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Summary.

This note, prepared for the N. A. C. A., contains a discussion of the meaning of vortices, so often mentioned in connection with the creation of lift by wings. The action of wings can be more easily understood without the use of vortices.

The conception of "vortices" in a moving fluid was originated by Helmholtz and during recent years it has been extended in connection with the investigations of the creation of the lift of airfoils by Lanchester, Prandtl and others. One individual vortex, as introduced by Helmholtz, is only an abstraction, standing in the same relation to "vorticity" as a mathematical line stands to a thin wire, or as a concentrated force or load stands to an area with a distribution of comparatively high pressure. Vorticity rather than a vortex, therefore, has to be the subject of study. It will further appear that neither the vortices nor the vorticity, said to create the lift of an airfoil, have any physical reality of their own. They are either quite fictitious, or at best stand for the summed-up effect of the vorticity about which nothing definite is known generally, according to what point of view is taken. During the war, when the study of aerodynamics received a strong impetus, the use of vortices for the description of the flow of the air was left chiefly to German investigators, in spite of the work of Lanchester. Just now, British investigators have taken up the study of the German results. Papers on vortices are often met with in the present British technical literature, but not all of them indicate that the author has attained to a full clearness of the subject. Some remarks on vortices from an aerodynamic standpoint seem quite desirable therefore at the present moment.

It is easy to explain what vorticity means as long as it remains of finite magnitude, as it always actually does. Picture any fluid, moving in any way consistent with its physical properties. Suppose the motion to be uniformly distributed, so that each of the velocity components u, v, w, which are functions of the space coordinates x, y, z (at right angles to each other) and parallel to them can be differentiated and possess finite differential coefficients. Now consider a very small portion of the fluid contained within a sphere. The smaller the diameter of the sphere chosen, the more uniform becomes the distortion and motion of this portion of the fluid. With vanishing diameter of the sphere one can speak of a definite velocity of the center of the sphere, say V, and of a definite angular velocity of the sphere about an axis in a certain direction. This angular velocity is closely allied to vorticity. Vorticity denotes a distribution of localized vectors through the entire space. The magnitude of the vorticity at the point referred to is equal to twice the angular velocity of the little sphere just spoken of. Its direction is parallel to the axis of rotation of the little sphere. Vortex lines, in analogy to stream lines, are defined to be such lines that at each point their direction coincides with the axis of vorticity at that point.

The vorticity, just as the velocity, thus possesses three components at each point. The component parallel to the X-axis, for instance, is equal to twice the component of the mean angular velocity with respect to this axis. Let the center of the sphere have the velocity components v and w, at right angles to the X-axis. Then the magnitude of these components in the vicinity of this center are, to a first approximation

$$v + dy \frac{\partial v}{\partial y} + dz \frac{\partial v}{\partial z}$$

and

$$w + dy \frac{\partial w}{\partial y} + dz \frac{\partial w}{\partial z}$$

It follows directly that the radius vector parallel to the Y-axis rotates with the angular velocity $\frac{\partial w}{\partial y}$ relative to the X-axis, and the radius vector parallel to the Z-axis rotates with the angular velocity $-\frac{\partial v}{\partial z}$. Hence it can be said that the mean angular velocity component with respect to the X-axis has the magnitude

$$\frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

and the vorticity component is twice as large:

$$f = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \tag{1}$$

Streamlines are defined to be such lines that at each point the direction of the line coincides with the velocity of the fluid at that point. One individual streamline, existing alone, is not conceivable. It could at best be defined as representing the flow of a fluid at rest except in the vicinity of this line where it flows as through a narrow pipe. The magnitude of this one isolated streamline should not be the actual velocity, as this would not give any useful and consistent information. It should rather be the delivery per unit of time, of fluid through a cross section containing the line having thus the dimension "velocity times area." A statement analogous to this can be made regarding single vortex lines. They never actually exist in the strict meaning of the word, vortex lines being characteristic of a uniform distribution of vorticity, and one of them, when existing alone, indicates anything else but a uniform distribution of vorticity. If the vorticity is confined to the immediate vicinity of a line, this line may be newly defined as an individual vortex line, and the strength of such a vortex is then better defined by the integral of all products of the elements of area of a cross section and the component of vorticity at right angles to them. Hence the dimension of the strength of an isolated vortex is "angular velocity times area" = "velocity times length."

In abstract mathematics the isolated streamline as well as the isolated vortex line are imagined to be strictly confined to the line. Then the velocity or vorticity, in their proper meaning become infinite if the strength of either line is finite. Any uniform distribution of velocity or vorticity can be described by reversing the mental process, considering them as a uniform distribution of infinitely many isolated streamlines or vortex lines of infinitely small strength. The reader will at once recall the magnetic and electric fields, where similar considerations are used.

There exists a state of singularity intermediate between uniform vorticity and an isolated vortex line, namely, the concentration of vorticity within a surface. Expressed in plain language, the fluid is slipping along these surfaces, there being a finite sudden drop between the velocity on both sides of the surface. No infinitely large velocities occur. If the surface be replaced by a thin layer of the thickness δ, and the velocity be supposed to drop gradually from V_1 to V_2 within this layer, the rate of change of velocity with respect to normals to the surface is seen to be $(V_1 - V_2)/\delta$. The smaller the thickness of the layer, the larger becomes the vorticity in the layer and when δ approaches zero, there can be said to be an infinitely large vorticity at points of the surface.

In view of this it is convenient to define such a measure for the entire vorticity contained within certain limited parts of the space, as will not break down if the vorticity itself becomes infinite along certain lines or surfaces. This is done by selecting a certain surface, and by computing the "flux" across it of all vortex lines,

$$\int f\, dS \tag{2}$$

where dS denotes the element of area of the surface and f the component of vorticity at right angles to the element. This measure is called Circulation and it can be computed in a much more convenient way than that indicated by the integral, as I now proceed to show.

According to definition the vorticity has the components

$$\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \quad \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}.$$

Over each very small element of area of the surface the vorticity can be supposed to be constant. Let the whole surface be composed of very small rectangular elements at right angles to one of the three axes of coordinates. One of them may have the sides dx and dy. Then its contribution to the circulation is

$$\int\int \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx\, dy,$$

extended over the area dxdy.

This can be written

$$\int (v + dv)dy - \int vdy - \int (u + du)dx + \int udx.$$

This now is a simple or line integral, obtained by integrating along the boundary of the element, passing around the element in a certain direction of turn, summing up the products of the elements of boundary and the components of the velocity parallel to it.

Now, if the same process of integration is repeated with the adjacent element, the two contributions of the common element of boundary cancel each other, as the directions of integration are opposed (Fig. 1). It is therefore only necessary to integrate along the boundary of the area made up of the two elements. This conclusion can be repeated, and it follows that the circulation of any finite surface is obtained by integrating the product of the boundary elements and the components of velocity parallel to them along its boundary. Hence the circulation of a certain flow with respect to a certain surface depends only on the magnitude and direction of the velocity along the boundary of the surface. It follows at once that the circulation with respect to a closed surface (like the surface of a body) is always zero. Now the circulation was computed in just the same way with respect to vortex lines as the flow through a surface would have been computed with respect to streamlines. Hence it follows that vortex lines behave like streamlines of an incompressible fluid, whatever the motion of the fluid may be. If the space be divided into "vortex tubes" of equal circulation, no such tube can ever end or begin, but every vortex tube either runs back to itself or to the boundaries of the space.

This continuity of the distribution of vorticity is the content of the first theorem of Helmholtz on vortices. It is not at all a physical law, but a mathematical truth, a necessary consequence of the definition of vorticity. The theorem is not obvious to a person not thoroughly mathematically trained. No measurement can ever upset this theorem, but on the contrary, it can only be a check to any measurement.

The second theorem of Helmholtz concerning vortices is a physical law applying to perfect fluids and gases, and this is therefore the more important one. It rests on the fact that no sphere can be put into rotation by forces acting on its surface and directed at right angles to it. Hence a spherical portion of a non-viscous fluid free of vorticity cannot attain to a vortex motion by forces acting from one particle to an adjacent particle, that is, by a distribution of pressure. Hence Helmholtz' second theorem: The vortex lines move with the matter of the fluid. No portion of the fluid changes its vorticity, except when under the action of suitable external forces. This theorem now is not self-evident, it is a physical law only and true only under the conditions stated: no viscosity and no external forces capable of producing vorticity. Gravity does not belong to these external forces. Helmholtz' second theorem will never be fully confirmed by experience as there is no perfect fluid known.

Any force exerted by an immersed body on the surrounding fluid is an external force, and hence the flow around any such body may be vortical. Examples of this kind are bodies moved through the fluid and encountering a drag or creating a lift, propellers and equivalent devices creating a thrust or acting as wind mills. The vorticity is then distributed in the surrounding fluid. Close to the surface of the body the fluid may or may not cling, according to whether it is a perfect or an actual fluid. In the latter case the layer along the surface moves with vorticity. An airship hull, for example, flying parallel to its axis, is surrounded by closed vortex rings at right angles to the axis and moving in the vicinity of the surface. The creation and existence of these vortices near the surface is of interest in connection with the study of air friction. Similar vortex rings exist in a fluid flowing through a circular tube. These vortices, however, have no direct bearing on the permanent creation of lift by airfoils, though they may be of interest for studying the initial creation of lift. These layers of vortical motion may as well be considered as constituting part of the body itself, surrounded by air in a non-vortical motion, and this alone demonstrates their relative unimportance for the flow at large.

Behind obtuse bodies, moved through air, there are often regions of turbulent and irregular, unsteady motion. These regions, in general, will contain vorticity. The vortices, which are most often mentioned in literature in connection with the wing action are not characterized by any turbulent motion, however.

As the absence of lift and drag amounts to the absence of vorticity in a perfect fluid, and contrariwise, it is pertinent to ask for the relation between the magnitude of the lift, and the circulation, if any such relation can be found. It exists, indeed, and this is the natural connection between lift and vortices. The relation can easily be obtained from the consideration of the momentum transferred from the wing to the surrounding air. Consider the two-dimensional case first. A wing with infinite span creates the lift L' per unit length of the span. Hence the momentum L' has to be transferred per unit of length and per unit of time from the wing to the air. Let us suppose there to be two (mathematical) vertical planes in front and in rear of the wing moving with these, and let us compute the vertical momentum transported by the air through these two planes per unit of time. The difference will be equal to the lift, for the pressure over these planes does not contribute to the lift, and the planes are supposed to extend to infinity, so that the influence of their ends can be neglected. The vertical momentum of the air, passing either of the planes, for one strip of unit width is $(uvds)\,\rho$, where ρ is the density of air, u the vertical velocity component and v the horizontal velocity component. The planes can be supposed to be so far away from the wing that v becomes constant and equal to the velocity of flight. The difference of these two integrals is the product of the velocity of flight, the density of air and the circulation taken around the wing along both planes (that is, lines in the two-dimensional picture) and two horizontal lines of connection at a great distance, where they do not contribute any appreciable amount. It follows, therefore, that the circulation around the wing is equal to

$$\Gamma = \frac{L'}{\rho V}$$

where Γ denotes the circulation

L' the lift per unit span

V the velocity of flight

ρ the density of air.

Any other closed line drawn around the wing gives the same circulation. For the entire circulation is the sum of the circulation of the inner closed line and of the ring-shaped space between it and the outer closed line. This ring-shaped space is now supposed to contain no additional wings, hence its circulation is zero.

The existence of this circulation indicates the existence of vorticity in the flow created by a wing of finite span when in action. Indeed, the pressure under the wing makes the air move outwards, and the suction on top makes it move inwards, so that the two portions of air over and under the wing, when meeting again, have opposite lateral velocity components and hence there is a surface behind the wing where slipping of air takes place. It is particularly strong near the ends, and suggests to the imagination the presence of concentrated vortices near the ends.

This, however, is an exaggeration of what actually happens.

In the two-dimensional flow there are no ends of the wing and hence there are no vortices near the ends. There is circulation without vorticity, and indeed vorticity is only a necessary consequence of circulation if the circulation is computed along the entire boundary, outside and inside (Fig. 2). It follows from this and from the consideration of the transfer of momentum that the vortices behind the wing do not create the lift. They are in their turn created by the lift. As the streamlines behind the wing are approximately known, the vortex lines behind the wing, coinciding with them, can be studied and are of some use for learning in detail the flow behind wings.

Whether there are vortices on the surface of the wing depends on whether the layer of viscosity is supposed to form part of the wing or part of the surrounding fluid. The former is more often supposed. Sometimes the vortices are said to be located "inside the wing." That is only a poetical illustration of the fact that there is circulation around the wing. These vortices inside the wing may even be used for computing certain characteristics of the flow, but that does not prevent them from being merely fictitious. It has also expressly to be understood that there are no lateral vortices under or above the wing at some distance from the surface and carrying it. One inventor proposed seriously to take the wing entirely away and to replace it by a pair of rotating discs on both ends of the wing. These discs would create a vortex connecting the two discs and hence a large lift. Such things of course are sophisms, and are the product of the absence of clear thought and insight.

The discussion so far has resulted in the statement that a lifting wing accelerates the air downwards and hence leaves a trail of air behind it, moving with vorticity. This motion can be very clearly understood without any reference to vorticity. The circulation stands in a close relation to the momentum transferred, and its knowledge is useful for the computation of the action of wing sections. But it is a wide step from the magnitude of the circulation around the wing to the study of the vorticity at each point. What then is the use of introducing the vortices, and why are they so frequently mentioned?

It is only fair to say that for a detailed study of the flow behind wings the vortex lines form a convenient means of describing the flow, and hence they are of some use for the scientific expert. This is not sufficient, however, to explain their publicity. The reason for the publicity of the vortices in connection with wing action is not any particularly great usefulness of this mathematical conception, but is chiefly a historical one. It happened that, at the time when there was ignorance about the explanation of the flight of heavier-than-air craft, the investigators studied the air flow by using vortices, which is perfectly correct, and they found the explanation by that method. It is now known that this explanation could also be found by simpler mathematical methods, but that must not diminish the respect due to those men who first found the correct result. The result itself, and not the way it was first obtained, merits respect. The explanation of the lift is now reduced to the consideration of the air motion behind the wing, becoming more and more two-dimensional as the distance behind the wing increases. This two-dimensional flow is now the chief object of study. Any two-dimensional flows, except trivial ones, when extending over the entire plane, contain regions of vorticity. But this vorticity has never been used intensively for the investigation of such flows. There is no longer any reason for the use of vortices in the computations in connection with airplane wings. The present publicity of vortices is chiefly due to the fact that in earlier papers on wing action this method is used and because there are always too many who cling to words appealing to the imagination, as vortices seem to do, instead of applying their common sense to scientific questions. Even at the birthplace of the vortices they are almost out of date now, in connection with the study of wings, and so will they be in England and in this country too, after the question has been thoroughly digested.

Figs.1,2.

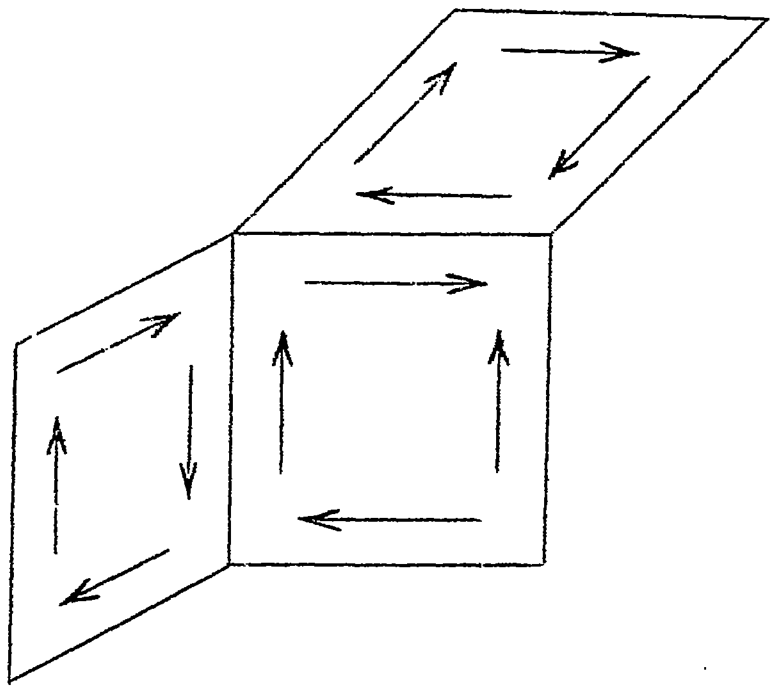

Fig.1

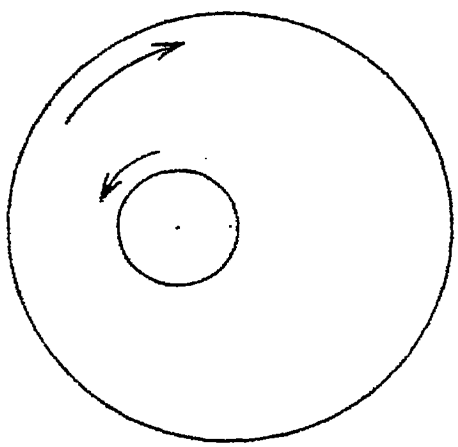

Fig.2