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PRACTICAL METHOD FOR BALANCING AIRPLANE MOMENTS.

By H. Hamburger.

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PRACTICAL METHOD FOR BALANCING AIRPLANE MOMENTS.\*

By H. Hamburger.

The present contribution is the sequel to a paper written jointly by Messrs. R. Fuchs, L. Hopf and H. Hamburger,\*\* and purposes to show how the methods therein contained can be practically utilized in computations. Furthermore, the calculations leading up to the diagram of moments for three airplanes, whose behavior in war service gave reason for complaint, are analyzed and their components given. Finally, it is shown what conclusions can be drawn from the diagram of moments in regard to the defects in these airplanes and what steps may be taken to remedy them. In order to avoid the necessity of continual reference to the former paper, the arguments developed therein will be repeated wherever required for clearness. The method previously given for calculating the tail moments has been considerably simplified and made more practical in accordance with the data furnished by L. Hopf.

1. Calculation of Wing Moments.

The calculations are based on the Göttingen monoplane wing-section tests. The coefficients are defined as follows:

\* Technische Berichte, Volume III, No. 7, pp. 292-302.

\*\* Technische Berichte, Volume II, No. 3, p.463.

$$C_n = \frac{\text{normal force}}{\text{dynamic pressure} \times \text{area}} ; C_t = \frac{\text{tangential force}}{\text{dynamic pressure} \times \text{area}}$$

$$C_m^o = \frac{\text{moment about the leading edge}}{\text{dynamic pressure} \times \text{area} \times \text{chord}}$$

Since the ratio between span and chord of the experimental model is usually 6 : 1, the first problem consists in allowing for the generally more favorable aspect ratio of the airplane wings, by altering the coefficients, obtained from the tests, in a determined ratio. This ratio can be estimated from tests made by Munk\* on wing models with the same section (101) and different aspect ratios. If, for instance, the aspect ratio of the airplane wings examined is 8 : 1, while the aspect ratio of the experimental model is 6 : 1, the coefficients of the air forces acting on the airplane become

$$C_n = C_n^{(1)} \frac{C_n(8)}{C_n(6)}, C_t = C_t^{(1)} \frac{C_t(8)}{C_t(6)}, C_m^o = C_m^{(1)} \frac{C_m^o(8)}{C_m^o(6)}$$

Here the values  $C_n^{(1)}$ ,  $C_t^{(1)}$  and  $C_m^{(1)}$  are the coefficients of the model,  $C_n(8)$ ,  $C_t(8)$  and  $C_m^o(8)$  and  $C_n(6)$ ,  $C_t(6)$  and  $C_m^o(6)$  are coefficients taken from Munk's paper,\*\* for models with aspect ratios of 8 : 1 and 6 : 1, respectively.

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\* Technische Berichte, Volume II, No. 2, p.220.

\*\* Idem. See tables 15 and 17 on pages 220 and 232.

Table I.

Airplane	$b_u$	$b_l$	$c_u$	$c_l$	$\frac{b_u}{c_u}$	$\frac{b_l}{c_l}$	$\zeta$
I	9.38	7.38	1.65	1.20	5.68	6.13	$-1^\circ$
II	12.38	11.00	1.50	1.50	8.25	7.34	$0^\circ$
III	12.18	11.00	1.74	1.74	7.00	6.32	$-0.8^\circ$

The suffixes  $u$  and  $l$  indicate respectively whether the coefficient belongs to the upper or lower wing. Furthermore,  $\alpha$  is the angle between the chord of the upper wing and the direction of the relative wind, that is, the angle of attack. In order to distinguish between the angles of attack of the upper and lower wings, we also use the symbols  $\alpha_u$  and  $\alpha_l$ .  $\zeta$  is the angle which the chords of the upper and lower wings make with each other and is called the "decalage."  $\zeta = \alpha_l - \alpha_u$ , that is,  $\zeta$  is positive when the upper wing has a smaller angle of attack than the lower wing.

In the tables, all the coefficients  $C_m^0$ ,  $C_n$ ,  $C_t$  refer to the angle of attack of the upper wing. If, for instance, airplane I has a decalage of  $\zeta = -1^\circ$  then  $C_{n1} = 0.582$  and corresponds to the angle of attack  $\alpha_u = 3^\circ$  of the upper wing and to the angle of attack  $\alpha_l = 2^\circ$  of the lower wing. As regards the moments of the wings about the center of gravity, it is characteristic that a large force acts on a short lever-arm, since, in a well-designed airplane, the center of gravity and the center of pressure (namely, the point of application of the

resultant of the air forces) nearly coincide. The numerical values are, therefore, very sensitive to any displacement of the center of gravity parallel to the chord of the wings. A method is given below, whereby all changes in the curve of moments, for a slight displacement in the center of gravity, can be quickly estimated.

Table II. Coefficients for Airplane I.

(Based on section 216)

$\alpha$	$-3^\circ$	$0^\circ$	$3^\circ$	$6^\circ$	$9^\circ$	$12^\circ$	$15^\circ$
$C_{mu}^\circ$	0.088	0.233	0.296	0.334	0.380	0.422	0.410
$C_{ml}^\circ$	0.024	0.198	0.279	0.321	0.364	0.410	0.423
$C_{mu}$	0.018	0.428	0.668	0.882	1.077	1.228	1.255
$C_{nl}$	-0.100	0.515	0.582	0.810	1.015	1.190	1.260
$C_{tu}$	0.0663	0.0486	0.0145	-0.0160	-0.0725	-0.1240	-0.01590
$C_{tl}$	0.0700	0.0560	0.0250	-0.0050	-0.0490	-0.1080	-0.1490

Table III. Coefficients for Airplane II.

(Based on section 159)

$\alpha$	$-3^\circ$	$0^\circ$	$3^\circ$	$6^\circ$	$9^\circ$	$12^\circ$	$15^\circ$
$C_{mu}^\circ$	0.113	0.216	0.262	0.317	0.361	0.394	0.394
$C_{ml}^\circ$	0.117	0.214	0.259	0.317	0.364	0.410	0.396
$C_{nu}$	0.091	0.432	0.668	0.908	1.149	1.289	1.278
$C_{nl}$	0.095	0.430	0.667	0.908	1.135	1.276	1.303
$C_{tu}$	0.0484	0.0336	0.0047	-0.0418	-0.1040	-0.1720	-0.1812
$C_{tl}$	0.0484	0.0335	0.0060	-0.0550	-0.1230	-0.1880	-0.1751

Table IV. Coefficients for Airplane III.

(Based on section 47)

$\alpha$	$-3^\circ$	$0^\circ$	$3^\circ$	$6^\circ$	$9^\circ$	$12^\circ$	$15^\circ$
$C_{mu}^0$	0.098	0.210	0.275	-0.324	0.379	0.435	0.444
$C_{ml}^0$	-0.006	0.186	0.260	0.315	0.367	0.425	0.449
$C_{nu}$	0.080	0.445	0.720	0.974	1.250	1.440	1.480
$C_{nl}$	-0.050	0.350	0.625	0.866	1.125	1.335	1.420
$C_{tu}$	0.0494	0.0365	0.0055	-0.0480	-0.1090	-0.1850	-0.2090
$C_{tl}$	0.0560	0.0450	0.0160	-0.0310	-0.0880	-0.1640	-0.2140

The moments of the wings are calculated separately. It follows, from the definition of  $C_{mu}^0$ , that the moment about the leading edge =  $q S_u c_u C_{mu}^0$ , when  $q$  is the dynamic pressure. The moments which have the tendency to tilt the airplane forward, that is, to make it nose-heavy, are here considered positive, while negative moments make the airplane tail-heavy. The moment about the leading edge must now be transformed, so that it will refer to the center of gravity. This is done by means of the formulas:

$$\frac{M_u}{q} = S_u (c_u C_{mu}^0 - x_u C_{nu} + y_u C_{tu}),$$

$$\frac{M_l}{q} = S_l (c_l C_{ml}^0 - x_l C_{nl} + y_l C_{tl}).$$

The meaning of  $x_u, x_l, y_u, y_l$ , which definitely fix the position of the center of gravity with regard to the mean leading edge of the upper and lower wings, can be taken from Figures 1 and 2. Here the mean leading edge when the wings are swept

back, as in airplane III, is taken as a straight line lying at right angles to the chord and bisecting the front edge of both wings, as shown in Figure 2. Let  $\bar{x}_u$  and  $\bar{x}_l$  denote the perpendicular distance of the apex of the upper (or lower) wing from the mean leading edge and we have

$$x_u = \bar{x}_u - \frac{b_u}{4} \tan \sigma_u \quad \text{and} \quad x_l = \bar{x}_l - \frac{b_l}{4} \tan \sigma_l$$

Table V.

Airplane	$S_u$	$S_l$	$\sigma$	$\bar{x}_u$	$\bar{x}_l$	$x_u$	$x_l$	$y_u$	$y_l$
I	13.82	8.32	0	--	--	0.768	0.220	-0.690	0.683
II	17.50	15.00	0	--	--	0.818	0.685	-0.935	0.700
III	20.40	15.80	$2\frac{1}{2}$	0.790	0.570	0.660	0.452	-1.170	0.590

Table VI.

Airplane I.

$\alpha$	$-3^\circ$	$0^\circ$	$3^\circ$	$6^\circ$	$9^\circ$	$12^\circ$	$15^\circ$
$\frac{M_u}{q}$	1.160	0.312	-0.622	-1.600	-2.770	-2.230	-2.450
$\frac{M_l}{q}$	0.820	1.720	1.860	1.690	1.500	1.300	1.074
Airplane II.							
$\frac{M_u}{q}$	0.885	-1.085	-2.765	-4.015	-5.265	-5.280	-4.98
$\frac{M_l}{q}$	2.160	0.735	-0.975	-2.780	-4.750	-5.860	-6.32
Airplane III.							
$\frac{M_u}{q}$	1.210	0.585	-0.051	-0.465	-0.763	0.489	0.857
$\frac{M_l}{q}$	0.972	3.038	2.835	2.184	1.233	0.635	0.229

$\frac{M_u}{q} + \frac{M_l}{q}$  gives the total moment of the wings  $\frac{M_T}{q}$

$x_u$  and  $x_l$  are reckoned positive toward the rear and  $y_u$  and  $y_l$  are positive upward, so that  $x_u$ ,  $x_l$  and  $y_l$  are nearly always positive and  $y_u$  negative. Table V contains the numerical values used and Table VI the results of the calculations.

The total moment is now subject to modification on account of the mutual influence of the air forces acting upon the individual wings, the effect being as follows:

$$\Delta M_u = \frac{c_l}{b_u} \frac{C_{Ll}}{4\pi} \left( 2mM_u + 57.3 n \frac{dM_u}{d\alpha} \right)$$

$$\Delta M_l = - \frac{c_u}{b_l} \frac{C_{Lu}}{4\pi} \left( 2mM_l + 57.3 [n + \ln(1 + \lambda^2)] \frac{dM_l}{d\alpha} \right)$$

$$M_T = M_u + \Delta M_u + M_l + \Delta M_l.$$

Here  $\lambda$  is used as an abbreviation for  $\frac{b_u + b_l}{2G}$ ,  $m$  and  $n$  are coefficients depending upon  $\lambda$  and on the stagger  $S$ . The exact formulas have been given by Betz.\* Table VII contains the computed values.

The final values for the expression  $\frac{\text{wing moment}}{\text{dynamic pressure}}$  are plotted in Figures 3 to 5.

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\* "Berechnung der Luftkräfte auf eine Doppeldeckerzelle (Calculation of the air forces on the wings of a biplane): Technische Berichte, Volume I, No. 4, p.106, note 1. For definite values of  $\lambda$  and  $S$  we may also take  $m$  and  $n$  from plate LXV, Figure 2, and plate LXVI, Figure 3.



Table 7.

Airplane I, Angle of Stagger $\zeta = 16^\circ$ , Gap $G = 1.42$ m (4.66 ft)							
$\alpha$	$-3^\circ$	$0^\circ$	$3^\circ$	$6^\circ$	$9^\circ$	$12^\circ$	$15^\circ$
$\frac{M_T}{q}$	1.980	2.032	1.238	0.090	-1.270	-0.930	-1.376
$\frac{M_T}{q}$	2.040	1.792	0.932	-0.183	-1.442	-1.551	-1.541
Airplane II, Angle of Stagger $\zeta = 5^\circ$ , Gap $G = 1.635$ m (5.36 ft)							
$\frac{M_T}{q}$	3.045	-0.350	-3.740	-6.795	-10.015	-11.140	-11.300
$\frac{M_T}{q}$	3.637	2.560	0.585	-0.694	-2.235	-2.436	--
Airplane III, Angle of Stagger $\zeta = 7^\circ$ , Gap $G = 1.78$ m (5.84 ft)							
$\frac{M_T}{q}$	2.182	3.623	2.784	1.719	0.4705	1.124	1.086
$\frac{M_T}{q}$	2.455	4.066	2.950	1.549	0.023	0.714	0.928

### 3. Calculation of Moments of Horizontal Tail Planes.

A new difficulty, of a totally different nature from those encountered in calculating the wing moments, arises in connection with the calculation of the moments of the stabilizer and elevator.\* The moment of the tail unit consists of the product of a long lever arm (distance of tail unit from center of gravity) and a small force (lifting force on the area of the tail which is small in comparison with the area of the wings). Furthermore, in consequence of the usual symmetry of section of the tail planes (as contrasted with the high cambered wing sections) the

\* Technische Berichte, Volume II, No. 3, p.464.

variation in the location of the center of pressure of the air forces on the horizontal tail surfaces is small and may, without serious error, be assumed to coincide with the elevator hinge.

If  $l$  is the distance of the center of gravity from the hinge of the elevator and  $F_H$  the force acting at right angles to the horizontal tail surfaces, then the moment produced by the tail is  $M_H = l F_H$ .

In order to introduce non-dimensional coefficients again, we may put  $F_H = q_H S_H C_{nH}$  and also  $\frac{M_H}{q_H} = l S_H C_{nH}$ .

Here  $S_H$  is the total area of the horizontal tail surfaces and  $q_H$  the dynamic pressure on the same, which differs from the dynamic pressure  $q$  on the wings on account of the slipstream, and which also causes the difference in equilibrium between flight under engine power and gliding flight. Since the propeller increases the dynamic pressure on the tail surfaces in engine-driven flight and diminishes it in gliding flight, the average is taken as  $q_H = q$ . Hence the total moment of the airplane, under these assumptions lies between the moments for engine-driven flight and gliding flight, as has already been confirmed by flight tests.\*

In order to judge the flying qualities of an airplane, it is generally sufficient to calculate the tail moments with the elevator deflection of  $\theta = 0$  and also with it free. In both cases the curve of the coefficient  $C_{nH}$  (and, therefore, of the moment of the tail) is considered approximately as a straight

\* Technische Berichte, Volume II, No. 3, Plate 240, Fig. 15.

line. The coefficient  $C_{nH}$  is first calculated as a function of the angle of deflection  $\alpha_H$  of the stabilizer. The straight line that represents this relation must pass through the origin in the  $\alpha_H, C_{nH}$  diagram since, owing to the symmetrical shape of the stabilizer and elevator sections, the normal force must be zero when these planes are at zero angle of attack. If we succeed in determining  $\frac{dC_{nH}}{d\alpha_H}$ , the tangent of the slope of the straight line, then it is clearly defined by  $C_{nH} = \frac{dC_{nH}}{d\alpha_H} \alpha_H$ . The angle  $\alpha_H$  is here always expressed in degrees.  $\frac{dC_{nH}}{d\alpha_H}$  is determined by interpolation, with the aid of Tables VIII and IX.

Table VIII.

Plan form of stabilizer and elevator	Ru. C III	Ro1. C II	B.E.	Ru. C I	Lvg. C II	Alb. C XII
$\frac{dC_{nH}}{d\alpha_H}$	0.0593	0.0558	0.0558	0.0523	0.0453	0.0418
Aspect ratio $\frac{b_H}{c_H}$	3.12	1.78	3.26	1.54	1.20	1.16

Table IX.

Rectangular plan form.

Aspect ratio $\frac{b_H}{c_H}$	1/1	2/1	3/1	4/1	6/1	8/1
$\frac{dC_{nH}}{d\alpha_H}$	0.0314	0.0437	0.0576	0.0698	0.0803	0.00925

The values in the first table were taken from the stabilizer and elevator measurements by Munk,\* by replacing the curves (there given for the normal-force coefficient  $C_{nH}$ , according to the method of least squares) with straight lines, whose slopes have been determined. The values given in the second table have been obtained in a similar manner from Foppl's thesis and are shown in Figure 6, in order to make the interpolation easier. The plan form of the stabilizer and elevator, most closely resembling that of the airplane to be calculated, is first chosen from Table VIII and the aspect ratio of the stabilizer and elevator, of the airplane in question, is further considered, in the sense that the value of  $\frac{dC_{nH}}{d\alpha_H}$ , taken from Table III, is altered proportionally by means of Table IX. If, for example, the tail unit is similar to that of Lvg C II, and has an aspect ratio of 1 : 2, we then put  $\frac{dC_{nH}}{d\alpha_H} = 0.0453 \times \frac{0.0437}{0.0332} = 0.0593$ . Here 0.0332 is the value of  $\frac{dC_{nH}}{d\alpha_H}$  for a rectangular plan form with the aspect ratio of the Lvg C II tail. In order to check this method of calculation, further experiments are now being instituted at Göttingen.

The calculation of  $\frac{dC_{nH}}{d\alpha_H}$  gives

for airplane	I,	0.0471
"	"	II, 0.0437
"	"	III, 0.0593.

In passing from the incidence  $\alpha_H$  of the tail to the incidence  $\alpha$  of the upper wing, it must be remembered that the sta-

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\* Technische Berichte, Volume I, No.5 pp.168-189, more especially Tables 157 and 158.

bilizer is generally set at a smaller angle to the axis of the crankshaft than the upper wing. The stabilizer and the chord of the upper wing are, therefore, at an angle  $\zeta_s$ , so that  $\alpha_H = \alpha - \zeta_s$ . Here the decalage is positive when, as is generally the case, the wing is set at a steeper angle than the stabilizer. If the value of  $\alpha_H$  is put in the equation for  $C_{nH}$ , it will be observed that, with a positive  $\zeta_s$ , the straight line is displaced a distance  $\zeta_{su}$  toward the right, parallel to itself, so that  $C_{nH} = \frac{dC_{nH}}{d\alpha_H} (\alpha - \zeta_s)$ .

The effect of the downwash is yet to be considered. The coefficient of the normal force, as modified by the downwash, may be denoted by  $C_{nH}^\epsilon$ . It follows, from the Prandtl theory, that the downwash disappears in any case for the incidence at which the lift becomes zero. This is the case, for example, for  $\alpha = -4.5^\circ$ . At this angle, therefore, the value of  $C_{nH}^\epsilon$  cannot be altered by the downwash. If the curve for  $C_{nH}^\epsilon$  plotted against  $\alpha$ , taking account of the downwash, may be approximately represented by a straight line, the latter must pass through the point P (Fig. 8) of the straight line  $C_{nH}$ , when the abscissa  $\alpha = -4.5^\circ$ . The straight line  $C_{nH}^\epsilon$  is, therefore, definitely fixed, when its inclination  $\frac{dC_{nH}^\epsilon}{d\alpha_H}$  has been determined. The inclination of the straight line is much reduced by the downwash and, on an average,  $\frac{dC_{nH}^\epsilon}{d\alpha_H} = 0.54 \frac{dC_{nH}}{d\alpha_H}$ .

A more exact value may be found by using the formula (derived from Prandtl's vortex theory)

$$\frac{dC_{nH}^e}{d\alpha} = (1 - \Delta) \frac{dC_{nH}}{d\alpha_H}$$

For biplanes we obtain

$$\Delta = 0.62 \frac{c_u}{b_u} \left( 1 + \sqrt{1 + \left(\frac{b_u}{2l}\right)^2} \right) + \frac{c_l}{b_u} \left( 1 + \sqrt{1 + \left(\frac{b_l}{2l}\right)^2} \right)^*$$

The greatest value of  $\Delta$ , which we have hitherto calculated for the usual types of airplanes, is 0.60 and the smallest is 0.40.

For monoplanes

$$\Delta = 0.73 \left\{ \frac{c}{b} \left( 1 + \sqrt{1 + \left(\frac{b}{2l}\right)^2} \right) \right\}$$

While, with a difference of incidence (or decalage)  $\zeta_s$ , the straight line of the tail moments, uninfluenced by the downwash, is displaced by an amount  $\zeta_g$ , parallel to itself, the parallel displacement of the line of moments of the tail without decalage, and under the influence of the downwash, is considerably greater than  $\zeta_g$ , as can be found from Figure 8. The results of the calculations are given in Table X.

In order to determine the straight line, which approximates the curve of moments of the tail with free elevator, as a function of the incidence of the upper wing, it must be remembered that the free elevator, when the section is symmetrical, adjusts itself automatically in the direction of the stabilizer, that is, it has zero angular motion,  $\theta = 0$ , when the moment on the tail is zero. With  $\alpha = 9.3^\circ$ , the moment of the tail vanishes, in Figure 8, for  $\theta = 0$ . The elevator is, therefore, completely without load for this incidence and consequently the stabilizer

\* Compare Technische Berichte, Volume II, No. 3, p.482.

is similarly without load for the same angle, when their section is symmetrical and the elevator is free, that is to say, the straight line of the moments of the horizontal tail surfaces also passes through zero, when the elevator is free. The inclination of this straight line,  $\frac{dC_{nH}^e}{d\alpha}$ , is determined by the values of  $\frac{dC_{nH}}{d\alpha}$  for rectangular surfaces having an aspect ratio of 1 to 3, which is plotted in Figure 7 against the fraction

$$\frac{S_e}{S_H} = \frac{\text{area of elevator}}{\text{total area of stabilizer and elevator}}$$

Table X.

Airplane	I	II	III
$S_H$	2.48	4.37	3.62
$l$	5.34	5.375	5.18
$\zeta_s$	$3.5^\circ$	$2^\circ$	$2^\circ$
$\Delta$	-0.478	-0.395	-0.465
$\frac{dC_{nH}^e}{d\alpha}$	-0.0246	0.0264	0.0318
Angle of attack for zero lift	} $-3^\circ$	$-4^\circ$	$-4^\circ$
Angle of attack for this angle $C_{nH}^e$	} -0.160	-0.158	-0.191
$\frac{d}{d\alpha} \frac{M_H}{q}$	0.326	0.619	0.597

The values have been taken from the tests made by Munk,\* by replacing the given curves with straight lines. For instance, with

\* Technische Berichte, Volume I, No. 5, p.168. plates CLXVIII to

a rectangular stabilizer and elevator having an aspect ratio of 3 : 1, divided in the proportion of  $\frac{S_e}{S_H} = \frac{2}{5}$ , the value 0.0576 for  $\frac{dC_{nH}}{d\alpha}$  is replaced by 0.0436, as shown by Figure 6, in passing from the case of  $\theta = 0$  to the case of the free elevator.

If it is assumed, as in the example on which Figure 8 is based, that the horizontal tail surfaces are divided in the ratio  $\frac{S_e}{S_H} = \frac{2}{5}$ , it may then be assumed that  $\frac{dC_{nH}^e}{d\alpha}$ , in passing from  $\theta = 0$  to the free elevator, varies in the same ratio as with a rectangular elevator having an aspect ratio of 1 : 3 and that, therefore,  $\frac{dC_{nH}^e}{d\alpha} = 0.025$  with a free elevator. The straight line of the tail moments is thus exactly defined for a free elevator by the inclination and by the point Q (Fig. 8). If the ratio  $\frac{S_e}{S_H}$  of the tail unit in question differs from  $\frac{2}{5}$ , then, instead of placing 2.43 in the numerator of the proportionality factor, the corresponding value must be taken from Fig. 7. With free elevators, the following calculated values were obtained for the three airplanes:

Airplane	$\frac{dC_{nH}^e}{d\alpha}$	$\frac{d}{d\alpha} \frac{M_H}{q}$
I	0.0174	0.230
II	0.0109	0.388
III	0.0213	0.400



### 3. Diagrams of Moments.

Three things are required in the diagram of moments:

I. In flat gliding and steep climbing, that is, with a mean angle of attack of the wings between  $6^\circ$  and  $9^\circ$ , the moments with a free elevator must be nearly in equilibrium, in order that no great deflection of the elevator may be required in this condition of flight.

II. If the mean slope  $\lambda$  of the curve of the wing moments has been determined by replacing the curve by a straight line by the method of least squares for values of  $\alpha$  between  $6^\circ$  and  $9^\circ$ , (Figs. 4 and 5) then the coefficient of static stability of the wing,  $\frac{d}{d\alpha} \frac{M_T}{q} = \tan \lambda$  (a quantity which is always negative) should not, except for the sign, differ greatly from the tangent of the slope of the straight line,  $\frac{d}{d\alpha} \frac{M_H}{q}$ , the coefficient of static stability for the tail. Large deflections of the elevator will otherwise be required, in order to equalize the moments for other conditions of flight, as, for instance, those with small angles of attack. As a rule, we seek to obtain moderate static stability, i.e., preferably the straight line for the tail rises a little more steeply than the wing curve falls, so

$$\frac{d}{d\alpha} \frac{M_T}{q} + \frac{d}{d\alpha} \frac{M_H}{q} > 0.$$

It would appear that a high degree of stability of airplanes is

preferred in England, since  $\frac{d}{d\alpha} \frac{M_T}{q}$  is, on an average, 40 to 45% greater than  $\frac{d}{d\alpha} \frac{M_T}{q}$  in British airplanes. Among German airplanes, the Albatros class is characterized by its high stability. Others prefer a more nearly neutral equilibrium and even slight instability, in the belief that this feature increases the maneuverability of the airplane. High stability can only be obtained by improving the tail unit. If it is sought to attain it by excessive forward displacement of the center of gravity, nose-heaviness generally results, a condition which has been observed in several German airplanes. Slight instability has been found in various airplanes which have given good service. Perfect neutrality, that is, exact mirror-image relation of the two straight lines, cannot actually be attained, since the inclination of the wing curve is quite sensitive to slight displacements of the center of gravity, even 2 cm (.787 in.), for instance, as always occurs during flight, owing to fuel consumption. The value of  $\frac{d}{d\alpha} \frac{M_T}{q}$  is changed 11% on the average by this displacement of the center of gravity.

III. The moments of the tail and wing must each be small for the most frequently occurring conditions of flight, i.e., with an angle of attack between  $6^\circ$  and  $9^\circ$ . Otherwise, the net moment of the wings and tail would be the difference between large quantities so that, for relatively small disturbances, large moments would appear, for which no reserve of control power would be available and which might prove fatal to the pilot. It

must, therefore, be regarded as imperative that the curve of the wing moments and also, in accordance with condition I, the straight line of the tail moment, should intersect the angle of attack between  $6^{\circ}$  and  $9^{\circ}$ .

1. In designing airplanes, these three points may be conveniently taken in the reverse order, that is, we may start with condition III and endeavor to place the center of gravity so that the moment of the wings disappears for an angle of attack between  $6^{\circ}$  and  $9^{\circ}$ , i.e., as close as possible to the center of pressure of the air forces at this angle of attack, for when these two points coincide, the wing moment vanishes.

2. The  $l \times S_H$  of the tail should, further, be so dimensioned that condition II is fulfilled and the straight line of the wing moment and that of the tail moment should, therefore, intersect at the desired angle, according to whether considerable stability or neutrality of equilibrium is preferred.

3. Lastly, it is important that the angle of décalage between the stabilizer and the upper wing be so chosen, that the straight line of the tail moments shall cut the axis of abscissas at the same point as the curve of the wing moments, whereby condition I will also be fulfilled.

The diagrams for the three airplanes computed will now be examined from these three points of view. In airplane I, as the diagram of moments (Fig. 3) shows, the center of gravity is in

approximately the correct position. Moving the center of gravity forward about 3 cm (1.18 in.) would perhaps improve the flying qualities somewhat, but this small deviation from the most favorable position is insignificant. Condition III is therefore essentially fulfilled. It is further seen that, as required by condition II, the curve of the wing moments descends almost as steeply as the curve of tail moments rises.

Complaints were received regarding the tail-heaviness of airplane I in engine-driven flight, which did not appear in gliding flight. The diagram, in fact, shows that the negative moments of the tail preponderate, since the moment curves of the wings and tail do not intersect on the axis. From this it may be concluded, that the difference of incidence between the stabilizer and the wings is about  $2^{\circ}$  greater than appears best for the flying qualities. The stabilizer might, therefore, be inclined about  $2^{\circ}$  more to the engine axis. Such small discrepancies can generally be corrected simply by bracing the wings and in this way altering the angle between the chord of the wings and the axis of the crankshaft. Complaint was made, however, in the case of this particular airplane, that on account of the unsuitable arrangement of the brace wires, the requisite adjustments could not be made in the manner indicated. After this defect was remedied, the flying qualities were perfectly satisfactory.

The fact that complaint of nose-heaviness in the above airplane was made in reference to flight under power and not in

gliding flight, is explained simply by the fact that the tail is less effective in gliding than in power flight, in which the dynamic pressure on the tail is increased by the slipstream.

It is often assumed that tail-heaviness, when it occurs only during power flight, is a consequence of the moment produced by the propeller thrust. The propeller thrust, during flight near the ground, with a mean incidence of  $6^{\circ}$  to  $9^{\circ}$ , may be estimated at 270 kg (595.3 lb.) and the dynamic pressure  $q$ , at  $52 \text{ kg/m}^2$  ( $10.65 \text{ lb/ft}^2$ ). Since the center of gravity is about 0.10 m above the axis of the crankshaft,

$$\frac{\text{moment of propeller thrust about center of gravity}}{\text{dynamic pressure}}$$

$$= - \frac{270 \times 0.1}{0.52} = - 0.52.$$

This value is considerably smaller at higher altitudes. The thrust with the same angle of attack amounts to only 206 kg (454.2 lb.) at an altitude of 4000 m (13123 ft.) and accordingly

$\frac{\text{moment}}{\text{dynamic pressure}} = 0.40$ . This value is, however, of little importance in comparison with the moments of the wings and tail, (Fig. 3).

Figure 4 is a characteristic example of the diagram of moments for a very tail-heavy airplane. In this case, none of the three conditions has been fulfilled. Firstly, the straight line of the wing moments intersects the angle of attack axis at  $0^{\circ}$ , instead of between  $6^{\circ}$  and  $9^{\circ}$ . Secondly, the fall of the curve

of moments is much steeper than the rise of the straight line of the tail moments. The airplane is, therefore, decidedly unstable. Lastly, the straight line of the tail moments and the curve of the wing moments do not intersect the angle of attack at the same point. The curve of the wing moments was originally calculated for a center of gravity which corresponded with the information given by the manufacturer. This gave a curve (also plotted in Fig. 4), which cuts the angle of attack axis in the manner required by condition III. The center of gravity, determined in this manner, was, however, 19 cm (7.48 in.) further forward than the center of gravity found in the completed airplane, to which Tables V-VII refer. Tests with various airplanes of this type gave further evidence that the center of gravity was always in about the same position, but that the difference in decalage between the stabilizer and the wings frequently amounted to only  $1^{\circ}$ , instead of  $2^{\circ}$ . The fact that these airplanes behaved fairly well in flat gliding flight in still air can be explained only by the small decalage, which was, indeed, still further reduced by the tautness of the brace wires and the resultant warping of the wings. Under all other conditions of flight, the airplane was very tail-heavy, and was considered extremely dangerous in gusty weather. These observations confirm the above theory, according to which, the defects, resulting from large individual moments of the wings and tail, become especially evident in such possible disturbances of the equilibrium as are met with in rough weather. In order to render airplane II

usable, the center of gravity must be shifted forward at least 19 cm (7.48 in.) and the dimensions and setting of the tail must, of course, be altered accordingly.

The opinions of the pilots were divided in regard to airplane III. Some contended that it behaved perfectly in the air, while others complained that it was nose-heavy. The diagram of moments (Fig. 5) shows two curves for the wing moments. The one is calculated with reference to the center of gravity of the fully loaded airplane. It cuts the angle of attack axis at  $9^{\circ}$  and condition III is, therefore, tolerably well satisfied. For this position of the center of gravity, the airplane also complies with conditions I and II, as shown by Figure 5.

The approximate formula, which is further explained below, has been used in calculating the curve of wing moments for the case where the fuel supply is exhausted and the airplane is only lightly loaded. The curve of the wing moments, for the thus altered position of the center of gravity, shows, in fact, a marked nose-heaviness and it may therefore be assumed that the pilots, whose opinions differed, flew the airplane under different loads. The defect may be remedied by shifting the center of gravity backward. A reduction in the amount of the sweep back of wings would have the same effect, since this is tantamount to advancing the mean leading edge of the wings.

#### 4. The Approximate Formula and its Application.

In the above calculations, the curve of the tail moments was at first represented by a straight line, while the wing moments for different angles of attack were specially calculated. In order, however, to facilitate the comparison of this curve with the straight line of the tail moments and in order to transform the wing moments quickly, when it is desired to refer them to a different center of gravity, a straight line was again used to represent the curve of the wing moments between  $0^\circ$  and  $9^\circ$ . If the angle between the straight line and the incidence axis is denoted by  $\lambda$ , we have

$$\tan \lambda = S c_m \frac{dC_{mT}}{d\alpha} \text{ and}$$

$$C_{mT} = \frac{\text{wing moment about center of gravity}}{\text{dynamic pressure} \times \text{total area of wings} \times \text{mean chord}} .$$

The mean chord  $c_m = \frac{c_u S_u + c_l S_l}{S_u + S_l}$  .  $\frac{dC_{mT}}{d\alpha}$  is a mean value for

the derivation of  $C_{mT}$  with respect to the angle of attack  $\alpha$ , again reckoned in degrees.

The position of the center of gravity behind the leading edge is denoted by its distance  $x$  from an assumed mean leading edge of the biplane, measured parallel to the chord of the upper wing and may be written

$$x = \frac{x_u S_u + x_l S_l}{S_u + S_l} .$$



If now  $C_{mT}^{\circ}$  is the coefficient of the total moment of the two wings about the mean leading edge lying in the chord of the upper wing (for which  $x = 0$ ,  $y_u = 0$ ), and if  $C_{nT}$  and  $C_{tT}$  are the coefficients of the normal force and the tangential force of the biplane, then we have

$$C_{mT} = C_{mT}^{\circ} + \frac{y_u}{c_m} C_{tT} - \frac{x}{c_m} C_{nT} \quad (1)$$

$$\frac{dC_{mT}}{d\alpha} = \frac{dC_{mT}^{\circ}}{d\alpha} + \frac{y_u}{c_m} \frac{dC_{tT}}{d\alpha} - \frac{x}{c_m} \frac{dC_{nT}}{d\alpha} \quad (2)$$

In order to devise an approximate formula for  $\frac{dC_{mT}}{d\alpha}$ , average values were calculated for  $\frac{dC_{mT}^{\circ}}{d\alpha}$ ,  $\frac{dC_{nT}}{d\alpha}$  and  $\frac{dC_{tT}}{d\alpha}$ , by replacing the known measurements for biplanes (within a range of  $\alpha$  from  $0^{\circ}$  to  $9^{\circ}$ ) by straight lines by the method of least squares, by determining the tangents of their slopes and by taking the mean.

In this way, we obtained the following mean values:

$$\frac{dC_{mT}^{\circ}}{d\alpha} = 0.00837, \quad \frac{dC_{tT}}{d\alpha} = 0.00942, \quad \frac{dC_{nT}}{d\alpha} = 0.0644. \quad \text{It is seen}$$

that the value of  $\frac{dC_{tT}}{d\alpha}$  is only a seventh part of the value of  $\frac{dC_{nT}}{d\alpha}$ . Hence  $\frac{dC_{mT}}{d\alpha}$  is seven times more sensitive to displacements of the center of gravity to the rear or to the front (that is, to a change of  $\frac{x}{c_m}$ ), than to displacements up or down (that is, to a change of  $\frac{y_u}{c_m}$ ). We may, therefore, conclude that stability in an airplane depends chiefly on  $\frac{x}{c_m}$  and, in fact, we

find that the numerical value of this expression lies between quite definite limits in airplanes which have given satisfaction. In airplanes with high stability,  $\frac{x}{C_m}$  falls between 0.32 and 0.36, and in neutral or slightly unstable airplanes, between 0.36 and 0.40. In airplanes where  $\frac{x}{C_m}$  is smaller than 0.32 or greater than 0.40, nose-heaviness or tail-heaviness must be expected. The values of  $\frac{x}{C_m}$ , for the three airplanes calculated, are given in Table II.

Table II.

Airplane	I	II		III	
		Actual center of gravity	c.g. according to manufacturer	Fully loaded airplane	Lightly loaded airplane
$\frac{x}{C_m}$	0.376	0.505	0.378	0.327	0.281

It would appear desirable to introduce a mean value for  $\frac{y_u}{C_m}$  also and to state the approximate formula so that  $\frac{dC_{mT}}{d\alpha}$  shall only depend upon  $\frac{x}{C_m}$ . The quantity -0.59 has been chosen as the mean value for  $\frac{y_u}{C_m}$ . The maximum observed deviation is 0.28 and the mean deviation is 0.103. In this manner we obtain the approximate formula

$$\frac{dC_{mT}}{d\alpha} = 0.014 - 0.0628 \frac{x}{C_m}$$

This formula, which differs somewhat from that given previously,\*

\* Technische Berichte, Volume II, No. 3, p.482.

has been derived from more extensive data and should, therefore, be more reliable.\* The corresponding formula for monoplanes is

$$\frac{dC_{mT}}{d\alpha} = 0.0172 - 0.0126 \frac{Y}{c} - 0.0756 \frac{X}{c} .$$

Since the difference in value of  $\frac{Y}{c}$  between a "parasol" monoplane and an airplane with low-set wings is too great, it is not here possible to introduce a mean value for  $\frac{Y}{c}$ . The greatest observed deviation of the value of  $\frac{dC_{mT}}{d\alpha}$  (obtained from the approximate formula for biplanes), from that obtained in the manner described, by taking the mean of the curves of the wing moments, amounts to 22%, the mean error being 12%. It is, therefore, obvious that this approximate formula can only serve as a guide and can, in no way replace exact calculation. The lack of exactness in the approximate formula is chiefly due to the mean values for  $\frac{dC_m^o}{d\alpha}$  and  $\frac{dC_t}{d\alpha}$ , in which the differences in the wing sections have much influence. The mean value for  $\frac{dC_{mT}}{d\alpha}$  is much more useful and may now be used for transforming the curve of wing moments, when it has been calculated for one center of gravity, to another center of gravity.

In order to make this procedure plain, we will use airplane II for an example. The moments of this airplane were calculated for the actual center of gravity, which lies 19 cm (7.48 in.) behind the center of gravity given by the manufacturer. The

\*  $\frac{S}{S_H} \frac{c}{l}$  is made nearly equal to the values given in Technische Berichte, Volume II, No. 3, p.483, in connection with the approximate formula. These values are somewhat too high. They vary, as a rule, between 2.4 and 3.4 and only rise to 5, when the tail design is very favorable.

equivalent straight line, found by the method of least squares, forms with the angle  $\lambda$ , whose tangent is  $-0.52$ . Then

$$\frac{dC_m}{d\alpha} = \frac{1}{S c_m} \tan \lambda = -0.0202. \quad \text{For the moment about the center of}$$

gravity .19 m (7.48 in.) further forward, we obtain, on the

$$\text{basis of formula (2), } \frac{dC_m'}{d\alpha} = -0.0202 + \frac{dC_{nT}}{d\alpha} \frac{0.19}{c} = -0.0105,$$

when we substitute for  $\frac{dC_{nT}}{d\alpha}$ , the approximate value 0.0628.

The prime in the coefficient  $C_m'$  is intended to designate that this coefficient refers to the new center of gravity. If the angle which the new straight line of the wing moments makes with the angle of attack axis is denoted by  $\lambda'$ , then

$$\tan \lambda' = - S c_m 0.0105 = - 0.512.$$

The direction of the new straight line, which represents the course of the wing moments with a fair approximation, has thus been calculated. In order to locate the straight line, we must have one more point. We therefore calculate the quantity  $M_T'$ , which is referred to the new center of gravity for an angle of attack of  $0^\circ$ . The moment about the old center of gravity was 0.086 for  $\alpha = 0^\circ$ . For the new center of gravity, we therefore obtain  $\frac{M_T'}{q}$  (for  $\alpha = 0$ ) =  $0.086 + S C_{nT} 0.19$ , in which  $M_T'$  is again to be referred to the new center of gravity. The value of  $C_{nT}$  for  $0^\circ$  can be easily calculated by the approximate formulas of Blasius and Hamburger.\* When the effect of the mutual influence is but slight, the results given by the approximate form-

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\* Technische Berichte, Volume II, No. 2, p.341.

ula (for  $\alpha = 0^\circ$ ) agree well with the results of the tests. The calculation gave  $C_{nT} = 0.39$ . Then  $\frac{M_{nT}'}{q}$  (for  $\alpha = 0$ ) = 2.49. The corresponding straight line is drawn in the diagram of the moments in Figure 4. For comparison with this straight line, which was found by an approximate calculation, the values of the curve were plotted, which were obtained by exact calculation of the wing moments about the new center of gravity. It is obvious that the agreement is exceedingly good.

The wing moments for airplane III are transformed according to the same method, when the center of gravity of the fully loaded airplane shifts to that of the lightly loaded airplane. For the coefficient of normal force, the calculation gave

$$C_{nT} = 0.36 \quad \text{for } \alpha = 0^\circ.$$

From the previous calculation, we obtained  $\frac{M_{nT}}{q} = 4.07$ , when  $\alpha = 0$ . Since the center of gravity of the lightly loaded airplane is 8 cm (3.15 in.) forward of the position for the fully loaded airplane, the wing moment referring to the new center of gravity, when  $\alpha = 0$ , becomes  $\frac{M_{nT}'}{q} = 4.07 + S C_{nT} 0.08 = 4.07 + 36.2 \times 0.36 \times 0.08 = 5.11$ .

The angle of inclination  $\lambda'$  of the straight line, which represents the wing moments about the new center of gravity, is found from the formula

$$\tan \lambda' = \frac{d}{d\alpha} \frac{M_{nT}'}{q} = \tan \lambda + S \frac{dC_{nT}}{d\alpha} 0.08.$$

If the curve for the wing moments previously found for air-

plane III is replaced by a straight line, we have  $\tan \lambda = 0.453$ . Putting the mean value  $\frac{dC_{aT}}{d\alpha} = 0.0628$  and  $S = 36.2$  in the formula for  $\tan \lambda'$ , we obtain  $\tan \lambda' = -0.271$ . A straight line with this slope, which cuts the axis of the ordinates at 5.11 (Fig. 5), must therefore represent the wing moments about the new center of gravity. For this airplane the question is further raised as to how the center of gravity must be shifted, in order that, with an average load, the nose-heaviness may be removed or (which amounts to the same thing) that the curve of the wing moments may cut the angle of attack axis at approximately  $\alpha = 7.3^\circ$ .

For this purpose, we start from the calculated curve of wing moments for the fully loaded airplane and denote by  $\delta$  the amount the center of gravity must be shifted. Here  $\delta$  must be positive when the center of gravity is moved backward and in the other instance, as in the previous transformation, it must be negative. We again assume this curve to be represented approximately by a straight line and denote its inclination to the axis of the abscissas by  $\lambda''$ . In order to calculate  $\delta$ , we seek to express  $\lambda''$  in two different ways by means of  $\delta$ . First, we have  $\tan \lambda'' = \frac{M_T''}{q}$  (for  $\alpha = 0$ ), when  $M_T''$  is taken as the wing moment about the new position of the center of gravity. According to equation (1), we have

$$\frac{M_T''}{q} \text{ (for } \alpha = 0) = \frac{M_T}{q} - S C_{nT} \delta \quad (3)$$

$$\text{Therefore, } \tan \lambda'' = \frac{-4.07 - 36.2 \times 0.36 \delta}{7.3} \quad (4)$$

On the other hand, by reference to equation (3),

$$\begin{aligned}\tan \lambda'' &= \frac{d}{dc} \frac{M_m}{q} = \tan \lambda - S \frac{dC_{nT}}{da} \delta = \\ &= -0.453 - 36.2 \times 0.0628 \times \delta\end{aligned}\tag{5}$$

By means of equations (4) and (5), we obtain a linear equation for  $\delta$ , which gives the value  $\delta = 0.026$ . We obtain the new straight line for the wing moments by calculating  $\tan \lambda''$  from equation (5) and  $\frac{M_m''}{q}$  (for  $\alpha = 0$ ) from equation (3). This straight line is also drawn in the diagram of moments in Figure 5. While the accurately calculated curve for the wing moments was referred to a center of gravity at a distance  $x_{u1} = 0.66$  m from the upper mean leading edge, the new curve of moments is referred to a center of gravity  $0.66$  m +  $0.028$  m =  $0.688$  m (2.26 ft.) from the upper leading edge.

The wings of airplane III had a marked sweep back  $\sigma = 2.5^\circ$ . Since a diminution of the sweep back corresponds to a shifting of the center of gravity backward, we are in a position to calculate how much this sweep back must be reduced, in order to make the nose-heaviness disappear. For the fully loaded airplane, the distance of the center of gravity from the mean upper leading edge is  $\bar{x}_{u1} = 0.79$  m (2.59 ft.) and for the lightly loaded airplane it is  $\bar{x}_{u1} = 0.71$  m (2.33 ft.), so that we have  $\bar{x}_{u1} = 0.75$  m (2.46 ft.) for an average load. As, however, the

distance should be  $\bar{x}_u = 0.683$ , the mean leading edge must be  
 $\frac{b_u}{2} \tan \sigma = 0.75 \text{ m} - 0.688 \text{ m} = 0.067 \text{ m}$  (.22 ft.) from the apex  
of the wings. Putting, as above,  $b_u = 12.18 \text{ m}$  (40 ft.), we  
obtain  $3.045 \tan \sigma = 0.067$  and therefore,  $\sigma = 1.25^\circ$ .

Translated by  
National Advisory Committee  
for Aeronautics.



Fig.1,2,3.

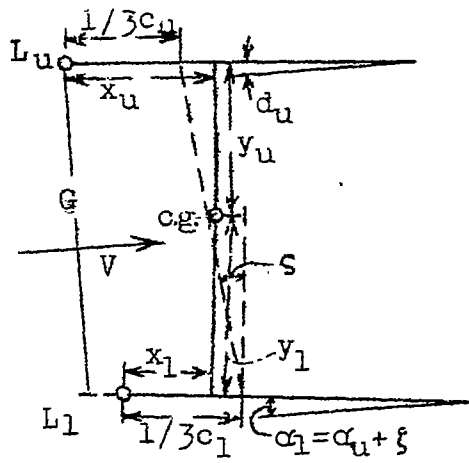


Fig.1

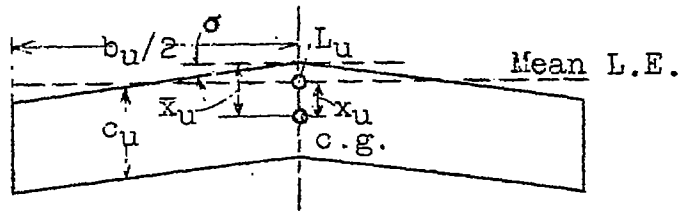


Fig.2

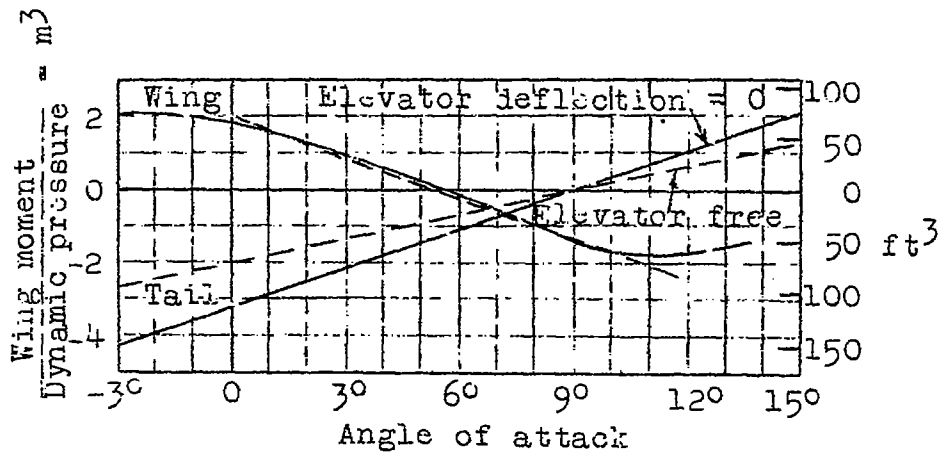


Fig.3 Diagram of moments for airplane I

Figs. 4,5.

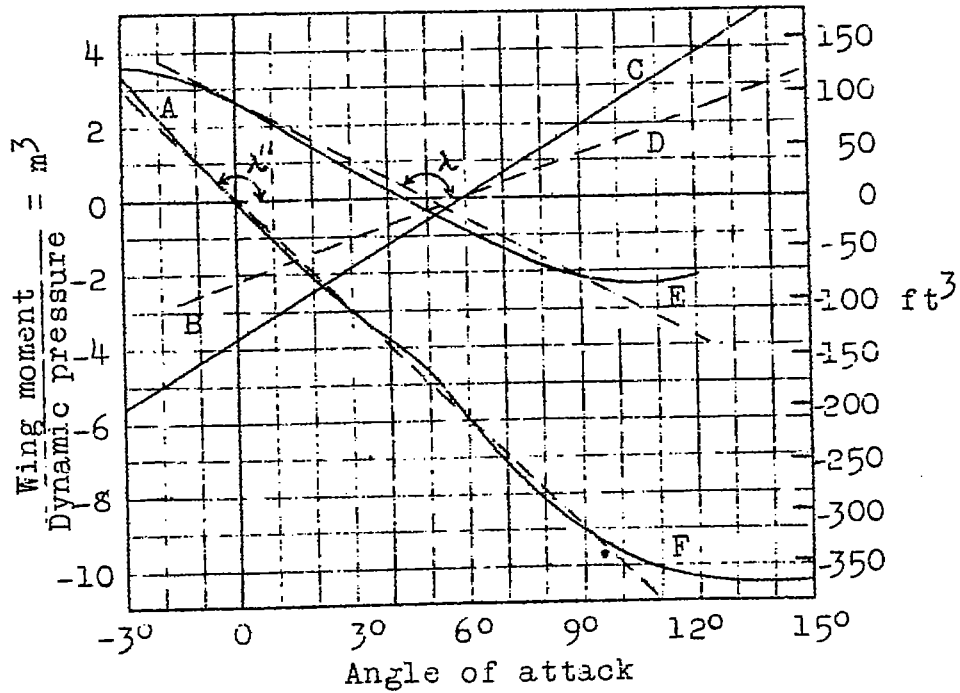
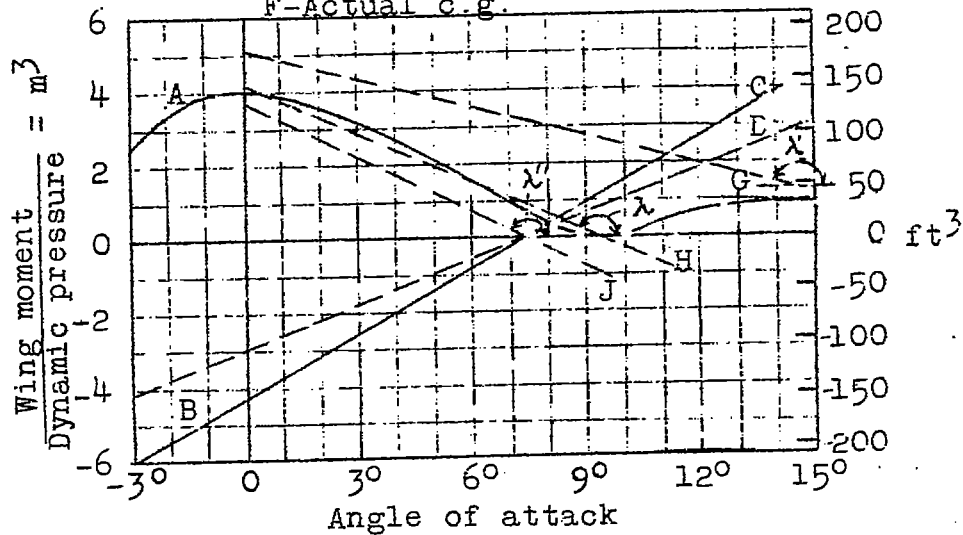


Fig.4 Diagram of moments for airplane II

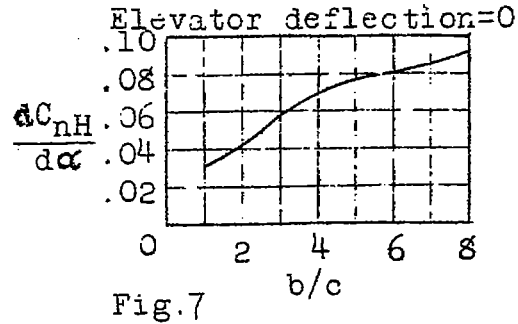
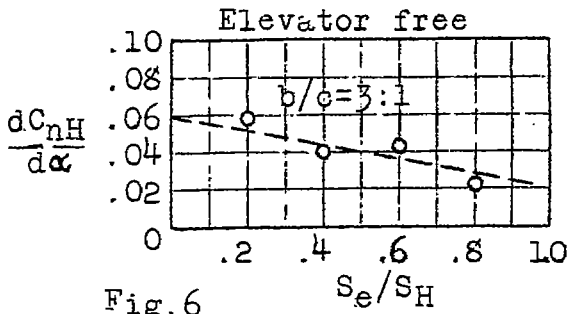
A-Wing  
 B-Tail  
 C-Elevator deflection=0  
 D-Elevator free  
 E-Position of c.g.  
 F-Actual c.g.  
 given by manufacturer



G-Light load  
 H-Full load  
 J-Best position of c.g.

Fig.5 Diagram of moments for airplane III

Figs. 6, 7, 8.



Normal force on tail of rectangular form.

- A-  $\theta=0, \zeta_e=0$ , Without downwash.
- B-  $\theta=0, \zeta_e=30^\circ$ , " "
- C-  $\theta=0, \zeta_e=0$ , With " "
- D-  $\theta=0, \zeta_e=30^\circ$ , " " "
- E- Elevator free

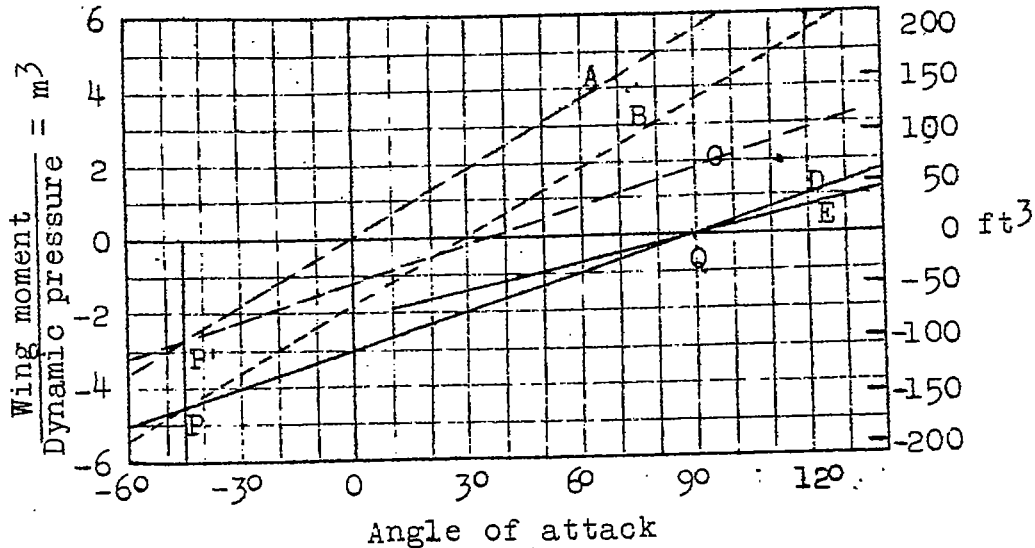


Fig. 8 Diagram of the normal force on the stabilizer and elevator.

- $C_{nH}$  = Coefficient of normal force on tail
- $\theta$  = Deflection of elevator
- $\zeta_e$  = Difference of incidence between upper wing and stabilizer