

# PROBABILISTIC COMPOSITE ANALYSIS

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## SUMMARY

Formal procedures are described which are used to computationally simulate the probabilistic behavior of composite structures. The computational simulation starts with the uncertainties associated with all aspects of a composite structure (constituents, fabrication, assembling, etc.) and encompasses all aspects of composite behavior (micromechanics, macromechanics, combined stress failure, laminate theory (and including structural response and tailoring (optimization)). Typical sample cases are included to illustrate the formal procedure for the computational simulation. The collective results of the sample cases demonstrate that uncertainties in composite behavior and structural response can be probabilistically quantified.

## INTRODUCTION

Probabilistic composite mechanics and probabilistic composite structural analysis are formal methods which are used to quantify the scatter that is observed in composite material properties and structural response. The observed scatter in composite material properties is the range of measured values in modulus, strength, thermal expansion coefficient, etc., while that in structural response is the range of measured values for displacement, frequency, buckling load, etc. The formal methods relate the scatter in the observed values to the corresponding scatter in the physical parameters which make up the composite and/or the composite structure. For example, these parameters include constituent material properties, fabrication process variables, structural component geometry, and any other variables which contribute to the composite behavior and/or structural response.

The development of these types of formal methods has been the subject of considerable research at NASA Lewis Research Center. This research has led to computational simulation methods for relating the scatter (uncertainties) in the composite properties or composite structural response to the corresponding uncertainties in the respective parameters (primitive variables) which are used to describe the composite in all its inherent scales: micro, macro, laminate and structural. The objective of this paper is to summarize salient features of these computational simulation methods and to present typical results to illustrate their applications.

Specifically, the paper covers (1) a brief description of the fundamental concepts, (2) probabilistic composite micromechanics, (3) probabilistic laminate theory, (4) probabilistic laminate tailoring, and (5) elementary probabilistic structural analysis.

## FUNDAMENTAL CONCEPTS

The fundamental concepts/assumptions in the probabilistic composite mechanics described herein are (1) the scatter in all the primitive variables, which describe the composite, can be represented by well known probabilistic distributions, (2) the values for the primitive variables can be randomly selected from the known distributions for a specific composite, (3) these values can be used in composite mechanics to predict composite behavior, (4) the whole process can be repeated many times to obtain sufficient information to develop the distribution of the ply property, composite property, or structural response. These concepts are analogous to making and testing a composite. The probabilistic distributions represent available materials that the composite can be made from. The composite mechanics represent the physical experiment and the processes repetition represents several experiments. Subsequent statistical analysis of the data is the same for both approaches.

The primitive variables which describe the composite are identified by examining the fabrication process. A schematic depicting the fabrication process for an aircraft wing top cover using top cover is shown in figure 1. The respective primitive variables with their corresponding probabilistic distributions and parameters are listed in Table 1. The use of these in composite mechanics and structural analysis are described in subsequent sections.

## PROBABILISTIC COMPOSITE MICROMECHANICS

The probabilistic simulation is performed by considering the ply as an assembly (equivalent laminate) of 15 subplies, where each subply is made from randomly selected properties from Table 1. The composite mechanics used in the simulation is that available in the Integrated Composite Analyzer (ICAN) (ref. 1). The structure of ICAN is schematically shown in figure 2.

Typical probabilistic results obtained for ply shear modulus and shear strength are shown in figure 3 and for ply longitudinal compressive strength in figure 4. Additional details are found in references 2 and 3.

## PROBABILISTIC LAMINATE THEORY

Probabilistic laminate theory consists of using probabilistic ply properties in the laminate theory equations. In the present simulation the probabilistic ply properties are available from the probabilistic micromechanics previously described. The simulation for laminate properties is performed using ICAN (fig. 2).

Typical probabilistic laminate properties for a quasi-isotropic ( $\theta = 45^\circ$  in fig. 5) laminate from graphite-fiber/epoxy composite are shown in figure 6 for laminate modulus ( $E_{xx}$ ). The mean value of the modulus is about 6.2 mspi, which is at 50 percent probability, while the range is from about 2 to 14 mspi. Corresponding results for the compressive strength are shown in figure 7 with a mean of about 80 ksi and a range from 45 to 140 ksi. Those for the thermal expansion coefficient are shown in figure 8 with a mean value of  $1.6 \mu\text{-in}/^\circ\text{F}$  and a range from 0.0 to about  $7.5 \mu\text{-in}/^\circ\text{F}$ . The ranges for the laminate

modulus and compressive strength include those measured ( table 5, and fig. 16 in ref. 4, ), although the scatter in the measured properties is considerably smaller than the simulation results. One reason may be that the scatter assumed for the primitive variables should be tightened. The important conclusion is that the computational simulation for probabilistic composite mechanics has sufficient flexibility to capture the observed scatter in composite properties. The probabilistic evaluation of laminate properties also provides sensitivities of the primitive variables. For the laminate modulus, these sensitivities are summarized in Table 2. The fiber modulus is the most sensitive. The details are described in a forthcoming report.

## PROBABILISTIC LAMINATE TAILORING

Computational simulation methods have been developed for tailoring laminates with probabilistic properties and subjected to probabilistic loads. The probabilistic properties are obtained as described in the previous two sections while the probabilistic loads are described by assuming appropriate probabilistic distributions. A flowchart of the tailoring procedure is shown in figure 9. Typical results obtained are shown in figure 10. As can be seen, laminate tailoring with probabilistic loads and strength yields heavier laminates for the same probability of failure. On the other hand, laminate tailoring for probabilistic loads with fixed strength probability is independent of laminate weight, lending credence to safety factor designs. The important conclusion is that computational simulation methods can be developed for probabilistic laminate tailoring. Additional details are given in reference 5.

## ELEMENTARY PROBABILISTIC COMPOSITE STRUCTURAL ANALYSIS

Computational simulation methods for elementary probabilistic composite structural analysis have been developed. These are based on the same fundamental concepts described previously, where the scatter in all of the primitive variables which describe the structure is expressed in terms of probabilistic distributions. Values from these distributions are then substituted in classical structural mechanics equations to evaluate the corresponding probabilistic distribution in the structural response.

The entire computational simulation is illustrated in figure 11, where the probabilistic buckling load of an eccentrically loaded composite cantilever is evaluated. The schematic with the eccentric load is shown. The equation for the buckling load is given under the schematic. The primitive variables (all the variables in the equation are listed under the equation). Respective probabilistic distributions for each of the primitive variables are shown. The probability density function (PDF) (frequency of occurrence) is shown at the top right of the figure; the corresponding cumulative distribution function (CDF) (probability of occurrence) is shown below the PDF. The sensitivity of the buckling load to primitive variables is given at the bottom right for two probability levels.

The important points to observe in figure 11 are (1) the mean load is approximately equal to the deterministic load which is calculated by using mean values for the primitive variables; (2) the thickness dominates the sensitivity at high probabilities (0.999) with all others being relatively small, while at low probabilities (0.02813) even though the thickness sensitivity still dominates those for modulus and length have doubled; and (4) the shear modulus has relatively negligible effect on the buckling load.

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As was already mentioned, figure 11 summarizes schematically the entire probabilistic structural analysis. For structures of practical interest such as aircraft wings and fuselages, the single equation for the structural analysis model is replaced by a finite element model, the primitive variables increase many fold, and several global responses are usually needed, as well as local stresses and strains. The important conclusion is that the uncertainties in composite structural behavior can be quantified by the probabilistic methods described herein. Several other examples of this cantilever are described in a forthcoming report. Parallel research activities at Lewis led to the development of probabilistic structural analysis methods for select components for the Space Shuttle Main Engine (ref. 6).

## CONCLUSIONS

Probabilistic composite mechanics and probabilistic structural analysis methods can be developed to quantify the uncertainties at all levels of composite behavior. These methods use probabilistic concepts in conjunction with composite mechanics and structural mechanics. Application of these methods to quantify scatter in ply properties, laminate properties, laminate tailoring, and elementary structural analysis demonstrate that uncertainties in composite behavior and composite structural response can be probabilistically quantified.

## REFERENCES

1. Murthy, P. L. N. and Chamis, C. C.: Integrated Composite Analyzer (ICAN) - User's and Programmers's Manual. NASA TP 2515, 1986.
2. Stock, T. A., Bellini, P. X., Murthy, P. L. N., and Chamis, C. C.: A Probabilistic Approach to Composite Micromechanics. NASA TM 101366, 1988.
3. Chamis, C. C. and Stock, T. A.: Probabilistic Simulation of Uncertainties in Composite Uniaxial Strengths. NASA TM 102483, 1990.
4. Chamis, C. C.: Mechanics of Composite Materials: Past, Present and Future. NASA TM 100793, 1988.
5. Thanedar, P. B. and Chamis, C. C.: Composite Laminate Tailoring with Probabilistic Constraints and Loads. NASA TM 102515, 1990.
6. Chamis, C. C.: Probabilistic Structural Analysis Methods for Space Propulsion System Components. NASA TM 88965, 1986.

Table 1. Constituent Input Distribution Parameters for ICAN.

	Units	Distribution Type	Parameter 1	Parameter 2
$E_{f11}$	Msi	Normal	$\mu = 31.0$	$\sigma = 1.5$
$E_{f22}$	Msi	Normal	$\mu = 2.0$	$\sigma = .10$
$G_{f12}$	Msi	Normal	$\mu = 2.0$	$\sigma = .10$
$G_{f23}$	Msi	Normal	$\mu = 1.0$	$\sigma = .05$
$v_{f12}$	---	Normal	$\mu = .20$	$\sigma = .01$
$v_{f23}$	---	Normal	$\mu = .25$	$\sigma = .01$
$\alpha_{f11}$	ppm/°F	Normal	$\mu = .2$	$\sigma = .01$
$\alpha_{f22}$	ppm/°F	Normal	$\mu = .2$	$\sigma = .01$
$\rho_f$	lb/in <sup>3</sup>	Normal	$\mu = .063$	$\sigma = .003$
$N_f$	---	Fixed	$\mu = 10,000$	$\sigma = 0$
$d_f$	inches	Normal	$\mu = .003$	$\sigma = .00015$
$C_f$	BTU/lb/°F	Normal	$\mu = .20$	$\sigma = 0.01$
$K_{f11}$	*	Normal	$\mu = 580$	$\sigma = 2.9$
$K_{f22}$	*	Normal	$\mu = 58$	$\sigma = 2.9$
$K_{f33}$	*	Normal	$\mu = 58$	$\sigma = 2.9$
$S_{ft}$	ksi	Weibull	$\beta = 400$	$\alpha = 40$
$S_{fc}$	ksi	Weibull	$\beta = 400$	$\alpha = 40$
$E_m$	Msi	Normal	$\mu = .500$	$\sigma = .025$
$G_m$	Msi	Normal	$\mu = .185$	$\sigma = .009$
$v_m$	---	Normal	$\mu = .35$	$\sigma = .035$
$\alpha_m$	ppm/°F	Normal	$\mu = 36$	$\sigma = 4$
$\rho_m$	lb/in <sup>3</sup>	Normal	$\mu = .0443$	$\sigma = .0022$
$C_m$	BTU/lb/°F	Normal	$\mu = .25$	$\sigma = .0125$
$K_m$	*	Normal	$\mu = 1.25$	$\sigma = .06$
$S_{mf}$	ksi	Weibull	$\beta = 15$	$\alpha = 5$
$S_{mc}$	ksi	Weibull	$\beta = 35$	$\alpha = 20$
$S_{ms}$	ksi	Weibull	$\beta = 13$	$\alpha = 7$
$B_m$	in/in/1% mois	Normal	$\mu = .004$	$\sigma = .0002$
$D_m$	in <sup>2</sup> /sec	Normal	$\mu = .002$	$\sigma = .0001$

\* = BTU·in/hr/in<sup>2</sup>/°F

Table 2. Nonzero Sensitivity Parameters for  $E_{xx}$  from FPI at  $\pm 0.3\sigma$  Away From Mean of  $\mu=5.744$  Msi.

Primitive Variable	Sensitivity Parameter
$E_{f11}$	0.624
$E_{f22}$	0.260
$G_{f12}$	0.130
$G_{f23}$	0.0
$v_{f12}$	0.360E-4
$v_{f23}$	0.360E-4
$E_{mp}$	0.036
$G_{mp}$	0.060
$v_{mp}$	0.386E-3
$k_f$	0.778
$k_v$	0.0

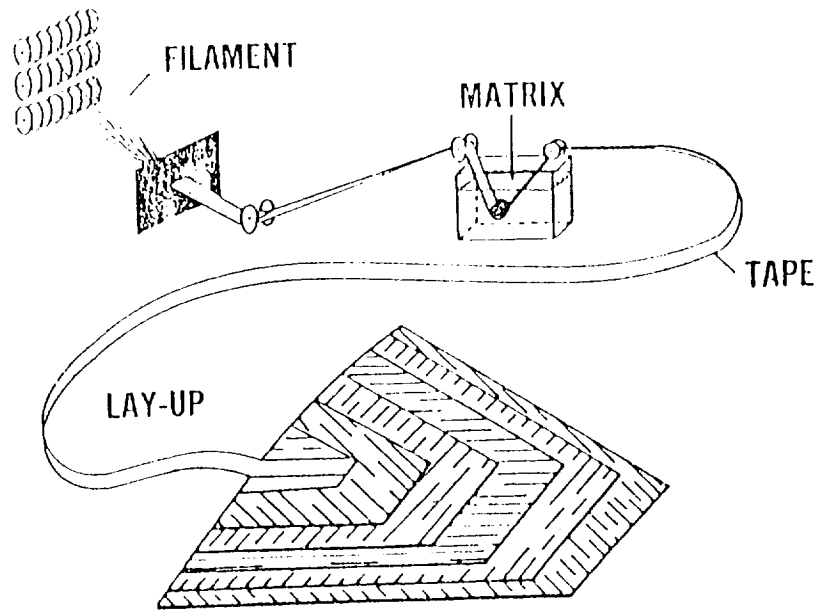


Figure 1. Schematic of Composite Fabrication Process

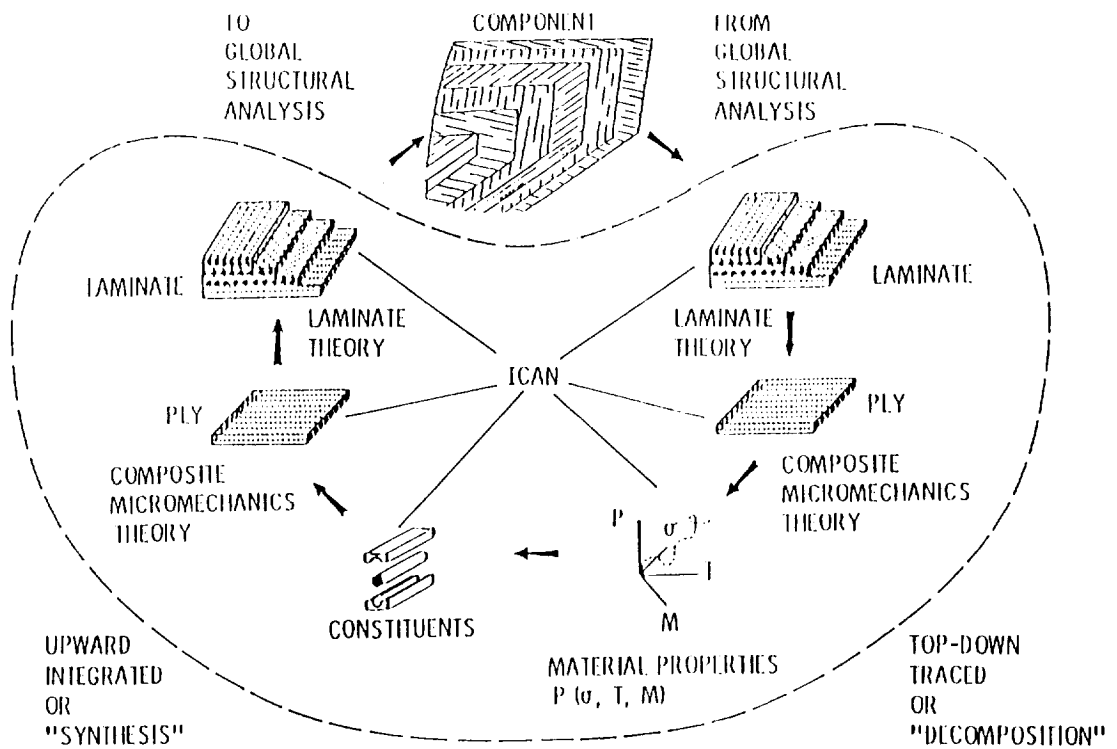


Figure 2. ICAN Intergrated Analyzer

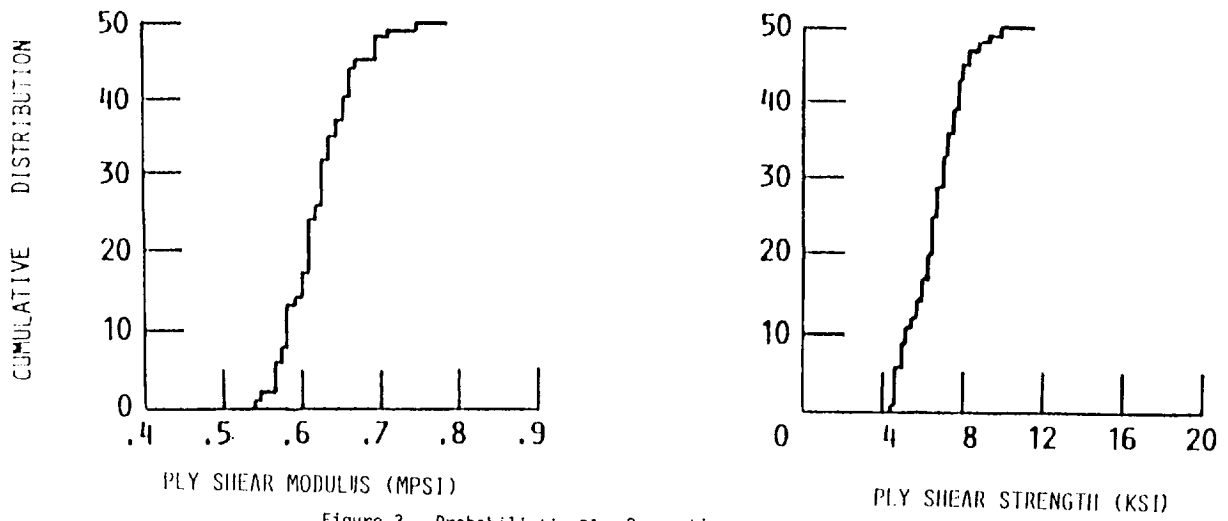


Figure 3. Probabilistic Ply Properties

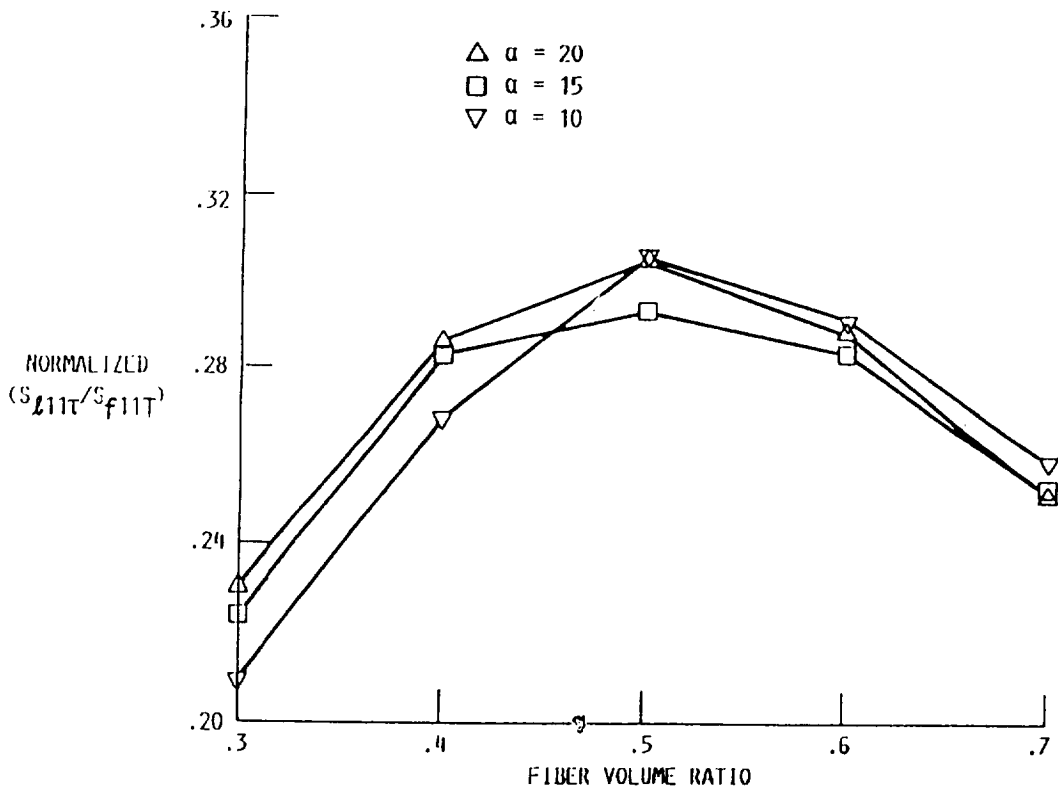


Figure 4. Probabilistic Ply Longitudinal Compression Strength

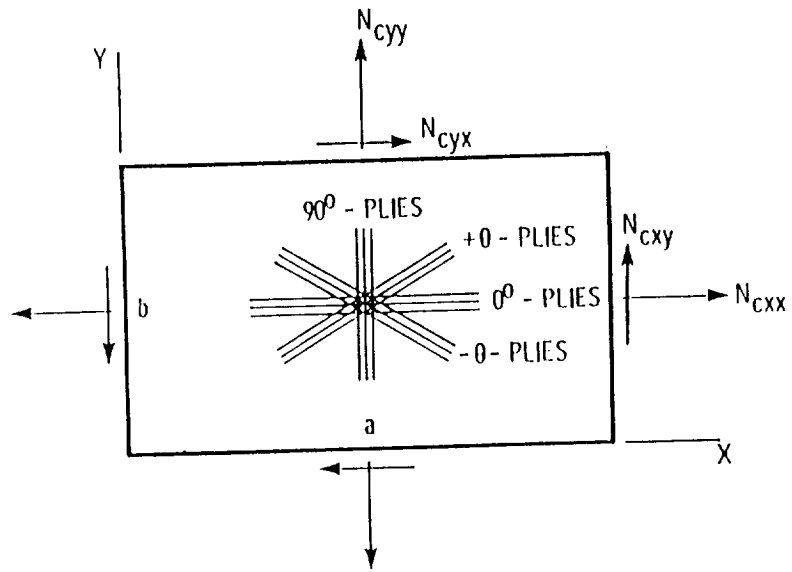


Figure 5. Schematic of Angleplyed Fiber Composite Panel Subjected To Combined In-Plane Loads

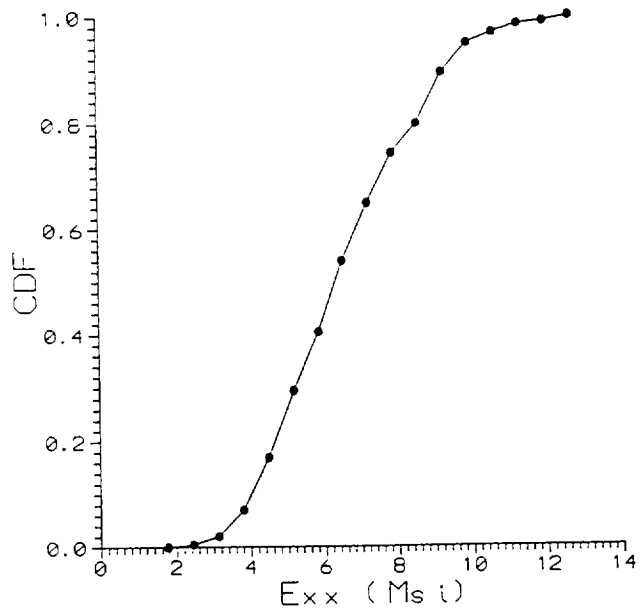


Figure 6. Laminate Modulus  $E_{cxx}$  -Quasi-isotropic Laminate (Graphite Fiber/Epoxy)



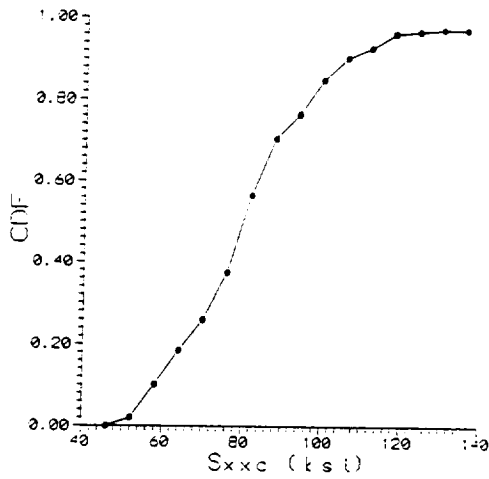


Figure 7. Longitudinal Compressive Strength of a Quasi-Isotropic Laminate (Graphite Fiber-Epoxy)

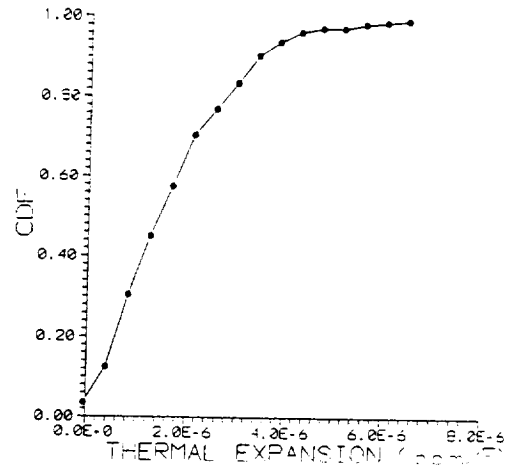


Figure 8. Thermal Expansion Coefficient of Quasi-Isotropic Laminate (Graphite Fiber-Epoxy)

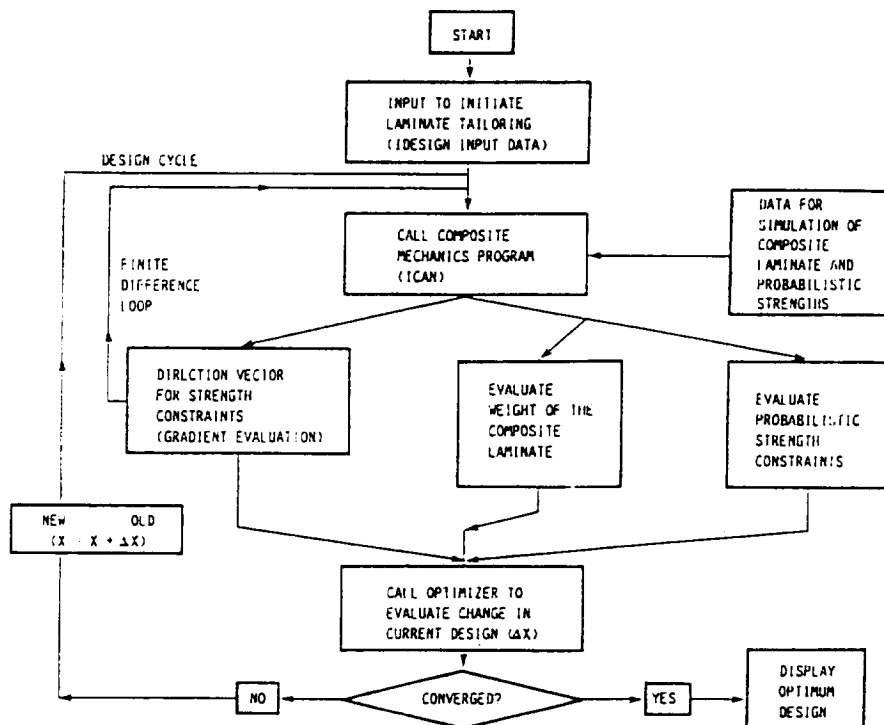


Figure 9. Flowchart For Composite Laminate Tailoring With Probabilistic Loads and/or Constraints.

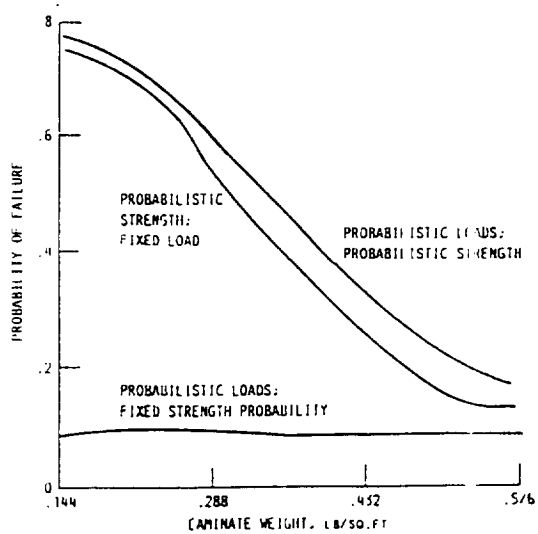


Figure 10. Laminate Optimum Weight For Specified Reliability.

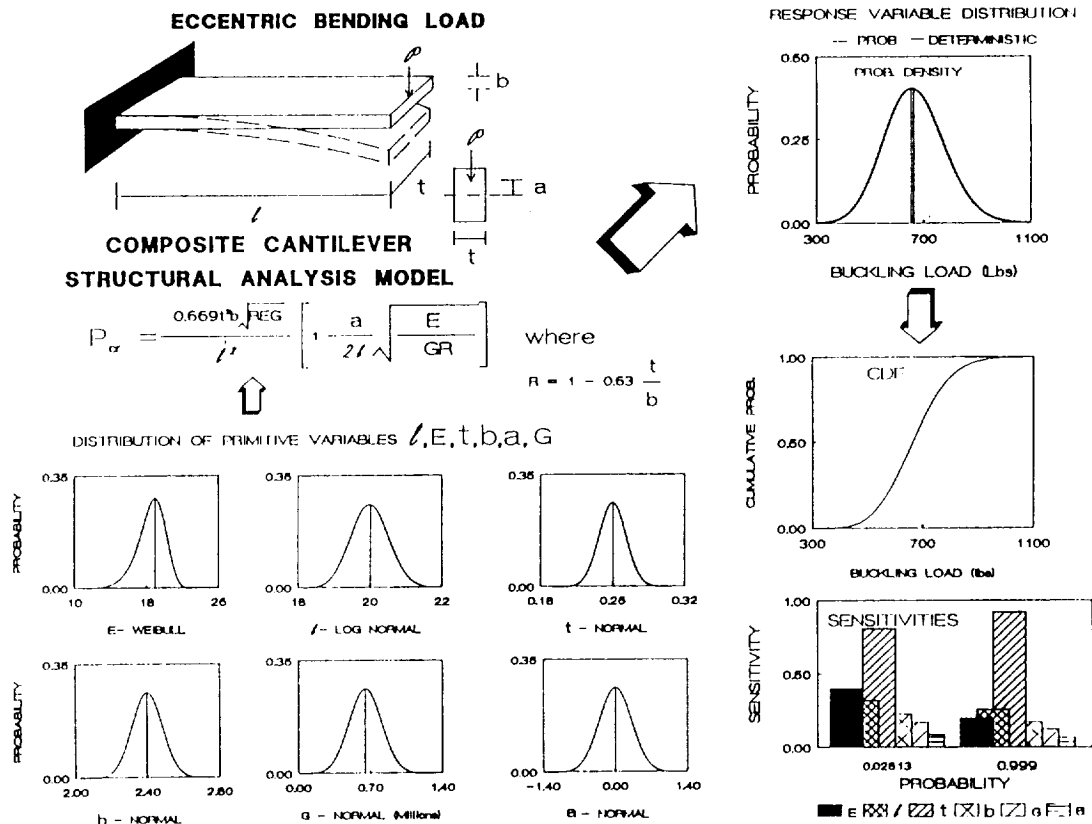


Figure 11. Probabilistic Structural Analysis.