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SOME REMARKS CONCERNING SOARING FLIGHT

By L. Prandtl

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The publication of the following details is due to the desire of the editor to have the problems of soaring flight treated on the occasion of the Rhone Soaring Flight Contest. In the main, these problems are only connected with matters which are more or less known. For anything which may be new, I am indebted to frequent talks with G. Madelung and A. Betz, who have given me their permission to publish these remarks.

By soaring flight - in contrast with gliding flight - is meant motorless flight without loss of height. According to the laws of the mechanics of flight, two sources of energy are available for soaring flight. One is, air currents having an upward trend; and the other is, irregularities in the natural wind. Under the latter head we can distinguish between two effective forms: (1) Great fluctuations in the strength (and also direction) of the wind, lasting several seconds; (2) The rapid fluctuation in wind direction which is commonly described as "turbulent wind." The fluctuations of long duration can only be utilized by effecting considerable changes in the velocity and height of the machine. For instance, by, in one single gust, climbing at the expense of relative velocity, and then, in the calm which follows, gliding with a downward acceleration. The procedure to follow in order to extract energy from the wind can be condensed into a sim-

* From "Zeitschrift fur Flugtechnik und Motorluftschiffahrt," July 30, 1921.

ple rule: <u>One must attempt to equalize the fluctuations in the wind</u>. In so doing, the energy in the wind fluctuations is obviously reduced, and the energy of the machine increased by a corresponding amount. Thus, for instance, one must present great resistance against a gust, small resistance against a lull; in an upward current the machine must be gradually elevated to increase the pressure on the wings, in a downward current depressed to decrease the pressure.

The fluctuations of short duration, on the other hand, can be utilized without any substantial change in velocity of the c.g. of the machine. As A. Betz has shown in a very instructive article in this journal in 1912,* useful effect is available in sufficiently great fluctuations in wind direction even with rigid and non-warped aeroplane wings. With elastically mounted or flexible wings, the force obtained from changes in wind direction, if these be sufficiently great, may assume perceptible proportions. The effect may be imagined as resembling that of the so-called fish-tail propellers. This name has been used to designate an arrangement by which the waves of the sea have been utilized for the production of power. The arrangement consists of a number of flexible plates, rigidly attached with their leading edge to the sides of a ship. As the waves rise and fall the plates bend like the tail of a fish, and produce power both when bent downwards and when bent upwards.

When we now come to the question of which of these sources of energy do soaring birds make use, the answer can - at least as re-

* A. Betz: "A contribution to the Explanation of Soaring Flight," Z.F.M., 1912, p.269.

gards our domestic birds - only be that, if not exclusively at any rate mainly, they take advantage of rising air currents. Rising air currents are always to be found in uneven country when a wind is blowing. They are also caused over the plains by meteorological influences. For their flying practice birds naturally seek the rising air currents, and as these are frequently not of very great extent in space, the birds have to circle in order to remain in the rising air currents. By observing the soaring of birds of prey one may frequently see that they suddenly lose height and then commence to flap their wings, continuing in flapping flight until they are seen suddenly to rise. From this moment onwards they recommence to soar. They have again found their rising current which they had previously lost. The soaring seagulls near a steamer make use of the air deflected by the steamer and commence to flap as soon as, for some reason or other, they have to leave their favorable position.

That a bird intentionally makes use of gusts does not appear to occur generally. In order to do so it would have constantly to make jumps up and down, which, so far as I am aware, it has not been observed to do. On the other hand, it does not appear unlikely that many birds utilize the rapid fluctuations in the wind besides, after the manner of the fish-tail principle. Probably, however, these forces suffice in no case to cover the entire work of flying, so that the axiom "no soaring without a wind with an upward trend" can probably be accepted as correct. It might be mentioned that in regard to sea birds, each individual wave gives the wind an upward deflection, of which the birds take advantage.*

In the case of human soaring flight, all the previously mentioned possibilities of making use of the energy in the air hold good in principle. The fish-tail effect, as I will call it for the sake of brevity, may possibly be utilized by a suitable form of flexible wing-sections, although I should advise experiments with crewless (unbemannten) models. It is quite probable that the wind fluctuations, which such wings would be designed to utilize, might frequently not be of such magnitude as to have a perceptible effect. In view of the probably inconsiderable gain to be expected one would not like to risk the uncertainty of the flying qualities of such a flexible aerofoil, which might easily lead to a useless, or even dangerous, machine. A crewless model, on the other hand, could more easily be sacrificed.

The utilization of great wind fluctuations is a question of the skill of individual fliers. It is conceivable now and then to extract a slight gain from these, but too much should not be expected from this source. There then remains, as the most important help to human soaring flight, air currents with an upward trend.

If it is desired to utilize to the fullest extent rising air currents, one must strive to build machines with a slow rate of descent (sinkgeschwindigkeit). One is then in a position to util-

* Since writing this article I find in Physikalischen Berichten, 1921, some notes by Everling on a work on soaring flight by E. H. Hankin (Proc. Cambr. Phil. Soc., 20 (1921), pp. 219-227), according to which soaring by turbulent wind only is said to be possible when the earth is strongly heated, as occurs in the tropics. Dr. Hankin describes observations of birds, dragon-flies and flying fish.

ize all rising air currents whose vertical component is greater than the rate of descent of the machine, provided that, at the same time, the upward slope of the wind is greater than the best gliding angle of the machine. On a slope which is steeper than the best gliding angle of the machine, it is moreover possible to soar in winds whose velocity is smaller than that of the lowest gliding speed of the machine, provided the rate of ascent of the wind is greater than the rate of descent of the machine. The machine will in this case soar out horizontally into the free air and will obviously rise in doing so. It would, however, in time get outside the region of the ascending wind, but if the slope has sufficient breadth the machine can be pointed diagonally to the slope and pass across it at the same height or climbing slowly, the sideways displacement giving sufficient speed for soaring. This method of flying can often be observed on seagulls over the beach. Under similar conditions the method should be possible of execution by human beings.

The conditions for the smallest possible rate of descent can be formulated. To begin with, the gliding angle ϵ is given by the well known relation

$$\tan \epsilon = \frac{C_w}{C_a} \qquad (1)$$

Further, the rate of descent, $v_z = v \sin \epsilon$. If we observe that the weight $G = C_a F \frac{1}{2} \rho v^2$, that is

$$v = \sqrt{\frac{2G}{\rho F C_a}} \qquad (2)$$

and that, owing to the fact that the angle is small, sin ϵ and tan ϵ can be exchanged, we find

$$v_z = \sqrt{\frac{2G}{\rho F} \frac{C_w}{C_a{}^{3/2}}} \tag{3}$$

Therefore, for a given wing loading, the rate of descent is smallest when the well-known efficiency ratio $C_a{}^3/C_w{}^2$ is a maximum. We must therefore aim at the smallest possible C_w with a large C_a. This is obtained on the one hand by a large aspect ratio, on the other by avoiding all sources of extra resistance. In order to obtain a large C_a one would have to choose a deeply cambered wing section, as is usual with fast motor-driven aeroplanes. If a polar diagram is available the maximum $C_a{}^3/C_w{}^2$ is easily found by trial. But if one wishes to obtain the results by calculation - which has the advantage that one is not obliged to start with a previously given aspect ratio - one may for instance proceed to do so as follows: - For the range of angles of incidence considered it is frequently possible to express the change of C_w with change of C_a by an equation of the following form:-

$$C_w = A\, C_a{}^2 + B \tag{4}$$

The coefficient A is mainly dependent upon the "induced drag," which in turn depends upon the aspect ratio. As, however, the "section drag" also usually shows an increase with greater angles of incidence, this can be included in the coefficient A by writing

$$A = \frac{F}{\pi\, b^2} + A \tag{5}$$

in which, as usual, b indicates span. B in equation (4) then indicates the constant portion of the "section drag," including all detrimental resistance of the machine. The second factor of equation (3) now becomes $C_w/C_a^{3/2} = A\,C_a^{1/2} + B\,C_a^{-3/2}$. A simple calculation gives for the minimum of this expression the relation $A\,C_a^2 = 3\,B$,

whence

$$C_a = \sqrt{\frac{3\,B}{A}} \tag{6}$$

As will be seen, the minimum of $C_w/C_a^{3/2}$ occurs when the first portion of the resistance in equation (4) is three times as great as the second. The total C_w then becomes equal to 4 B, and consequently

$$\frac{C_w}{C_a^{3/2}} = \frac{4}{3}\sqrt[4]{3\,A^3 B}. \tag{7}$$

In making this calculation one must, of course, make certain that one obtains from equation (6) a value of C_a which lies within the limits for which formula (4) is valid. Should this not be possible the greatest value of C_a for which formula (4) is valid may be taken. It might be of interest to elucidate these calculations by a numerical example, the figures for which are taken from model tests. Assuming an aspect ratio F/b^2 of 1/10, and further $A^1 = 0.01$, and $B = 0.025$ we obtain $A = 0.0318 + 0.01 = 0.0418$.

Thus

$$C_a = \sqrt{\frac{3 \times 0.025}{0.0418}} = 1.34,$$

which figure is attainable with deeply-cambered aerofoils. C_W becomes $= 4\ B = 0.1$. This gives $C_a^3/C_W^2 = 240$. If we assume a wing loading of 9 kg/m² (1.85 lbs/sq.ft.) we get

$$\sqrt{\frac{2}{\rho} \times \frac{G}{F}} = \sqrt{16 \times 9} = 12$$

Thus the rate of descent becomes $12/\sqrt{240} = 0.775$ m/s^2.
If a value of C_a of 1.2 only were attainable, the other constants remaining as before, C_W would become

$$C_W = 0.0418 \times 1.2^2 + 0.025 = 0.0855,$$

giving $C_a^3/C_W^2 = 236$. As $C_a = 1.2$ is still comparatively near the optimum value of 1.34, C_a^3/C_W^2 is thus but little smaller than the maximum value of 240. One can therefore also in cases like that just dealt with confidently use formulae (6) and (7).

For $C_a = 1.2$ the gliding speed v is found from equation (2) to be approximately 11 m/s and the gliding angle is $0.0855 : 1.2 = 1 : 14$. On a wide slope of 1 : 5 the rate of descent would by a wind of 4 m/s, be smaller than the rate of ascent of the air. Soaring across the slope would, of course, require a transverse velocity of $\sqrt{11^2 - 4^2} = 10.25$ meters per second.

[In order to assist those English readers who are not familiar with the German system of expressing lift, resistance, etc., it might be mentioned that in the equations printed above C_W is the drag coefficient, and C_a the lift coefficient. They are converted into the "absolute" units employed by the N. P. L. by being divided by two. Thus when Professor Prandtl speaks of a lift coefficient of 1.2, this corresponds to an "absolute" lift coefficient of 0.6. The velocity is, of course, stated in meters per second, and the letter F is used to denote wing area. b denotes wing span and G the weight (Gewicht).