TECHNICAL NOTES

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DETERMINATION OF CLIMBING ABILITY.

By H. Blasius.

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1. Statement of the Problem.- The vertical distribution of the pressure, temperature and density of the atmosphere varies from day to day. Thus, rates of climb on different days can not be compared directly, but must be corrected with reference to a standard rate of diminution of air density, with increasing altitude.

The following problem, therefore, has to be solved:- An airplane has climbed on a certain day under prevailing atmospheric conditions as shown by the barograph. How would the same airplane climb in a standard atmosphere? This problem has already been dealt with by Everling (Technische Berichte, Volume I, No. 2, p.40, and No. 6, p.247), using the monthly and yearly mean of the vertical temperature distribution. Von Mises solved the problem by arithmetical methods (Zeitschrift für Flugtechnik und Motorluftschiffahrt, 1917, p.173).

In the following discussion, the conditions are examined which shorten or lengthen the climbing time. In establishing the corrected barogram, computation seems more practical than graphical treatment.

2. The Basis of the Answer to the question is summed up in the remark that lift, drag, propeller thrust and torque and engine power, depend only on the density of the air and do not change with the

* Technische Berichte, Volume III, No.6, pp. 193-198, (1918).

pressure and temperature, provided the density remains constant. Accordingly, the rate of climb $w = \frac{dz}{dt}$, is a function of γ only and is, on the other hand, constant for constant γ, independently of p and T. Hence, w, as a function of γ, is the true measure of the rate of climb. Accordingly, the transformation of the barogram of the day to the standard barogram must be made so that points of equal density on the two diagrams (Figs. 4 and 5) will be brought into correspondence for mutual comparison.

3. Transformation of the Altitudes.- For this purpose, it is necessary to know the vertical temperature gradient, as well as the barogram of the day, from which the density can be calculated for each point on the curve. In the standard atmosphere, a given value of γ will, in general, be found at a different altitude than in the atmosphere of the day.

The altitude at which, on the average, a definite value of γ exists (that is, the ordinate in the standard barogram), is here called the "standard altitude" or $z_{standard}$ (Table IV). On the other hand, the altitude given by the barogram of the day is called the "nominal altitude" or $z_{nominal}$. The latter is really only a measure of the pressure p, and is the altitude at which ordinarily the pressure prevails which is given in Table V. Since pressure and density are different every day, neither the nominal altitude $z_{nominal}$, nor the standard altitude $z_{standard}$, gives the true altitude of the day z_{today}, which the airplane attained in the atmosphere of the day. There is no need to know this altitude, since the density of the air and not the altitude is the measure of the airplane's performance. Only the differential coefficient of z_{today} is of importance, since the rate of climb is given by $\frac{d\, z_{today}}{dt}$.

The density γ is calculated, accordingly, for a number of points on the barogram of the day (Fig. 4)*, for approximate values of the nominal altitude, from the measure of the pressure $z_{nominal}$ and from the absolute temperature of the day, T_{today}. The standard altitudes $z_{standard}$ correspond to these densities. In this way the ordinates of the standard barogram (Fig. 5) are calculated for the selected points on the barogram of the day, in which the following temperatures correspond to the individual points.

$Z_{standard}$	Sea-level	1000	2000	3000	4000	5000	5800 m
Temperature °C	20°	14°	8°	-2°	-5°	-14°	-18.5°

We have now to transform the observed climbing times, between the selected points, to the climbing times that the same airplane with the same climbing speed would make in standard atmosphere.

4. The Standard Atmosphere is computed on the following assumptions. At sea-level, z = 0,-

* Figure 4 is the first figure in this note.

Table IV - Relation between Density, Temperature and Standard Altitude.

$z_{standard}$ m	T °C absolute	γ kg/m³
-500	285.5	1.312
0	283.0	1.250
500	280.5	1.188
1000	278.0	1.127
1500	275.5	1.069
2000	273.0	1.013
2500	270.5	0.960
3000	268.0	0.910
3500	265.5	0.862
4000	263.0	0.815
4500	260.5	0.770
5000	258.0	0.729
5500	255.5	0.689
6000	253.0	0.651
6500	250.5	0.614
7000	248.0	0.579
7500	245.5	0.546
8000	243.0	0.515

The temperature 10° or T_0 = 283°C absolute;

The density γ_0, = 1.25 kg/m³

It follows that the pressure p_0, = 10.365 kg/m³ and that the barometric pressure B_o, = 762 mm mercury.

Assume the temperature drop to be $\theta = \frac{5^\circ}{1000 \text{ m}}$

These assumptions are sufficient for the calculation of p, T, γ as functions of z.

From

$$dp = -\gamma \, dz,$$

$$dT = -\theta \, dz,$$

$$p = R \, T \, \gamma,$$

in which, for air, $R = 29.3 \ \frac{\text{m}}{\text{degrees}}$ and, by integration, we obtain the formulas:

$$\frac{p}{p_o} = \left(1 - \frac{\theta z}{T_o}\right)^{\frac{1}{R\theta}} = \left(1 - \frac{z}{56600}\right)^{6.83} = \frac{p}{10363}$$

$$\frac{\gamma}{\gamma_o} = \left(1 - \frac{\theta z}{T_o}\right)^{\frac{1-R\theta}{R\theta}} = \left(1 - \frac{z}{56600}\right)^{5.83} = \frac{\gamma}{1.250}$$

$$T = T_o - \theta z = 283 - \frac{5}{1000} z$$

$$\frac{p}{p_o} = \left(\frac{\gamma}{\gamma_o}\right)^{\frac{1}{1-R\theta}} = \left(\frac{T}{T_o}\right)^{\frac{1}{R\theta}} = \left(\frac{\gamma}{\gamma_o}\right)^{1.172} = \left(\frac{T}{T_o}\right)^{6.83}$$

in accordance with which Mises constructed his standard atmosphere.

Table V - Relation between Pressure and Nominal Altitude.

$z_{nominal}$ m	Barometric pressure in mm of mercury	p kg/m^3
-500	808.0	10988
0	762.0	10363
500	717.2	9754
1000	674.6	9175
1500	634.2	8625
2000	595.9	8104
2500	559.7	7612
3000	525.3	7144
3500	492.7	6701
4000	461.9	6282
4500	432.8	5886
5000	405.2	5511
5500	379.2	5157
6000	354.6	4823
6500	331.5	4508
7000	309.6	4211
7500	289.0	3930
8000	269.5	3665

Table V gives the relation between p and z, that is, the basis for calibrating the barograph chart (Everling, Technische Berichte, Volume I, No.6, p.255). The relation between γ and z fixes a standard altitude for each value of γ (Table IV). The equations are first written for an arbitrary temperature gradient θ. To a constant θ, there corresponds a polytropic distribution of pressure and density

$$\frac{p}{p_o} = \left(\frac{\gamma}{\gamma_o}\right)^n \quad \text{with} \quad n = \frac{1}{1 - R\theta}.$$

For $\theta = 0$, we have an isotherm with $n = 1$. For $\theta = \frac{5^o}{1000\ m}$ $n = 1.172$. For $\theta = \frac{10^o}{1000\ m}$, we have an adiabatic curve with $n = 1.41$.

5. Time Correction.- After calculating, as in section 3, the ordinates $z_{standard}$, of the standard barogram for a series of points on the barogram of the day, it is further necessary to transform the observed climbing times Δt, between these points, into the climbing times $\Delta\tau$ of the standard barogram. This is done in accordance with the fundamental principle that, in corresponding altitude intervals (i.e., with equal values of γ), the climbing speeds must also be equal.

From

$$\frac{dz_{today}}{dt} = \frac{dz_{standard}}{d\tau}$$

it follows that

$$\frac{d\tau}{dt} = \frac{dz_{standard}}{dz_{today}}.$$

$dz_{standard}$ and dz_{today} are altitude intervals corresponding to equal values of γ, since the ordinate $z_{standard}$ is so calculated as to correspond to the value of γ in actual flight. $d\tau$ and differ, therefore, in so far as the distance of the layers, between which γ varies by $d\gamma$, is different in the standard atmosphere from what it is in the atmosphere of the day. Should this distance between layers, for a given $d\gamma$, be greater in the atmosphere of the day than in the standard atmosphere, then, with equal climbing speed, Δt is also greater than $\Delta\tau$ and the observed Δt must, therefore, be diminished, in order to arrive at the standard climbing time, $\Delta\tau$. The distance between layers, to which $d\gamma$ corresponds, depends on the state of the atmosphere, especially on the temperature gradient, and can be calculated therefrom. The formula for the time correction is written thus:

$$\frac{d\tau}{dt} = \frac{-\left(\frac{dz}{d\gamma}\right)_{standard}}{-\left(\frac{dz}{d\gamma}\right)_{today}}$$

because, according to the above, the ratios $\frac{z}{\gamma}$ belong together and we can calculate the numerator from the equations of the standard atmosphere and the denominator from the state of the atmosphere of the day, since dz_{today} can be expressed by $-\frac{dp}{\gamma}$

From

$$dp = -\gamma dz$$

$$dT = -\vartheta dz$$

$$p = R T\gamma$$

it follows that

$$dp = R T d\gamma + R\gamma d T$$

$$-\gamma dz = R T d\gamma - R\vartheta\gamma dz$$

$$-\frac{dz}{d\gamma} = \frac{RT}{(1 - R\vartheta)\gamma} .$$

This formula is valid for the numerator and the formula for $\frac{d\tau}{dt}$ is valid for the denominator. When we put $T = T_{standard}$, with known γ from Table V and $\theta = \frac{5^{\circ}}{1000\ m}$, the numerator is given by

$$\left(\frac{dz}{d\gamma}\right)_{standard} = \frac{1.172\ R\ T_{standard}}{\gamma}$$

and when we put $T = T_{today}$ and θ equal to the prevailing temperature gradient, which differs for each altitude interval, the denominator is given by

$$\left(\frac{dz}{d\gamma}\right)_{today} = \frac{R\ T_{today}}{(1 - R\theta)\gamma} .$$

It follows that $\frac{d\tau}{dt} = 1.172\ (1 - R\theta)\ \frac{T_{standard}}{T_{today}}$. If R is taken at 29.3 $\frac{m}{deg.}$ and, on the contrary, θ is expressed in $\frac{deg.}{km}$, then the product $R\theta$ must be divided by 1000. The calculated values of $1.172\ (1 - R\theta)$ are given in Table VI. By this formula, the time intervals Δt of Fig. 4 are transformed to the time intervals $\Delta\tau$ between the corresponding points in Fig. 5, the temperatures and the temperature gradient being known for each interval.

The difference between this and the method of Mises, based on the same principle, is that Mises obtains the differentials by the construction of tangents to the barogram, while here they are calculated from the thermodynamic relations. In Everling's work (Technische Berichte, Volume I, No. 2, p.36), the distance between layers, and hence the transformation of the times, are obtained from the observed mean monthly values of γ.

large and if the temperature of the day is above the standard. The explanation follows from the comparison of the state of both atmospheres. As a consequence of the equal rates of climb, the times corresponding to equal $d\gamma$ vary as the distance between layers for the same $d\gamma$. (section 5).

Starting from the same conditions (p, T and γ), Figure 6 shows the decrease of γ with the altitude z, for both large and small temperature gradients. The pressure drops in both cases were initially equal, since we started from equal values of γ at the lower level. The temperature higher up is lower for larger values of θ, than for smaller values and γ is, therefore, greater in the former case. Therefore, γ decreases more slowly with larger θ. The distances between layers for equal values of $d\gamma$ are, therefore, greater than standard with larger θ and less than standard with smaller θ. Δt is larger than Δ^{τ} with greater θ.

Another presentation of the same idea may be made as follows: On comparing the distances between layers with equal values of γ in the atmosphere of the day and in the standard atmosphere, we find that $\frac{dp}{dz}$ is the same at both points compared, since the values of γ are equal. The distances between layers dz, for equal dp are thus equal and, therefore, the ratio of the values of $\frac{dz}{dp}$ is equal to the ratio of the values of $\frac{dp}{d\gamma}$ in both atmospheres. But $\frac{dp}{d\gamma}$ is the slope of the polytrope (section 4) which, with the gases in the same condition, is proportional to the exponent of the polytrope $n = \frac{1}{1 - R\theta}$ (Fig. 7). With adiabatic distribution ($\theta = 10 \frac{deg.}{km.}$), the slope $\frac{dp}{d\gamma}$ is 1.41 times the isothermal gradient, while with standard distribution ($\theta = 5 \frac{\text{deg.}}{\text{km.}}$), it is 1.173 times the isothermal gradient. The values of $\frac{dz}{d\gamma}$ are, likewise, in the same ratio as the distances between layers for equal $d\gamma$. If the temperature gradient of the day is $\binom{\text{greater}}{\text{less}}$ than the standard or if it approaches the $\binom{\text{adiabatic}}{\text{isothermal}}$ gradient, then the distance between layers for equal $d\gamma$ for the day are $\binom{\text{greater}}{\text{less}}$ than the standard and the dt of the day is, therefore, $\binom{\text{greater}}{\text{less}}$ than the standard $d\tau$. The effect of the first factor in $\frac{d\tau}{dt}$ is thus made clear.

If the initial conditions differ (always with the same γ), then the slope of the polytrope increases with the temperature (Fig. 7). If, then, $T_{standard}$ is greater than T_{today}, $\frac{dp}{d\gamma}$ and $\frac{dz}{d\gamma}$ and likewise the distances between layers in the standard atmosphere are greater than in the atmosphere of the day and $d\tau$ is greater than dt. This is the physical meaning of the formulas in section 5.

For example, in the layer between 2000 and 3000 m in Fig. 4, Δt is greater, while in the interval 3000 to 4000 m, Δt is less than $\Delta\tau$ in the corresponding layer in Fig. 5, since θ is equal to $\frac{10^\circ}{1000\text{ m}}$, in the first case, and to $\frac{3^\circ}{1000\text{ m}}$ in the second.

7. <u>Transformation of the Barogram</u>.- The formula for $\frac{d\tau}{dt}$ serves to investigate the extent to which the variations in the times depend on the thermodynamic state of the atmosphere and, since the calculated factor given in Table VI has a preponderating influence on $\frac{d\tau}{dt}$, the sign and magnitude of the time correction between

points of equal densities can be determined from it. For the practical transformation of the barograms it is, on the contrary, more advantageous to calculate the climbing speed w from observations.

We have

$$w = \frac{dz_{today}}{60\ dt}$$

w being expressed, as customary, in m/sec and t in minutes. Now dz_{today} depends on the pressure drop, and the pressure p is measured by the barometric pressure B, in mm of mercury. Then:

$$dz_{today} = - \frac{dp}{\gamma} = - \frac{13.6\ d\ B}{\gamma}$$

$$w = - \frac{13.6\ d\ B}{60\ \gamma\ dt} = - \frac{0.227\ d\ B}{\gamma\ dt}\ .$$

Table VII - Calculation of Rate of Climb from a Barogram.

$z_{nominal}$	Time t	Δt	Pressure B	- Δ B	Temperature	T absolute	Specific gravity γ	γ_{mean}	Rate of climb w
m	min.	min.	mm Hg	mm Hg	°C	°C	kg/m³	kg/m³	m/sec
Ground	0		760.8		20	293	1.204		
		3.5		86.2				1.149	4.86
1000	3.5		674.6		14	287	1.094		
		3.2		78.7				1.040	5.36
2000	6.7		595.9		8	281	0.986		
		4.0		70.6				0.943	4.24
3000	10.7		525.3		-2	271	0.900		
		5.2		63.4				0.850	3.26
4000	15.9		461.9		-5	268	0.800		
		5.8		56.7				0.764	2.89
5000	31.7		405.2		-14	259	0.727		
		8.1		40.8				0.696	1.64
5800	29.8		364.4		-18.5	254.5	0.666		

Table VII was calculated by this method, which corresponds to the graphical method of Mises. To each $z_{nominal}$ there is a corresponding time t, given by the barogram, a barometric pressure B (Table V) and a temperature observed in flight. From these, γ is obtained by the equation of condition

$$\gamma = \frac{0.464\ B}{T}$$

To calculate the rate of climb

$$w = -\frac{0.227\ \Delta B}{\gamma \Delta}.$$

the differences Δt and $-\Delta B$ being calculated, as also the mean values of γ in the altitude intervals. Then w is obtained as a function of γ (Fig. 8). From Table IV, a $z_{standard}$ corresponds to each γ. From this, the second subdivision of the abscissa, on the upper edge of Fig. 8, has been inserted.

If we now desire to determine the climbing time $\Delta\tau$ in the standard atmosphere, we select altitude intervals, e.g., of 500 or 1000 m, in approximate values of $z_{standard}$, establish the value of w at the middle of each altitude interval, from Fig. 8, and obtain in minutes

$$\Delta t = \frac{\Delta z_{standard}}{60\ w_{average}} = \frac{16.67}{w} \text{ for 1000 m intervals}$$

$$= \frac{8.33}{w} \text{ for 500 m intervals,}$$

as in the case in Table VIII. A continuous summation of the $\Delta\tau$ values gives the climbing time τ for altitude $z_{standard}$. This again gives the normal barogram (Fig. 5) up to a constant difference in the abscissas. In contradistinction to sections 5 and 6, the values of $\Delta\tau$, as calculated here, do not apply between the points transferred from Fig. 4, but directly between the integral values of $z_{standard}$.

Table VIII - Calculation of Climbing Times for the Standard Barogram.

Standard height intervals	w	$\Delta\tau$	τ	$z_{standard}$
km	m/sec	min	min	km
			0	1
1 ÷ 2	5.23	3.18		
			3.18	2
2 ÷ 3	4.45	3.75		
			6.93	3
3 ÷ 4	3.35	4.98		
			11.91	4
4 ÷ 5	2.95	5.67		
			17.58	5
5 ÷ 5.5	1.90	4.38		
			21.96	5.5

Summary.

In order to transform the barograms obtained on a given day to the normal barogram the same airplane would give when flying in an atmosphere equivalent to the standard, the temperature must be measured simultaneously. Only then can the specific gravity γ be calculated from the pressure and temperature, thus giving the true measure of the climbing speed. The standard altitude is so determined that the same density γ prevails in the standard stmosphere

and in the atmosphere of the day. The climbing times are transformed in proportion to the thickness of the layers corresponding to equal $d\gamma$, so that the climbing speeds remain unaltered. The ratio of the thicknesses of the layers follows from the thermodynamic equations for the decrease of density with altitude, for which purpose the temperature gradient is recognized as essential. Precisely this circumstance makes it impossible to base calculations on monthly averages, since, while the temperature itself may differ greatly from the average, the temperature gradient must still more be regarded as varying daily. The vertical temperature distribution must, therefore, be measured daily, preferably in the airplane itself, in order to make the transformation at all possible. Since the density enters into the climbing ability and not the pressure of the atmosphere, it would be better to use, instead of the barometer, an apparatus that would indicate γ and not p. Such a "density recorder" would measure and indicate the volume of a segregated mass of air having the same pressure and temperature as the surrounding atmosphere. It should, however, be expressly noted, that the recording of γ does not render superfluous the simultaneous recording of temperature and pressure, since the transformation of the times depends on the distance between the layers, i.e., on the temperature gradient, or on $\frac{dp}{d\gamma}$.

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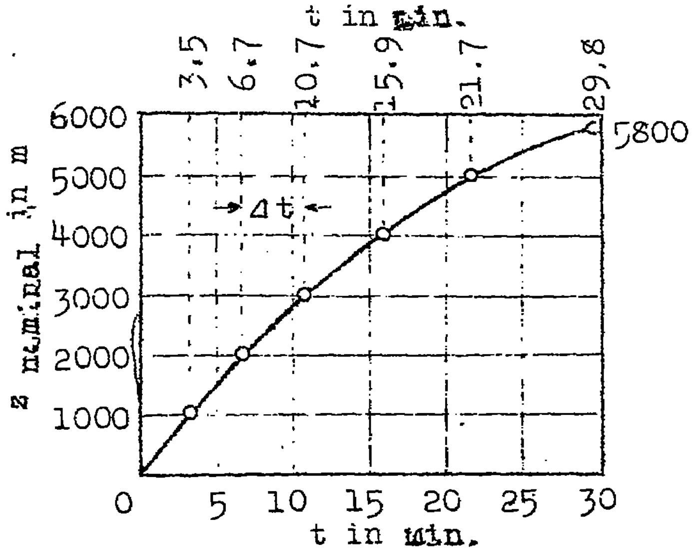

Fig. 4 Barogram of the day of flight

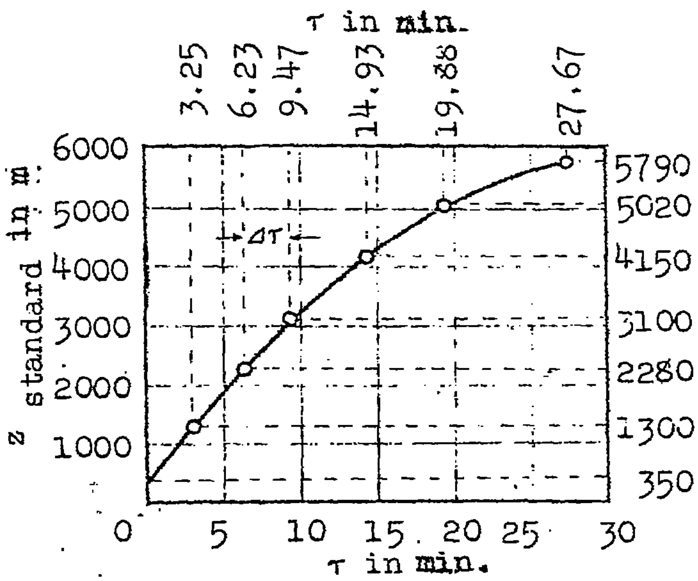

Fig. 5 Barogram of the standard day

Figs. 6, 7, & 8

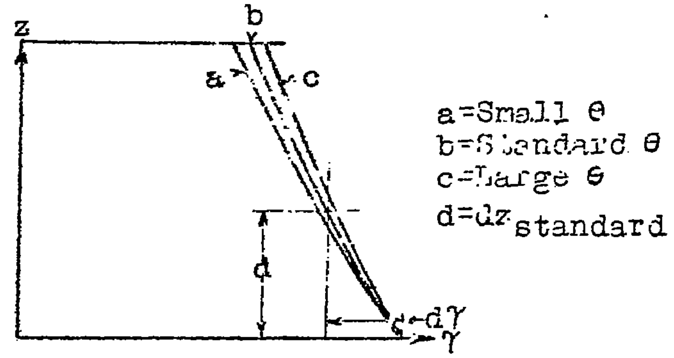

Fig. 6 Drop of γ with height with varying lapse rates

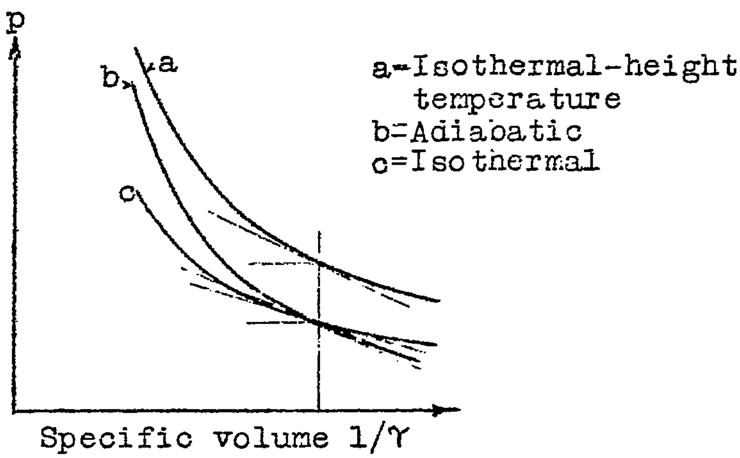

Fig. 7 Comparison of slopes of curves of thermodynamic state

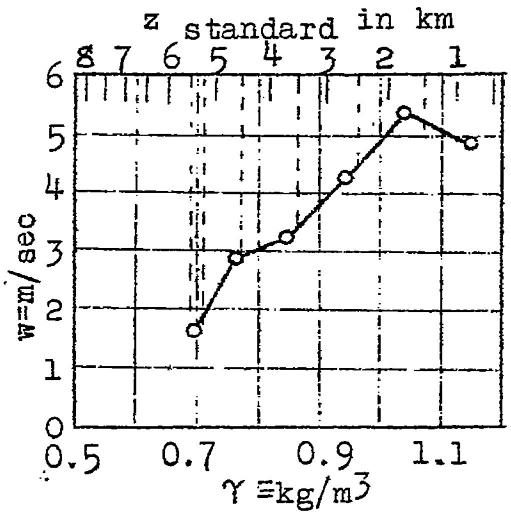

Fig. 8 Climbing speed as a function of γ