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PROPELLER DESIGN

PRACTICAL APPLICATION OF THE BLADE ELEMENT THEORY - I

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PRACTICAL APPLICATION OF THE BLADE ELEMENT THEORY - I.

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Summary

This report is the first of a series of four on propeller design and contains a description of the blade element or modified Drzewiecke theory as used in the Bureau of Aeronautics, U. S. Navy Department. Blade interference corrections are used which were taken from R.& M. No. 639 of the British Advisory Committee for Aeronautics. The airfoil characteristics used were obtained from tests of model propellers, not from tests of model wings.

A short method is also shown in which the forces on only one blade element are considered in order to obtain the characteristics of the whole propeller.

The methods described have proved satisfactory in use.

Introduction

The U. S. Navy method of aircraft propeller design is based on data obtained from a combination of the blade element theory, model propeller tests in a wind tunnel, and tests of full scale propellers in flight. The data are plotted as curves for propellers of standard Navy form, making the actual operations in designing a propeller very short and simple. For the analysis or design of special propellers not conforming to the standard, the modified blade element theory is used, with airfoil section characteristics which give resultant powers and efficiencies checking the standard model data.

Although the blade element theory is well known to aeronautical engineers, the accuracy of the results obtained through its use depends upon corrections which can be obtained only through experience. It is the intention to present in this report a modified form of the theory with data which makes its application sufficiently accurate for practical use.

Discussion and Development of the Theory

In the simple blade element or Drzewiecke theory the propeller blade is considered as made up of a number of small elements and the forces acting on each are found. From the summation of the forces on the elements the resultant forces on the whole blade are determined.

No account is taken in the simple theory of the inflow or increase in slip-stream velocity which takes place in front of the propeller. The fact that the blades interfere with each other in a manner similar to the interference of the wings of a multiplane with backward stagger is also neglected. These factors are taken care of in this report by blade interference corrections to the lift and drag coefficients of the airfoil sections obtained from R.& M No. 639 of the British Advisory Committee for Aeronautics (Reference 1).

Airfoil characteristics as found from tests on model wings at low air velocities do not apply to propeller sections which are under entirely different conditions. Propeller airfoil section characteristics have been calculated from model propeller test data (Fig. 6), as will be explained in Technical Note No. 236. These are used in calculations of propeller performance and give powers and efficiencies corresponding to the tip speed of the model propellers (about 350 ft./sec.). The powers are then modified according to the particular tip speed of the full sized propeller (Reference 2).

The symbols used in the following development are given in Table I. The lift in pounds on a section of the propeller blade of length Δr and of width b is given by the expression $\rho \frac{b(V_r)^2 C'_L \Delta r}{2}$, where V_r is the resultant velocity of the section (Fig. 5).

The total force on the section due to both lift and drag is $\rho \frac{b(V_r)^2 K \Delta r}{2}$ and is in the direction of K in Fig. 5, where $K = \frac{C'_L}{\cos(\gamma - \epsilon)}$. As $\cos(\gamma - \epsilon)$ is in all cases between .995 and 1, the expression for the force on the blade element may be taken as $\rho \frac{b(V_r)^2 C'_L \Delta r}{2}$.

The thrust is the component of this force in the direction of the propeller axis or

$$d\ T = \rho \frac{b(V_r)^2 \ C'_L \cos(\Phi+\gamma) \ dr}{2} \tag{1}$$

where $\tan \gamma$ is taken as $\tan \epsilon$ plus D/L of the airfoil section at the corrected angle of attack α'. This is an approximation but is correct within the limits of accuracy of the design. Substituting $V_r = \frac{V}{\sin\Phi}$ and expressing b and r in terms of the diameter, equation (1) becomes

$$dT = \rho V^2 D^2 \times \frac{C'_L}{2 \sin^2\Phi} \times \frac{b}{D} \times \cos(\Phi+\gamma) \ d\left(\frac{r}{D}\right). \tag{2}$$

Let $K_p = \frac{C'_L \times b}{2 \sin^2\Phi \times D}$ and $T_c = K_p \cos(\Phi+\gamma)$.

Then $dT = \rho V^2 D^2 \ T_c d\left(\frac{r}{D}\right)$ and the total thrust for the propeller (of B blades) is

$$T = \rho V^2 D^2 B \int_0^{1/2} T_c d\left(\frac{r}{D}\right). \tag{3}$$

In like manner the expression for torque is

$$Q = \rho V^2 D^3 B \int_0^{1/2} Q_c \ d\left(\frac{r}{D}\right) \quad \text{where} \quad Q_c = K_p \times \frac{r}{D} \times \sin(\Phi+\gamma). \tag{4}$$

The horsepowers are then found from the expressions:

$$T.HP. = \frac{TV}{550} \qquad Q.HP. = \frac{2\pi\ QN}{60 \times 550} = \frac{QN}{5255} \qquad \eta = \frac{T.HP.}{Q.HP.}$$

T_c and Q_c are found for each section, and the integrations in equations (3) and (4) are performed graphically with the aid of a planimeter.

Single Section Method of Analysis.

It was noticed that for wood propellers of Navy standard blade form (Fig. 11) the torque and thrust grading curves used in the above graphical integration always reached their maxima at approximately 75 per cent of the radius; also these curves were approximately similar for all pitch ratios and slips. This suggested that if a constant relation could be found between the ordinate Q_c, at the 75 per cent radius and the total area, $\int_0^{1/2} Q_c \, d\left(\frac{r}{D}\right)$, it would be necessary to analyze for this one station only and multiply by a constant factor to get the value of $\int_0^{1/2} Q_c \, d\left(\frac{r}{D}\right)$, and $\int_0^{1/2} T_c \, d\left(\frac{r}{D}\right)$ could be found in a similar manner.

The torque and thrust grading curves for a series of standard wood propellers were therefore calculated by means of the full blade element theory using blade interference corrections and airfoil characteristics as found in the McCook Field high speed wind tunnel. The series included various pitch ratios, slips, blade widths and thicknesses. As the curves are all approximately horizontal at the 75 per cent station, any slight shifting of the peak has practically no effect on the value of the ordinate.

The actual ratios of Q_c for 75 per cent radius divided by $\int_0^{1/2} Q_c \, d\left(\frac{r}{D}\right)$ have a constant value of .272 within ±2%. For thrust, the ratio is constant within the same limits at .266.

If the torque and thrust coefficients at 75 per cent radius are designated as Q'_c and T'_c respectively, the expressions for horsepower become

$$Q.HP. = \frac{.272 \rho V^2 D^3 B Q'_c N}{5255}$$

$$T.HP. = \frac{.266 \rho V^3 D^2 B T'_c}{550}$$

$$\eta = \frac{T.HP.}{Q.HP.} = .156 \times \frac{V}{ND} \times \frac{T'_c}{Q'_c}$$

As $T'_c = K_p \cos(\Phi+\gamma)$ and $Q'_c = .375 K_p \sin(\Phi+\gamma)$, the expression for efficiency can be further reduced to

$$\eta = \frac{.416 \times \frac{V}{nD}}{\tan(\Phi+\gamma)}$$

This short method of analyzing the 75 per cent radius station only can be applied with good results to any conventional wood propeller having a plan form tapering toward the tip, and approximately uniform pitch, but is strictly applicable to none but standard Navy wood propellers.

Application

Blade Element Theory:

The application of the blade element theory can best be shown by means of an example. Consider a propeller 10 ft. in diameter with a uniform geometric pitch of 7 ft. and blade widths and thicknesses as shown in Table II, revolving at 1800

R.P.M. on an airplane having forward speed of 129 M.P.H.

For the section at 75 per cent of the radius the blade width is .66 ft. and the camber or $\frac{h_U}{b}$ is .107.

$$\tan \text{ blade angle } \Phi_\beta = \frac{\text{Pitch}}{2\pi r} = \frac{7}{2\pi \times 3.75} = .2975$$

Hence the blade angle, $\Phi_\beta = 16.6^\circ$.

For 129 M.P.H., $V = \frac{129 \times 88}{60} = 189$ ft./sec.

$$\tan\Phi = \frac{V}{2\pi rn} = \frac{189}{2\pi \times 3.75 \times 30} = .267$$

Hence the path angle, $\Phi = 15.0^\circ$.

The apparent angle of attack,

$$\alpha = \Phi_\beta - \Phi = 16.6^\circ - 15.0^\circ = 1.6^\circ.$$

From Fig. 6, $C_L = .530$ for $\frac{h_U}{b} = .107$ and $\alpha = 1.6^\circ$.

Fig. 7 shows the blade interference correction to the lift coefficient. The correction depends on the angle Φ, C_L, the radius, blade width, and number of blades. The last three are used in the form of a coefficient S, where $S = \frac{2\pi r}{Bb}$. For our example $S = \frac{2\pi \times 3.75}{2 \times .66} = 17.8$. From Fig. 7, the correction to the lift coefficient ∂C_L, is .058, where $\Phi = 15^\circ$, $C_L = .530$, and $S = 15$. When $S = 20$, $\partial C_L = .032$. Using linear interpolation, when $S = 17.8$,

$$\partial C_L = .058 - (.058 - .032)\left(\frac{17.8 - 15}{20 - 15}\right) = .044.$$

The corrected lift coefficient is then

$$C'_L = C_L - \partial C_L = .530 - .044 = .486.$$

In like manner, from Fig. 8, the correction to the angle of attack is found to be $\epsilon = .64 - (.64 - .40)\left(\frac{17.8 - 15}{20 - 15}\right) = .5^\circ$.

The corrected angle of attack

$$\alpha' = \alpha - \epsilon = 1.6^\circ - .5^\circ = 1.1^\circ.$$

From Fig. 6, the L/D for a standard propeller section of $\frac{h_U}{b} = .107$ at 1.1° is 16.8.

$$\tan\gamma = \frac{D}{L} + \tan\epsilon = .0595 + .0087 = .0682 \quad \text{and} \quad \gamma = 3.9^\circ$$

$$K_p = C'_L \times \frac{b}{D} \times \frac{1}{2\sin^2\Phi} = \frac{.486 \times .66 \times 1}{10 \times 2 \times .2588^2} = .239$$

$$Q_c = K_p \times \frac{r}{D} \times \sin(\Phi+\gamma) = .239 \times \frac{3.75}{10} \times \sin 18.9^\circ$$

$$= .239 \times .375 \times .3239$$

$$= .0290$$

$$T_c = K_p \cos(\Phi+\gamma) = .239 \times .9461 = .226.$$

The values of Q_c and T_c are found for each section (Table II) and plotted against r/D (Fig. 10). These are called the torque and thrust grading curves. The area under the torque grading curve is $\int_0^{1/2} Q_c\, d\left(\frac{r}{D}\right) = .0081$.

From equation (4) the torque is

$$Q = \rho V^2 D^3 B \int_0^{1/2} Q_c\, d\left(\frac{r}{D}\right)$$

$$= .00237 \times 189^2 \times 10^3 \times 2 \times .00805 = 1365 \text{ ft.lb.}$$

$$Q.HP. = \frac{QN}{5255} = \frac{1365 \times 1800}{5255} = 467.$$

From equation (3) the thrust is

$$T = \rho V^2 D^2 B \int_0^{1/2} T_c \, d\left(\frac{r}{D}\right)$$

$$= .00237 \times 189^2 \times 10^2 \times 2 \times .0620 = 1050 \text{ lb.}$$

$$\text{T.HP.} = \frac{TV}{550} = \frac{1050 \times 189}{550} = 361.$$

The efficiency $\eta = \frac{\text{T.HP.}}{\text{B.HP.}} = \frac{361}{467} = .772$

The torque horsepower found above is the power which the propeller would absorb if it were operating alone (without body interference) at a tip speed of about 250 ft./sec. In this case the propeller being 10 ft. in diameter and revolving at 1800 R.P.M., the tip speed =

$$\pi nD = .0524 \, ND = .0524 \times 1800 \times 10$$

$$= 944 \text{ ft./sec.}$$

If it is operating in front of an average fuselage at 944 ft./sec., tip speed, the Q.HP. should be increased by about 15 per cent to give the actual power absorbed by the propeller (Reference 2) or B.HP. = 1.15 × 467 = 537.

Single Section Method:

In the short method Q_c is found for the section at 75 per cent radius only and called Q'_c. Then since the ratio $\frac{\int_0^{1/2} Q_c \, d\left(\frac{r}{D}\right)}{Q'_c}$ has a constant value of .272,

$$\int_0^{1/2} Q_c \, d\left(\frac{r}{D}\right) = .272 \times Q'_c = .272 \times .029 = .0079$$

$$Q = \rho V^2 D^3 B \int_0^{1/2} Q_c \, d\left(\frac{r}{D}\right) = .00237 \times 189^2 \times 10^3 \times 2 \times .0079 = 1342$$

$$\text{Q.HP.} = \frac{QN}{5255} = \frac{1342 \times 1800}{5255} = 460$$

$$\text{and} \quad \text{B.HP.} = 1.15 \times 460 = 530.$$

$$\text{The efficiency} = \frac{.416 \times \frac{V}{nD}}{\tan(\Phi + \gamma)} = \frac{.416 \times \frac{189}{30 \times 10}}{\tan 18.9^\circ}$$

$$= \frac{.416 \times .63}{.3424} = .765.$$

References

1. Wood, R. McKinnon; Bradfield, F. B.; Barker, M. : Multiplane Interference Applied to Air Screw Theory. Brit. Advisory Com. for Aeron. R.& M. No. 639. (1919)

2. Weick, Fred E. : Propeller Scale Effect and Body Interference. N.A.C.A. Technical Note No. 225. (1925)

TABLE I.

Symbols

D - Diameter of propeller in feet.

P - Geometrical pitch of propeller in feet.

r - Radius of any section of propeller, in feet (Fig. 1).

b - Blade width at any section, in feet (Fig. 1).

c - Maximum blade width, in feet (Fig. 1).

B - Number of blades in propeller.

Q - Torque of propeller, in foot pounds.

T - Thrust of propeller, in pounds.

Q_c - Torque coefficient of any section of propeller.

T_c - Thrust coefficient of any section of propeller.

V - Velocity of advance in ft./sec.

M.P.H. - Velocity of advance in mi./hr.

n - Revolutions of propeller per second.

N - Revolutions of propeller per minute.

B.HP. - Brake horsepower of engine.

T.HP. - Thrust horsepower.

Q.HP. - Torque horsepower.

η - Efficiency

Φ_β - Blade angle in degrees (Fig. 2).

Φ - Angle which path of blade makes with plane perpendicular to propeller axis.

- = arc tan $\frac{V}{2\pi}$ rn (Fig. 3).

α - Apparent angle of attack of section = $\Phi_\beta - \Phi$ (Fig. 3).

ϵ - Interference correction for angle of attack (Fig. 5).

α' - Corrected angle of attack = $\alpha - \epsilon$ (Fig. 5).

C_L - Absolute lift coefficient of airfoil at angle of attack α (Fig. 6).

∂C_L - Interference correction for lift coefficient (Fig. 7).

C'_L - Corrected lift coefficient = $C_L - \partial C_L$ (Fig. 5).

L/D - Ratio of lift to drag for the airfoil section.

h_U - Maximum upper camber of section (Fig. 4).

h_U/h_L - Standard blade ratios (Fig. 9).

h_L - Maximum lower camber of section (Fig. 4).

h/b - Camber ratio of any section.

γ - Angle between resultant force on blade element and a line perpendicular to V_r.

ρ - Mass density of air. This may be taken as .00237 for sea level and standard atmosphere.

Tip speed - The distance traveled by the tip of the propeller in unit time in plane of rotation.

Tip speed - = πnD = .0524 ND ft./sec.

TABLE II.

Analysis of Standard Navy Wood Propeller.

Diameter = 10 ft. R.P.M. = 1800 M.P.H. = 129
Pitch = 7 ft. n = 30 r.p.s. V = 189 ft./sec.

r/R	.30	.45	.60	.75	.90
b (ft.)	.788	.833	.788	.660	.450
h_U/b	.200	.167	.133	.107	.090
h_L/b	.058	.007			
r (ft.)	1.5	2.25	3.0	3.75	4.5
$2\pi r$	9.42	14.13	18.84	23.55	28.25
$2\pi rn$	283	434	565	707	848
$S = \frac{2\pi r}{Bb}$	6	8.5	12	17.8	31.4
$\tan\Phi_\beta = \frac{P}{2\pi r}$	.743	.495	.372	.2975	.248
Φ_β	36.6°	26.4°	20.4°	16.6°	13.9°
$\tan\Phi = \frac{V}{2\pi rn}$	.668	.446	.334	.267	.223
Φ	33.8°	24.0°	18.5°	15.0°	12.6°
$\alpha = \Phi_\beta - \Phi$	2.8°	2.4°	1.9°	1.6°	1.3°
C_L	.760	.802	.656	.530	.430
∂C_L	.100	.096	.066	.044	.024
$C'_L = C_L - \partial C_L$	.660	.706	.590	.486	.406
ϵ	2.4°	1.8°	1.0°	.5°	.2°
$\alpha' = \alpha - \epsilon$	.4°	.6°	.9°	1.1°	1.1°
$L/D\ (\alpha')$	14.5	15.4	16.3	16.8	16.5
$D/L\ (\alpha')$	.0690	.0649	.0613	.0595	.0606

Table II (Cont.)

Analysis of Standard Navy Wood Propeller.

Diameter = 10 ft. R.P.M. = 1800 M.P.H. = 129

Pitch - 7 ft. n = 30 r.p.s. V = 189 ft./sec.

$\tan \epsilon$	.0419	.0314	.0175	.0087	.0035
$\tan\gamma = D/L + \tan \epsilon$	.1109	.0963	.0788	.0682	.0641
γ	6.3°	5.5°	4.5°	3.9°	3.7°
$\Phi + \gamma$	40.1°	29.5°	23.0°	18.9°	16.3°
$\sin \Phi$	.5563	.4067	.3173	.2588	.2181
b/D	.0788	.0833	.0788	.0660	.0450
C'_L	.660	.706	.590	.486	.406
$K_p = C'_L \times \frac{b}{D} \times \frac{1}{2\sin^2\Phi}$	.0841	.1805	.2310	.2390	.1920
$\sin(\Phi+\gamma)$	.6441	.4924	.3907	.3239	.2807
r/D	.15	.225	.30	.375	.45
$Q_c = K_p \times \frac{r}{D} \times \sin(\Phi+\gamma)$	.0081	.0200	.0271	.0290	.0242
$\cos(\Phi+\gamma)$	.7649	.8704	.9205	.9461	.9598
$T_c = K_p \cos(\Phi+\gamma)$	.0643	.1573	.2130	.2260	.1850

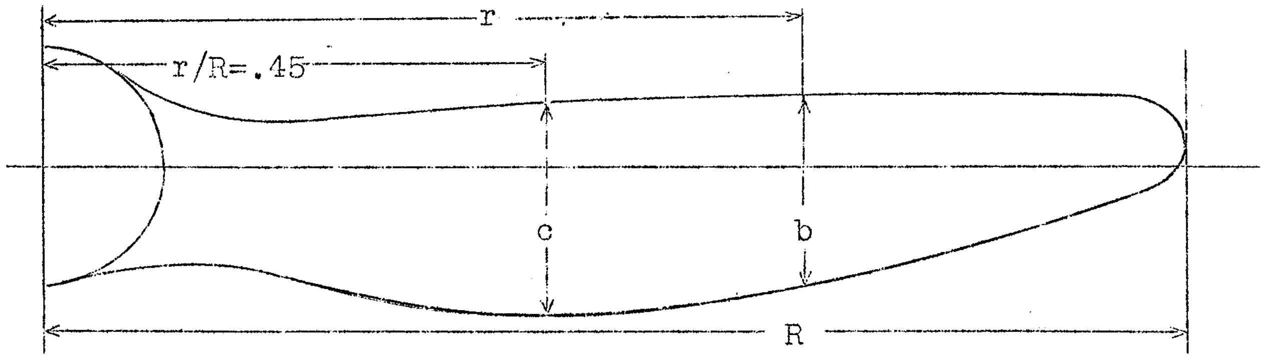


Fig.1

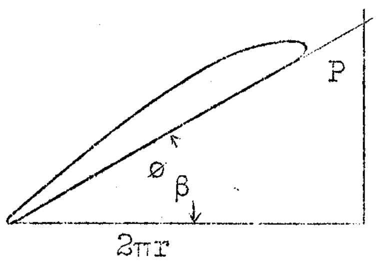


Fig.2

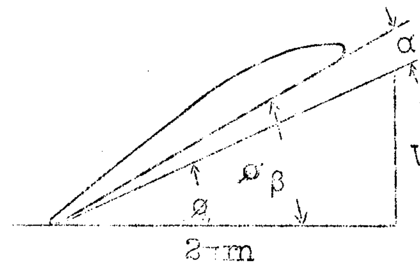


Fig.3

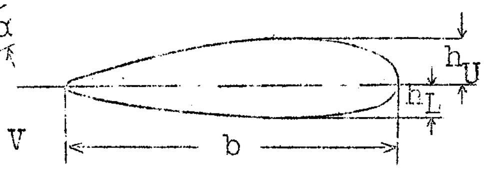


Fig.4

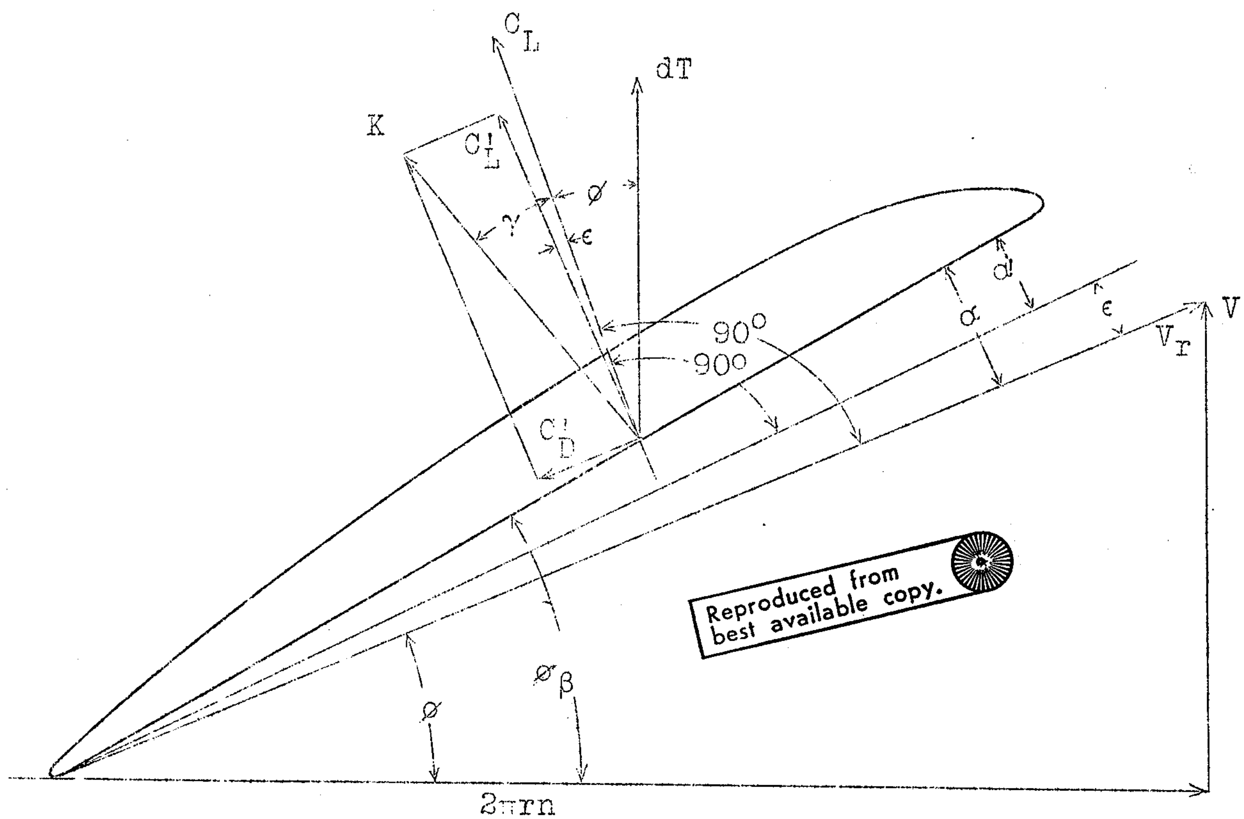


Fig.5

Diagrams used in the development of the modified blade element theory.

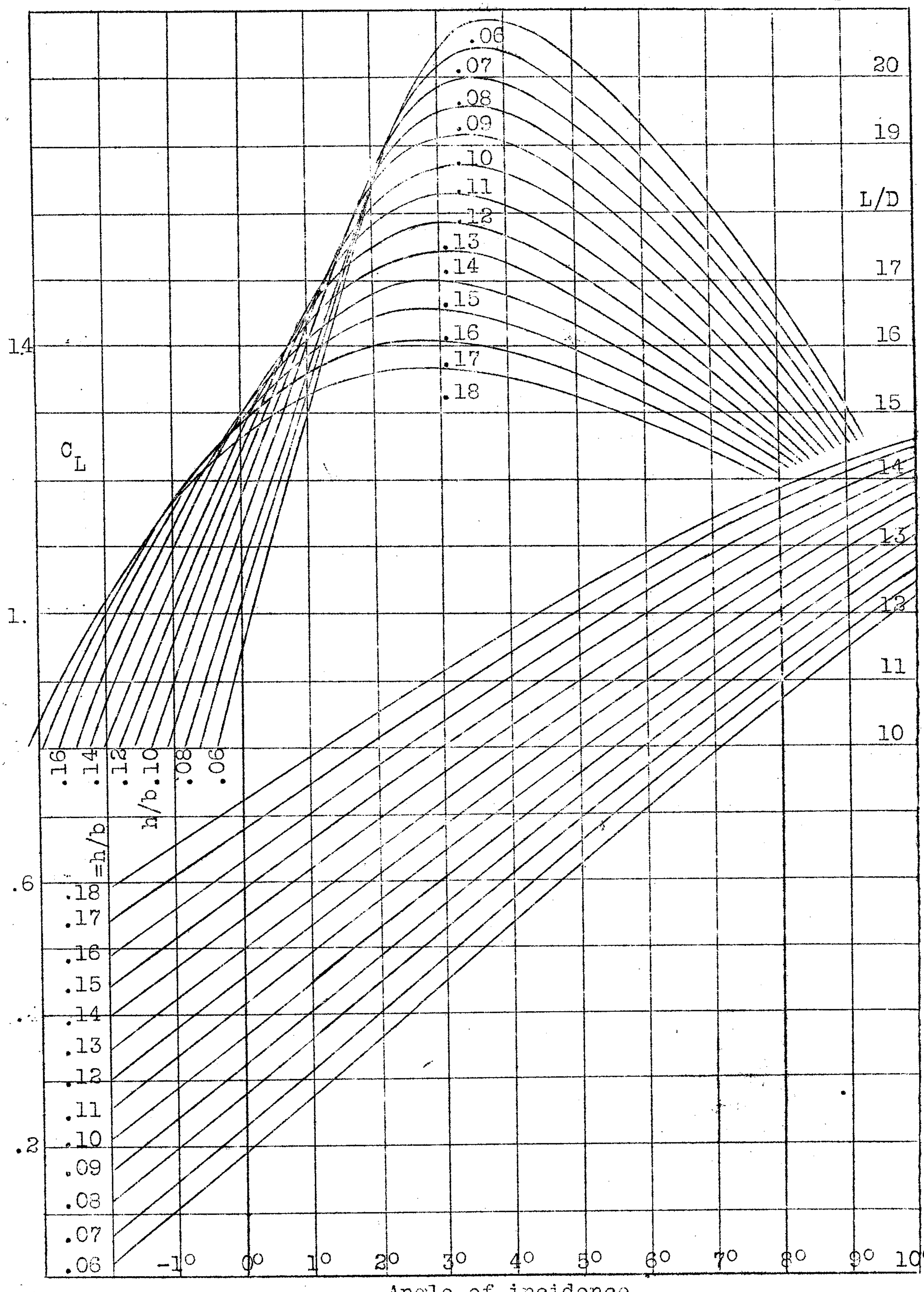


Fig.6 L/D and C_L curves from model Navy prop.data.expanded.

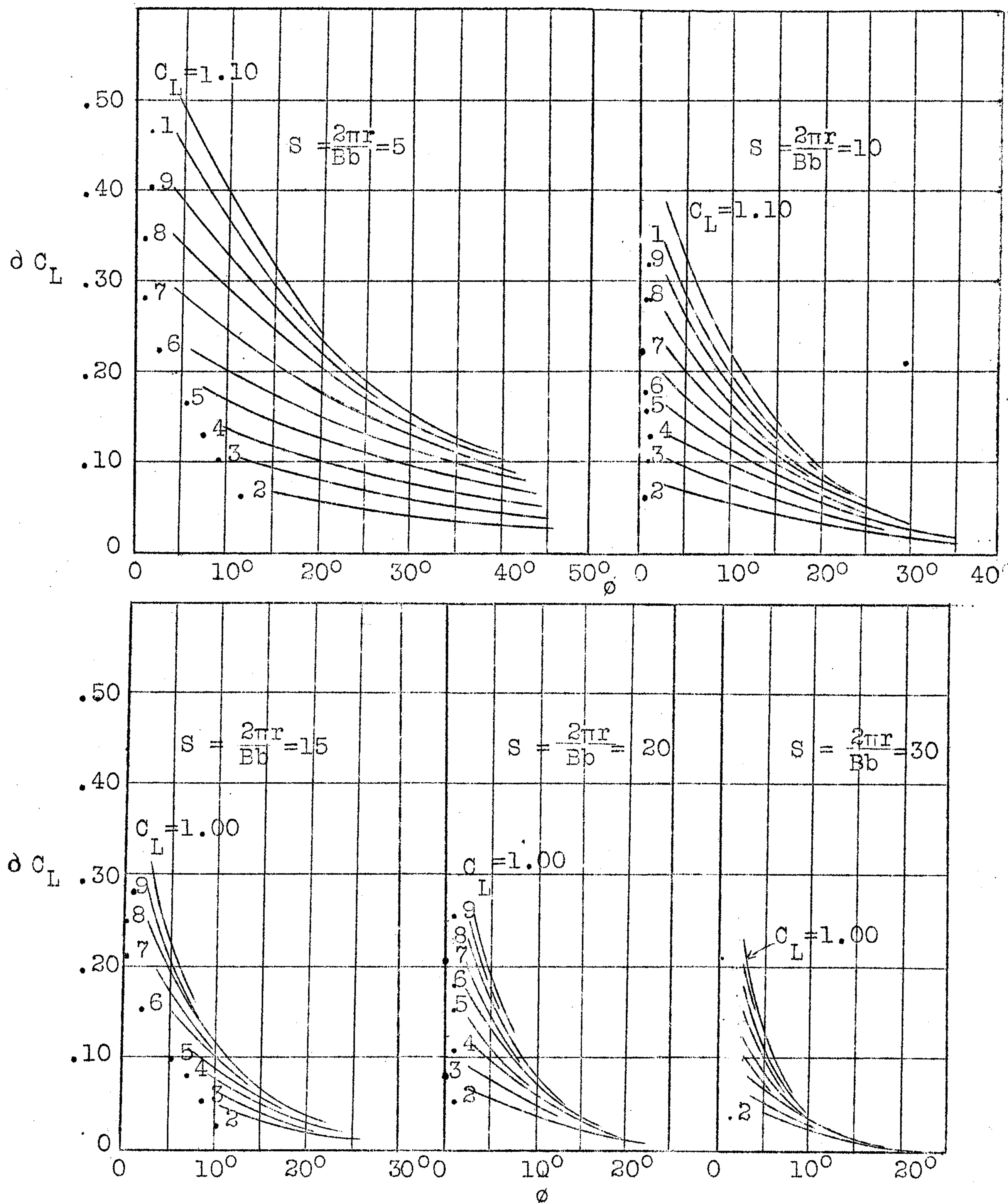
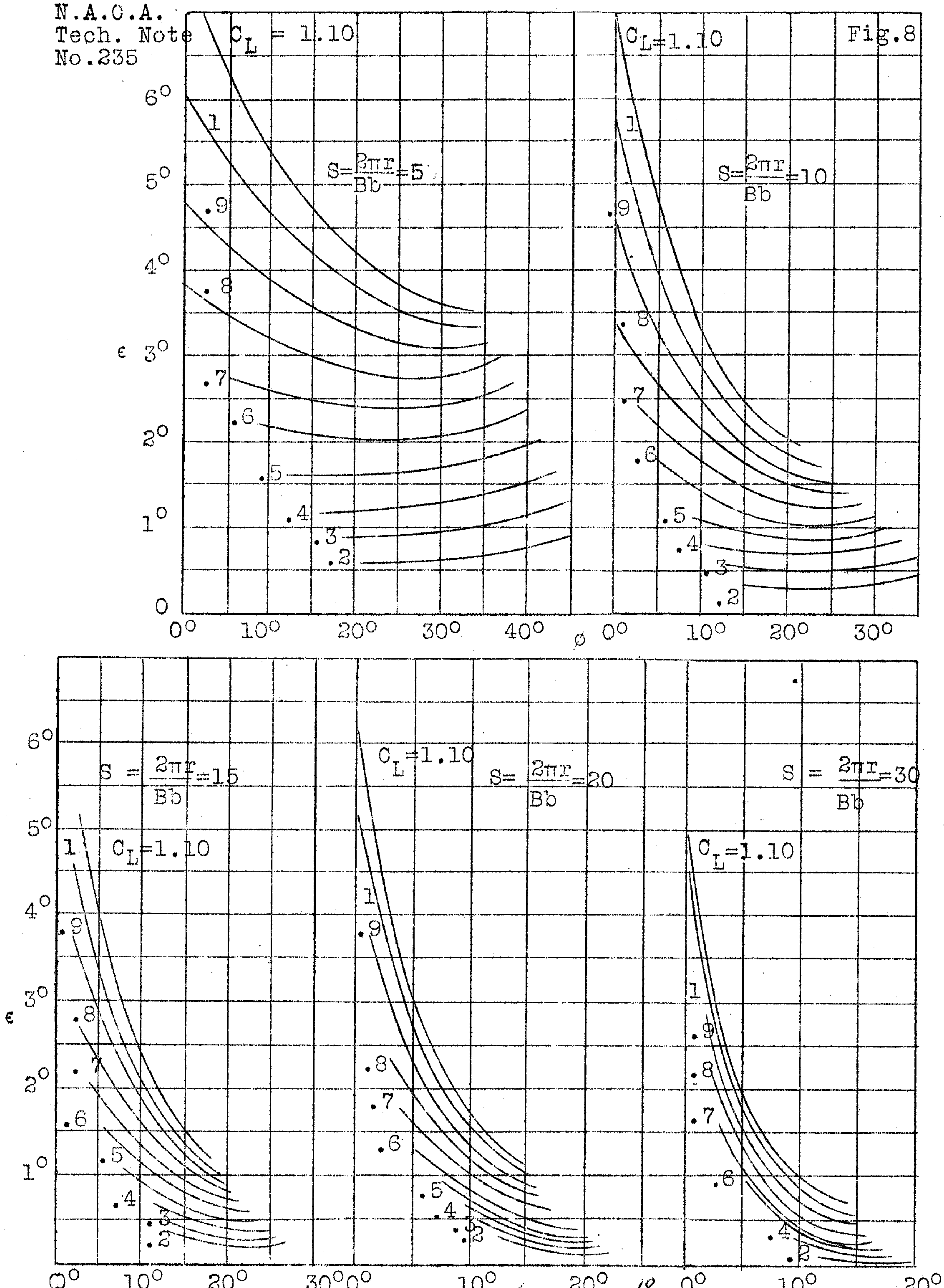


Fig.7 Lift correction, ∂ C_L.

Fig.8


C_L = 1.10
$S=\frac{2\pi r}{Bb}=5$
C_L=1.10
$S=\frac{2\pi r}{Bb}=10$
$S=\frac{2\pi r}{Bb}=15$
C_L=1.10
C_L=1.10
$S=\frac{2\pi r}{Bb}=20$
$S=\frac{2\pi r}{Bb}=30$
C_L=1.10
ε
φ
0°
10°
20°
30°
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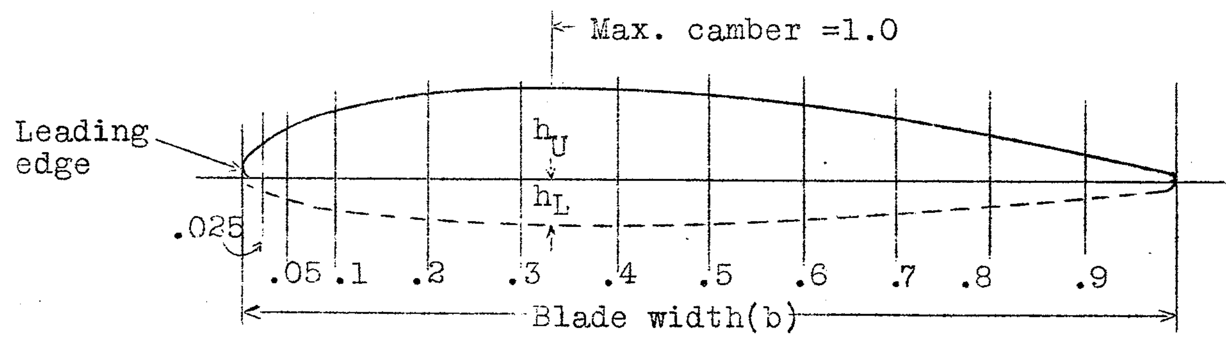

Station	LER	.025	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	TER
Ordinate	.10	.41	.59	.79	.95	.998	.99	.95	.87	.74	.56	.35	.077

Fig.9 Navy standard blade section.R.A.F.No.6 Modified, flat face.

$$\int_0^{1/2} T_c \, d_{(r/D)} = 0.01 \times 6.20 \text{ sq.in.} = .0620$$

$$\int_0^{1/2} Q_c \, d_{(r/D)} = 0.001 \times 8.05 \text{ sq.in.} = .00805$$

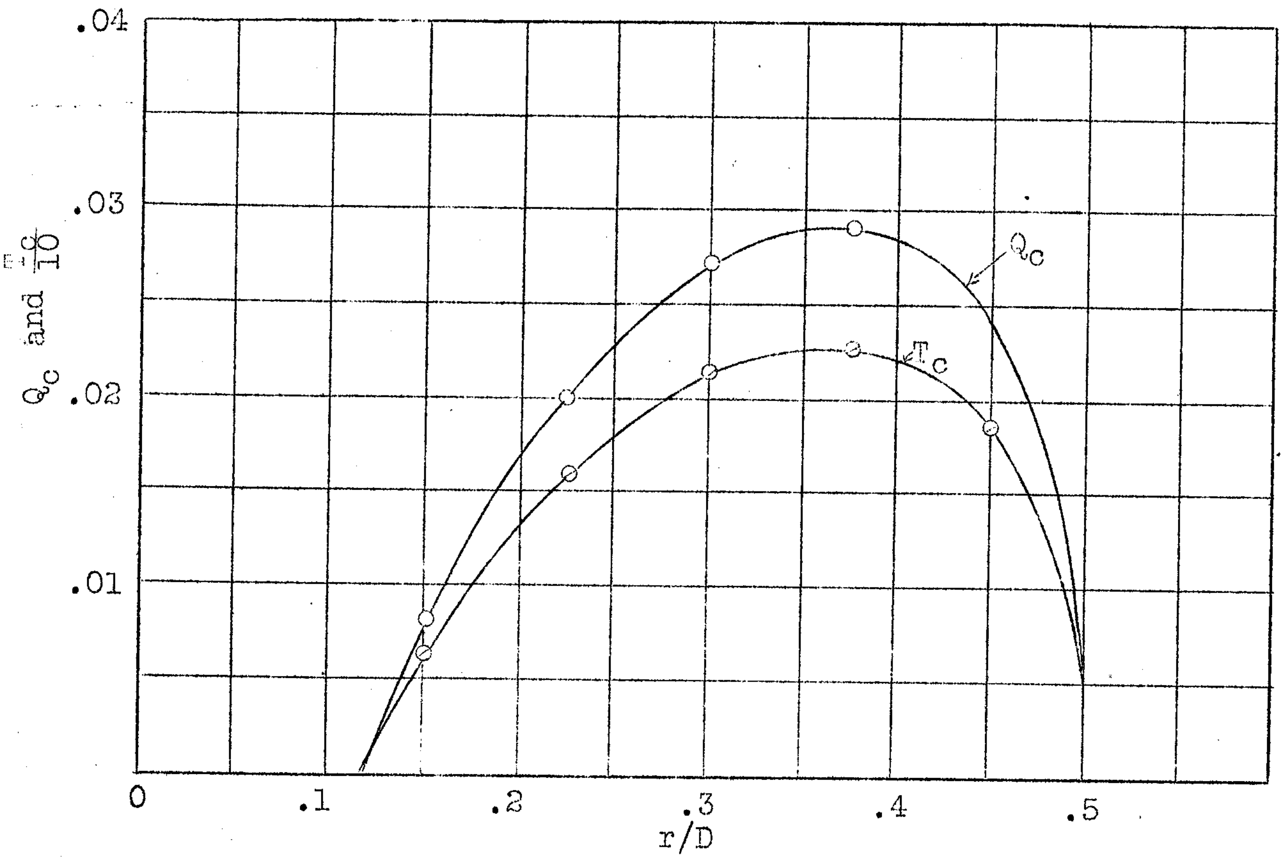

Fig.10 Thrust and torque grading curves.

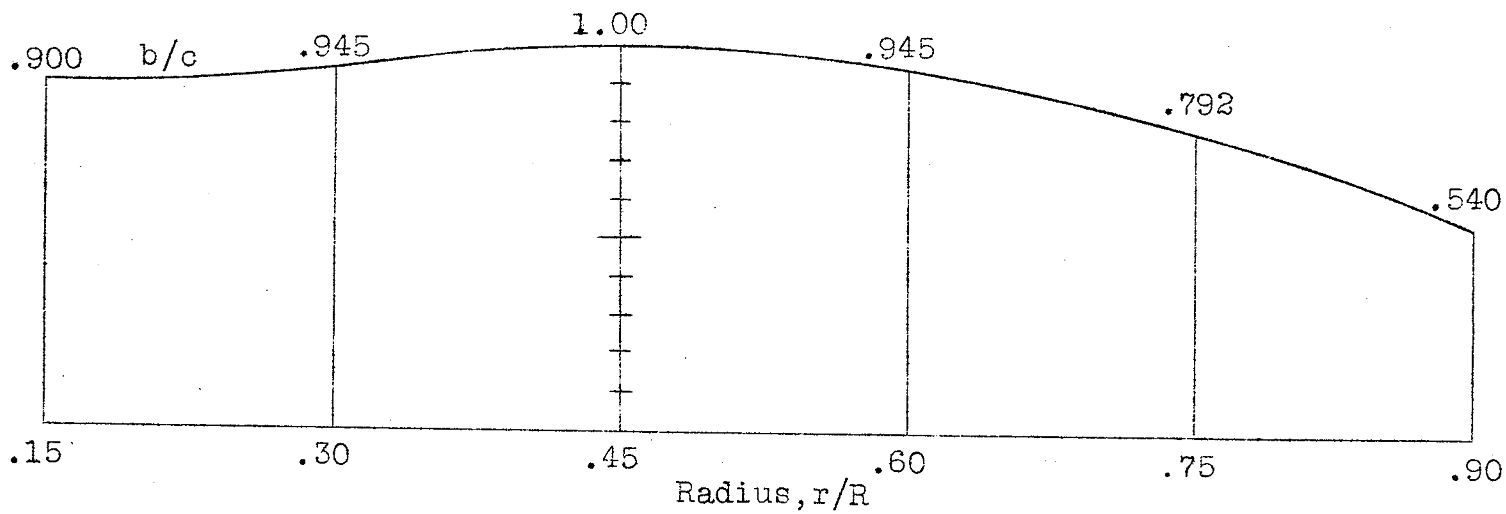

Fig.11 Standard Navy plan form for wooden propellers.