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PROBABILISTIC STRUCTURAL ANALYSIS ALGORITHM DEVELOPMENT FOR COMPUTATIONAL EFFICIENCY

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The PSAM (Probabilistic Structural Analysis Methods) program, funded by NASA Lewis Research Center, is developing a probabilistic structural risk assessment capability for the SSME components. PSAM is currently in the seventh year of a two-phase, ten-year contract. An advanced probabilistic structural analysis software system, NESSUS (Numerical Evaluation of Stochastic Structures Under Stress), is being developed as part of the PSAM effort to accurately simulate stochastic structures operating under severe random loading conditions.

A central part of the NESSUS system is a finite element analysis (FEA) module. FEA is generally known to be computer intensive. Thus, the conventional Monte Carlo method, which requires a large number of simulations (i.e., a large number of deterministic computer runs), is too time-consuming to be practical for probabilistic FEA analysis. One of the major challenges in developing the NESSUS system is the development of the probabilistic algorithms that provide both efficiency and accuracy. The main probability algorithms developed and implemented in the NESSUS system are efficient, but approximate in nature. In the last six years, the algorithms have improved very significantly.

In probabilistic FEA analysis, a good index for measuring the computational efficiency is the number of deterministic solutions required for the user-selected performance function. To minimize this number, denoted as M, the first approach taken by PSAM was to generate a response surface over a "wide" range (say, $\pm$ 3 standard deviations for each random variable). Once the response surface is generated, a fast probability integration (FPI) algorithm [Ref. 1] can be used. In practice, when the number of random variables is not small, M might be too large, and the FEA part tends to dominate the total computational time. Moreover, the response surface approach does not generally provide sufficient accuracy unless an expensive iterative procedure is applied to update the response surface at focused regions [Ref. 2].

To improve the efficiency, the concept of FPI was applied directly to guide the FEA to develop a good approximate performance function. The basic concept is to use the initial information from the conventional mean value first order (MVFO) solutions to identify regions that are probabilistically more likely for the given performance function values, then move the FEA to these regions. MVFO requires $(n + 1)$ deterministic solutions, where n is the number of random variables, and provides approximate mean and standard deviation for the performance function. However, in probabilistic structural analysis, it is more desirable and often necessary to have knowledge in the whole distribution function (CDF). The advanced mean value (AMV) method was developed to provide the guidance for the FEA "move," and for efficiently generating performance CDF based on the MVFO solution [Refs. 3, 4]. The AMV method has been found to be quite effective for a wide variety of engineering problems. Further procedures requiring more M were also developed to improve the accuracy of the AMV method. In summary, $M = n + 7$ is believed to be the minimum number required to obtain reasonably accurate probabilistic output that includes the performance CDF, and the probabilistic sensitivity factors for the input random variables.

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In addition to the AMV-based methodology development, the probability algorithms have also been improved for problems with closed-form performance functions. The original FPI algorithm [Ref. 1] has proved to provide a good approximate solution. However, the drawback is that it tends to run into numerical problems when the input random variables are highly non-normal or have very large coefficient of variations. To solve the problem and to further improve the accuracy, the FPI algorithm has been enhanced recently by combining the linearization concept developed in the original FPI with the fast convolution method [Ref. 5]. The fast convolution theorem provides an exact CDF solution if the performance function can be expressed by a sum of random variables. The combined analysis procedure includes three steps: (1) establish a linear or quadratic performance function based on the AMV-based procedure, (2) transform the quadratic function into a linear function (if the function is quadratic), (3) apply the convolution theorem to compute the performance CDF. In the last step, a procedure based on the discrete, fast Fourier transform (FFT) technique has been developed to speed up the convolution calculations. In summary, the current NESSUS probabilistic analysis procedure combines the AMV-based method with the fast convolution method. The AMV-based procedure generates linear or quadratic performance functions, and the fast convolution method takes the polynomial performance functions and generates probability solutions.

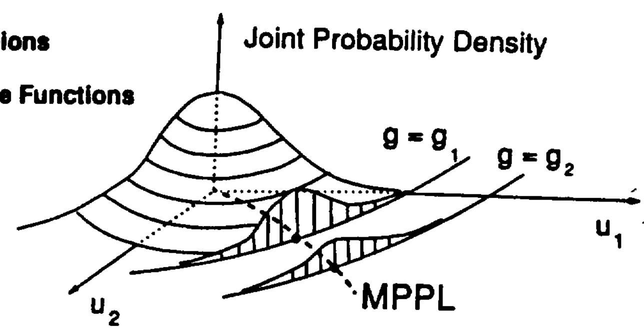
The PSAM program is moving into the area of system risk assessment. The methodology currently under development includes system reliability analysis that deals with multiple failure modes and multiple components. Here, the challenge is to accurately and efficiently evaluate probabilities associated with joint and conditional events. An efficient adaptive importance sampling method is being implemented in the NESSUS system. The method was originally developed for probabilistic rotordynamics analysis under a project funded by the NASA Marshall Space Flight Center [Ref. 6]. It is anticipated that other system reliability analysis tools will be developed and implemented in the NESSUS system based on a fault-tree type analysis framework.

References

1. Wu, Y.-T. and Wirsching P.H., "A New Algorithm for Structural Reliability Estimation," *Journal of Engineering Mechanics*, ASCE, Vol. 113, No. 9, pp. 1319-1336, September 1987.
2. Y.-T. Wu and P.H. Wirsching, "Advanced Reliability Methods for Fatigue Analysis," *Journal of Engineering Mechanics*, ASCE, Vol. 110, No. 4, April 1984.
3. Wu, Y.-T., O.H. Burnside, and T.A. Cruse, "Probabilistic Methods for Structural Response Analysis," *Computational Mechanics of Reliability Analysis*, W.K. Liu and T. Belytschko (eds.), Elmepress International, Ch. 7, 1989.
4. Wu, Y.-T., H.R. Millwater, and T.A. Cruse, "An Advanced Probabilistic Structural Analysis Method for Implicit Performance Functions," *AIAA Journal*, Vol. 28, No. 9, pp. 1663-1669, 1990.
5. Wu, Y.-T. and T.Y. Torng, "A Fast Convolution Procedure for Probabilistic Engineering Analysis," *Proceedings of the First International Symposium on Uncertainty Modeling and Analysis*, IEEE, December 1990.
6. Y.-T. Wu, T.Y. Torng, H.R. Millwater, A.F. Fossum, and M.H. Rheinfurth, "Probabilistic Methods for Rotordynamics Analysis," *Proceedings of the SAE Aerospace Atlantic 1991 Conference*, April 1991, (to appear).

Establish CDF Using Fast Probability Integration Algorithm

- Define Limit-State
- Approximate Performance Function At One or More Probability Significant Regions
- Compute Probability Based on Approximate Functions



ADVANCED MEAN VALUE (AMV) METHOD

- **Conventional Mean Value First-Order (MVFO) method**

First-order Taylor's series expansion at mean values:

$$Z = a_o + \sum a_i X_i (\equiv Z_1)$$

Valid for small standard deviations.

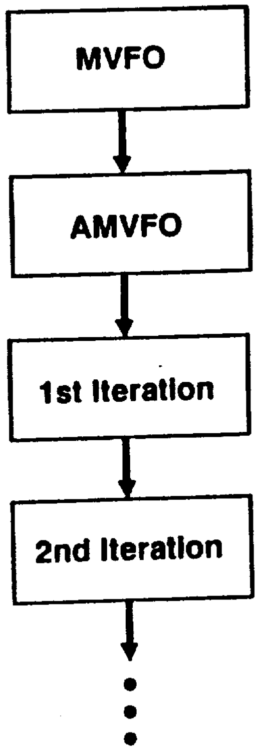
- **Advanced Mean Value (AMV) First Order Method**

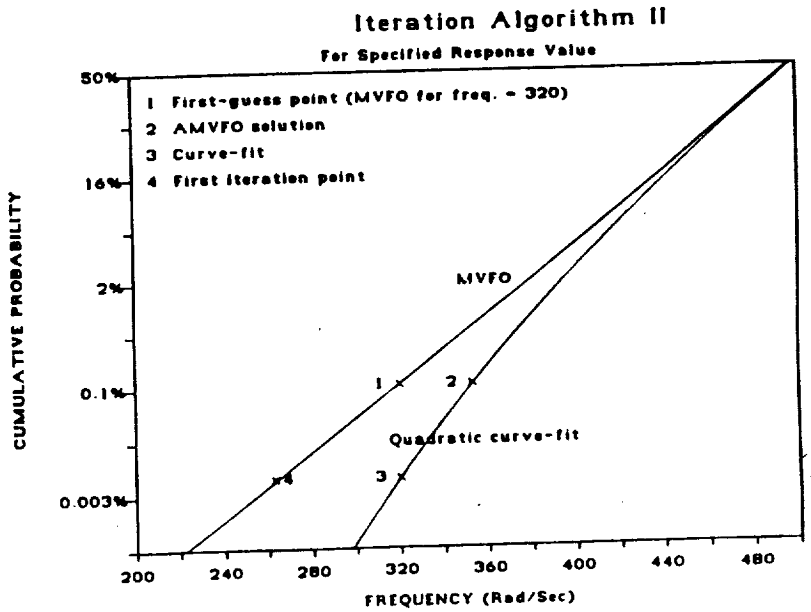
$$Z^* = Z_1 + H(Z_1)$$

Features:

- $H(Z_1)$ introduced to minimize truncation error.
- Iteration procedure available to find $H(Z_1)$.
- Can be used to detect non-monotonic functions.

AMV - Based Iteration Procedure

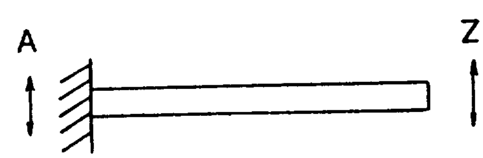




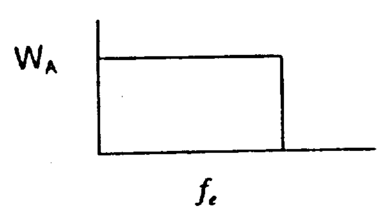
AMV EXAMPLE: RANDOM VIBRATION

Monotonic Performance Function

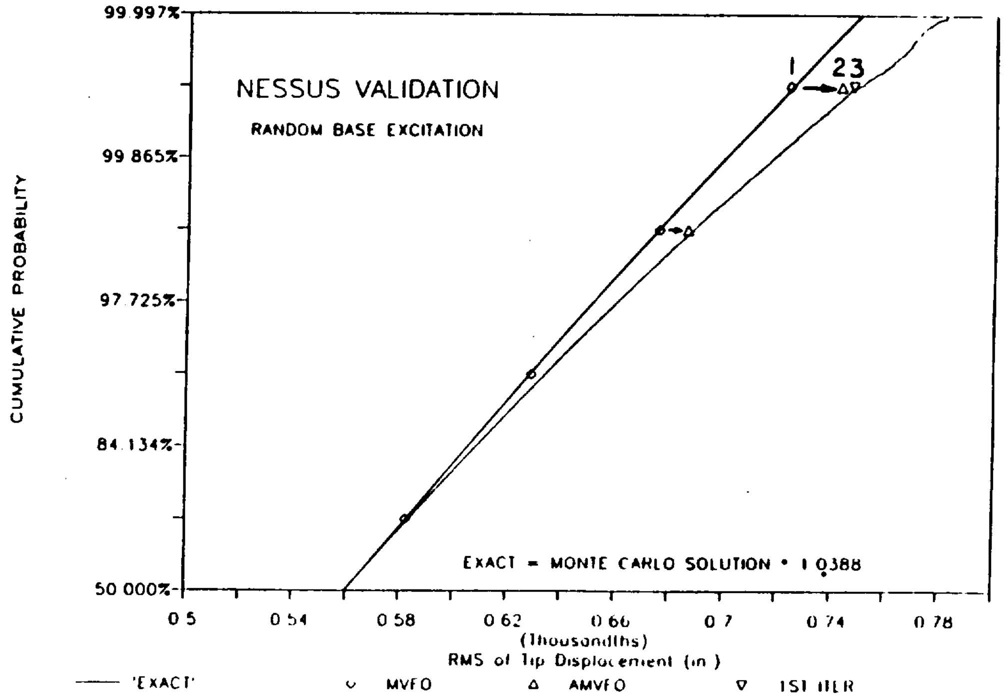
$$RMS = \sqrt{\frac{1.707 L^6 W_A \rho^{1.5}}{E^{1.5} t^3 \xi}}$$



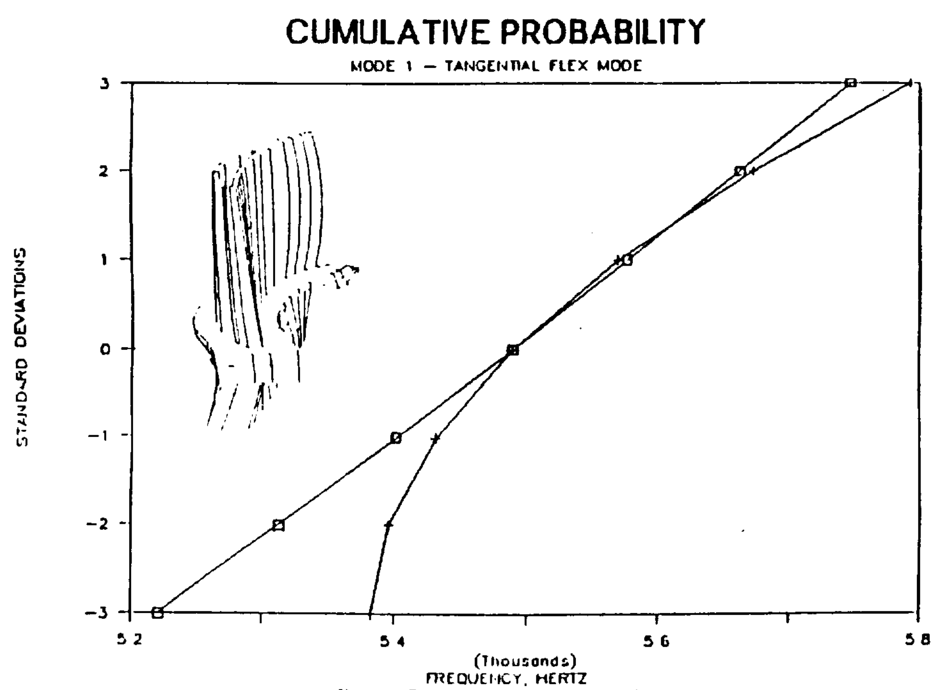
Variable	Mean	COV	Distribution
E	10E+6 lb/in^2	0.03	Lognormal
L	20.0 in.	0.01	Normal
t	0.98 in.	0.01	Normal
ξ	0.05	0.10	Lognormal
W_A	1.0 in/sec^2-rad	0.10	Lognormal
ρ	2.5E-4 lb-sec^2/in^4	0.02	Normal



AMV EXAMPLE: RANDOM VIBRATION



Turbine Blade Verification Example



Fast Convolution Procedure

1. Dependent Non-normal RVs to Independent Std. Normal RVs.
2. Find Most Probable Point and Construct Second-order Approx.
3. Eliminate Product Terms by Orthogonal Transformation.
4. Transform to Linear Polynomial.
5. Apply Convolution Theorem.

Fast Convolution

- SUM OF INDEPENDENT RANDOM VARIABLES

$$Z = X_1 + X_2 + \ldots + X_i + \ldots + X_n$$

- CHARACTERISTIC FUNCTION

$$\Phi(\omega) = \int_{-\infty}^{\infty} f(x) e^{i\omega x} dx$$

$$\Phi_Z(\omega) = \Phi_{X_1}(\omega)\Phi_{X_2}(\omega)\Phi_{X_n}(\omega)$$

- USE FAST FOURIER TRANSFORM TO COMPUTE $\Phi(\omega)$
- USE INVERSE FTT TO COMPUTE PDF OF Z
- FOR <u>NON-LINEAR Z</u>, USE <u>MPP</u>, <u>QUADRATIC APPROXIMATION</u> AND <u>LINEARIZATION</u>

Linearization

$$g_2(\mathbf{u}) = a_o + \sum_{i=1}^{n} a_i(u_i - u_i^*) + \sum_{i=1}^{n} b_i(u_i - u_i^*)^2 + \sum_{i=1}^{n}\sum_{j=1}^{i-1} c_{ij}(u_i - u_i^*)(u_j - u_j^*)$$

- **QUADRATIC APPROXIMATION AND LINEARIZATION**

$$g(X) \cong a_o + \sum_{i=1}^{n} a_i x_i + b_i x_i^2$$

$$c_o = a_o - \frac{1}{4}\sum_{i=1}^{n}\frac{a_i^2}{b_i}$$

$$g(Y) = c_o + \sum_{i=1}^{n} b_i Y_i$$

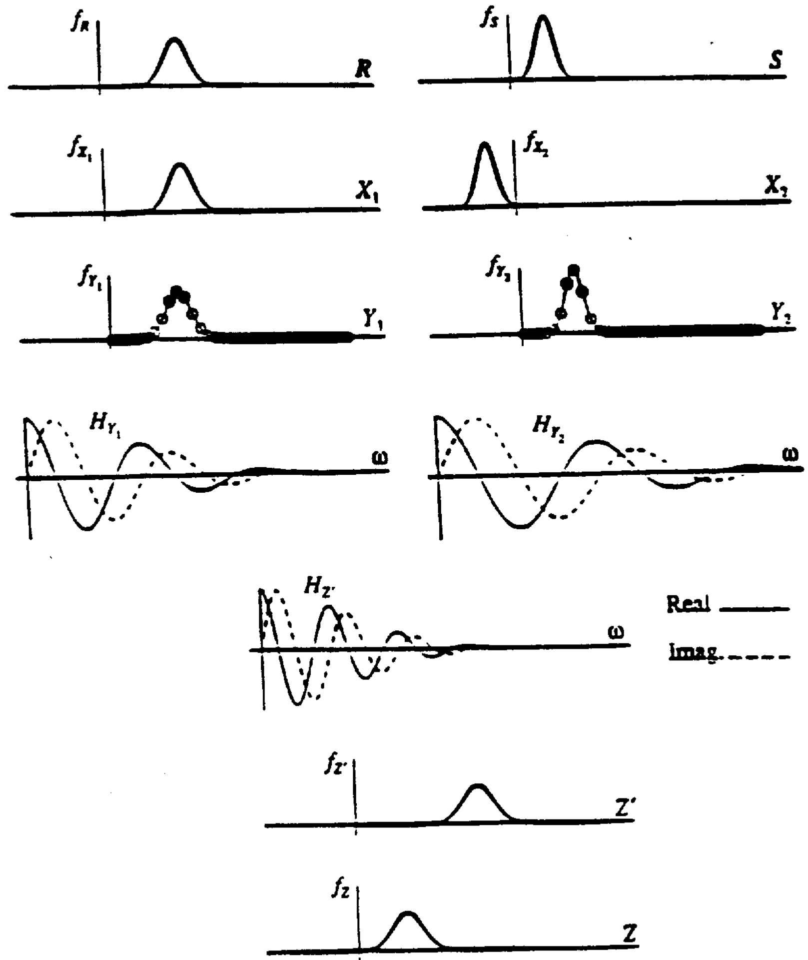
$$Y_i = \left[X_i + \frac{a_i}{2b_i}\right]^2 = [X_i + A_i]^2$$

- **LOG-TRANSFORMATION**

$$g(X) \cong a_o + \sum_{i=1}^{n} a_i \ln x_i + b_i(\ln x_i)^2$$

Fast Convolution Example

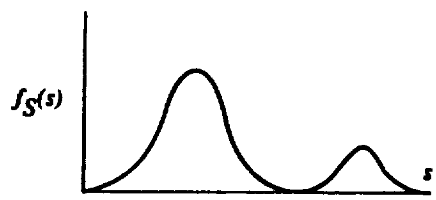
Z = R - S



Fast Convolution Validation Problem

Random variable S has bi-modal pdf:

$$f_S(s) = \left(\frac{a}{b}\right)\phi\left(\frac{s-\mu_1}{\sigma_1}\right) + \left(\frac{b-a}{b}\right)\phi\left(\frac{s-\mu_2}{\sigma_2}\right)$$



Limit state function:

$$g = R - S = 0$$

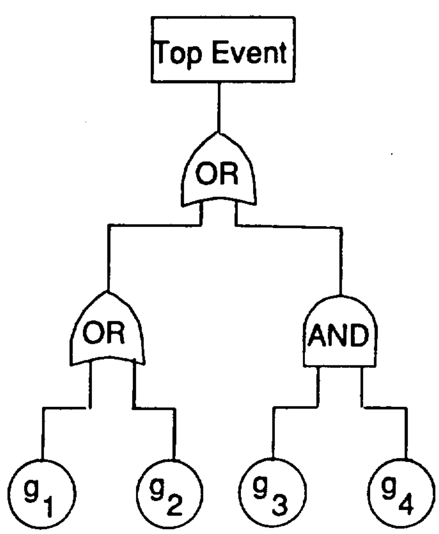
Strength: R~Lognormal~(20., 5.)

Stress: S~Bi-modal~$(\mu_1, \mu_2, \sigma_1, \sigma_2)$ = **(10, 2, 40, 2)**

Probability of failure:

a	b	Exact	Previous Method	Improved Method
19	20	6.331E-2	8.29E-2	6.285E-2
99	100	2.307E-2	Numerical Problem	2.347E-2

SYSTEM RELIABILITY ANALYSIS METHODOLOGY

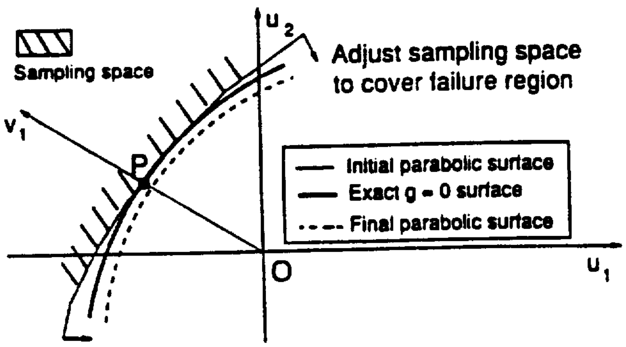


→ COMPUTE:

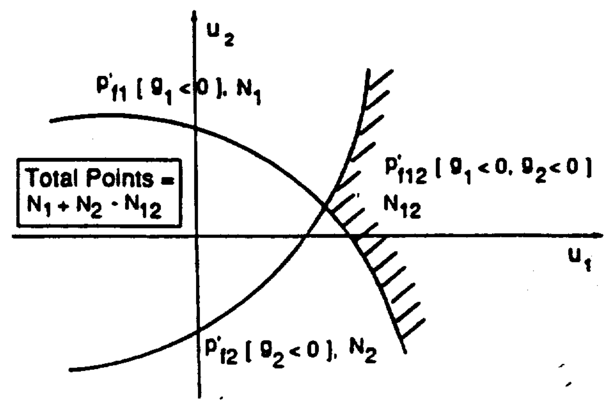
$P[(g_1 < 0) \cup (g_2 < 0)$
$\cup ((g_3 < 0) \cap (g_4 < 0))]$

ADAPTIVE IMPORTANCE SAMPLING METHOD FOR SYSTEM RELIABILITY ANALYSIS

METHODOLOGY



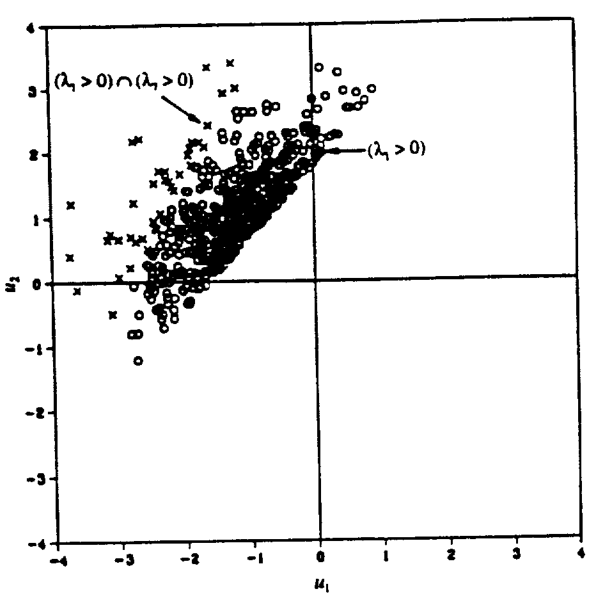
The Concept of the Curvature-Based Adaptive Importance Sampling Method



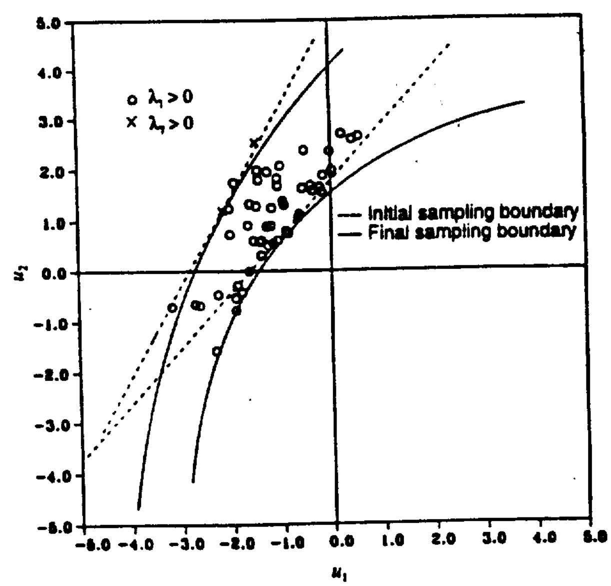
The Concept of Generating Importance Sampling Points for Multiple Limit States

ADAPTIVE IMPORTANCE SAMPLING METHOD FOR SYSTEM RELIABILITY ANALYSIS

ROTORDYNAMICS EXAMPLE



Probability of Instability Sampling Points for $\lambda_1 > 0$ and $\lambda_r > 0$ in the u-Space (5000 Monte Carlo Samples)



Curvature-Based Importance Sampling Points for $\lambda_1 > 0$ and $\lambda_r > 0$ in the u-Space