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PROBABILISTIC FRACTURE FINITE ELEMENTS

W.K. Liu, T. Belytschko, and Y.J. Lua
Northwestern University
Evanston, Illinois 60201

The Probabilistic Fracture Mechanics (PFM) is a promising method for estimating the fatigue life and inspection cycles for mechanical and structural components. Since failure process depends on random defects and may be influenced by random variables, the probabilistic aspects of the problem should be taken into account. The main sources of uncertainty are attributed to the randomness in defect sizes, growth law parameter, loads, and material properties.

The Probabilistic Finite Element Method (PFEM), which is based on second-moment analysis developed by Liu et al. [1-2], has proved to be a promising, practical approach to handle problems with uncertainties. As the PFEM provides a powerful computational tool to determine first- and second- moment of random parameters, the second-moment reliability methods can be easily combined with PFEM to obtain measures of the reliability of the structural system. The fusion of the PFEM and reliability analysis for brittle fracture and fatigue has been reported by Besterfield et al. [3-4]. In order to model the crack-tip singularity, an enriched element is employed. As the stress intensity factors are directly included as unknowns along with the nodal displacements, this approach simplifies the development of the sensitivity analyses which are required in the first-order reliability methods. In addition to the uncertainties in loads, material properties and component geometry, the randomness in the crack length, crack location and orientation and fatigue crack growth parameters are also considered. Based on the first-order reliability analysis, a constrained optimization problem to calculate the reliability index for brittle fracture and fatigue has been formulated. The performance of this method is demonstrated through the Mode I fatigue crack growth reliability analysis (see Figs.1-2).

The method is also being applied to fatigue crack growth. Due to the combined effects of external loading, unsymmetrical component geometry and crack geometry, cracks rarely grow in a straight line. In a finite element method, a scheme for remeshing is required during the crack growth. In order to avoid the numerical complexity in remeshing and provide highly accurate solutions in both crack path and the relation between the crack length and stress intensity factors, an alternative approach based on the Boundary Integral Equation Method (BIEM) is developed for a multi-region embedded with a crack. The applicability of this method is demonstrated by the following two problems: 1) a rectangular plate with a circular hole under simple tension, where a crack emanates from the hole at the maximum stress point (see Figs 3a-3b), 2) a rectangular plate with an inclined edge crack under simple tension (see Figs 4a-4b). The first-order reliability analysis for a curved fatigue crack emanated from a hole is also formulated by a constrained optimization problem. The effects of uncertainty on the crack path and fatigue life are investigated. The Monte Carlo simulation is employed to check the accuracy of the first-order reliability analysis.

Uncertainties in the material properties of advanced materials such as polycrystalline alloys, ceramics and composite are commonly observed from experimental tests. This is mainly attributed to intrinsic microcracks, which are randomly distributed as a result of the applied load and the residual stress. In order to quantify the inherent statistical distribution, a stochastic damage model (see Fig.5) has been proposed most recently by Lua et al. [5-6]. The model, based on macrocrack-microcrack interaction, incorporates uncertainties in locations, orientations and numbers of microcracks. Due to the high concentration of microcracks near the macro-tip, a higher order analysis based on traction boundary integral equations is formulated first for an arbitrary array of cracks. The effects of uncertainties in locations, orientations and numbers of microcracks at a [illegible] macro-up are analyzed quantitatively in [5-6] by using the BIEM in conjunction with the computer simulation of the random microcrack array. The statistical nature of the fracture toughness is compared to the Neville function in Figs. 6a and 6b for both dilute and highly concentrated microcracks. This model can also serve as a semi-empirical tool for predicting the fracture toughness based on a statistical characterization of the geometric parameters of microcracks, which can be obtained experimentally.

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[5] Y. J. Lua, W. K. Liu and T. Belytschko, A stochastic damage model for the rupture prediction of a multi-phase solid: Part I: Parametric studies, submitted to *Int. J. Fracture Mech.*.

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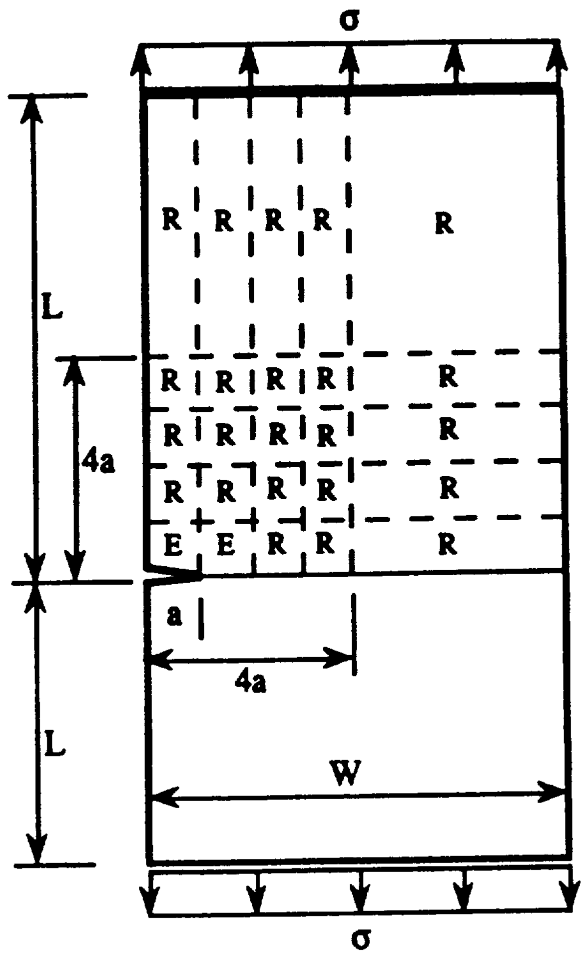

Parameter	Mean	Standard Deviation	Percent
Length (L)	10.0 in	0.0	0.0
Width (W)	4.0 in	0.0	0.0
Thickness (t)	1.0 in	0.0	0.0
Young's Modulus (E)	30,000.0 ksi	0.0	0.0
Poisson's Ratio (ν)	0.3	0.0	0.0
Applied Stress (t)	12.0 ksi	3.0 ksi	25.0
Initial Crack Length (a_1)	0.01 in	0.01 in	100.0
Final crack Length (a_f)	0.1 in	0.01 in	10.0
Fatigue Parameter (D)	1.0×10^{-10}	3.0×10^{-11}	30.0
Fatigue Parameter (n)	3.25	0.08	2.5
Note: 1 in. = 0.0254 m 1 ksi = 6.89 kPa			

Figure 1. Problem Statement for Single Edge-Cracked Beam with an Applied Load.

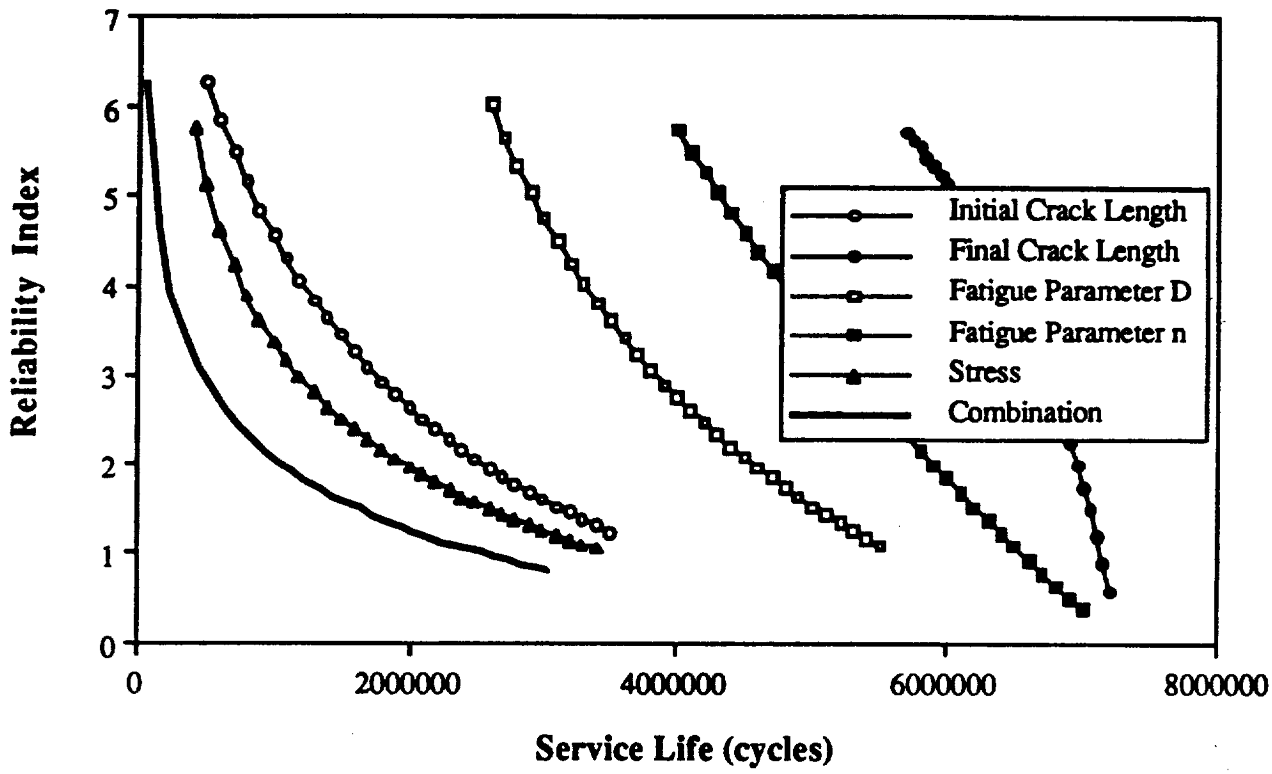

Figure 2. Reliability Index for the FEM Solution Comparing the Effects of Uncertainty in the Individual Variables and their Combined Effect.

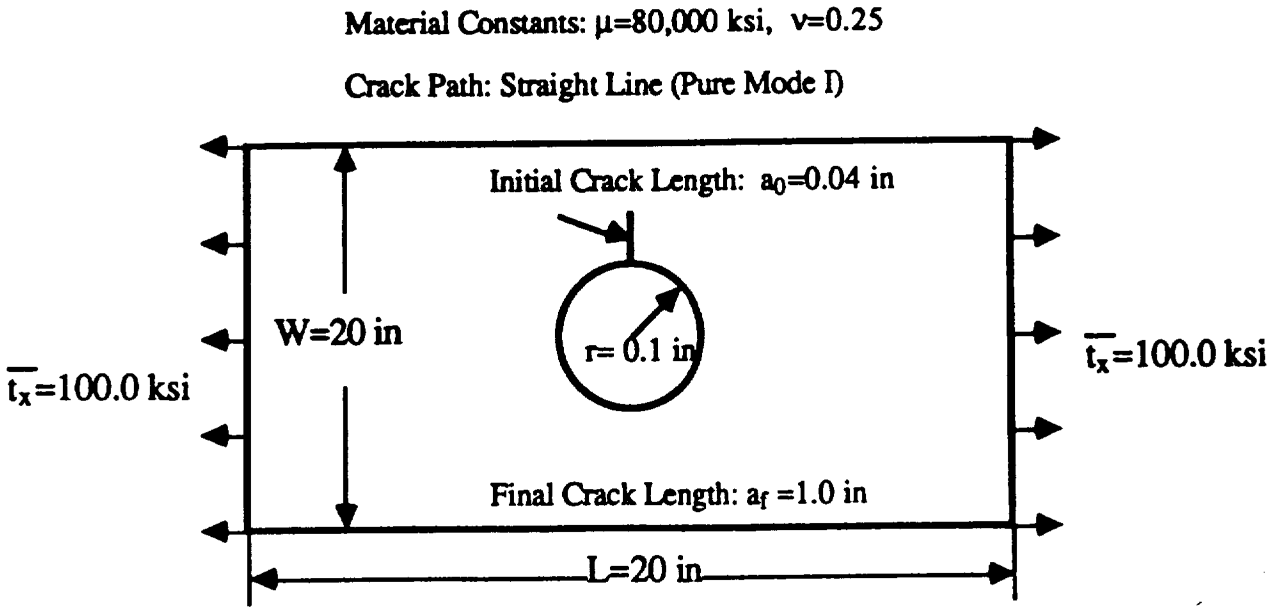

Figure 3a. Problem Statement for a Rectangular Plate with a Crack Emanating from a Hole under Uniaxial Tension.

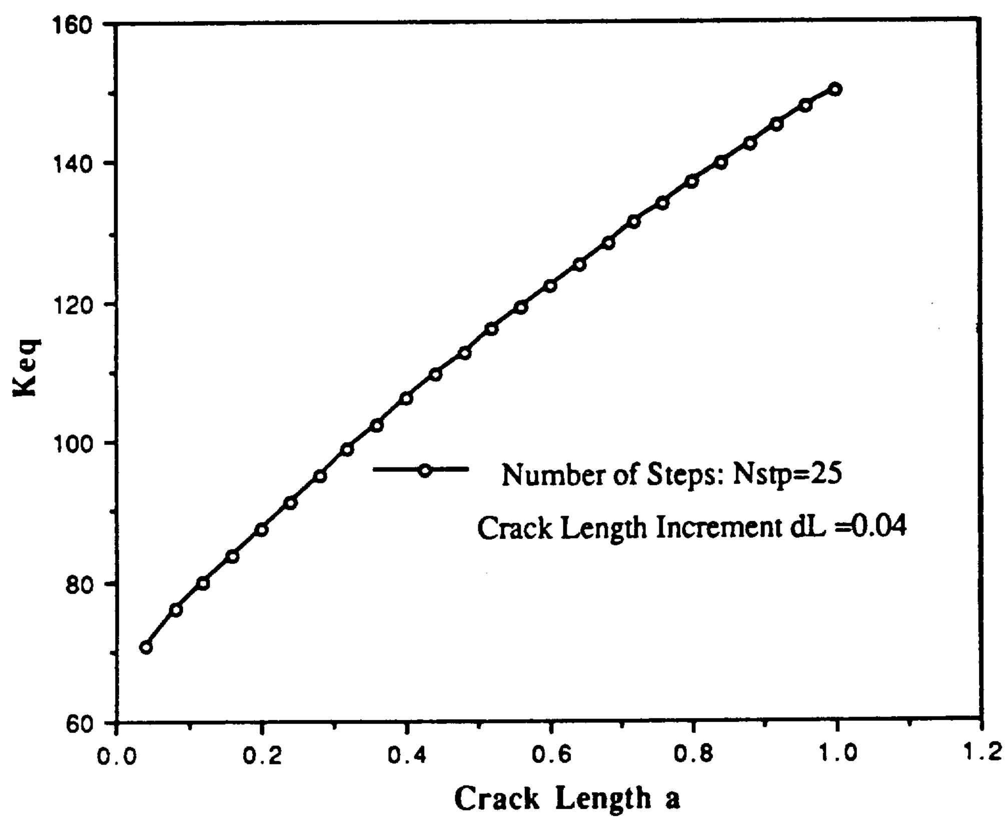

Figure 3b. Relation between the Equivalent Stress Intensity Factor Keq and the Crack Length for the Problem of a Rectangular Plate with a Crack Emanating from a Hole under Uniaxial Tension.

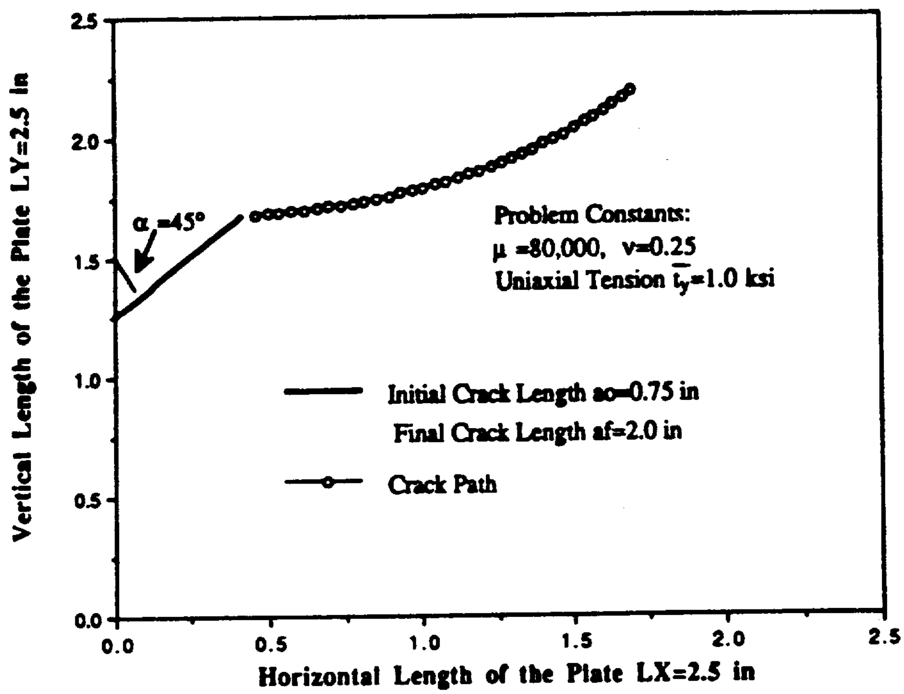

Figure 4a. Crack Path for an Inclined Edge-Cracked Plate under Uniaxial Tension.

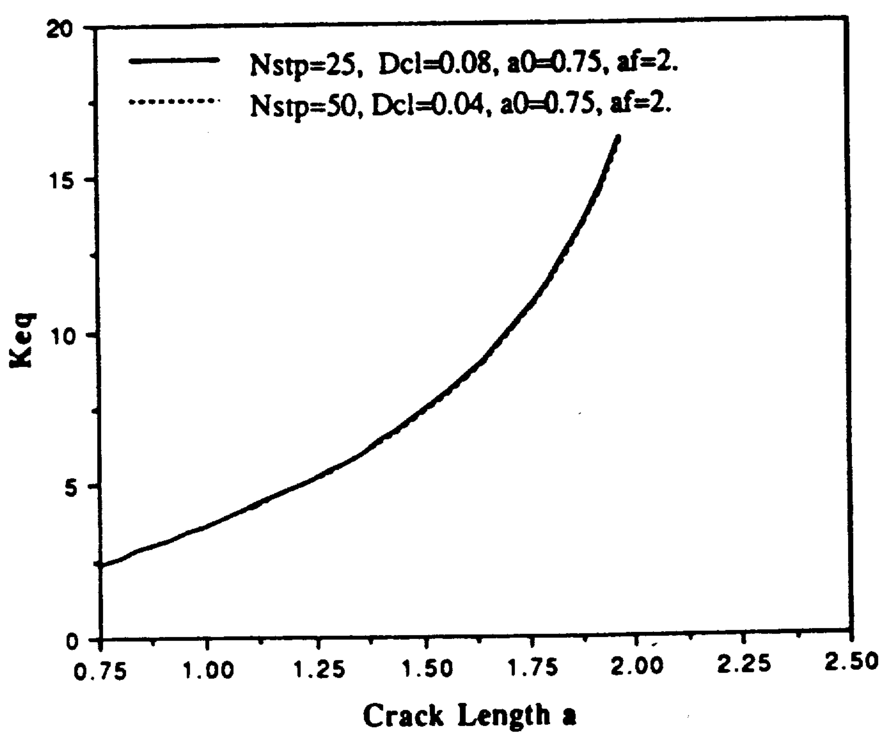

Figure 4b. Relation between the Equivalent Stress Intensity Factor Keq and Crack Length a for an Inclined Edge-Cracked Plate under Uniaxial Tension.

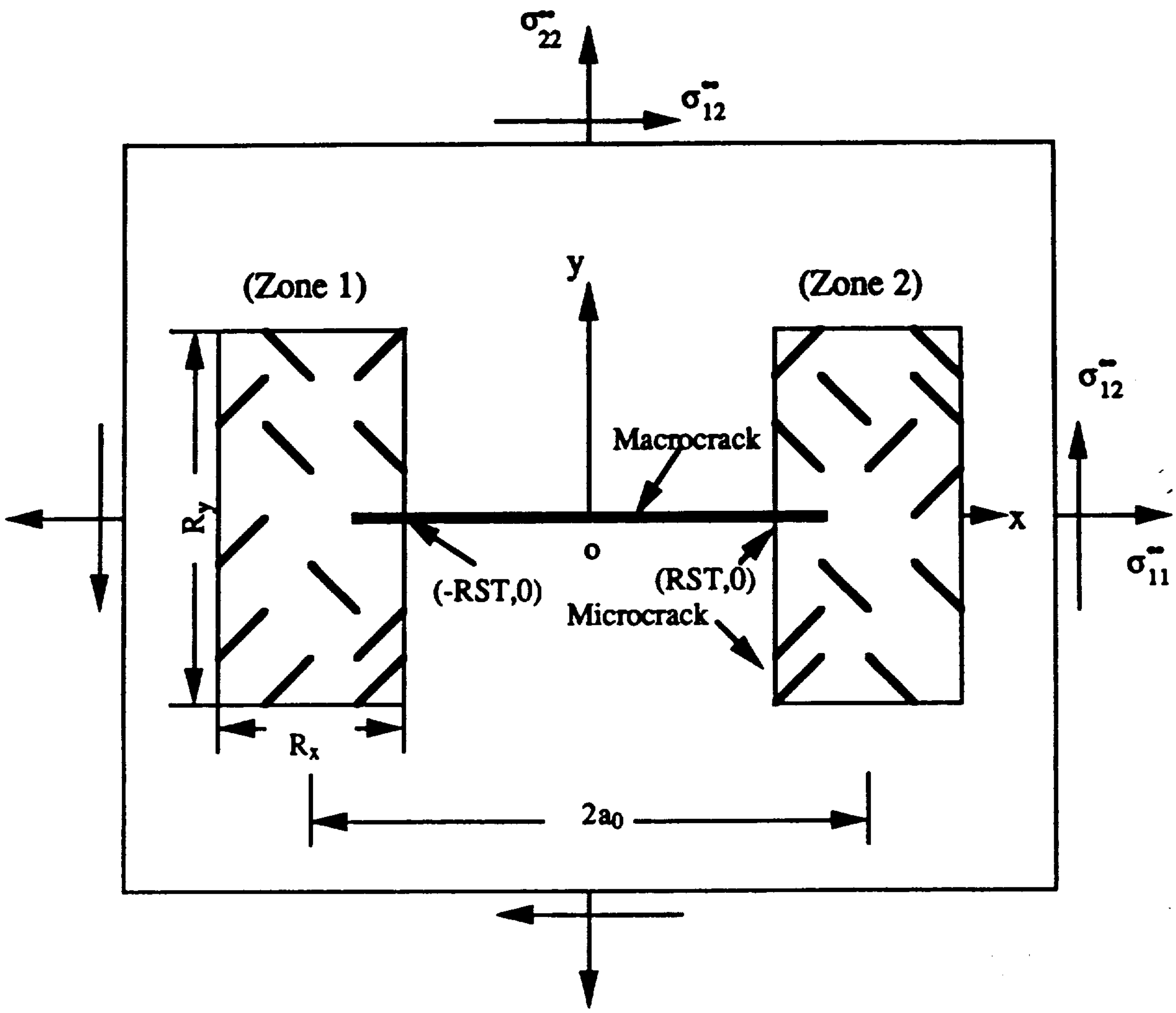

Figure 5. A Damage Saturation Model for a Brittle Multi-Phase Solid.

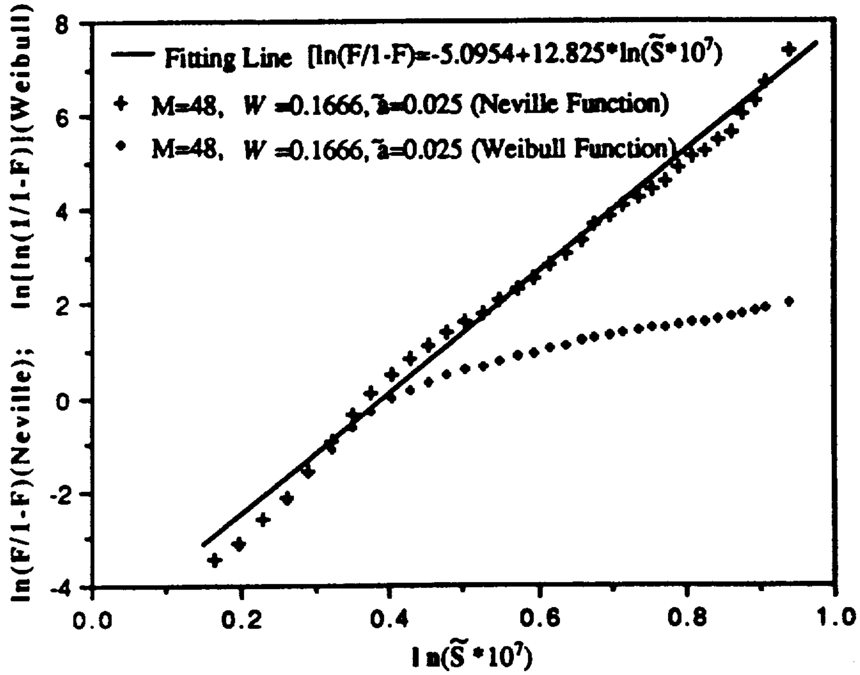

Figure 6a. Neville and Weibull Plots of the Intensity of Strain Energy Density $\tilde{S}$ for Low Microcrack Concentration $W = 0.1666$.

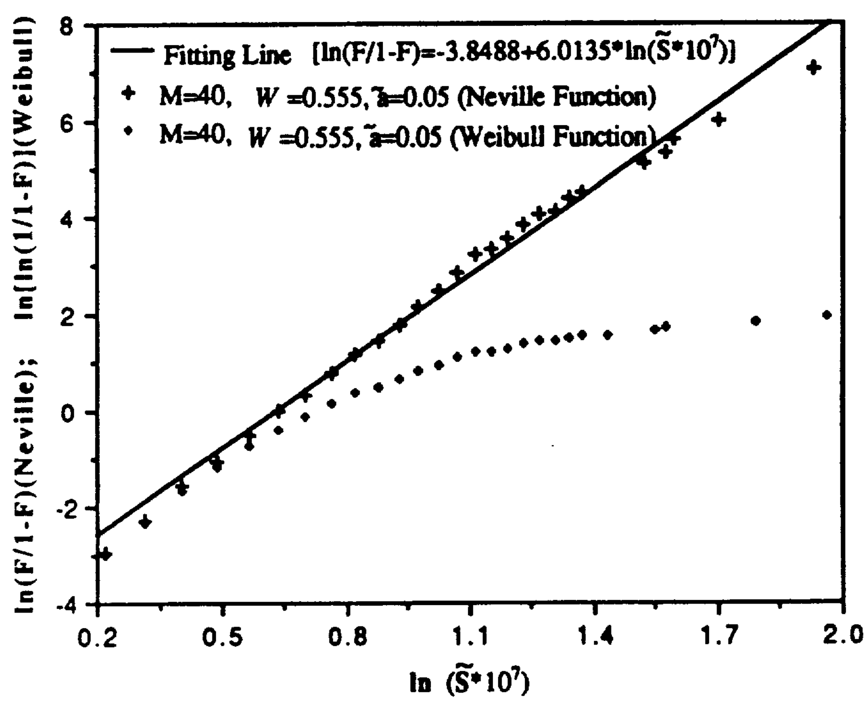

Figure 6b. Neville and Weibull Plots of the Intensity of Strain Energy Density $\tilde{S}$ for Microcrack Concentration $W = 0.5555$.