TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 181

INTERFERENCE OF MULTIPLANE WINGS HAVING ELLIPTICAL LIFT DISTRIBUTION.

By H. von Sanden.

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In calculating the self-induction of a wing surface, elliptical lift distribution is assumed; while in calculating the mutual induction or interference of two wing surfaces, a uniform distribution of lift along the wing has hitherto been assumed. Whether the results of these calculations are substantially altered by assuming an elliptical lift distribution (which is just as probable as uniform distribution) is examined in the present communication.

Let the span of two rectangular, unstaggered wings, normal to the plane of symmetry be taken as $b = 2l$ and their gap as G. Let the lift on the lower wing be elliptically distributed.** The eddies passing off from the trailing edge of the upper wing produce a vertically downward positive acceleration, which, at a distance of ϵ from the center of the wing, amounts to

$$D(\epsilon) = \frac{L}{2\pi^2 \rho V l^2} \int_{-l}^{+l} \frac{(x - \epsilon) x dx}{\{G^2 + (\epsilon - x)^2\}\sqrt{l^2 - x^2}}$$

* From Technische Berichte, Volume III, No. 7, pp. 291-2. (1918). (Communication from the Bavarian Airplane Works.)

** Gremmel "Die aerodynamischen Grunglagen des Fluges," (The Aerodynamical Basis of Flight," p.119.

After introducing a new variable integration u, defined by $\frac{x}{l}$ = sin u, we obtain

$$D(\epsilon) = \frac{L}{2\pi^2 \rho V l^2} \int_{-\pi/2}^{+\pi/2} \frac{l \sin u \, (l \sin u - \epsilon)}{G^2 + (l \sin u - \epsilon)^2} du.$$

The integral has been graphically determined for

$\frac{\epsilon}{l}$ = 0, 0.2, 0.4, 0.6, 0.8, 1.0 and

$\frac{2l}{G} = z$ = 4.8, 12.5. For the average value

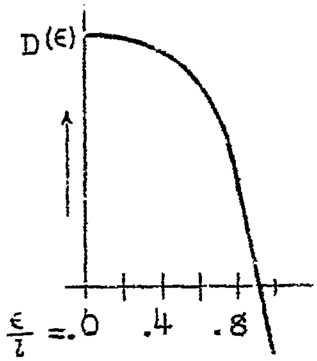
$$D_m = \frac{1}{l} \int_0^l D(\epsilon) \, d\epsilon = \frac{L}{2\pi^2 \rho V^2 l^2} f$$

we obtained the values of f given below, together with the corresponding average values of $\bar{D}_m = \frac{L}{2\pi^2 \rho V^2 l^2} \bar{f}$ (for comparison) and the proportionate differences for a uniform distribution of lift.

z	f	$\bar{f}$	Difference
4	1.16	1.11	4.5%
8	1.71	1.64	4.2%
12.5	1.98	1.98	0.0%

Within the limits of the values of z occurring in practice, the difference, therefore, is insignificant. For greater values, it increases without limit, since for z = ∞ and $\bar{D}_m = \infty$, while $\bar{D}_m = \frac{L}{2\pi^2 \rho V^2 l^2} \times 3.15$.

It is worth noting in considering the acceleration D (ε), which is plotted in the accompanying figure for z = 8, that



with elliptical distribution of lift on the lower wing, D(ε) is negative at the ends of the upper wing, that is, the acceleration is here directed upward, so that the actual angle of attack of the upper wing becomes larger at the ends. The turning up of the wing tips, therefore, appears justifiable.

Translated by
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for Aeronautics.