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THE LATERAL FAILURE OF SPARS

Stevens Bromley and William H. Robinson, Jr.

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By Stevens Bromley and William H. Robinson, Jr.

In the design of spars it often happens that the airfoil section will permit the use of a deeper spar than necessary for sufficient strength if the depth-breadth ratio is to be kept within the limits conventional in beams, limits generally observed as permitting the use in computation of the ordinary beam formula:

$$f = \frac{M\,y}{I}$$

In the case of rectangular sections

$$f = \frac{M\,\frac{d}{2}}{\frac{bd^3}{12}} = \frac{6\,M}{bd^2}$$

where

b = spar breadth

d = spar depth

From the above it is observed that the strength of a spar varies as the square of its depth. Since the weight of a spar varies as the first power of the depth, the maximum strength-weight ratio will be gained by the use of as deep a spar as possible, other factors being equal.

* Work done as a thesis in aeronautical engineering at the Massachusetts Institute of Technology.

When a spar of very large depth-breadth ratio is subjected to bending, for a time it acts as a beam according to beam formulae. At some point as the load is increased, however, that part of the spar under compression from the bending begins to buckle as a column and deflects laterally. More lateral deflection accompanies any further increase in the load until a maximum load is reached and the beam fails, a load considerably below that directly computed from the beam formula.

The fibers of the spar under maximum tension from the bending remain straight. As viewed from the end there is no apparent distortion of the sectional form at any point, but a simple torsion.

If the stress-strain diagram be plotted, it is found to be of the general shape shown in Fig. 1.

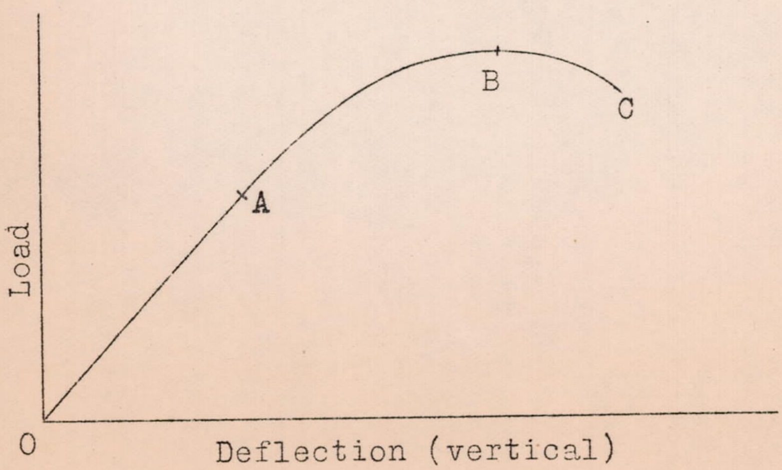


Fig.1

From O to A the spar acts like an elastic material. At A, where the calculated stress is still well below the elastic limit, lateral deflection sets in and continues until the specimen fails either under compression from the primary bending or a combination of primary and lateral bending, or under tension or, most probably, until the excessive deflection causes secondary structural members to fail and the structure to disintegrate.

But little research has been made on this subject so far as could be ascertained. There are, however, a few sets of tests which covered some parts of the present work.

As for the mathematical analysis of this subject, there has been one treatment specifically directed to the attention of the aeronautical engineer. In "Flight," May 30, 1918, there appeared a note by J. Prescott, M.A., D.Sc., entitled "The Sideways Buckling of Loaded Beams of Deep Section." The details of his work are not available, but the method implied suggested a mechanical analysis of the question rather than any experimental work. To quote Prescott, "The buckling load depends on the flexural rigidity for sideways bending, and on the torsional rigidity of the beam." The latter is true in that a beam could not buckle without twisting (Reference 1).

Prescott published a very interesting formula by which the ultimate load can be computed, and which takes, for a simply loaded beam, the form

$$P = \frac{16.94\sqrt{E\,I\,N\,K}}{L^2}$$

where

P = concentrated load at the center.

L = length of the beam.

E = modulus of elasticity.

I = smallest moment of inertia of the section.

N = modulus of rigidity.

KN = torsional rigidity.

From the theory of the torsion of prisms, approximately:

$$K = \frac{3\, b^3\, h^3}{10\,(b^2 + h^2)} \quad \text{(Reference 2)}$$

b = breadth of beam.

h = depth of beam.

In all cases Prescott considered the load applied at the center line of the beam.

Nature of Experiments

Prescott's theory prescribes conditions of loading, and in order to check this formula experimentally, special precautions were taken to insure these conditions. For the determination of the relation between depth-breadth ratio and lateral failure the specimens were supported at the ends so that they were free to deflect in their own plane, and partially free to deflect laterally. The ends were mounted on rollers so that there could be no external horizontal forces applied to the beam.

In studying the effect of span only three specimens were used, each being tested at several different lengths. The apparatus was as described above, and the span (l) was shortened by moving both end yokes toward the center. This was possible because failures by lateral collapse occurred at stresses below the elastic limit of the material, and repeated tests could therefore be made on a single specimen.

For the determination of the effect of load applied at more than one point, undamaged specimens were tested with loads at the third points.

The wood used in these tests was western spruce, kiln dried, but only of fair quality. Although all specimens came from the same source and apparently from the same tree, some of the grain slopes were excessive.

The sizes selected were such as to fit the apparatus. Three specimens of each size were used. Proportions were varied through a range wide enough to insure that both lateral and direct failures would occur. In general, the dimensions of the sections within any group were varied so that the section modulus would remain substantially constant.

All specimens were 48 inches in length, except the ones used for the span tests, which were 58 inches long. All told, 54 tests were made on 27 specimens.

The load was transmitted to the specimen through a yoke and distributed by steel and wooden blocks over a portion of the length of the beam sufficient to prevent local crushing. The effect of this distribution of load, reducing the maximum bending moment by approximately one percent, is negligible when compared to factors such as the variation in the wood, etc. The yokes fitted closely along the sides of the specimens to prevent lateral deflection at any point of load application, being further filled in with paper shims. Some of the load was therefore transmitted through the sides of the specimen. The supporting yokes were similar, but usually no distribution of the load was necessary at those points.

When lateral deflection set in it continued until the beam of the testing-machine dropped. Beyond this point no more load could be applied, the beam simply distorting further and further. The point at which the beam dropped therefore gave the maximum load.

If the specimen failed in tension the failure load was recorded. If it showed evidence of crushing, more load was applied until the beam failed in tension or a maximum load was reached.

In testing some of the heavier specimens crushing appeared at the end supports. Wooden blocks 1/2" x 3" x 1/16" were used for wider distribution of the load with these supports, and when heavier loads yet were applied steel ones 1" x 5" x 1/2" were introduced. The wooden blocks are tabulated as 1 - 1, steel 3 - 3.

Correction of Data

In the correction of the data for the modulus of rupture, the following assumptions have been made:

(1) That specific gravity is a function of percent summer growth and rate of growth, and that a correction for specific gravity will include the two latter.

(2) That moisture content, grain slope, and specific gravity, while affecting the modulus of rupture, do not alter the tendency to fail laterally. This assumption means that lateral failure is governed only by the dimensions of the specimen and the manner of loading.

(3) That moisture, grain slope, and specific gravity affect the modulus of rupture the same, whether the specimen fails laterally or not.

The above assumptions apply also to the modulus of elasticity corrections. The methods of correction used were drawn from Bulletin No. 70 and Project Report No. 2284 of the Forest Products Laboratory. Views of the Laboratory and testing-machine are shown in Figs. 2 and 3. From this corrected modulus of rupture a corrected maximum bending moment (M_c) was obtained. This was further corrected (M_c') to a standard sectional area by multiplying by the three halves power of the ratio between the sectional area of the specimen and a standard value.

Standard Values

The average moisture content of the specimens was 7.36%, the average specific gravity .396. All results were corrected to these values and to a zero grain slope.

Discussion of Results

In the plots which were constructed all values to the left of the dotted vertical line which has been drawn represent tension or compression failures, while those to the right represent lateral failures. There were no overlaps, the division between the two types of failure in terms of depth-breadth ratio being sharply defined.

Fig. 4 is a plot of the depth-breadth ratio against corrected modulus of rupture for beams of constant span with a concentrated load at the center. The low points at depth-breadth ratio of about 4 and 10 represent single specimens, presumably of poorer than average material.

The curve best representing the points on this plot is, it will be observed, one nearly horizontal to the left of the dotted line and dropping sharply down to the right in the lateral failure region. Fig. 5 is a like plot for the span tests and is quite similar.

Fig. 6 is a plot of the depth-breadth ratio vs. the corrected modulus of rupture for the tests with third-point loading. The curve is of like trend as the two preceding. The specimen indicated by the cross and arrow failed in tension instead of laterally. The failure occurred in a region of sapwood, presumably before lateral deflection had started. The mean curve in Fig. 6, so far as it extends, is almost identical with that in Fig. 4.

In like manner Fig. 7 has been plotted for corrected bending moments reduced to a constant sectional area of 2.46 sq.in., the average for the specimens. It will be observed from Fig. 7 that the moment reaches a maximum at a depth-breadth ratio of about 7. It would therefore be inadvisable to permit the ratio to exceed this figure in a beam, however great the depth that might be available. In a wing spar, however, a still larger ratio would be permissible because of the added lateral support given by the ribs. There is no explanation excepting a defective specimen, as to why the bending moment should again fall off at depth-breadth ratio of 4.

Fig. 9 was plotted to show the modulus of elasticity variation with depth-breadth ratio for the single-load tests.

Fig. 10 is a plot of depth-breadth ratio vs. span-breadth ratio (l/b) for all the specimens loaded at the middle point. The number adjacent to each point represents in approximate thousands of pounds per square-inch, the modulus of rupture of that specimen. The dotted line is drawn through the point which represents specimen 9 B of 30-inch span which failed in compression and laterally at the same time. A negatively sloping line such as the dotted one shown, divides the causes of failure precisely, those above and to the right being lateral failures and those below and to the left being either tension or compression failures.

An attempt was made to check Prescott's formula for a beam simply loaded and failing laterally:

$$P = \frac{16.94}{L^2} \sqrt{E\ I\ N\ K}$$

Three representative tests were chosen, and in all cases the specimen failed at a much lower load than that computed from Prescott's formula, ranging from one-half to one-fifth the computed value (N being taken as 90,000). This is somewhat surprising, as tests on steel beams have shown excellent agreement with the figures given by the formula. The discrepancy so may be due to the homogeneous and isotropic nature of the metal and the quite different structure of the wood. Such other lateral failure tests as have previously been made on wood seem to agree with this work in making the importance of lateral failure appear greater in practice than the theory would indicate.

Conclusions

From Fig. 4 it is seen that the strength of the specimen as denoted by its modulus of rupture increases as the depth-breadth ratio decreases. From Fig. 5 the modulus of rupture increases as the span-depth ratio decreases.

From Fig. 10 it is observed that the tendency to fail laterally does not bear a constant relation to the modulus of rupture.

The conclusion from these tests is that after the critical span or depth-breadth ratio has been reached, the modulus of rupture varies approximately inversely as the first power of the span and of the depth-breadth ratio.

The direction of lateral deflection is alternate between successive supports by theory and all tests. For this reason we believe that rib spacing along the spar is more important in reducing lateral deflection than the distance between supports at the strut points. Furthermore, we believe that within the limits of modern design any increase in distance between strut points can well be compensated for by spacing the ribs closer together, providing the ribs do furnish lateral support.

Table I.

Characteristics of the Specimens

Specimen	b	h	h/b	y/I	I	Slope	%SG	%M	RG	SG
1A'	.53	6.00	11.32	.315	9.54	50	40	10.20	18	.397
1C'	.50	5.88	11.76	.347	8.47	200	25	5.26	8	.373
1A"	.53	5.98	11.27	.316	9.46	100	60	11.11	28	.382
2A	.51	4.97	9.75	.476	5.22	50	35	5.15	7	.362
2B	.51	4.94	9.69	.482	5.12	30.3	50	8.23	24	.396
2C	.50	4.90	9.80	.500	4.89	21.8	50	5.26	30	.415
3A	.48	3.72	7.75	.893	2.06	15.9	30	6.39	25	.412
3B	.47	3.70	7.87	.933	1.99	11.0	40	5.82	28	.433
3C	.48	3.71	7.73	.908	2.04	6.9	50	6.05	33	.425
4A	.71	5.00	7.04	.338	7.40	71.7	15	7.07	9	.405
4B	.72	5.00	6.94	.333	7.50	33.3	40	8.94	18	.380
4C	.73	4.99	6.84	.330	7.55	25.0	50	8.70	12	.392
5A	.73	3.99	5.32	.503	3.97	9.1	20	6.84	30	.387
5B	.74	3.98	5.38	.512	3.89	10.5	40	6.61	32	.380
5C	.75	3.98	5.31	.505	3.94	8.3	40	6.61	39	.384
6A	.75	2.92	3.90	.936	1.56	10.5	30	11.11	40	.384
6B	.75	2.95	3.93	.917	1.61	50.0	45	6.38	40	.390
6C	.74	2.91	3.94	.957	1.52	8.0	50	6.83	28	.402
7A	.74	2.00	2.70	2.03	.494	33.3	40	6.38	7	.364
7B	.75	2.01	2.68	1.99	.506	18.2	60	13.62	14	.452
7C	.76	2.03	2.67	1.92	.528	100.0	60	11.72	28	.418
8A	.35	5.88	16.8	.497	5.92	66.7	30	5.26	10	.391
8B	.35	5.90	16.8	.492	6.01	66.7	30	5.26	9	.385
8C	.35	5.89	16.8	.495	5.95	200.0	30	5.54	10	.399
9A	.37	3.00	8.12	1.80	.833	200.0	25	6.95	18	.391
9B	.40	3.00	7.70	1.67	.900	100.0	30	6.95	21	.391
9C	.38	3.00	8.01	1.78	.843	67.0	25	6.95	22	.407

Symbol	Significance
b	Breadth of specimen - inches
h	Depth of specimen - inches
h/b	Depth-breadth ratio
y/I	Section modulus - (inches)$^{-3}$
I	Moment of inertia of section - (inches)^{4}
Slope	Number of inches for 1-inch rise of grain
%SG	Percent summer growth
%M	Percent moisture
RG	Rate of growth - rings per inch
SG	Specific gravity

Table II.

Single Load Tests

Specimen	Failure Load	Manner	Apparent E	f	h/b
1A'	1660	lat.	1062	6140	11.32
1C'	1450	lat.	873	5920	11.76
1A"	1925	lat.	1043	7150	11.27
2A	1060	lat.	1220	5900	9.75
2B	1515	lat.	1310	8600	9.69
2C	1565	lat.	1230	9200	9.80
3A	830	lat.	1330	8700	7.75
3B	830	lat.	1280	9100	7.87
3C	770	lat.	1220	8200	7.73
4A	2240	ten.	1360	8900	7.04
4B	2360	com.	1280	9300	6.94
4C	2320	com.	1440	9000	6.84
5A	1580	ten.	1190	9300	5.32
5B	1500	ten.	1300	9050	5.38
5C	1660	ten.	1240	9850	5.31
6A	870	ten.	1470	9570	3.90
6B	690	ten.	1342	7420	3.93
6C	930	ten.	1356	10410	3.94
7A	450	com.	1570	10730	2.70
7B	460	com.	1990	10760	2.68
7C	420	com.	1310	9490	2.67
8A	600	lat.	1220	3500	16.8
8B	510	lat.	1125	2950	16.8
8C	720	lat.	1383	4190	16.8

Load is maximum scale reading in pounds.

Lat. signifies lateral failure.
com. " compression failure.
ten. " tension failure.

E is Modulus of Elasticity calculated from plot made as the specimen was loaded - pounds/square-inch.

f is apparent modulus of rupture figured from the load given here - pounds/square-inch.

h/b is the depth-breadth ratio of the specimen.

Table III.

Original Test Data

Span Tests (Single Load)

Span	Failure Load	Failure Manner	Apparent E/1000	Apparent f	h/b	Specimen
57	230	lat.	1830	5910	8.12	9A
57	260	lat.	1750	6180	7.50	9B
57	245	lat.	2055	6210	8.01	9C
51	270	lat.		6200		9A
51	290	lat.		6170		9B
51	290	lat.		6580		9C
45	370	lat.		7480		9A
45	440	lat.		8250		9B
45	375	lat.		7510		9C
40	440	lat.		7920		9A
40	520	lat.		8670		9B
40	490	lat.		8720		9C
35	570	lat.		8970		9A
35	680	lat.		9920		9B
35	600	lat.		9360		9C
30	810	lat.		10930		9A
30	960	lat.		12000		9B
30	910	lat.		12150		9C
25	970	com.		10900		9A
25	1120	com.		11670		9B
25	1025	com.		11400		9C

Load is the maximum scale reading in pounds.

Lateral failure is signified by lat.
Compression " " " com.

E is modulus of elasticity in pounds per square-inch calculated from plot made as the specimen was loaded with 57-inch span.

f is apparent modulus of rupture, figured from the load given here - pounds/square-inch.

h/b is the depth-breadth ratio of the specimen.

Table IV.

Two Point Loading Tests

Specimen	Failure Load	Failure Manner	Apparent f	h/b	Span	a	Chips
8A	910	lat.	3540	16.8	47	15.67	
8B	950	lat.	3660	16.8	47	15.67	
8C	950	lat.	3690	16.8	47	15.67	
3A	1065	lat.	7450	7.75	47	15.67	
3B	1160	lat.	8490	7.87	47	15.67	
3C	870	*	6200	7.73	47	15.67	
2A	1770	lat.	6600	9.75	47	15.67	1-1
1A'	3380	lat.	7540	11.32	44	14.17	3-3
1C'	2560	ten.	6300	11.76	44	14.17	3-3

* tension at a knot.

Load is the maximum scale reading in pounds.

lat. signifies lateral failure.
ten. " tension "

f is the apparent modulus of rupture figured from the loads given here - pounds/square-inch.

h/b is the depth-breadth ratio.

Chips noted are the ones used to prevent crushing at the supports.

a is the arm used in computing the moment in calculating the modulus of rupture - inches.

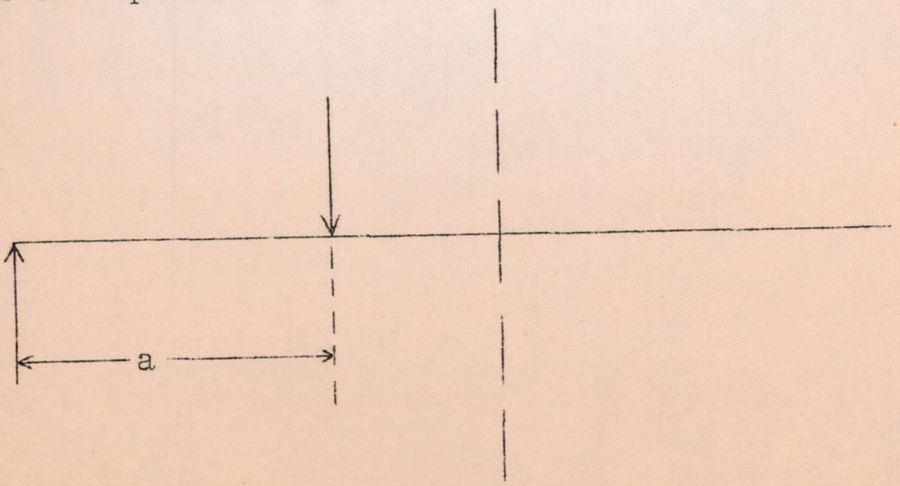


Table V.

Depth Breadth Tests Corrected Values

Specimen	f_c	M_c	M_c'	E_c	h/b
1A'	6970	22100	15000	1173000	11.32
1C'	5800	16700	12800	858000	11.76
1A"	8660	27400	18500	1235000	11.27
2A	5770	12100	11600	1234000	9.75
2B	9210	18700	18000	1345000	9.69
2C	8620	17200	17400	1219000	9.80
3A	9010	10100	14900	1358000	7.75
3B	9380	10100	15500	1397000	7.87
3C	10880	12000	17800	1621000	7.73
4A	8550	25300	14600	1345000	7.04
4B	10350	30900	17500	1441000	6.94
4C	10060	30500	17000	1508000	6.84
5A	11710	23300	17400	1556000	5.32
5B	11170	21900	16700	1628000	5.38
5C	12660	25100	18800	1676000	5.31
6A	12970	13400	15500	1966000	3.90
6B	7230	7900	9000	1321000	3.93
6C	12860	13400	15800	955000	3.94
7A	11400	5600	10400	1677000	2.70
7B	11960	6000	10900	1757000	2.68
7C	10190	5300	9400	1417000	2.67
8A	2880	5800	7200	1151000	16.8
8B	2500	5100	6300	1074000	16.8
8C	3450	7000	8700	1301000	16.8

f_c is the corrected modulus of rupture in pounds per square-inch, the sum of the apparent modulus of rupture from Table II and the corrections.

M_c is the maximum bending moment calculated from f_c in pound-inches.

M_c' is M_c corrected to a constant sectional area of 2.46 square inches, in pound-inches.

E_c is the corrected modulus of elasticity.

h/b is the depth-breadth ratio.

Table VI.

Span Tests Corrected Values

Specimen	f_c	M_c	M_c''	E_c	Span	h/b
9A	5900	3280	10070	1799000	57	8.12
9B	6170	3700	10700	1719000	57	7.50
9C	5760	3230	9790	2072000	57	8.01
9A	6190	3440	10550		51	
9B	6160	3690	10650		51	
9C	6130	3440	10470		51	
9A	7470	4150	12720		45	
9B	8240	4933	14250		45	
9C	7060	3960	12000		45	
9A	7910	4390	13500		40	
9B	8660	5190	15000		40	
9C	8270	4640	14070		40	
9A	8960	4980	15300		35	
9B	9910	5940	17150		35	
9C	8910	5000	15150		35	
9A	10920	6070	18600		30	
9B	11990	7180	20700		30	
9C	11700	6570	20200		30	
9A	10890	6050	18600		25	
9B	11660	6980	20150		25	
9C	10950	6150	18650		25	

f_c is the corrected modulus of rupture in pounds per square-inch, the sum of the apparent modulus of rupture from Table III and the corrections.

M_c is the maximum bending moment in pound-inches calculated from f_c.

M_c'' is M_c corrected to a constant sectional area of 2.46 square-inches, in pound-inches.

E_c is the corrected modulus of elasticity.

h/b is the depth-breadth ratio.

Table VII.

Two-Point Loading Tests Corrected Values

Specimen	f_c	M_c	M_c'	h/b
8A	2920	5870	7290	16.8
8B	3210	6530	8120	16.8
8C	2950	5950	7400	16.8
3A	7760	8700	11950	7.75
3B	8770	9400	13400	7.87
2A	6470	13600	13200	9.75
1A'	8370	26600	20600	11.32
1C'	6180	19200	16050	11.32

f_c is the corrected modulus of rupture in pounds per square-inch, the sum of the apparent modulus of rupture from Table IV and the corrections.

M_c is the maximum bending moment in pound-inches calculated from f_c.

M_c' is M_c corrected to a constant sectional area of 2.46 square-inches, in pound-inches.

h/b is the depth-breadth ratio.

References

1. A. G. M. Michell: Elastic Stability of Long Beams Under Transverse Forces. Phil. Mag. 1899, p. 298.
J. Prescott: Buckling of Deep Beams. Phil. Mag. 1918, p. 297; 1920, p. 194.
A. E. H. Love: A Treatise on the Mathematical Theory of Elasticity, 2d edition, p. 400.

2. J. Prescott: Buckling of Deep Beams. Phil. Mag. 1918, p. 297; 1920. p. 194.

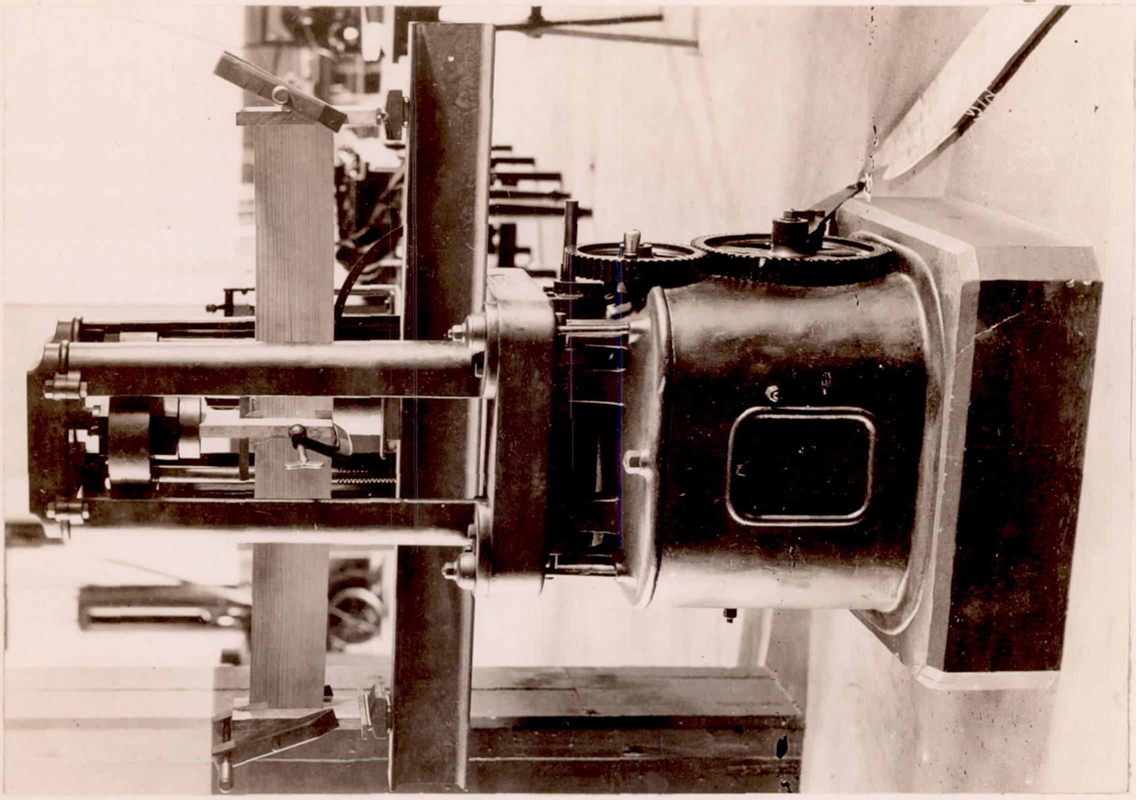


Fig.3

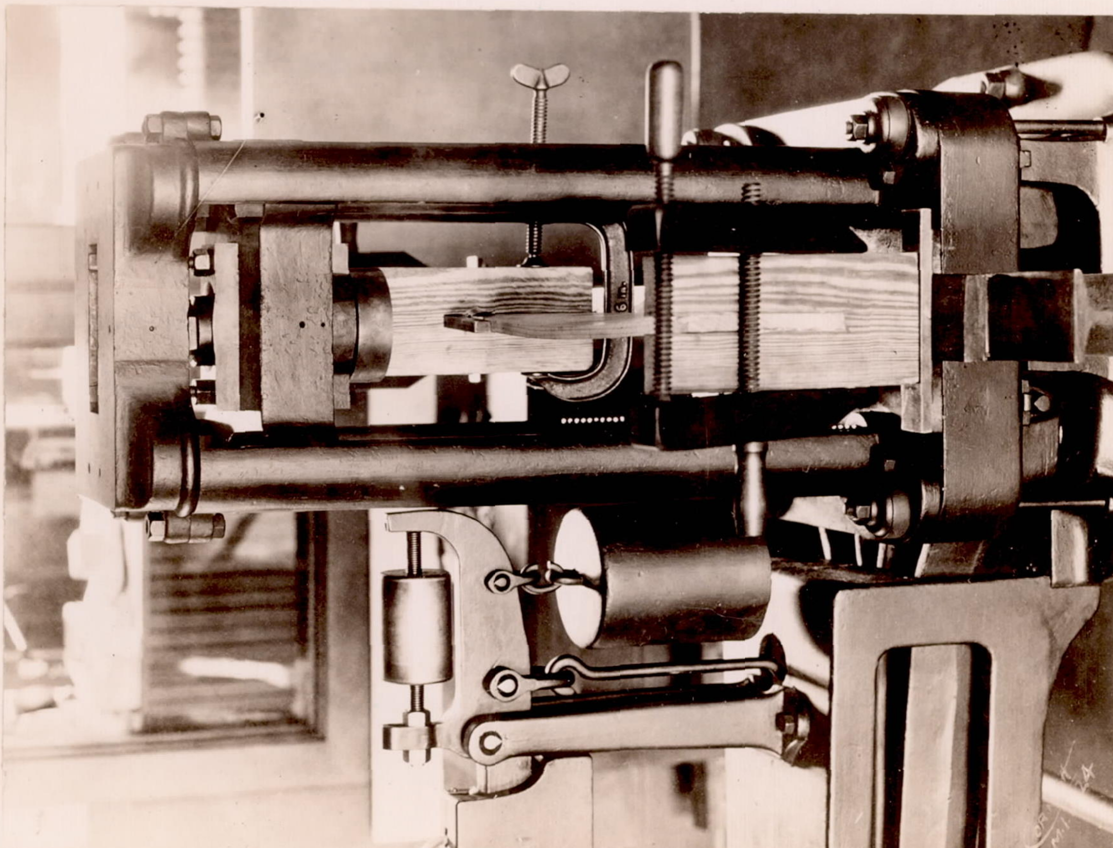
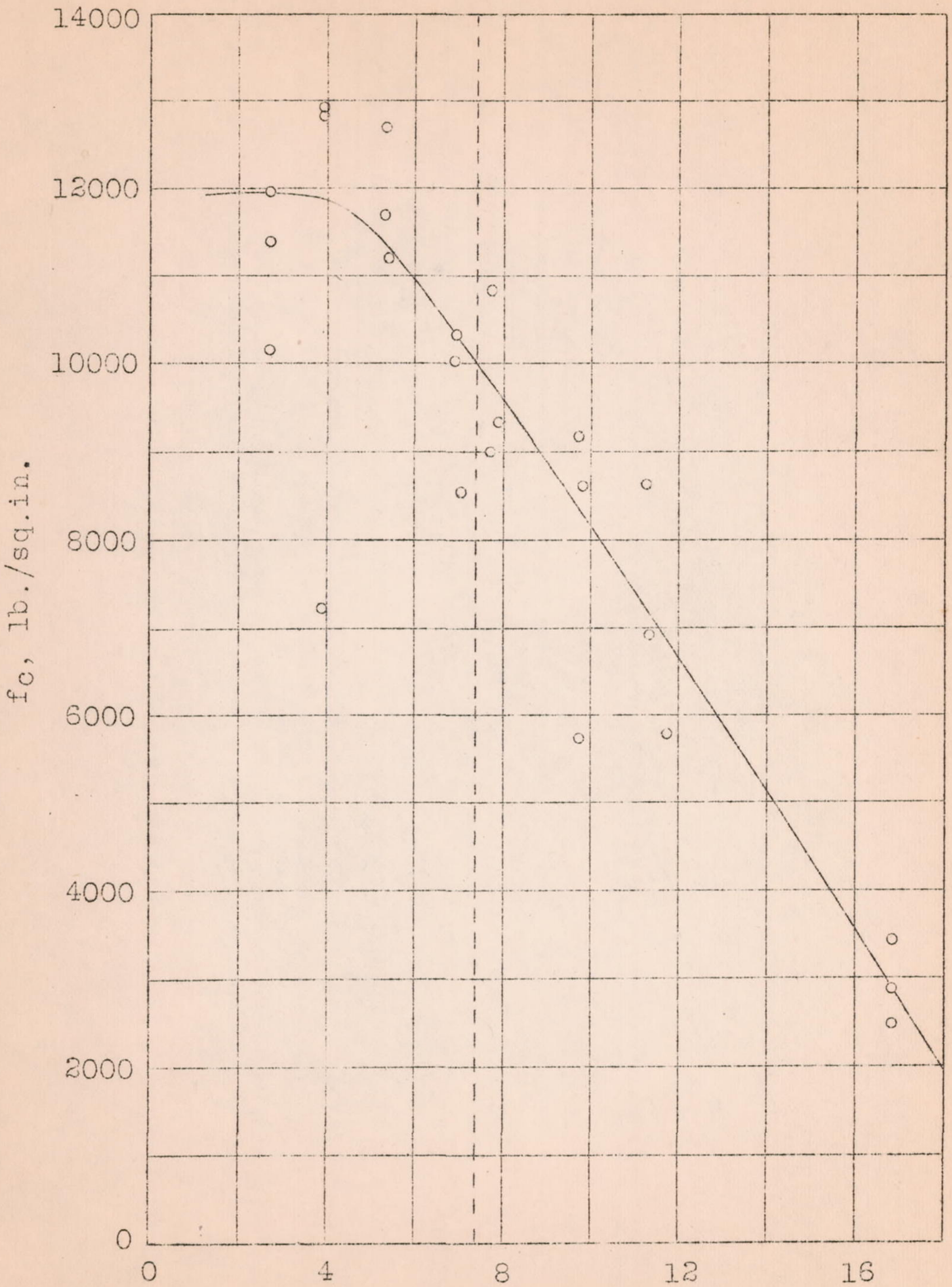


Fig.2

8658 A.S.



Span = 48"
Average section modulus = 2.0 in.^{3}

Fig.4 Depth-breadth ratio vs. corrected modulus of rupture.

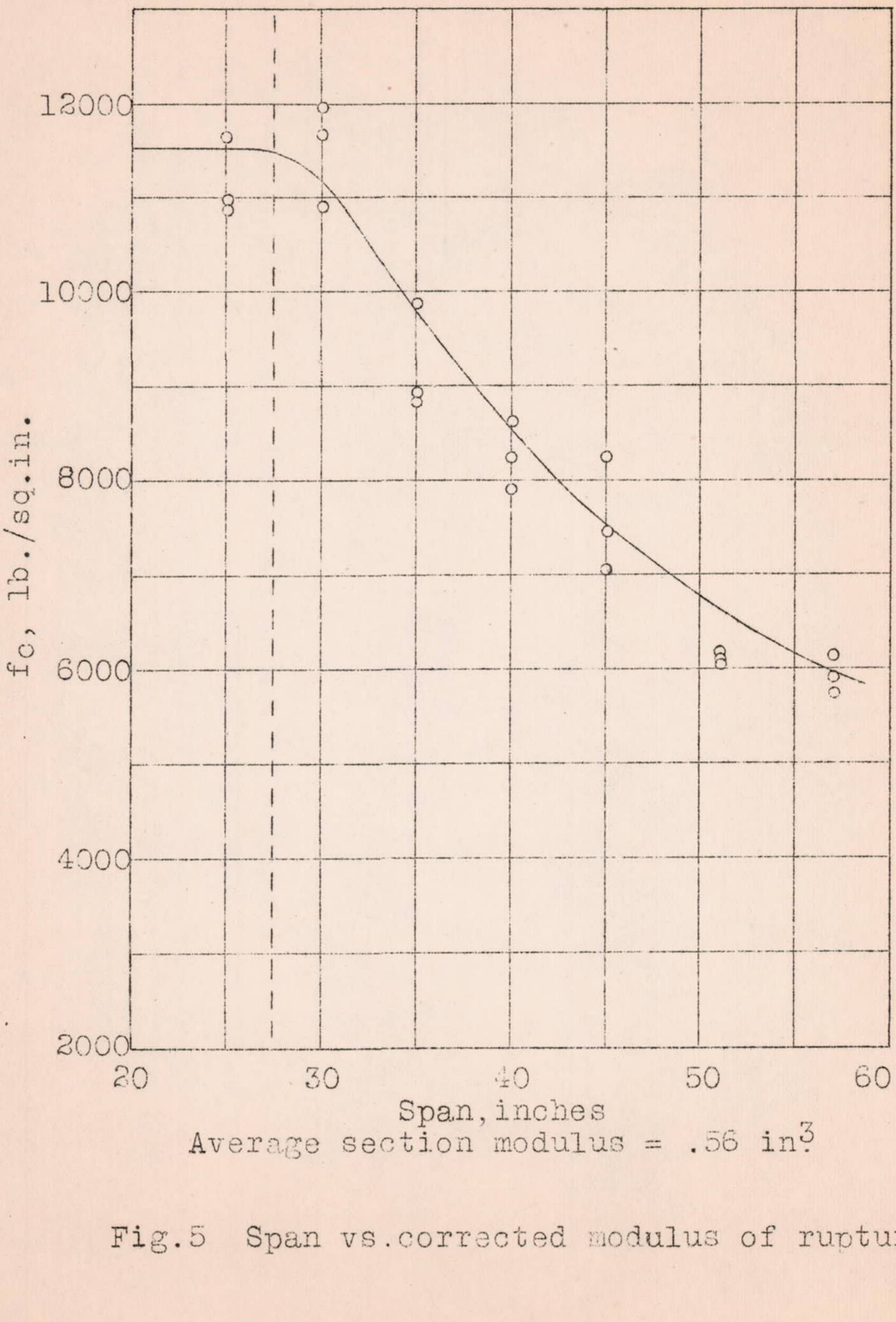


Fig.5 Span vs. corrected modulus of rupture.

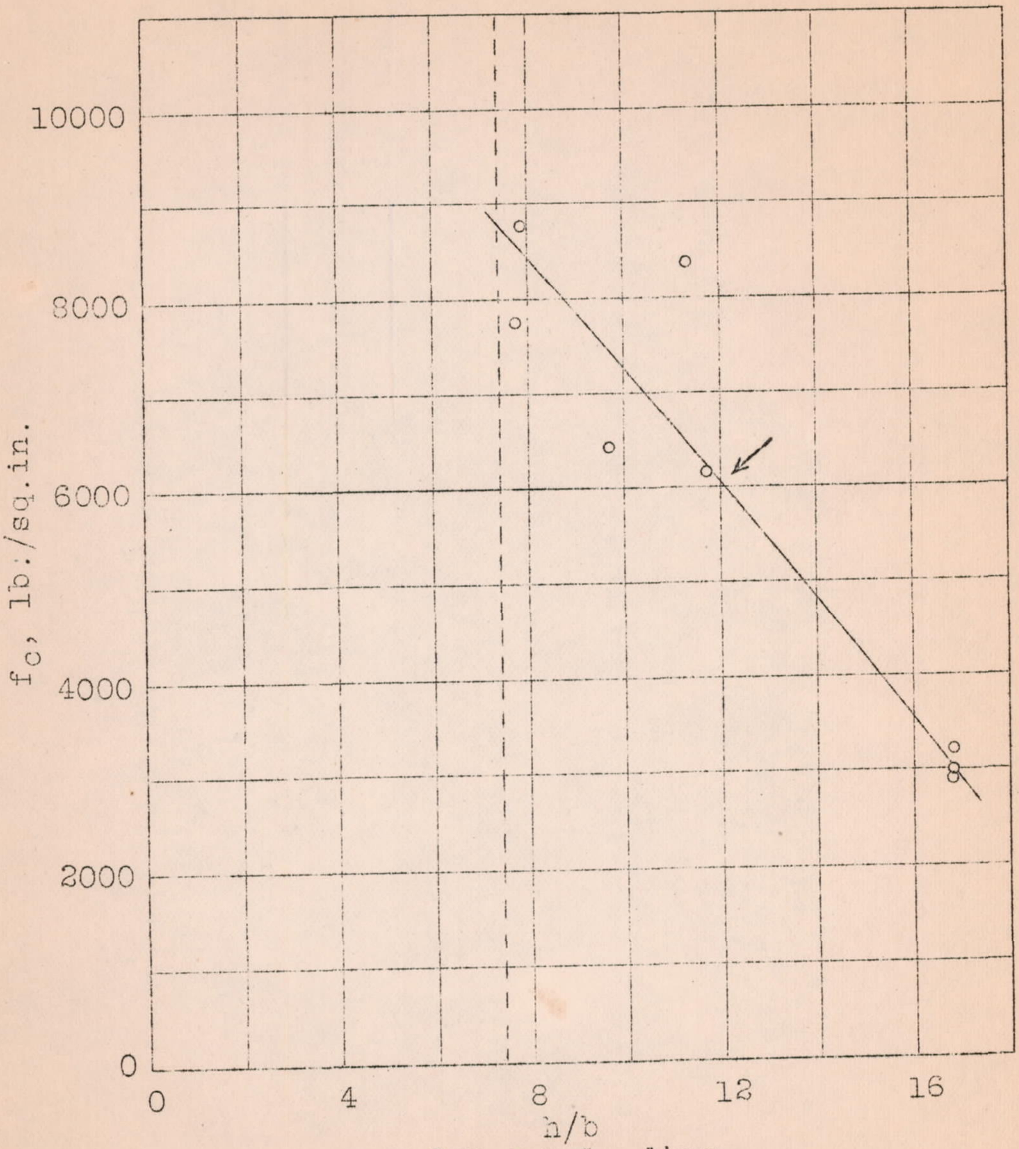


Fig.6 Depth-breadth ratio vs. corrected modulus of rupture.

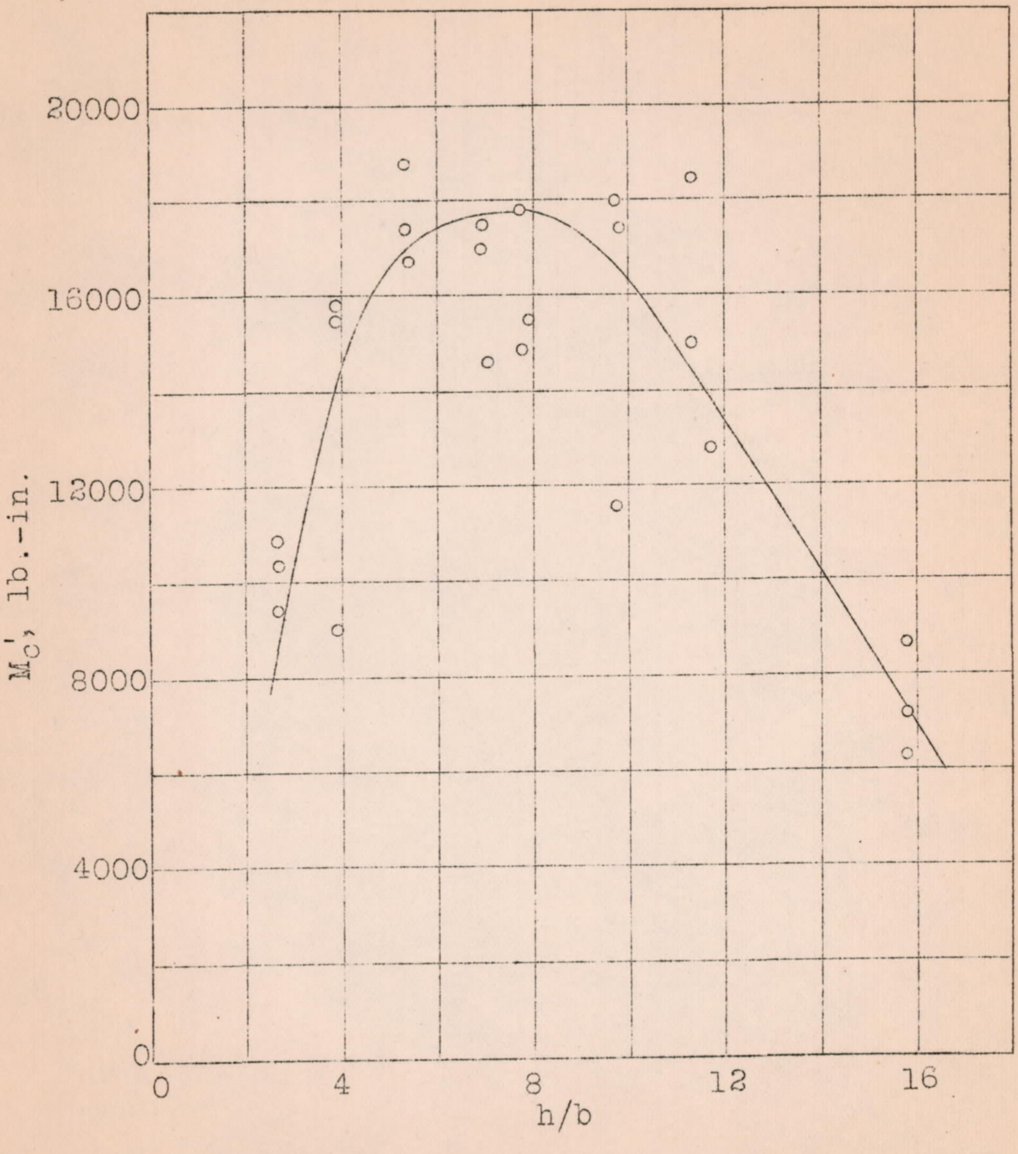


Fig.7 Depth-breadth ratio vs. corrected maximum moment.

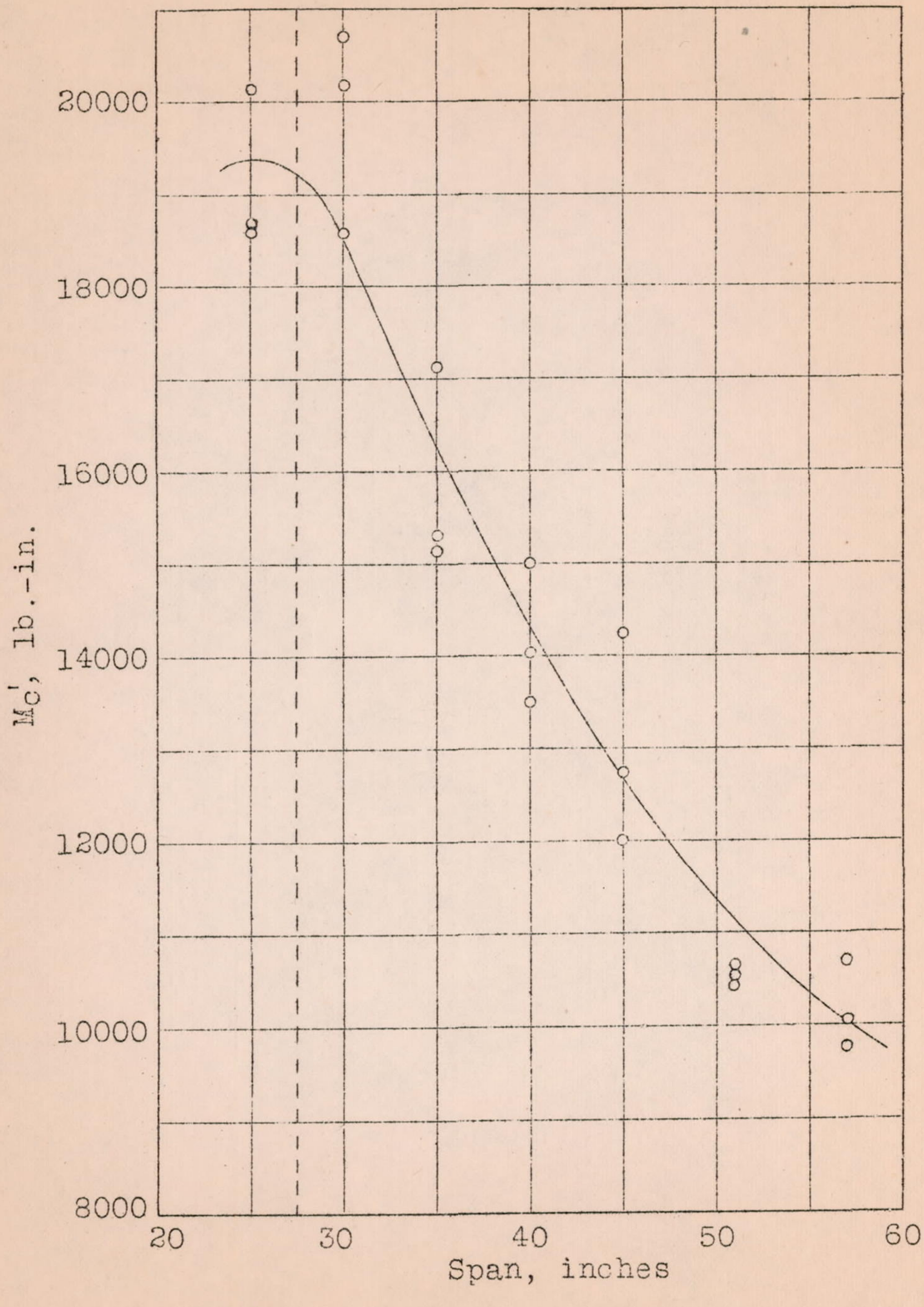


Fig.8 Span vs. corrected maximum moment.

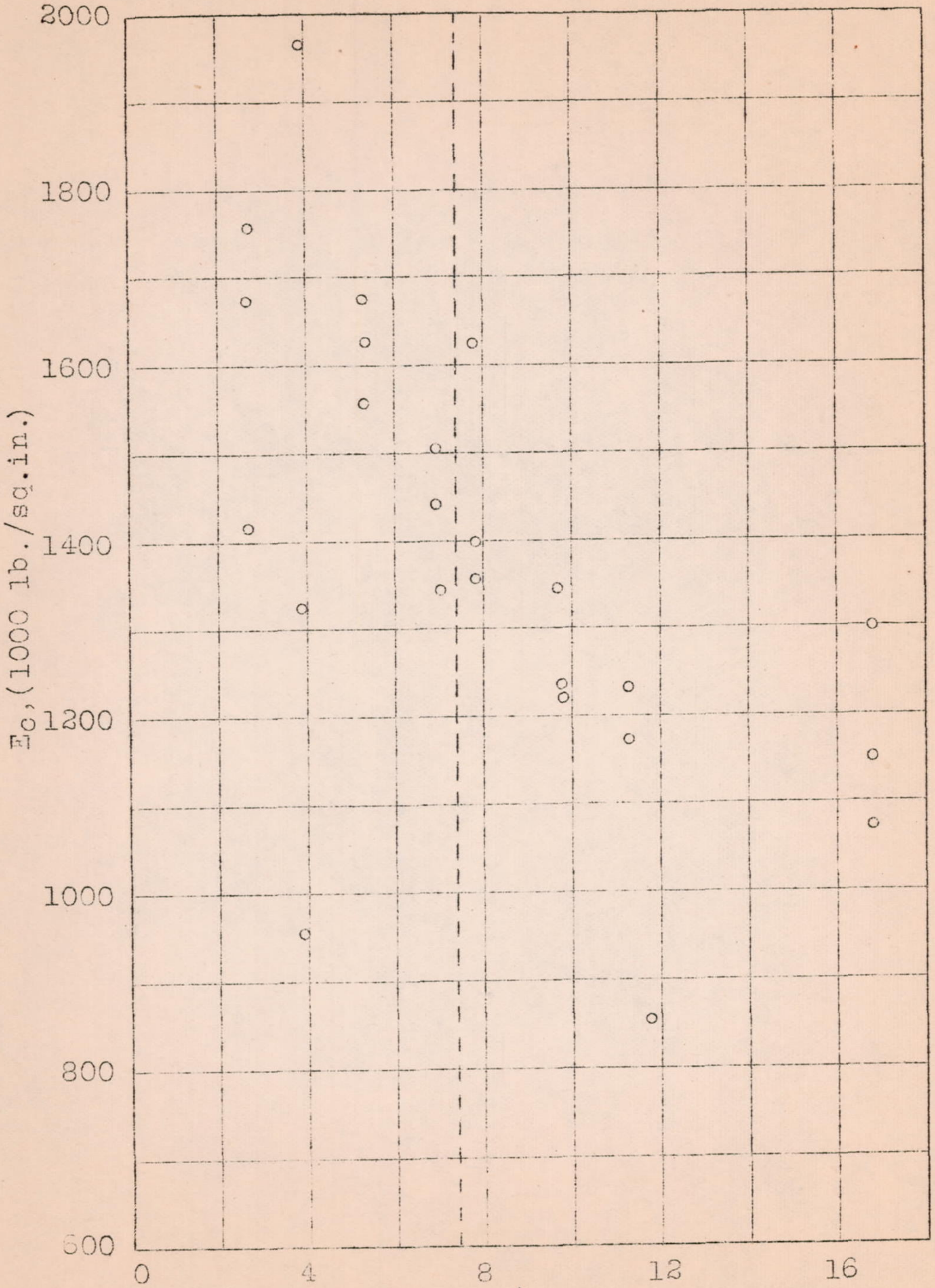


Fig.9 Depth-breadth ratio vs. corrected modulus of elasticity.

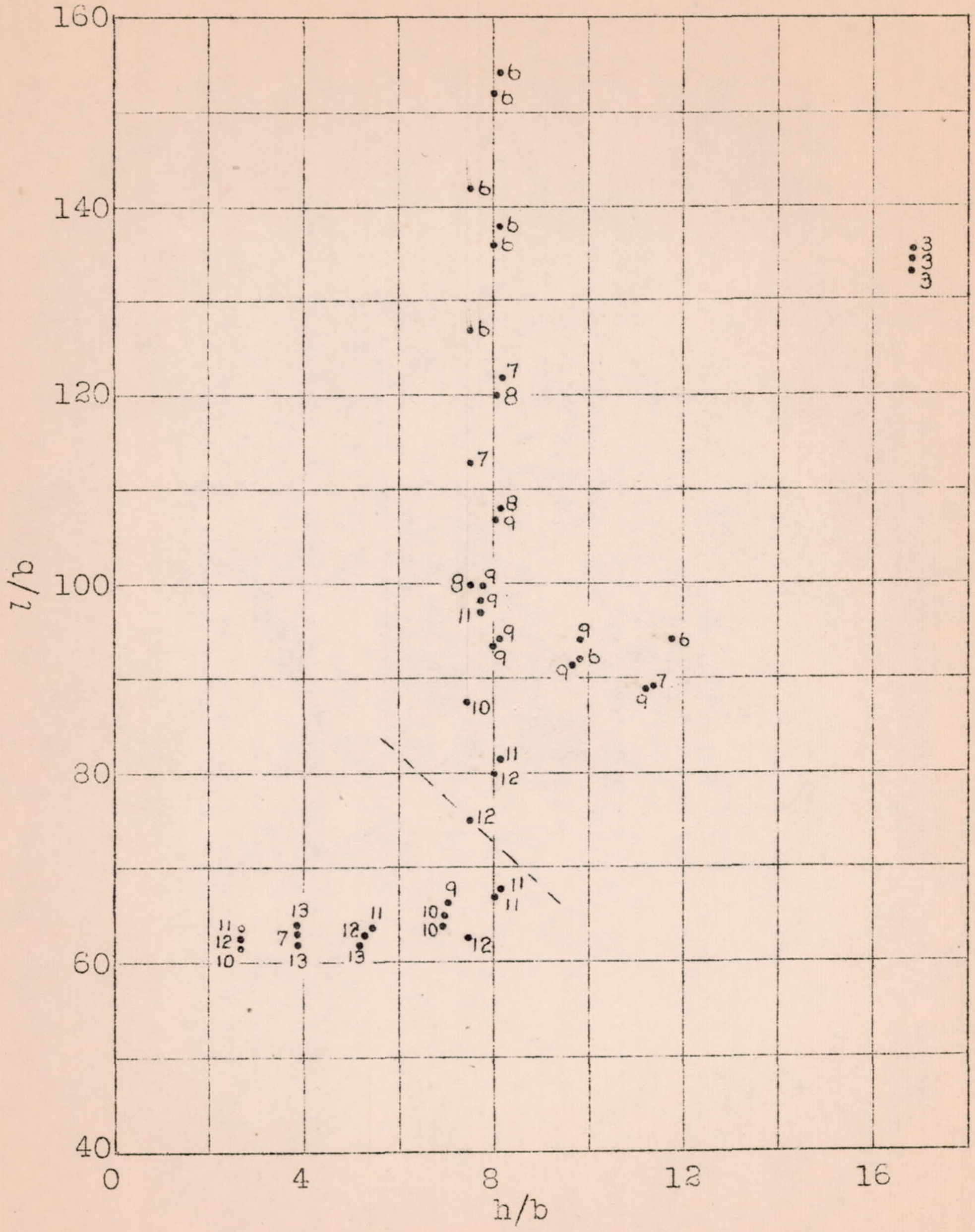


Fig.10 Debth-breadth ratio vs. span-breadth ratio.