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THE DRAWING OF EXPERIMENTAL CURVES.

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INTRODUCTION.

The occasion often arises in research work when it is necessary to determine the location of a line or surface from experimental data. These data are usually in the form of a series of ordinates measured from a base line or plane. To locate an unknown curve with exactness would of course require an infinite number of ordinates. What we desire to know practically, however, is the number of ordinates required to obtain a certain probable precision in drawing a line or surface. It is inefficient to draw the curve more exactly than the precision of the ordinates themselves and there is undoubtedly a great deal of time and money being wasted by experimenters because this fact is not taken into consideration. As this is a question of great importance in the determination of the pressure distribution over aerofoils or bodies it is felt that a discussion of the problem will be of interest.

THE BEST REPRESENTATIVE CURVE.

The first consideration is the method to be used in drawing the most probable curve through a given set of points, assuming that there are no discontinuities. Considering the points given in Fig. 1, it is obvious that we may draw through them an infinite number of curves, but experience tells us that we can approach the most probable curve by several methods. The simplest is to draw straight lines between the points (curve a), a method which may be used legitimately only to show the sequence of points when the vertical scale is exaggerated as, for example, when plotting daily averages of temperature where the curve can not be considered as a representation of a continuous physical variation. Another common example of this type of plotting is the daily fluctuation in stock prices. When, however, this straight line method is used to connect points which are supposedly taken from a smooth curve it shows that the experimenter does not fully understand the subject. For example, suppose a series of temperature readings of a heating and cooling furnace is taken. The temperature obviously varies in an even and regular manner whereas the straight line plot would indicate an instantaneous change from heating to cooling at the maximum temperature.

The next method of connecting the points (curve b, Fig. 1) consists in passing through them a perfectly elastic curve, as is usually done by ship draftsmen. This method undoubtedly gives the most representative line if the curvature is slight compared with the spacing of the points, or if the curvature is uniform. In a case like Fig. 1, however, curve b is not the best representative curve as the maximum is too flat and there is an apparent oscillation shown which does not really exist.

Curve c in Fig. 1 is intended to be the best representative curve through the given points. It is seen to lie nearly midway between the other curves. Unfortunately there can be no rule laid down for drawing truly representative curves, their careful execution being entirely dependent on a trained eye, which can be acquired only by long experience.

AREA UNDER CURVES.

The practical problem that comes up in the investigation of pressure distribution, consists in finding out how many ordinates are required to give the total load on the surface within a prescribed degree of accuracy. Let us take the typical curve shown in Fig. 2 and determine with what accuracy it can be reproduced, using respectively 3, 5, 9, and 17 evenly spaced ordinates. As stated before it is impossible to lay down a rule for locating the best representative curve, so the method used in this case to determine the most probable curve through any given number of ordinates consisted in giving independently to each of four men (a, b, c and d), two draftsmen and two engineers, all familiar with curve drawing, a set of three points. After each one had drawn their curve through these points, two more were supplied and the curves were redrawn; and so on up to 17 ordinates. The area under these curves was found by means of a planimeter, giving the following results:

Number of Ordinates	Percentage deviation in area from true curve.					
	a	b	c	d	mean	max.
3	-38	-55	-37	-38	42	55
5	+ 5	- 8	+ 4	- 3	5	8
9	+ 2	- 8	+ 3	- 1	3	8
17	- 2	0	+ 3	+ 1	1	3

The mean deviation taken without consideration of the sign is plotted in Fig. 3 and takes the form of a rectangular hyperbola. It seems reasonable to suppose that the error in determining a curve with a given number of ordinates will be inversely proportional to the minimum radius of any part of the curve. As the curve in question had a minimum radius of 1/32 of the base line there are shown the values for curves with a minimum radius of 1/16 and 1/64 of the base line. This assumption may not be strictly true, but it seems to hold for the few cases which it has been applied to.

It has been assumed in the previous discussion that the ordinates were evenly distributed along the base line. If we know, as is usually the case, the general shape of the curve to be determined, it is possible to greatly increase the accuracy with a given number of ordinates by using an uneven spacing. To obtain an even spacing of points along the curve the spacing on the base line must obviously be proportional to the cosine of the slope angle. As the straight portion of the curve however can be determined with greater precision than the curved parts for a given spacing of the points along the curve, it is also necessary to space the ordinates in proportion to the radius of curvature. The final spacing of the coordinates along the base line should then be proportional to $r \cos \theta$, where r is a radius of curvature and θ is the angle of slope at any point.

As the exact shape of any curve is not known beforehand the ordinates may usually be placed sufficiently closely by eye

Take for example, a typical wing pressure curve as shown in Fig. 4, for which we desire to obtain the area with a probable precision of 1%. The problem is to find how many ordinates we need and how they should be spaced. Referring to Fig. 3 it is seen that 10 stations are required when the minimum radius of curvature is 1/16 of the base line, as occurs in this case. By distributing these stations as outlined above we should be able to halve the number of ordinates required. The final spacing of the ordinates is shown in Fig. 4.

In order to test out these conclusions a set of the points were given to each of four men who drew curves through them. The areas under the resulting curves were found and the deviation from the true area was as follows:

Case	a	b	c	d	mean	max.
Percentage Deviation.	0.0	-0.5	-0.7	-1.3	.6	1.3

The mean deviation is .6% and the maximum 1.3%, which checks very well with the predicted precision. An uneven spacing of the ordinates should only be resorted to when the general form of the curve is certain, for if it is applied to an unknown curve there is a probability of introducing a much larger error than if the spacing had been even.

As a practical example let us consider the case of finding the average load on the rib used in an airplane tail. We will assume that the form of the pressure curve is unknown and the the minimum radius will be greater than 1/32 of the length of the rib. The probable error in measuring the pressure at any point is 5% of the average pressure. Referring to Fig. 3, with an even distribution of the ordinates it is seen that 7 ordinates are sufficient to draw the curve so that its area will be precise to 5%. Let us take a similar case of the load on a wing rib where we know approximately the shape of the curve and that the minimum radius is greater than 1/16 of the length of the rib. Again referring to Fig. 3 it is seen that 3 ordinates are sufficient for this case.

THE SHAPE OF CURVES.

As shown in the preceding paragraphs it is remarkable how closely the area under a given curve can be determined by using a few ordinates. When it comes to the question of the exact form of the curve however we require a larger number of ordinates as we may have one portion of the curve too low and another too high and yet in determining the area these errors will cancel out. There is especial liability to error where the slope of the curve is steep. From the curve in Fig. 2 let us determine the error at any point in percentage of the mean ordinate from the same series of curves used previously for areas. The results are tabulated below.

Number of ordinates	a	b	c	d	mean	[illegible]
3	155	170	150	150	156	170
5	130	150	135	136	138	150
9	40	80	28	36	46	80
17	25	13	23	5	16	25

It is evident from these figures that the maximum probable error in the height of an ordinate is about ten times the error in the total area so that in order to determine a curve so that none of its ordinates may be more than a certain percentage in error will require three or four times as many ordinates as for the same error in the area. It should be noted that the error in the ordinates is greatest where the curve is steep, so that even though the vertical error is large the distance between the drawn curve and the true curve may be very slight.

LOCATING THE HEIGHT OF A PEAK.

It often happens that we are interested in determining the exact height of the peak of a curve, or to find out how many ordinates are needed to locate this height with a given precision. The curve shown in Fig. 5 was drawn and two separate sets of ordinates were taken from it, one being represented by the circles, and the other by the crosses. Both sets of ordinates were given to four men separately, and each drew a representative curve through them. As will be noticed, one set of ordinates has a point at the peak of the curve, whereas, the other has the points equally divided about the peak. The separation of the ordinates in each case was equal to one-half the width of the peak. The results obtained are given in the following table:

	a	b	c	d	mean	max.
Ordinate at peak..	0	0	+0.5	0	0.1	0.5
Ordinates on either side of peak.	+4.0	+6.5	-0.7	-2.7	3.5	6.5

It is seen that the mean deviation from the true height of the peak in the first case is only 0.1 of a per cent, whereas, in the second case it is 3.5%. By halving the number of ordinates the probable error in reading the height of the peak could be reduced to below 1%.

MOMENT OF AN AREA.

It is often necessary to determine the moment of an area enclosed by a curve about some point. In this case to obtain a maximum precision from a given number of ordinates the spacing of the ordinates besides following the rule given previously should also be placed inversely as the distance from the point about which the moment is taken. For example, if we wish to determine the moment about the hinge of an elevator by means of the pressure upon that member taken from a series of holes along a rib we should space the holes more closely near the trailing edge of the elevator than at the hinge.

DISCONTINUOUS CURVES.

It hasbeen assumed in the previous discussion that the curves were continuous. The condition sometimes arises however when a quantity will suddenly change from one value to another, as for example the lift force on thick aerofoils. Considering a simple case asin Fig. 6, it is evident that the break may come anywhere between the adjacent points so that the maximum error in area introduced by the discontinuity will be:

$$\pm \frac{1}{2} \; \frac{x\,y}{A} \times 100$$