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PRELIMINARY STUDY OF THE DAMPING FACTOR IN ROLL.

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Summary.

The following paper was submitted by the writers as a thesis to the Department of Aeronautical Engineering, at the Massachusetts Institute of Technology. It constitutes a general theoretical discussion of the damping factor in roll, together with the results of wind tunnel tests on the continuous rolling of a U.S.A.-30 airfoil. Two general formulas are derived for the damping of roll, each of which contains unavoidable indeterminate functions. Certain of these functions have been evaluated from the test data. Of chief interest is the deduction that the actual damping as experienced in flight differs from the damping as theoretically calculated ny a function of the wing-tip pressure distribution, which is in turn largely influenced by the form of the wing-tip and by the rolling velocity. Finally, it has been shown that in the damping equations $\frac{dC_L}{d\alpha}$ may be substituted for $\frac{dC_Z}{d\alpha}$, even under full flight conditions, without serious error.

PART I.

THEORETICAL DISCUSSION.

Introduction.

The damping of an airplane in roll is an exceedingly complex problem and one which it may never be possible to solve completely, but if we can form some idea of the magnitude of the principal factors involved and of their importance in practical flying, a mathematically complete solution can be dispensed with. Our interest in the damping coefficient is mainly as a guide to the determination of stability and as an indication of the forces encountered in maneuvers.

The damping coefficient itself is made up of many components, arising from the several elements concerned: wings, fuselage, tail, etc. As long, however, as we confine our tests to the complete airplane, we have no means of analyzing the source of the magnitude of the basic elements of damping. The chief contributor to these elements, the wing, is itself affected by a series of complications. Dihedral, stagger, taper and aspect ratio are only a few of the complicating factors. In view of the foregoing, therefore, this report will be confined principally to the straight rectangular wing of constant chord and constant section.

Theory of Dimensions.

Probably the simplest method of attacking the problem is by the theory of dimensions. We may express the damping coefficient of a rectangular wing of constant section as a function of several variables, thus:

$$L_T = \frac{L}{p} = f\left(\frac{dC_z}{d\alpha}, u, p, c, z, \rho, b\right)$$

L_T = the total damping coefficient arising from all causes taken together, as distinguished from L_p, the damping coefficient of roll due to roll.

L = the total rolling torque in lb-ft.

p = angular velocity of roll in radians per second.

$\frac{dC_z}{d\alpha}$ = rate of change of the coefficient of normal force with angle of attack (i.e., the slope of the normal force curve). The units of $d\alpha$ are radians.

u = air speed in feet per second.

c = wing chord in feet.

b = wing span in feet.

z = perpendicular distance from the axis of rotation to the mid-point of the wing chord, in feet.

ρ = mass density of the air.

The term $\frac{dC_z}{d\alpha}$ is used instead of the angle of attack because the same angle of attack does not give the same results for different wing sections, or for the same wing section tested at different values of VL. Thus we eliminate both scale effect and the effect of different wing contours. $\frac{dC_z}{d\alpha}$ is the slope of the curve of that component of force perpendicular to the line of steady flight, (the trajectory of the center of gravity) and should not be confused with $\frac{dC_L}{d\alpha}$ which is the slope of the lift curve as read from a wind tunnel plot and relates to the force perpendicular to the relative wind, whatever direction that may take.

It is usually assumed that the torque caused by a force on an elemental area dS, is proportional to $C_z dS$ or to $(dC_z/d\alpha)(\Delta\alpha)dS$, and it would therefore seem necessary to introduce a $\Delta\alpha$ term into the foregoing equation. $\Delta\alpha$ is the difference between the real angle of attack at the point in question and that at the plane of symmetry. However, since $\Delta\alpha$ is always small, $\Delta\alpha \approx \tan(\Delta\alpha) = py/u$, y being the distance from the plane of symmetry along the span, and since both p and u are already expressed in the equation it will not be necessary to add any $\Delta\alpha$ term.

Evaluating the various elements in the above equation by the theory of dimensions, we get:

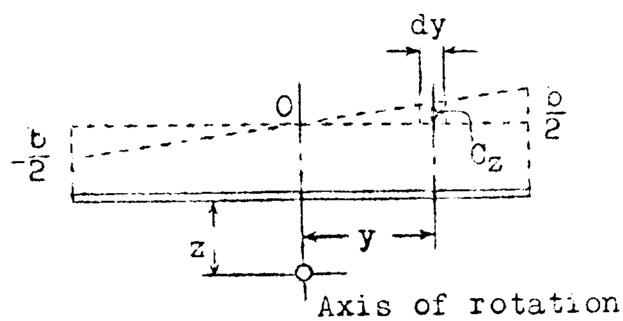
$$L_T = k\left[\frac{dC_Z}{d\alpha}\, ub^4\right] f'\left(\frac{pb}{u}\right) f''\left(\frac{b}{c}\right) f'''\left(\frac{z}{b}\right) \qquad (1.$$

The first of these indeterminate functions contains the ratio of the linear velocity of rotation of the wing-tip to the wind velocity which amounts to a particular value of $\tan(\Delta\alpha)$ or $\Delta\alpha$, approximately. The second contains the aspect ratio. The third contains the ratio of the height of the wing above the rotational axis to the span of the wing. Note that all of these functions are dimensionless ratios.

Before we can make any use of this equation it will be necessary to examine the three indeterminate functions more fully, and check the results of our theory with the experimental data. To this end a more elaborate, if less convenient theory has been developed.

General Theory.

The damping coefficient in roll is usually calculated theoretically by assuming the wing to be made up of many minute elements, the lift on each of which contributes an element of rolling moment. Thus in the accompanying sketch,



$$L' = 2\int_0^{b/2} (\text{Lift})\ (\text{Arm})$$

$$= 2\int_0^{b/2} (\Delta c_z)V^2 y\, dS$$

L' = the theoretical rolling torque.

dS = element of area = chord × dy = c(dy).

V = resultant wind velocity where

$$V^2 = u^2 + (pR)^2 \text{ or } V^2 = u^2 + p^2\ y^2 + p^2\ z^2$$

C_z = normal force coefficient = $\frac{dc_z}{d\alpha}$ ($\Delta\alpha$)
where $\frac{dc_z}{d\alpha}$ is the slope of the normal force curve, and is assumed constant,

while $\Delta\alpha$ is the change of angle of attack along the wing from the plane of symmetry. ΔC_z is the difference between the value of C_z at the wing center line and that at any point, y along the span.

Therefore:

$$L' = 2\int_0^{b/2} \frac{dC_z}{d\alpha} (\Delta\alpha) \, (u^2 + p^2 y^2 + p^2 z^2) \, (y) \, (c\, dy)$$

At any point y along the wing $\Delta\alpha \tan^{-1} py/u$, and since $\Delta\alpha$ is necessarily small we may take $\Delta\alpha = py/u$, where $\Delta\alpha$ is in radians, without appreciable error.

Then:

$$L' = 2\int_0^{b/2} \frac{dC_z}{d\alpha} \left(\frac{cpy^2}{u}\right) (u^2 + p^2 y^2 + p^2 z^2) dy$$

Integrating, and collecting terms, we obtain

$$L' = \frac{dC_z}{d\alpha} (pcb^3) \left[\frac{1}{12} \left(u + \frac{p^2 z^2}{u}\right) + \frac{1}{80} \left(\frac{p^2 b^2}{u}\right)\right] \qquad (2.$$

This formula neglects the irregular pressure distribution at the wing-tip, or any change in that distribution due to the rolling motion. If we generalize the equation of roll so as to include the tip effects and to include the effects of the other rotary and resistance derivatives as well, we get the total rolling torque L, thus

$$L = L' + [f(t)] \, b + L_v v + Y_v v z + Y_p p z \qquad (3.$$

where L' is the theoretical rolling moment due to roll obtained by equation #2; $f(t)$ is an indeterminate function of the tip pressure distribution, and produces roll by acting in the direction of lift, at a distance from the axis of rotation which is some fraction of the span b; L_v is the rolling moment due to side-slip, wherein the side-slip is introduced by the fact that the wing is rotating at a normal distance z from the axis of roll; v is the amount of the side-slip velocity; and Y_v and Y_p are the lateral forces due to side-slip and to roll respectively, which produce roll by acting along the wing span at an arm z with respect to the axis of rotation.

If we substitute $v = pz$ in equation #3 (where p is in radians per unit time) and divide through by p, we obtain

$$\frac{L}{p} = \frac{L'}{p} + [f(t)]\frac{b}{p} + L_v z + Y_v z^2 + Y_p z$$

and since torque divided by the corresponding velocity gives the coefficient, we have the general coefficient of roll expressed thus:

$$L_T = L_p + [f(t)_p]\, b + [L_v + Y_p]\, z + Y_v z^2 \qquad (4.$$

In this equation the $f(t)_p$ serves as a correction factor for the theoretical damping of roll (L'_p), which might be expected to be too large since it neglects the falling off of the lift at wing-tips. The terms L_v, Y_v, and Y_p are the experimental values for the wing in question. If now we divide equation #2 by p, we get an expression for L'/p (i.e., L'_p) which may be substituted into equation #3. Collecting terms, we have the completely general equation:

$$L_T = \frac{1}{12}\left(\frac{dC_z}{d\alpha}\right) cb^3 u + \frac{1}{80}\left(\frac{dC_z}{d\alpha}\right)\frac{p^2 cb^5}{u}$$

$$[f(t)_p]\ b + [L_v + Y_p]\ z + [Y_v + \frac{1}{12}\left(\frac{dC_z}{d\alpha}\right)\frac{p^2 cb^3}{u}\ z^2] \quad (5.$$

In this equation the three terms involving $\frac{dC_z}{d\alpha}$ and the term $[f(t)_p]\ b$ together make up what is usually called L_p, damping of roll due to roll. The first of these $\frac{1}{12}\left(\frac{dC_z}{d\alpha}\right)cb^3u$, represents the damping obtained by the element theory, if the resultant airspeed is everywhere taken equal to the speed of flight. The second term represents the added damping obtained if we take the airspeed as the resultant of the speed of flight and the normal speed due to rotation, and, under full flight conditions with the maximum probable velocity of roll, is about 2% of the first term. The last term $\frac{1}{12}\left(\frac{dC_z}{d\alpha}\right)p^2cb^3z^2$ represents the further increment of roll added by considering the transverse component of airspeed across the wing, and amounts at most to approximately 0.25% of the first term. The other terms have already been discussed.

Discussion of the General Theory.

Obviously the general equation is too complicated for convenience and will have to be simplified by assumption. If we neglect the two smaller terms in $\frac{dC_z}{d\alpha}$ and thereby introduce an error of not more than 2 1/2%, the equation becomes:

$$L_T = \frac{1}{12}\left(\frac{dC_z}{d\alpha}\right)cb^3u + [f(t)_p]\ b + [L_v + Y_p]\ z + Y_v\ z^2 \quad (6.$$

Further simplifications will depend upon the specific conditions.

If z is small, both of the last terms disappear; if z is material, Y_V is usually negligible; L_V and Y_p increase with dihedral, but the latter is apt to be unimportant. In the case of the rectangular wing of constant section, all three terms L_V, Y_V and Y_p, can probably be neglected and the equation takes the form

$$L_T = \frac{1}{12} \frac{dC_z}{d\alpha} cb^3 u + [f(t)_p] b \qquad (7.$$

It will be noted in this case that $L_T = L_p$ since the correction factor $[f(t)_p] b = L_p - L'_p$.

A study of equations Nos. 5, 6 and 7, will shed some light on the variation of L with changes in aspect ratio and wing area. In equation #5 the first term and the last term involving $\frac{dC_z}{d\alpha}$ depend upon cb^3, while the second term depends on cb^5. The third term $[f(t)_p] b$, is rather difficult to analyze. Being a function of the tip pressure distribution, $f(t)_p$ evidently depends upon the chord; also the extent to which this irregular distribution extends inward will presumably be a function of the chord rather than of the span. We might say, then, that $[f(t)_p] b$ depends on $c^2 b$. However, the question arises as to whether the form of the tip distribution does not depend upon the normal component of the wing-tip velocity, which, in turn depends directly on the span. In other words, does not $[f(t)_p] b$ depend primarily upon $c^2 b^2$? The latter seems more reasonable. Of the remaining terms, L_V depends upon the area, the amount of the dihedral, and the span, or on cb^2. Y_p and Y_V do not depend upon the span to any appreciable extent, but rather on c^2. Summing up, therefore, it appears that the most important term depends on cb^3; the term $[f(t)_p]\,b$, which may amount to 30% of the total depends either on c^2b or c^2b^2; the term involving $\frac{1}{80}\left(\frac{dC_z}{d\alpha}\right)$, which comprises about 2% of the total, depends on cb^5; and the almost negligible terms Y_p and Y_V depend on c^2.

Applying the formula

$$c^x\, b^y = \left[(bc)^{\frac{x+y}{2}}\right] \left[\left(\frac{b}{c}\right)^{\frac{y-x}{2}}\right] = \left[S^{\frac{x+y}{2}}\right] \left[R^{\frac{y-x}{2}}\right]$$

where S and R are area and aspect ratio respectively, it follows that the cb^3 terms depend on S^2R; the c^2b^2 term on S^2 alone; the c^2b, if we choose to use it, on $S^{3/2}\,R^{-1/2}$; the cb^5 term on $S^3\,R^2$; and the two c^2 terms on SR^{-1}. At first sight the term involving $S^3\,R^2$ would assume undue importance. Actually this term also involves p^2 (the square of the rolling velocity) which obviously decreases at about the same rate, or, perhaps, faster, than S^3 increases in actual flight, so the proportion of 2% of L_T which was obtained for that term for typical flight conditions on a 2000-pound airplane, will probably not be exceeded for airplanes of any size or proportion.

Neglecting these less important terms, we come to a study of equation #7, wherein the first term depends upon S^2R and the second upon S^2 or $S^{3/2}\,R^{-1/2}$ depending on how we consider $f(t)_p$. It seems most reasonable to take $[f(t)_p]\,b$ as dependent upon c^2b^2, (or on S^2) which has the added advantage of bringing in the area without fractional exponents, and checks the b^4 term obtained by the theory of dimensions in equation #1. It has been found by the N.A.C.A.,* however, that in going from model test to full flight, L_T increases less rapidly than S^2, so there is something to be said for the alternative supposition. In either event, the mean exponent of the aspect ratio is bound to be less than unity, probably around .8, since about 70% of L_T depends upon aspect ratio to the first power. This also has a bearing on equation #1.

The Slope of the Normal Force Curve.

Throughout the discussion we have used the term $\frac{dC_Z}{d\alpha}$ rather than $\frac{dC_L}{d\alpha}$. If we take lift as perpendicular to the relative wind and the "z" force as perpendicular to the line of flight, we have by the familiar transition

$$C_Z = C_L \cos\alpha + C_D \sin\alpha$$

Differentiating,

$$\frac{dC_Z}{d\alpha} = \frac{dC_L}{d\alpha}\cos(\Delta\alpha) - C_L \sin(\Delta\alpha) + C_D \cos(\Delta\alpha) + \frac{dC_D}{d\alpha}\sin(\Delta\alpha) \qquad (8.$$

$(\Delta\alpha)$ here represents the change in angle of attack from the value at the wing center-line. $(\Delta\alpha)$ is zero at the center-line. At that point, therefore, $\frac{dC_Z}{d\alpha} = \frac{dC_L}{d\alpha} + C_D$.

A full-scale example has been worked out in Fig. 1, for a U.S.A.-30 wing of 60 ft. span and 10 ft. chord, turning at 1.5 radians per second, which is certainly an exaggerated case. It will be seen that the $\frac{dC_Z}{d\alpha}$ curve follows the $\frac{dC_L}{d\alpha}$ curve very

* N.A.C.A. Report #167

closely. Above $\alpha = -6^{\circ}$ the deviation is less than 3 1/2% througn-out the entire range. This means that for all present-day air-planes under ordinary conditions $\frac{dC_Z}{d\alpha} = \frac{dC_L}{d\alpha}$. For auto-rotation the more exact form is required, and a new curve of normal force must be plotted for each angle of attack considered, and a graphi-cal solution must be made if the roll is very rapid.

Experimental Results.

As a conclusion to the theoretical discussion, the results of the experiments may be summarized. The indeterminate functions of $\left(\frac{z}{b}\right)$ and $\left(\frac{pb}{u}\right)$ in equation #1 were investigated for an aspect ratio of 6. It was found, as might be expected, for the plain rectangular wing of constant section, that L_T was independent of $\left(\frac{z}{b}\right)$, at least within the experimental error. This follows from equation #6, since L_V, Y_p, and Y_V are known to be small. The other indeterminate $f'\left(\frac{pb}{u}\right)$, takes the form of $k_1\left(\frac{pb}{u}\right)^n$, where k_1 and n vary practically as straight line functions with $\frac{dC_Z}{d\alpha}$. The equation then takes the form $L_T = k_1 \left(\frac{dC_Z}{d\alpha}\right) ub^4 \left(\frac{pb}{u}\right)^n$. Values for k_1 and n are plotted in Fig. 2. The full lines rep-resent decreasing values of $\frac{dC_Z}{d\alpha}$ and cover the range of an aver-age lift curve from maximum steepness up to nearly maximum lift. The dotted lines represent decreasing values of $\frac{dC_Z}{d\alpha}$, which cover the range of the lift curve below maximum steepness (i.e., in gen-eral, below $\alpha = 0^{\circ}$) where the lift curve tends to bend upwards from a straight line. The point of intersection is at the maximum value of $\frac{dC_Z}{d\alpha}$. Obviously, with wings having a different maximum value of $\frac{dC_z}{d\alpha}$, the intersection would take place at some other point, and presumably the dotted line would be displaced vertically to correspond. The fact that these lines intersect at $n = 0$, for this particular wing, is probably accidental. However, the full lines, which cover the normal range of flight angles, should apply equally well to any plain rectangular wing of constant section and aspect ratio 6. The curves show very clearly a departure from a linear relation between rolling moment and rolling velocity when the rolling velocity is high.

A value of k_1 can be calculated from the element theory on the assumption that n is equal to 0. Using equation #2 and neglecting all terms within the bracket except the first, we have:

$$L_p = \frac{1}{12} \left(\frac{dC_z}{d\alpha}\right) cb^3$$

where the units are homogeneous throughout. In plotting the curves of Fig. 2, $\frac{dC_z}{d\alpha}$ was taken in lb/sq.ft/mile per hour/degree, and u in M.P.H. This introduces a correction factor of 57.3 × 15/22. A further factor of 1/6 is introduced by the substitution of b^4 for cb^3. The total value of the calculated coefficient, on a basis comparable with that used in Fig. 2, is therefore, 57.3 × 15/22 × 1/6 × 1/12, or .52. The experimental constant will be seen to approach the calculated one closely at small values of $\frac{dC_z}{d\alpha}$, but it falls far below at high values of the slope (corresponding to angles of attack well below that of maximum lift).

PART II.

EXPERIMENTAL DETERMINATION OF THE DAMPING OF ROLL.

Method of Test.

The tests were carried out in the 4-foot wind tunnel at the Massachusetts Institute of Technology, on an 18" x 3" wood model of the U.S.A.-30 wing at a wind speed of 40 M.P.H. A 12" spindle 1/2" in diameter was mounted axially of the tunnel between a pair of conical bearings. Each bearing was supported by three wires to the side of the tunnel, so attached as to keep the bearings seated snugly on the ends of the spindle. The spindle was provided with a pair of slots 2" apart, through which passed two 1/8" diameter rods, the rods being screwed into the wing model at mid-span, one behind the other. Special counter-weights were mounted at the opposite ends of the rods. Thus, by loosening two set-screws opposite the spindle slots, both the angle of attack of the wing and its distance from the axis of rotation could be altered. Finally, a light flexible cord was wrapped three times around the spindle and the ends carried out through the bottom of the tunnel to a pair of weights, which supplied the driving torque to keep the model in continuous rotation. The mounting is shown in Fig. 6.

Runs were made at various torques for five angles of attack (-4.5°, 0°, 6°, 12°, and 18°) and at 0° for four positions of the wing relative to the axis. Each run was repeated in the reverse direction and the results averaged to remove any error due to warp in the wing. The speed of rotation was observed for each value of torque by counting the revolutions of the wing against a stop-watch. The net torque was then obtained by subtracting a friction and windage correction which has been found independently by experiment. Torque in foot-lb divided by angular velocity in radians per second gave the damping coefficient. Tests were also made to determine the usual characteristics of the wing and are given in Fig. 3. From these tests the values of $\frac{dC_z}{d\alpha}$ were obtained.

Analysis of Results.

In order to determine the unknown function of z/b in the dimensional equation #1, values of L_T were plotted against z/b in Fig. 4 for three values of pb/u. Unfortunately, it was not possible to make pb/u exactly constant, since in making the experiment the speed of rotation for a given torque could not be foretold. However, groups of values were selected in which pb/u is essentially constant and since all values for all groups lay within 5% of the average, which is within the error of the experiment, the evidence seemed sufficient to indicate that L is practically independent of z/b for straight rectangular airfoils. A few tests were made at $\alpha = 12^\circ$ for various values of z/b to determine the effect of a change in $\frac{dC_z}{d\alpha}$ on z/b. These results were slightly more erratic than those at 0°, but nevertheless bore out the fact that L_T is independent of z/b. This is what the theory led us to expect, knowing that L_v, Y_p, and Y_v are small.

To evaluate the pb/u term in equation #1, the runs for each angle of attack were plotted on logarithmic paper, using pb/u as abscissae and $L_T/(dC_Z/d\alpha)ub^4$ as ordinates. It was found that each set of points lay nearly along a straight line. Accordingly, from the intercepts of this line the values of k_1 and n given in Fig. 3, were obtained. These values were plotted against the slope of the lift curve instead of the angle of attack so as to be of more general application.

It is interesting to compare the theoretical and experimental values of L. Fig. 5 shows a typical comparison. The bottom line represents the experimental values of L_T and the top line the theoretical values of the conventional L_p obtained from equation #2. Since both theory and experiment agree that for the straight rectangular wing the extraneous terms L_V, Y_p, etc., are negligible, it follows that the only difference between the theoretical L_p and the actual L_T should be the term $[f(t)_p]\,b$, (i.e., the tip loss correction). With this in mind the central line in Fig. 5 was obtained by solving graphically for L_p and assuming the conventional tip pressure distribution of 1/2 the running load at the tip, tapering up to full load at .8 of a chord-length inboard from the tip. Evidently, then, the tip pressure distribution is altered by the rolling motion. This is not unexpected.

The comparison shown in Fig. 5 gives the greatest deviation which was found. As the angle of attack is increased, the deviaation becomes less, until near maximum lift the experimental value becomes the greater of the two, as already noted in Part I, in connection with the discussion of Fig. 2. Except for the varying distortion of the tip loading no adequate explanation accounts for this peculiarity.

Conclusion.

Finally, it must be recalled that the foregoing is in the nature of a theoretical discussion, and that the experimental data represents only a single wing on which we cannot afford to generalize too much. We must have further data. Specifically, we require wind tunnel tests on wings with dihedral and taper, on biplane combinations, and on different tip forms; we require free flight tests on various airplanes, both large and small, especially monoplanes. Without these additional data very little tangible progress can be made.

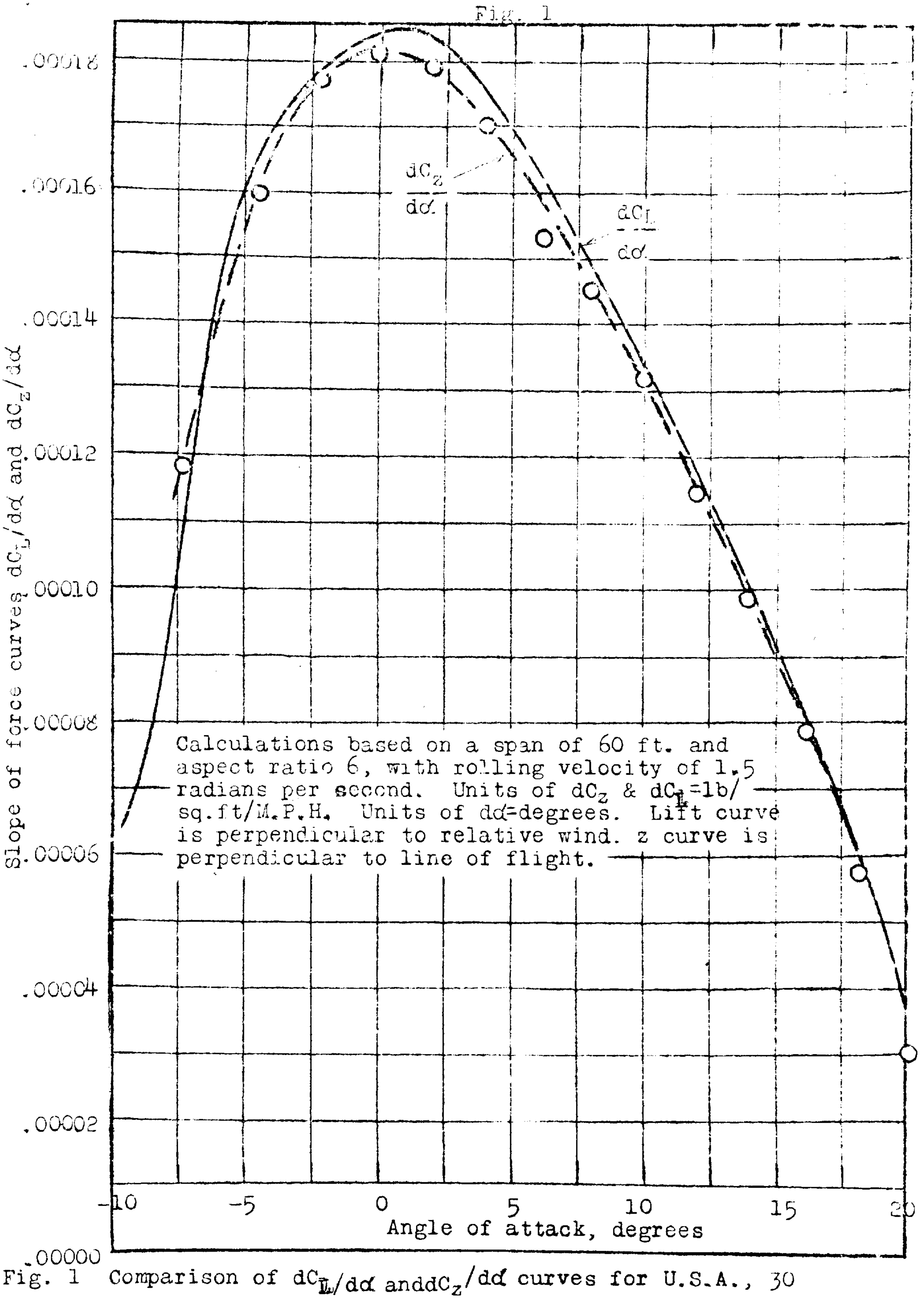
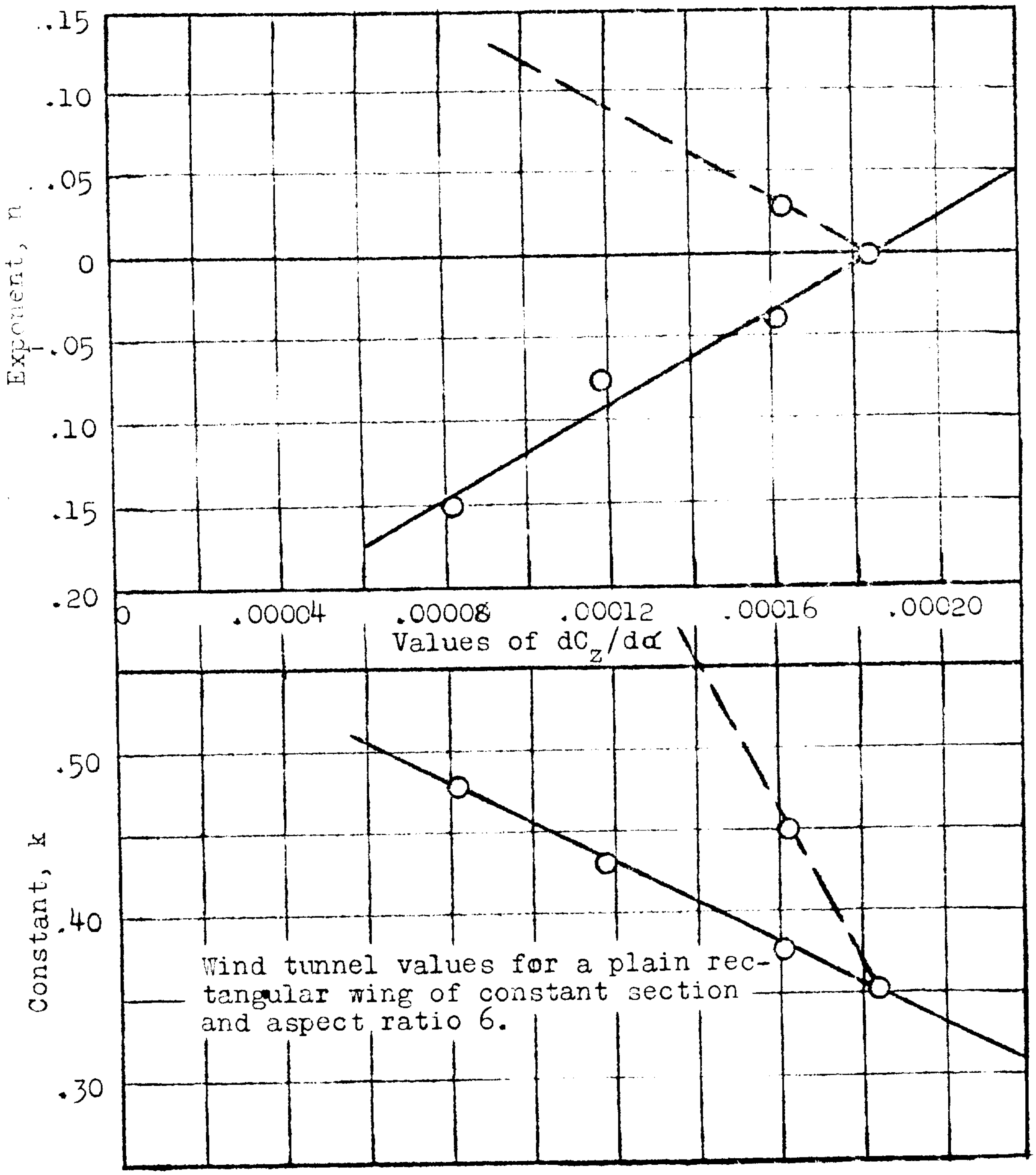


Fig. 1 Comparison of $dC_L/d\alpha$ and $dC_z/d\alpha$ curves for U.S.A., 30

Fig. 2



Full lines for decreasing values of $dC_z/d\alpha$. Dotted lines for increasing values. dC_z in lb/sq.ft/M.P.H. Units $d\alpha$ in degrees. b in feet. u in M.P.H. p in radians/sec k & n non-dimensional.

Fig. 2 Total damping factor in roll, $L_T = k_1 \left(\frac{dC_z}{d\alpha}\right) u b^4 \left(\frac{pb}{u}\right)^n$

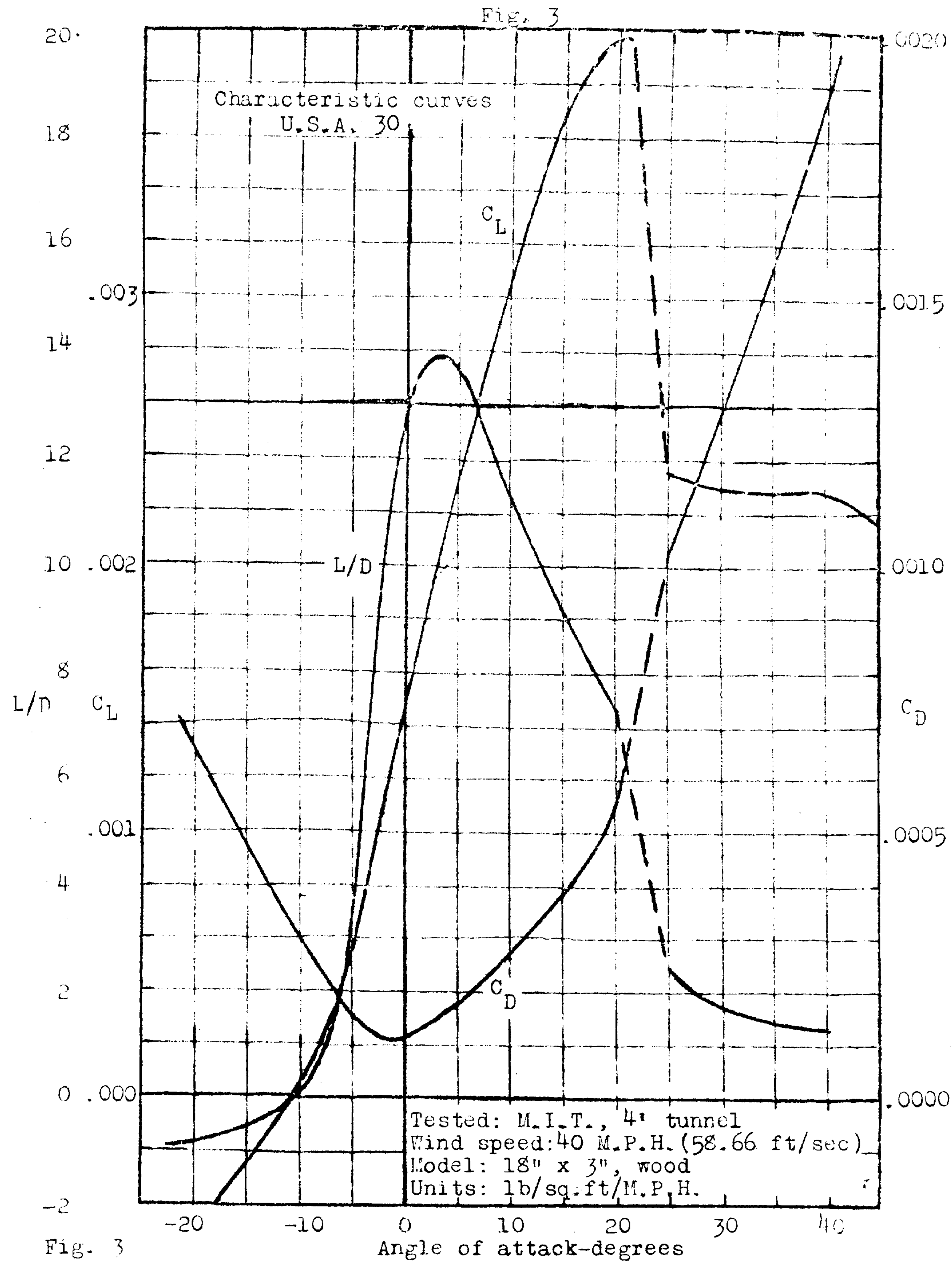
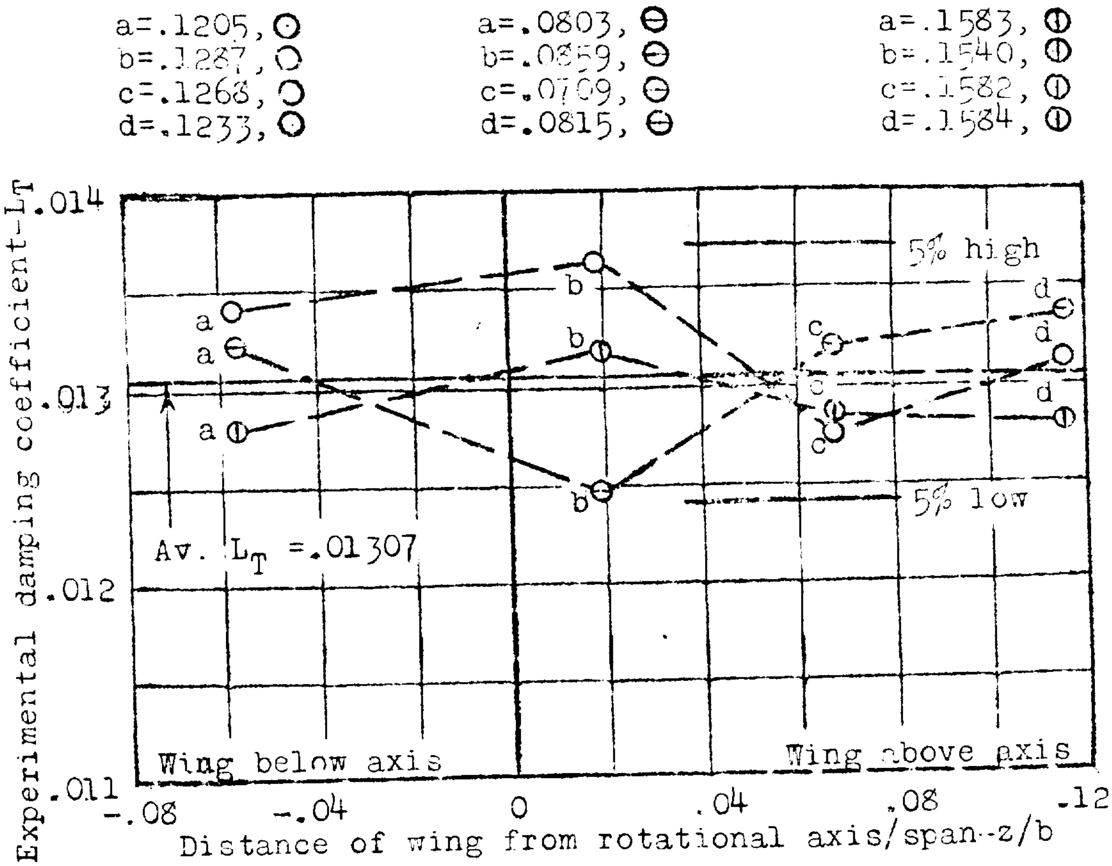


Fig. 3
Characteristic curves
U.S.A. 30
C_L
L/D
C_D
L/D
C_L
C_D
20
18
16
14
12
10
8
6
4
2
0
-2
.003
.002
.001
.000
.0020
.0015
.0010
.0005
.0000
-20
-10
0
10
20
30
40
Tested: M.I.T., 4' tunnel
Wind speed: 40 M.P.H. (58.66 ft/sec)
Model: 18" x 3", wood
Units: lb/sq.ft/M.P.H.
Angle of attack-degrees

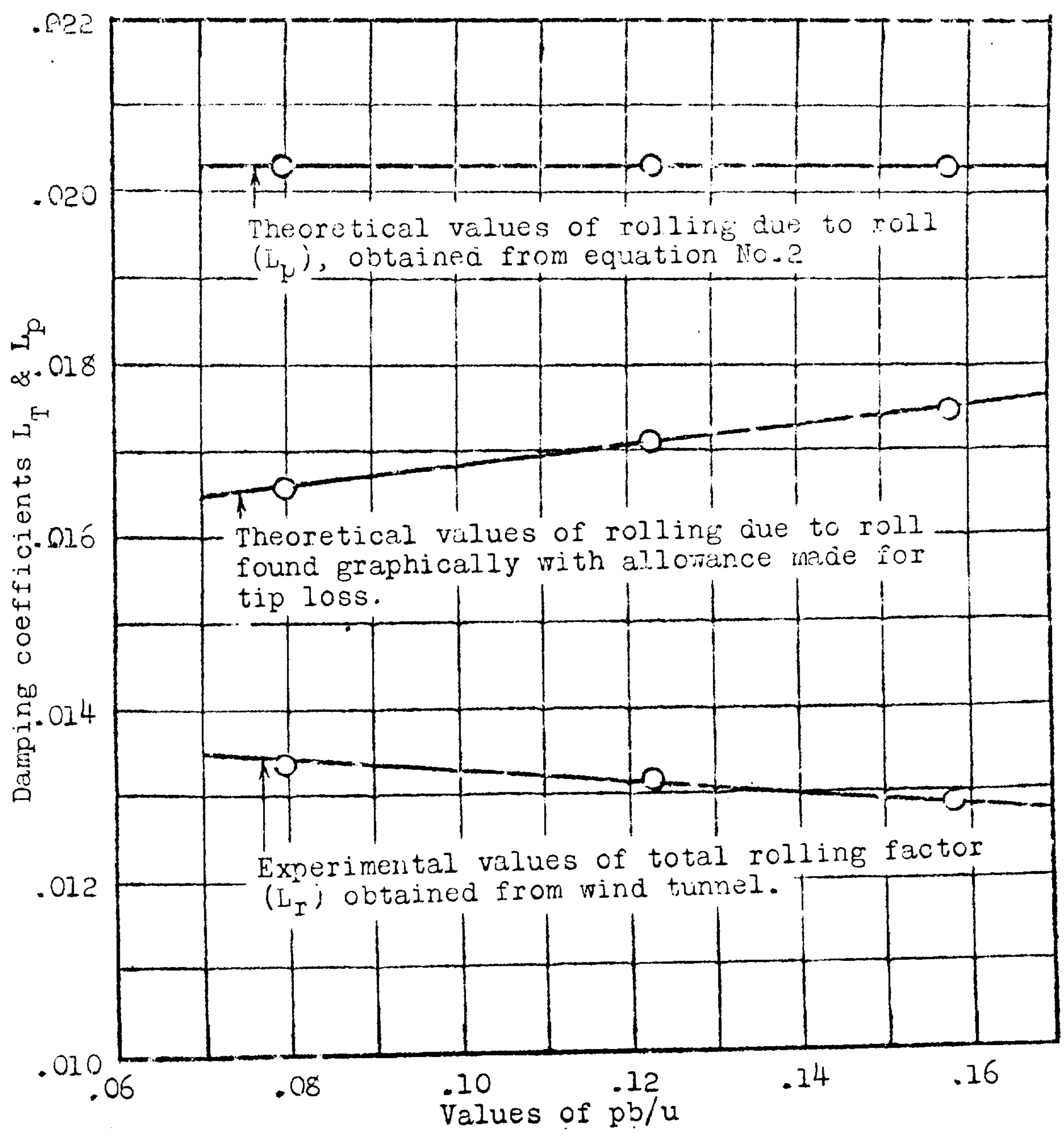
Fig. 3



The numbers above each point give values of pb/u. Groups of essentially constant values of pb/u are connected by dotted lines.

Fig. 4 Plot of wind tunnel test L_T vs z/b. Wing section, U.S.A., 30. Angle of attack, 0°

Fig. 5



$dC_z/d\alpha$=.000184 lb/sq.ft/M.P.H./degree=.00491 lb/sq.ft/f.p.s./radian

Fig. 5 Comparison of theoretical and experimental values of the damping factor in roll.

Fig. 6

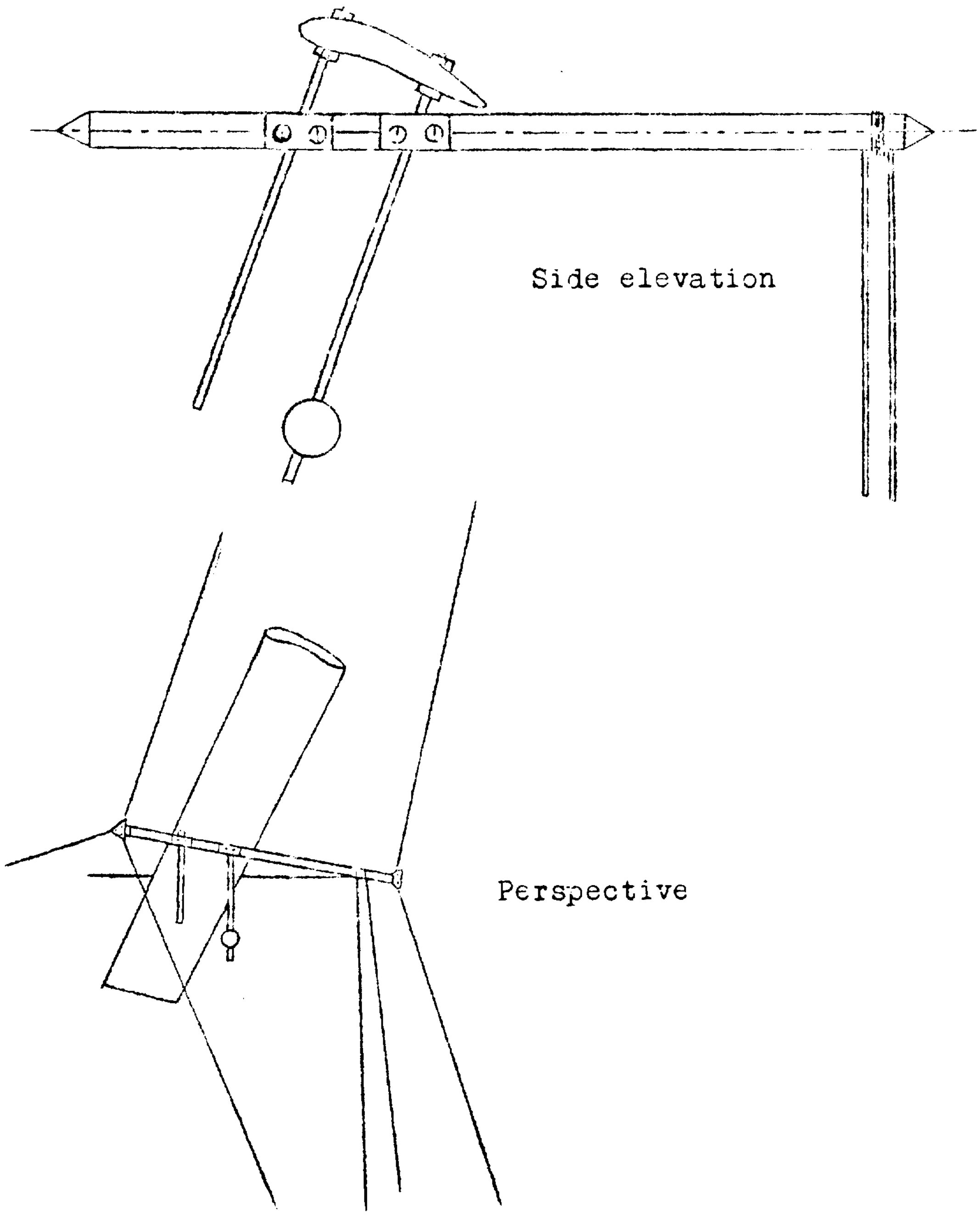


Fig. 6 Set-up of apparatus