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ANALYTICAL DETERMINATION OF THE LOAD ON A TRAILING EDGE FLAP

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Summary

This report presents a theoretical analysis of the lift on a trailing edge flap. An analytical expression has been derived which enables the computation of the flap load coefficient. The theoretical results seem to show a fair agreement with the meager experimental results which are available.

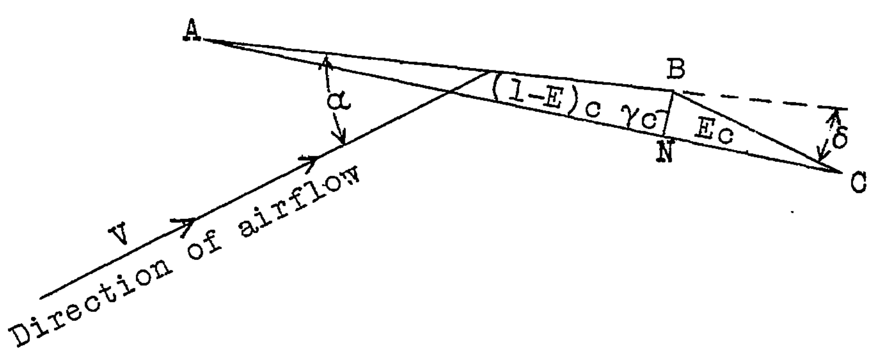
Introduction

Theoretical relationships for an airfoil with a hinged flap have been developed by different authors and presented in several publications, a list of which is contained in the references of this report. These relationships consist of analytical expressions which measure the effect of a displaced flap upon the lift, pitching moment, and hinge moment coefficients. A summary of the theoretical methods used and the results obtained is given in Reference 6. This reference contains, also, a discussion of the effect of aspect ratio and shows that the parameters determined therein are independent of aspect ratio. There is, however, one phase of the problem which has not been presented, namely, the flap load.

The purpose of this report is to extend the existing theory of Glauert to an investigation of the load on a trailing edge flap. An analytical expression for the coefficient of lift on the flap is derived in the following pages.

Analysis

For the purpose of this investigation a thin symmetrical section is chosen as a basis for calculation. Such a section may be reduced essentially to the diagram below, where the broken line A B C is the mean camber line of the section. The diagram and notation are those used by Glauert with two exceptions, namely, the use of δ instead of η for flap angle and α instead of α' for angle of attack of forward part of airfoil. These changes are in keeping with American usage.



α = angle of attack of forward part of airfoil,

δ = angle of flap displacement,

E = ratio of flap chord to total chord,

c = total chord (approximately AC),

Ec = flap chord (BC or NC),

γc = height of the hinge B above the base line AC,

from which it follows that

$$\delta = \frac{\gamma}{E} + \frac{\gamma}{1 - E} \quad \text{(for small angles).}$$

The coordinate axes are chosen so that the X-axis coincides with AC and the origin lies at the leading edge, namely, the point A. As will be observed later, it is necessary to replace the variable x with the variable θ where

$$x = \tfrac{1}{2} c \, (1 - \cos \theta) \tag{1}$$

so that θ passes along the airfoil from 0 to π. From the diagram and equation (1) it is easily found that the hinge is located by φ where

$$\begin{aligned} \cos\varphi &= -\,(1 - 2E) \\ \sin\varphi &= 2\sqrt{E\,(1 - E)} \end{aligned} \tag{2}$$

It is shown in Reference 1 that the lift force on an airfoil is given by

$$L = \int_0^c \rho \, V \, k \, d\,x \tag{3}$$

where $k\,d\,x$ is the element of vorticity distribution taken along the airfoil and is assumed to be of the form

$$k\,d\,x = c\,V\,[A_0\,(1 + \cos\theta) + \sum_1^\infty A_n \sin n\theta \sin\theta]\,d\,\theta \tag{4}$$

The values of A_0 and A_n are found in Reference 3:

$$A_0 = \alpha + \frac{\pi - \varphi}{\pi}\,\delta \qquad (5)$$

$$A_n = \frac{2 \sin n\varphi}{n\pi}\,\delta$$

The lift force L_f, on the flap may now be expressed by means of (3) and (4) and the substitution of the proper limits

$$L_f = \rho V^2 c \int_\varphi^\pi [A_0(1 + \cos\theta) + \sum_1^\infty A_n \sin n\theta \sin\theta]\, d\theta \qquad (6)$$

Before integration of this expression, it is necessary to rewrite it in the form (the reason will be apparent upon integration)

$$L_f = \rho V^2 c \int_\varphi^\pi [A_0(1+\cos\theta) + A_1 \sin^2\theta + \sum_2^\infty A_n \sin n\theta \sin\theta] d\theta \qquad (6')$$

Integration and substitution of the values of the coefficients leads finally to the result

$$L_f = \frac{1}{2} \rho V^2 c [2(\pi - \varphi - \sin\varphi)\alpha + \frac{2}{\pi}\left\{(\pi - \varphi)^2 + \frac{1}{2}\sin\varphi \sin 2\varphi + \sum_2^\infty \frac{\sin n\varphi}{n}\left(\frac{\sin \overline{n+1}\,\varphi}{n+1} - \frac{\sin \overline{n-1}\,\varphi}{n-1}\right)\right\}\delta] \qquad (7)$$

Also from the theory of two-dimensional motion

$$L_f = C_{L_f}\, E\, c \frac{\rho V^2}{2} \qquad (8)$$

By inspection of equation (7) we may write the lift coefficient for the flap in the form

$$C_{L_f} = d_1\, \alpha + d_2\, \delta \qquad (9)$$

where, from (7)

$$d_1 = \frac{2}{E}(\pi - \varphi - \sin\varphi)$$

and

$$d_2 = \frac{2}{\pi E}[(\pi - \varphi)^2 + \frac{1}{2}\sin\varphi \sin 2\varphi + \sum_2^\infty \frac{\sin n\varphi}{n}\left(\frac{\sin \overline{n+1}\varphi}{n-1}\right. \qquad (10)$$

$$\left.\frac{\sin \overline{n-1}\varphi}{n-1}\right)]$$

Equation (9) may be written in the more convenient form (as will be shown later)

$$C_{L_f} = n_o\, C_L - n\, \delta \tag{11}$$

by means of the expression for C_L

$$C_L = a_o\,(\alpha + k\,\delta) \quad \text{(Reference 6)} \tag{12}$$

where a_o is the slope of the lift curve for infinite aspect ratio and k is given by

$$k = \frac{1}{\pi}\,(\pi - \varphi + \sin\varphi). \tag{13}$$

Using 2π, the theoretical value of a_o, and equations (10) and (13), the values of n_o and n may be calculated as follows:

$$n_o = \frac{2}{\pi(1 + \cos\varphi)}\,(\pi - \varphi - \sin\varphi)$$

$$n = \frac{4}{\pi(1 + \cos\varphi)}\left[\sin^2\varphi\,(1 + \cos\varphi) + 2\sum_2^\infty\left(\frac{\sin\varphi\,\sin n\varphi\,\cos n\varphi}{n^2 - 1} - \frac{\cos\varphi\,\sin^2 n\varphi}{n(n^2 - 1)}\right)\right].$$

A general summation of the series term in the expression for n has not been found, hence an approximate method of calculation is necessary for each size of flap desired.. Values of n_o and n have been calculated for different values of φ from 0 degrees to 180 degrees at 15-degree intervals and plotted against computed values of E in Figure 1. Sufficient terms in the series have been taken to obtain reliable results to three decimal places.

Thus far the analysis has been confined to two-dimensional flow and the final result is an expression for the lift coefficient on the flap, namely,

$$C_{L_f} = n_o \, C_L - n \, \delta$$

If the usual assumptions are made regarding the effect of changing the aspect ratio, the expression for C_{L_f} written in the form above will not be affected by aspect ratio. In other words, it is assumed that the section characteristics remain the same if the effective angle of attack and the shape of the section remain the same. Since the lift coefficient is a measure of the effective angle of attack, the flap load coefficient is independent of aspect ratio at a given value of the flap angle and lift coefficient. The equation as written above may, therefore, be applied to a wing of any aspect ratio, and the values of the parameters n_o and n, as determined for the infinite wing (Figure 1), may be applied directly to find the flap load on a finite wing.

Comparison with Experimental Results.- In order to check the accuracy of the theoretical results presented herein, it would be necessary to have accurate measurements of actual flap loads. Such data are not easily obtainable. However, there are available some pressure distribution measurements made on a symmetrical R.A.F. 30 airfoil with trailing edge flap which have been published in Reference 6. The results at a Reynolds Number of 3.56×10^5 have been cross-faired to obtain the experimental curves which are shown with the theoretical lines in Figure 2.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., Oct. 13, 1930.

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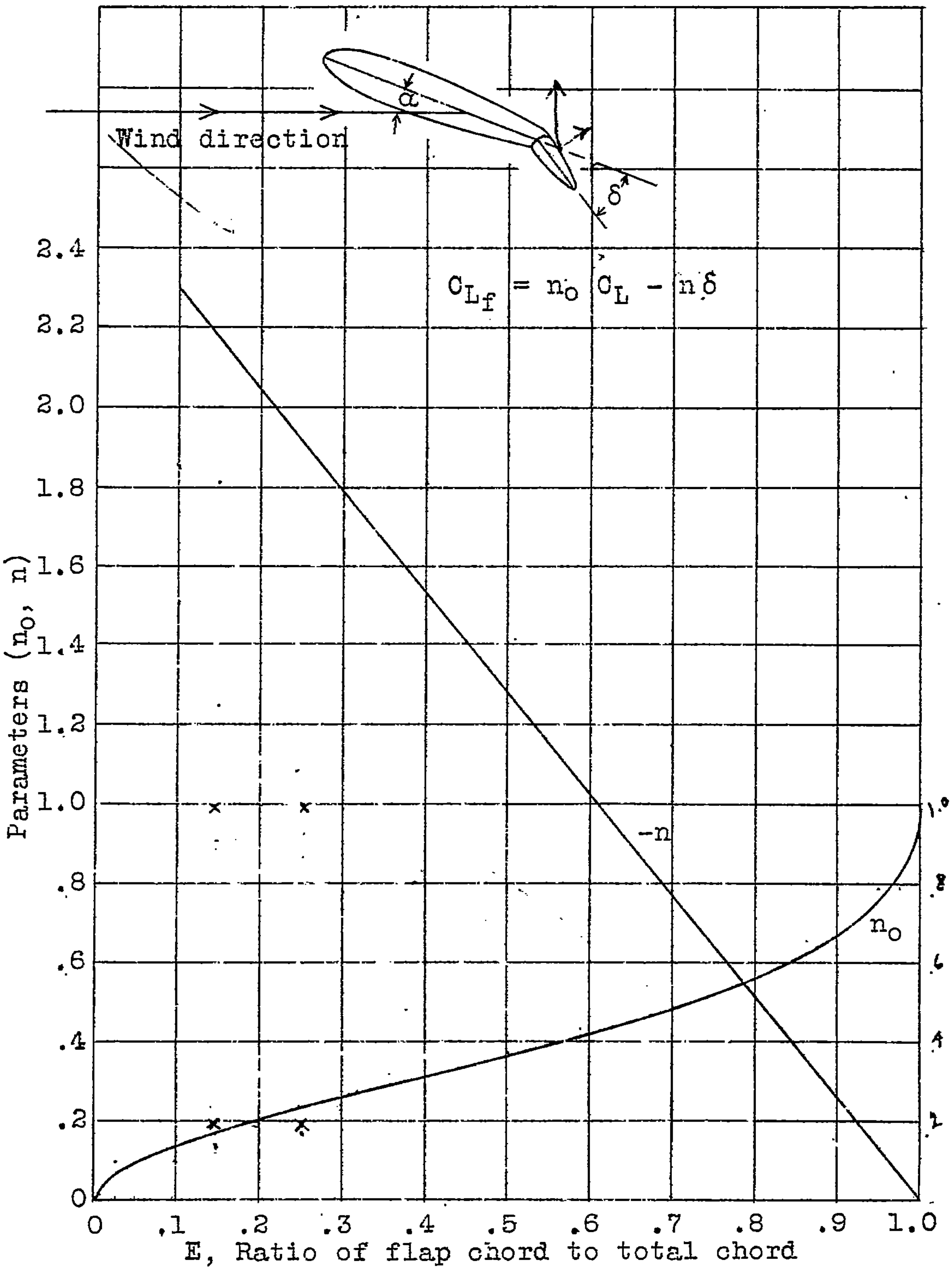

Fig.1 Theoretical parameters.

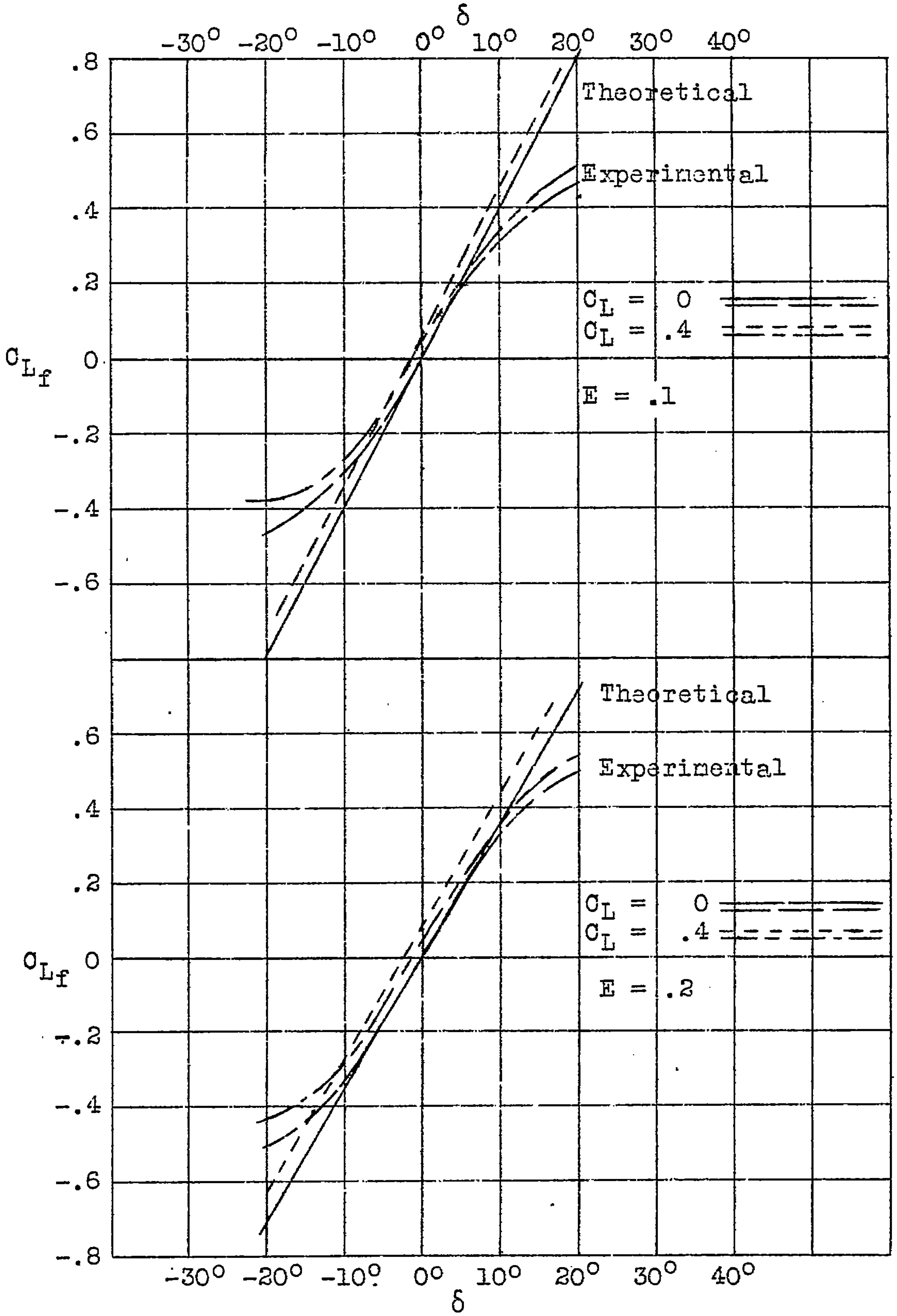

Fig.2 Comparison of theoretical and experimental coefficients.