (3.5), (3.6) while equations (3.1), (3.4) become

$$\dot\omega_1 = -G(\phi, \omega_1),$$
$$\dot\phi = \omega_1.$$

Consequently, after a finite time $\omega_1 = 0$, $\phi = 0$; and thus the state $(0,0,0,0.5\pi,0,0)$ is transferred to the state $(0,0,0,0,0,0)$ in finite time.

In summary, the feedback control strategy outlined above can be implemented by sequential switching between the following feedback functions.

Maneuver 1. Apply

$$u_1^1(x) = -a_1\omega_2\omega_3 - k\,\mathrm{sign}\omega_1,$$
$$u_2^1(x) = -a_2\omega_3\omega_1 - k\,\mathrm{sign}\omega_2,$$

until $(\omega_1,\omega_2,\omega_3) = (0,0,\varpi_3)$ for some value ϖ_3; then go to Maneuver 2.

Maneuver 2. Compute

$$\omega_1^* = \left(\frac{3k|\varpi_3|}{2|a_3|}\right)^{\frac{1}{3}}, \quad \omega_2^* = -\left(\frac{3k|\varpi_3|}{2|a_3|}\right)^{\frac{1}{3}}\mathrm{sign}\varpi_3\ \mathrm{sign}a_3;$$

apply

$$u_1^2(x) = -a_1\omega_2\omega_3 - k\,\mathrm{sign}(\omega_1 - \omega_1^*),$$
$$u_2^2(x) = -a_1\omega_3\omega_1 - k\,\mathrm{sign}(\omega_2 - \omega_2^*),$$

until $(\omega_1,\omega_2,\omega_3) = (\omega_1^*,\omega_2^*,\frac{\varpi_3}{2})$; then go to Maneuver 3.

Maneuver 3. Apply

$$u_1^3(x) = -a_1\omega_2\omega_3 - k\,\mathrm{sign}\omega_1,$$
$$u_2^3(x) = -a_2\omega_3\omega_1 - k\,\mathrm{sign}\omega_2,$$

until $(\omega_1,\omega_2,\omega_3) = (0,0,0)$, i.e., $(\omega_1,\omega_2,\omega_3,\phi,\theta,\psi) \in M_1$; then go to Maneuver 4.

Maneuver 4: Apply

$$u_1^4(x) = -G(\phi, \omega_1),$$
$$u_2^4(x) = 0,$$

until $(\omega_1,\omega_2,\omega_3,\phi) = (0,0,0,0)$, i.e., $(\omega_1,\omega_2,\omega_3,\phi,\theta,\psi) \in M_2$; then go to Maneuver 5.

Maneuver 5: Apply

$$u_1^5(x) = 0,$$
$$u_2^5(x) = -G(\theta, \omega_2),$$

until $(\omega_1,\omega_2,\omega_3,\phi,\theta) = (0,0,0,0,0)$, i.e., $(\omega_1,\omega_2,\omega_3,\phi,\theta,\psi) \in M_3$; then go to Maneuver 6.

Maneuver 6: Apply

$$u_1^6(x) = -G(\phi - 0.5\pi, \omega_1),$$
$$u_2^6(x) = 0,$$

until $(\omega_1,\omega_2,\omega_3,\phi,\theta) = (0,0,0,0.5\pi,0)$, i.e., $(\omega_1,\omega_2,\omega_3,\phi,\theta,\psi) \in M_4$; then go to Maneuver 7.

Maneuver 7: Apply

$$u_1^7(x) = 0,$$
$$u_2^7(x) = -G(\psi, \omega_2),$$

until $(\omega_1,\omega_2,\omega_3,\phi,\theta,\psi) = (0,0,0,0.5\pi,0,0)$; then go to Maneuver 8.

Maneuver 8: Apply

$$u_1^8(x) = -G(\phi, \omega_1),$$
$$u_2^8(x) = 0,$$

until $(\omega_1,\omega_2,\omega_3,\phi,\theta,\psi) = (0,0,0,0,0,0)$.

This feedback control strategy achieves attitude stabilization of the spacecraft by executing a sequence of maneuvers. This strategy is discontinuous and nonclassical in nature. Justification that it stabilizes the complete spacecraft dynamics to the equilibrium attitude (at the origin) in finite time, under the ideal model assumptions, follows as a consequence of the construction procedure. A computer implementation of the feedback control strategy can be easily carried out.

5. Attitude Stabilization of an Axially Symmetric Spacecraft with Two Control Torques

From the analysis made in Section 3, we find that the complete dynamics of a spacecraft controlled by two control torques supplied by gas jet actuators, as described by equations (3.1)-(3.6), fail to be controllable or even accessible if the uncontrolled principal axis is an axis of symmetry of the spacecraft, i.e., if $J_1 = J_2$. Due to the lack of controllability, the control algorithm proposed in [7] is not applicable to this case. In this section we concentrate on the case where the uncontrolled principal axis of the spacecraft is an axis of symmetry, i.e., $J_1 = J_2$. In particular we ask the question: what restricted control and stabilization properties of the spacecraft can be demonstrated in this case? Our analysis begins by demonstrating that, under appropriate restrictions of interest, the spacecraft equations can be expressed in a reduced form. Controllability and stabilizability properties for this case follow from an analysis of the reduced equations.

Consider the equations (3.1)-(3.6) describing the motion of a spacecraft controlled by input torques supplied by gas jet actuators about only two of its principal axes. It is assumed that the uncontrolled principal axis is an axis of symmetry of the spacecraft. From equations (3.1)-(3.6) and $J_1 = J_2$ we have

$$\dot{\omega}_1 = a_1\omega_2\omega_3 + u_1 , \tag{5.1}$$

$$\dot{\omega}_2 = a_2\omega_1\omega_3 + u_2 , \tag{5.2}$$

$$\dot{\omega}_3 = 0 , \tag{5.3}$$

$$\dot{\phi} = \omega_1 + \omega_2\sin\phi \tan\theta + \omega_3\cos\phi \tan\theta , \tag{5.4}$$

$$\dot{\theta} = \omega_2\cos\phi - \omega_3\sin\phi , \tag{5.5}$$

$$\dot{\psi} = \omega_2\sin\phi \sec\theta + \omega_3\cos\phi \sec\theta . \tag{5.6}$$

If $\omega_3(0) \neq 0$ then ω_3 cannot be transferred to zero using any control function. If we assume that $\omega_3(0) = 0$, then $\omega_3 \equiv 0$. Under the restriction $\omega_3(0) = 0$, the reduced spacecraft dynamics for this case are described by

$$\dot{\omega}_1 = u_1 , \tag{5.7}$$

$$\dot{\omega}_2 = u_2 , \tag{5.8}$$

$$\dot{\phi} = \omega_1 + \omega_2\sin\phi \tan\theta , \tag{5.9}$$

$$\dot{\theta} = \omega_2\cos\phi , \tag{5.10}$$

$$\dot{\psi} = \omega_2\sin\phi \sec\theta . \tag{5.11}$$

The following results can be easily shown. The proofs of Theorem 5.1 and Theorem 5.2 are similar to the proofs of Theorem 3.1 and Theorem 3.2 respectively in [13]. Theorem 5.3 follows from the results in [5], [9] and [12].

Theorem 5.1: The reduced dynamics of an axially symmetric spacecraft controlled by two pairs of gas jet actuators as described by equations (5.7)-(5.11) are strongly accessible.

Theorem 5.2: The reduced dynamics of an axially symmetric spacecraft controlled by two pairs of gas jet actuators as described by equations (5.7)-(5.11) are small time locally controllable at any equilibrium.

Theorem 5.3: The reduced dynamics of an axially symmetric spacecraft controlled by two pairs of gas jet actuators as described by equations (5.7)-(5.11) cannot be asymptotically stabilized to an equilibrium using a time-invariant continuous feedback control law.

The implications of the properties stated above are as follows. For all initial conditions that satisfy $\omega_3(0) = 0$, the axially symmetric spacecraft controlled by two pairs of gas jet actuators as described by equations (5.1)-(5.6) can be controlled to any equilibrium attitude. However, any time-invariant feedback control law that asymptotically stabilizes the spacecraft to an isolated equilibrium attitude must necessarily be discontinuous. Thus arbitrary reorientation of the spacecraft can be achieved if $\omega_3(0) = 0$; if $\omega_3(0) \neq 0$, reorientation of the spacecraft to an equilibrium attitude cannot be achieved.

Conveniently, it turns out that sequential execution of the maneuvers defined as Manuevers 3 through 8 in the previous section transfers any initial state of the reduced spacecraft dynamics (5.7)-(5.11) to the origin in finite time. The physical interpretation of the manuevers is the same as described previously; the overall feedback control strategy is as follows.

Maneuver 1. Apply

$$u_1^1(x) = -k\,\text{sign}\omega_1 ,$$

$$u_2^1(x) = -k\,\text{sign}\omega_2 ,$$

until $(\omega_1,\omega_2) = (0,0)$; then go to Maneuver 2.

Maneuver 2: Apply

$$u_1^2(x) = -G(\phi, \omega_1) ,$$

$$u_2^2(x) = 0 ,$$

until $(\omega_1,\omega_2,\phi) = (0,0,0)$; then go to Maneuver 3.

Manuever 3: Apply

$$u_1^3(x) = 0 ,$$

$$u_2^3(x) = -G(\theta, \omega_2) ,$$

until $(\omega_1,\omega_2,\phi,\theta) = (0,0,0,0)$; then go to Maneuver 4.

Maneuver 4: Apply

$$u_1^4(x) = -G(\phi - 0.5\pi, \omega_1) ,$$

$$u_2^4(x) = 0 ,$$

until $(\omega_1,\omega_2,\phi,\theta) = (0,0,0.5\pi,0)$, then go to Maneuver 5.

Maneuver 5: Apply

$$u_1^5(x) = 0 ,$$

$$u_2^5(x) = -G(\psi, \omega_2) ,$$

until $(\omega_1,\omega_2,\phi,\theta,\psi) = (0,0,0.5\pi,0,0)$; then go to Maneuver 6.

Maneuver 6: Apply

$$u_1^6(x) = -G(\phi, \omega_1) ,$$

$$u_2^6(x) = 0 ,$$

until $(\omega_1,\omega_2,\phi,\theta,\psi) = (0,0,0,0,0,0)$.

This feedback control strategy achieves attitude stabilization of the spacecraft, in the sense described previously, by executing a sequence of maneuvers. This strategy is discontinuous and nonclassical in nature. A computer implementation of the feedback control strategy can be easily carried out.

Notice that according to equation (2.4), the condition that $\omega_3 = 0$ implies that

$$-(\sin\phi)d\theta + (\cos\theta\cos\phi)d\psi = 0 ;$$

this represents a nonintegrable invariant of the spacecraft motion. Therefore the reduced spacecraft dynamic equations define a nonlinear control system of the form studied in [4]. An alternate discontinuous control strategy which achieves attitude stabilization of the spacecraft is presented in [13].

6. Simulation

We illustrate the results of the paper with an example of a nonaxially symmetric spacecraft with principal moments of inertia $J_1 = 100\ Kg.\ M^2$, $J_2 = 250\ Kg.\ M^2$, and $J_3 = 350\ Kg.\ M^2$. There is no control torque about the third principal axis and two control torques, generated by gas jet actuators, are applied about the other two principal axes. The spacecraft has an initial orientation described by the Euler angles $\phi^0 = -\pi$, $\theta^0 = 0.25\pi$, and $\psi^0 = -0.5\pi$ radians, and an initial angular velocity given by $\omega_1^0 = 0.3$, $\omega_2^0 = -0.3$, and $\omega_3^0 = 0.1$ radians per second. A computer implementation of the feedback control strategy described in Section 4 was used to asymptotically stabilize the spacecraft to the origin. The value of k is chosen to be 1. Fig. 1, Fig. 2 and Fig. 3 show the time responses on the Euler angles, angular velocities and the control torques respectively. At $t = 0.3$ seconds, which is the end of Maneuver 1 of the algorithm, ω_1 and ω_2 are both zero while $\omega_3 = \bar{\omega}_3 = 0.1039$ radians per second. At $t = 1.73$ seconds, which is the end of Maneuver 3 of the algorithm, $\omega_1 = \omega_2 = \omega_3 = 0$, and $\phi = -2.59$, $\theta = 0.37$ and $\psi = -1.913$ radians. The subsequent maneuvers described by Steps 4 through 8 results in $\phi = \theta = \psi = \omega_1 = \omega_2 = \omega_3 = 0$ as shown in Fig. 1 and Fig. 2. It might be observed from Fig. 3 that until 1.73 seconds, which is the end of Maneuver 3, the control torques u_1 and u_2 are both applied to bring the spacecraft to rest. But once the spacecraft is brought to rest, the subsequent maneuvers are such that only one of the control torques is nonzero in any interval of time. Thus ω_3 remains zero at all time beyond 1.73 seconds, and ω_1 and ω_2 vary so that only one is nonzero at any time interval beyond 1.73 seconds. Since the feedback control strategy for the reorientation of an axially symmetric spacecraft is similar to the feedback control strategy for the reorientation of a non-axially symmetric spacecraft, we do not consider a seperate example to illustrate this case.

7. Conclusion

The attitude stabilization problem of a spacecraft using control torques supplied by gas jet actuators about only two of its principal axes has been considered. If the uncontrolled principal axis is not an axis of symmetrv of the soacecraft, the complete spacecraft dynamics cannot be asymptotically stabilized to an equilibrium attitude using continuous feedback. A discontinuous feedback control strategy was constructed which stabilizes the spacecraft to an equilbrium attitude in finite time. If the uncontrolled principal axis is an axis of symmetry of the spacecraft, the complete spacecraft dynamics cannot be stabilized. The reduced spacecraft dynamics cannot be asymptotically stabilized using continuous feedback, but again a discontinuous feedback control strategy was constructed which stabilizes the spacecraft (in the reduced sense) to an equilibrium attitude in finite time. The results of the paper show that although standard nonlinear control techniques do not apply, it is possible to construct a stabilizing control law by performing a sequence of maneuvers.

One of the advantages of the development in this paper is that feedback control strategies are constructed which guarantee attitude stabilization in a finite time. The total time required to complete the spacecraft reorientation is the sum of the times required to complete the sequence of maneuvers described. From the analysis provided, it should be clear that the time required to complete each maneuver depends on the single positive parameter k in the corresponding control law. There is a trade off between the required control levels, determined by the selection of k, and the resulting times to complete each of the maneuvers and hence the total time required to reorient the spacecraft. In particular, the time to reorient the spacecraft from a given initial state to the origin can be expressed as a function of the value of the parameter k and of the initial state.

For each of the two attitude stabilization problems considered, we have presented one example of a sequence of maneuvers which achieves the desired spacecraft attitude stabilization. There are many other maneuver sequences, and corresponding feedback control strategies, which will also achieve the desired attitude stabilization of the spacecraft. But each such strategy is necessarily discontinuous.

We have demonstrated the closed loop properties for the special feedback control strategies presented. Our analysis was based on a number of assumptions which are required to justify the mathematical models studied. Further robustness analysis is required to determine effects of model uncertainities and external disturbances. Unfortunately, such robustness analysis is quite difficult since the closed loop vector fields are necessarily discontinuous. Perhaps, feedback control strategies which stabilize the spacecraft attitude, different from ones presented in this paper, would provide improved closed loop robustness.

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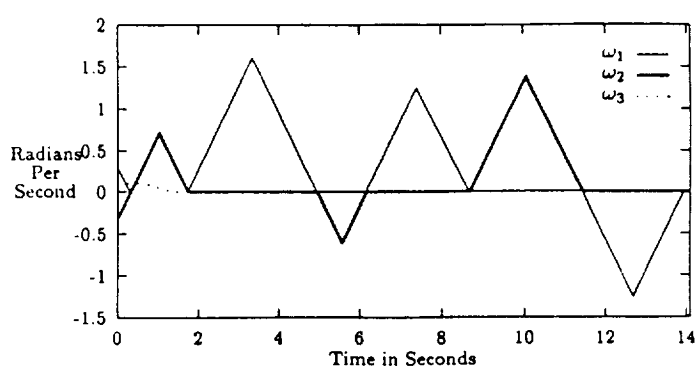

Figure 2: Plot of Angular Velocities.

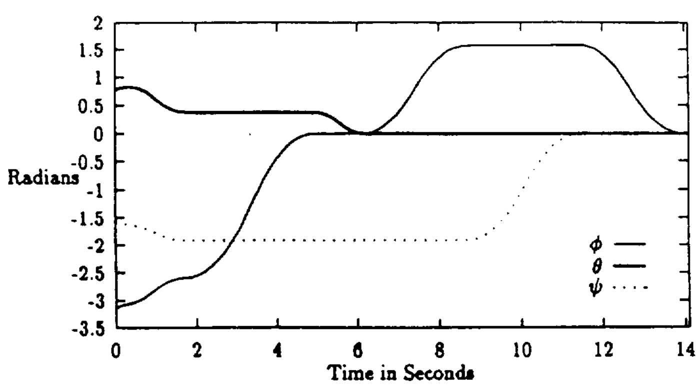

Figure 1: Plot of Euler Angles.

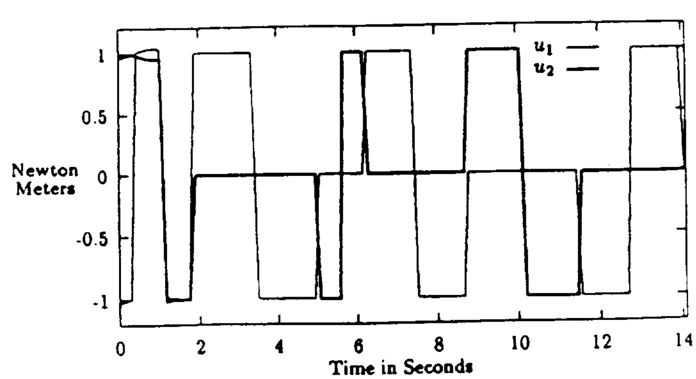

Figure 3: Plot of u_1 and u_2.