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No. 413

THE COMPRESSIVE STRENGTH OF DURALUMIN COLUMNS OF EQUAL ANGLE SECTION

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SUMMARY

This report presents a chart giving the compressive strength of duralumin columns of equal angle section. The data used in the construction of the chart were obtained from various published sources and were correlated with theory in the range where secondary failure occurred.

Appendices are included giving excerpts from Army and Navy specifications for duralumin and approximate formulas for the properties of the equal angle section.

INTRODUCTION

In the present trend toward all-metal airplane construction there is an increasing need on the part of designers for charts giving the compressive strength of various open sections now frequently used as compression members in trusses and as stiffeners in stressed-skin structures. At present, the only column chart in general use for open sections is the one for duralumin channels first published by the Army Air Corps (reference 1) and republished in the book entitled "Airplane Structures," by Niles and Newell.

In an effort to compile additional charts of this type, the National Advisory Committee for Aeronautics at Langley Field, Va., made a study of the results of numerous column tests reported in technical literature. In the course of this study it was observed that the test data of references 2, 3, and 4 for equal duralumin angles were fairly consistent and that the column curves were generally similar to theoretical curves plotted from a formula given in reference 5. By the introduction of an empirical constant, the theoretical curves were made to fit the points plotted from the tests. The present report develops a column chart for equal duralumin angles based upon the test data and the theory of the above-mentioned references.

Because the strength properties of the materials used were not given in any of the references containing test data, some difficulty was experienced in reducing the test results to the results of a specification material. In this connection, valuable assistance in the form of comments and data was obtained from Dr. L. B. Tuckerman and Captain S. N. Petrenko of the Bureau of Standards. The column chart, as finally constructed, is intended to apply, insofar as the strength properties are concerned, to material which conforms to Army Specification 57-187-1 - Type B material, Navy Specification 44T21 - Type B material, or to Navy Specification 47A3a - Type 3 material. Excerpts from these specifications are given in Appendix A.

For the convenience of designers, a list of approximate formulas for the section properties of equal angles is given in Appendix B.

SYMBOLS

Symbols Used in the Following Discussion

b, width of leg, in. (See fig. 1)
t, thickness of leg, in. (See fig. 1)
l, length of column, in.
ρ, radius of gyration, in.
S, stress, lb. per sq.in.
c, coefficient of end fixity.
E, Young's modulus, lb. per sq.in.
σ, Poisson's ratio.
λ, half-wave length of wrinkle in an outstanding flange or leg of the angle, in.

Additional Symbols Used in Appendix B

A, area of cross section, sq.in.
x, distance of c.g. from leg, in. (See fig. 1)
I, moment of inertia of cross section, in.^{4}

TYPES OF COLUMN FAILURE

Columns may fail in any of the following ways: by compression; by bending; by local wrinkling of some thin part; by twisting about a longitudinal axis, which axis may or may not coincide with the centroidal axis of the column; or by any combination of the above types of failure.

Compression.- Compression failure is characterized by a plastic flow of the material in the column. Ultimate strengths corresponding to this type of failure depend upon the dimensions of the column and the stress-strain curve for the material, but compression failure is usually assumed to occur at or near the yield-point stress.

Bending.- An Euler, or long, column which deforms into a continuous curve from end to end is considered to have failed by bending. The stress at which this bending takes place is given by the well-known Euler column formula

$$S = \frac{c\,\pi^2 E}{\left(\frac{l}{\rho}\right)^2} \qquad (1)$$

Local wrinkling and twisting.- A column of large torsional rigidity, but with very thin parts, may fail by a local wrinkling of one of the thin parts into waves of length 2λ. If this thin part is an outstanding flange or leg (see fig. 2) perfectly fixed at its base, the minimum critical wrinkling stress is (reference 5, eq. 2)

$$S = 1.16\ E\ \frac{t^2}{b^2} \qquad (2)$$

It is pointed out further in reference 5 that on account of elastic giving and imperfect fixity at the base of the flange or leg, the coefficient in equation (2) is considerably reduced in any practical case.

If the thickness of the heavy leg in Figure 2 is reduced, the torsional rigidity is reduced and the angle column twists appreciably under load. On account of this twisting the conditions in the thin leg become analogous to those in a flange, or leg, with reduced fixity. In the limiting case when the thicknesses of the two legs are the same, the fixity at the base of each leg becomes zero: λ approaches the length of the column and we have what is commonly referred to as failure by twisting. The critical stress for the condition of zero fixity at the base of a compression flange, or leg, is (reference 5, eq. 90)

$$S = \left[\frac{\pi^2}{12(1-\sigma^2)}\frac{b^2}{\lambda^2} + \frac{1}{2(1+\sigma)}\right] E \frac{t^2}{b^2} \qquad (3)$$

TEST DATA AND DISCUSSION

The column tests upon equal duralumin angles reported in references 2, 3, and 4 were made with pin ends. Representative portions of these data have been plotted in Figure 3 and will be summarized relative to the types of failure.

Compression.- In the tests of references 2, 3, and 4 no mention was made of a compression type of failure. Examination of the test data plotted in Figure 3 indicates that this type of failure might have occurred at stresses of from 34,000 to 36,000 pounds per square inch.

Bending.- Except in the range where secondary failure occurred, long columns failed at the Euler load. It will be observed that the test points in Figure 3 scatter somewhat but are, in general, above the Euler curve. These discrepancies are probably caused by a slight amount of friction in the pin ends and by the fact that E was, in some cases, higher than 10^7 pounds per square inch, the value for which the Euler curve was drawn.

Combinations of compression, bending, wrinkling, and twisting.- Any type of column failure which is not clearly either compression or bending comes under this general heading. A detailed discussion of the test data concerning the exact types of failure would be interesting but cannot be made because practically no information on this subject was given in the reports. Consequently, further discussion of the type of failure will be avoided and curves will be derived representing the test data. These curves will be based upon theoretical formulas where possible.

In Figure 3, below values of l/ρ equal to 80, the results of tests on columns designated H, O, and P lie approximately on a straight line. These tests ($\frac{b}{t}$ = 7.9 to 9.9) represent two types of failure changing from bending failure at about $l/\rho \doteq 80$ to compression failure at approximately $l/\rho \doteq 30$. From these data it is concluded that a single straight line and Euler curve will closely define the strength of a duralumin column of equal angle section up to values of b/t of 10 or perhaps 12.

Plotting the critical twisting stress as given by equation (3) with the following substitutions

$$\lambda = l$$

$$b \doteq \frac{\rho}{0.204} \qquad \text{(Appendix B, eq. 10.)}$$

$$\sigma = 0.3$$

$$E = 10^7 \text{ lb. per sq.in.,}$$

the dotted curves in Figure 3 are obtained for values of $\frac{b}{t}$ of 14, 17, 19, 20, and 25. Although these theoretical curves for twisting failure agree fairly well with the test data at large values of l/ρ they give conservative stresses at low values of l/ρ. It was found by inspection that the test data could be represented very well by equation (3) if λ were assumed equal to $\frac{l}{2.62}$ instead of l. The solid curves in Figure 3 for the same values of $\frac{b}{t}$ were drawn with $\lambda = \frac{l}{2.62}$. All of the experimental results plot fairly well along these curves except the points for the column designated S-3, which plot below the curve for $\frac{b}{t} = 14$. These low points may be caused by differences in the properties of the material used in the tests of references 2 and 3, or by inapplicability of the formulas for elastic failure in this region of the chart. Because no stress-strain curves were given, no definite idea of the relative importance of these two factors could be formed for this case.

An idea of the importance of knowing the strength properties of the column material may be obtained by comparing the results of tests upon duralumin tubes with the results of tests upon duralumin angles plotted in Figure 3.. It is not likely that a higher "form-factor," i.e., higher average stresses, would be developed in a duralumin column of angle section than in a duralumin tube tested with the same l/ρ, neither of which fails locally. This apparent absurdity must, therefore, be caused largely by a difference in the properties of the materials. Consequently, in the construction of the column chart for equal duralumin angles the several sets of test data must be corrected for differences in the properties of the materials.

In Figure 3 the curves representing failure by twisting intersect the Euler curve rather than become tangent to it. This intersection indicates an abrupt change in the type of elastic failure and at the lower stresses is borne out by the test data. (Column N.) At the higher stresses, above the stress at which the straight line having b/t labeled "10 or less" becomes tangent to the Euler curve, the transition from failure by twisting to failure by bending (the Euler type) is actually not so abrupt as the crossing of the theoretical curves would indicate. (Columns S-3 and L.) This phenomenon is caused by inelastic behavior of the material in the columns and must be taken into account in the construction of the column chart for values of b/t below 18.

Some recent tests made by the Bureau of Standards upon sheet duralumin furnished by the National Advisory Committee for Aeronautics showed a marked difference in the shape of the stress-strain curves taken normal and parallel to the direction of rolling. (See figs. 4 and 5.) Therefore, if angles are made from sheet material the direction of the grain in the angle column must be taken into account in the construction of the column chart.

The importance of including stress-strain curves with the results of column tests cannot be too strongly emphasized. Without them no account can be taken of the strength properties or the inelastic behavior of the material in the analysis of test results. The Bureau of Standards has repeatedly shown the necessity of including stress-strain curves of the material with the results of all tests on structures if the results are to have a lasting value. The failure to include these curves or to make any reference to the strength properties of the material in references 2, 3, and 4 has greatly reduced the value of the results, except in the range of elastic failure.

CONSTRUCTION OF COLUMN CHART

In the absence of tests on angle columns in which the properties of the material are known it will be assumed that for columns constructed of any material, angle columns with values of b/t up to 12 will fail at the same stress as a tube of the same l/ρ. Upon the basis of this assumption the results of the Bureau of Standards tests on tubes which conform to Army Specification 57-187-1 - Type B material and to Navy Specification 44T21 - Type B material will be used to give the upper limit of the strength of duralumin columns of angle section having the same l/ρ as the tubes, and the column chart will be constructed for material having the strength properties required by these specifications.

In Figure 6 a stress of 33,000 pounds per square inch is taken as representing the probable minimum yield point in compression in the direction of the grain for duralumin which just meets the strength requirements of the above-mentioned Army and Navy specifications. This value was obtained from the tests made at the Bureau of Standards on duralumin tubes, the results of which have not yet been published. Also, a straight line tangent to the Euler curve and passing through a point representing a stress of 31,000 pounds per square inch at $l/\rho = 30$ is used instead of the straight line obtained directly from the tests on angles. This line, shown as a dashed line in Figures 3 and 6, represents fairly well the results of the above-mentioned Bureau of Standards tests in this region. In view of the uncertainty of the test data on angles, it was not thought worth while in this paper to use the curve which more closely represents the Bureau of Standards data. This line, as well as the horizontal line drawn at a stress of 33,000 pounds per square inch, is assumed to apply to equal angle columns only when the direction of the grain is parallel to the axis of the column.

When the direction of the grain is normal to the axis of the column, which may sometimes be the case, the solid line having b/t labeled "13 or less - - -" and drawn tangent to the Euler curve from a stress of 34,500 pounds per square inch at $l/\rho = 0$ should be used together with the horizontal line drawn at a stress of 28,000 pounds per square inch. These lines were obtained from the results of the Bureau of Standards tests by making a reduction to allow, approximately, for the lower strength properties when the stress is normal to the direction of the grain.

In drawing the curves of twisting failure allowance was made for the effect of inelastic behavior of the material and for possible differences in the properties of the materials in the tests by assuming $\lambda = l/2$ in equation (3). For values of b/t less than 19 the transition from the curves of failure by twisting to the curve of failure by bending (the Euler curve) is effected by drawing straight lines tangent to the curves of twisting failure from a point on the Euler curve at a stress of 11,500 pounds per square inch. This point is the point of tangency of the Euler curve and the straight line labeled "13 or less - - -." For values of b/t greater than 13 no distinction regarding the direction of the grain in the angle is made in the column chart.

In drawing the Euler curve and the curves of twisting failure it was assumed that $E = 10^7$ pounds per square inch and that $\sigma = 0.3$.

Compression members in aircraft structures usually have some end fixity. A column chart for angles having $c > 1$ would therefore be of more general value. Such a chart has been constructed for $c = 2$ and is shown in Figure 7. In the construction of this chart the straight lines representing the upper limits of the column chart were drawn tangent to the Euler curve from their respective intercepts on the axis $l/\rho = 0$ in Figure 6, but the curves representing secondary failure at larger b/t ratios were plotted using the same equation as in Figure 6.

LIMITATIONS OF THE COLUMN CHART

Because no stress-strain curves were included in the reports containing test data on the strength of angle columns, it is impossible to establish the accuracy of the column chart of Figure 6 for the lower b/t ratios. It is estimated, however, that for material which conforms to the specifications of Appendix A the errors are probably within 10 per cent.

The column chart of Figure 7 must be understood to be approximate only. Whether the degree of end fixity in an airplane structure is such that the chart for $c = 2$ may be used is a matter to be determined by special tests. Figure 7 has only been included because a column chart for $c > 1$ might be of more general value than that for $c = 1$.

The column charts of Figures 6 and 7 have been constructed for single angles and the l/ρ values have been calculated using the minimum radius of gyration. Therefore if two angles are riveted together, greater strength may be developed. If angles are riveted to sheets, as might be done in stressed-skin structures, greater or less strength will be developed, depending upon the conditions of failure in the sheet.

RECOMMENDATIONS

It is appreciated that angles are, in general, not as efficient as other sections for compression members in trusses and for stiffeners in stressed-skin structures. For this reason other sections of various shapes are frequently used. The test results now available, although covering a wide variety of shapes, are very scattered and incomplete. Consequently, a complete and systematic investigation should be made to establish design charts and formulas for the more common sections. When such tests are made the results should be correlated with the stress-strain curves for the materials and these curves included in the report presenting the results. Unless such procedure is followed, the value of the results will be greatly reduced.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., March 17, 1932.

APPENDIX A

Specifications

	Army 57-187-1 Type B	Navy 44T21 Type B	Navy 47A3a Type 3
Tensile strength, lb. per sq.in. (minimum)	55,000	55,000	55,000
Yield point, lb. per sq.in. (minimum)	40,000	40,000	42,000

The yield point is defined in these specifications as the stress at which the test specimen shows an extension of 0.002 inch per inch in excess of what would be computed from Young's modulus of elasticity for the alloy (10^7) and the usual formula.

For the purposes of this paper, it is assumed that these specifications are met only when test specimens are taken parallel to the direction of the grain.

APPENDIX B

Approximate Formulas for the Properties of the Equal Angle Section

Exact formulas for the section properties of angles are given in the various handbooks. If the thickness of the legs be assumed to be small compared to the width, approximate formulas for the section properties may be derived. These formulas are, for the equal angle section (see fig. 1 and list of symbols):

$$A = 2\,b\,t \qquad (4)$$

$$x = \frac{b}{4} = 0.25\,b \qquad (5)$$

$$I_{1-1} = I_{2-2} = \frac{5}{24}\,b^3\,t = 0.208\,b^3\,t \qquad (6)$$

$$I_{3-3} = \frac{b^3 t}{12} = 0.0833\ b^3 t \tag{7}$$

$$I_{4-4} = \frac{b^3 t}{3} = 0.333\ b^3 t \tag{8}$$

$$\rho_{1-1} = \rho_{2-2} = \sqrt{\frac{5}{48}}\ b = 0.323\ b \tag{9}$$

$$\rho_{3-3} = \sqrt{\frac{1}{24}}\ b = 0.204\ b \tag{10}$$

$$\rho_{4-4} = \sqrt{\frac{1}{6}}\ b = 0.408\ b \tag{11}$$

In the derivation of the above formulas it was assumed that the angle was square at the corner, as in Figure 1. Usually the corner is rounded or carries a small fillet, but the error resulting from the use of the approximate formulas will be small in most practical cases $\left(\frac{b}{t} > 8\right)$.

When determining the total load, it is recommended that the approximate formulas for ρ be used to obtain the allowable stress, but that this stress be multiplied by the actual area rather than by the area as given in equation (4). This method is analogous to that recommended by R. A. Miller for duralumin channels. (Reference 1.)

REFERENCES

1. Miller, R. A.: Compressive Strength of Duralumin Channels. A.C.I.C. No. 598, Matériel Division, Army Air Corps, 1928.

2. Aeronautical Research Committee (British): Report on Materials of Construction Used in Aircraft and Aircraft Engines. pp. 70-71. His Majesty's Stationery Office, London, 1920.

3. Lee, John G.: Tests on Duralumin Columns for Aircraft Construction. T.N. No. 208, N.A.C.A., 1924.

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5. Trayer, George W., and March, H. W.: Elastic Instability of Members Having Sections Common in Aircraft Construction. T.R. No. 382, N.A.C.A., 1931.

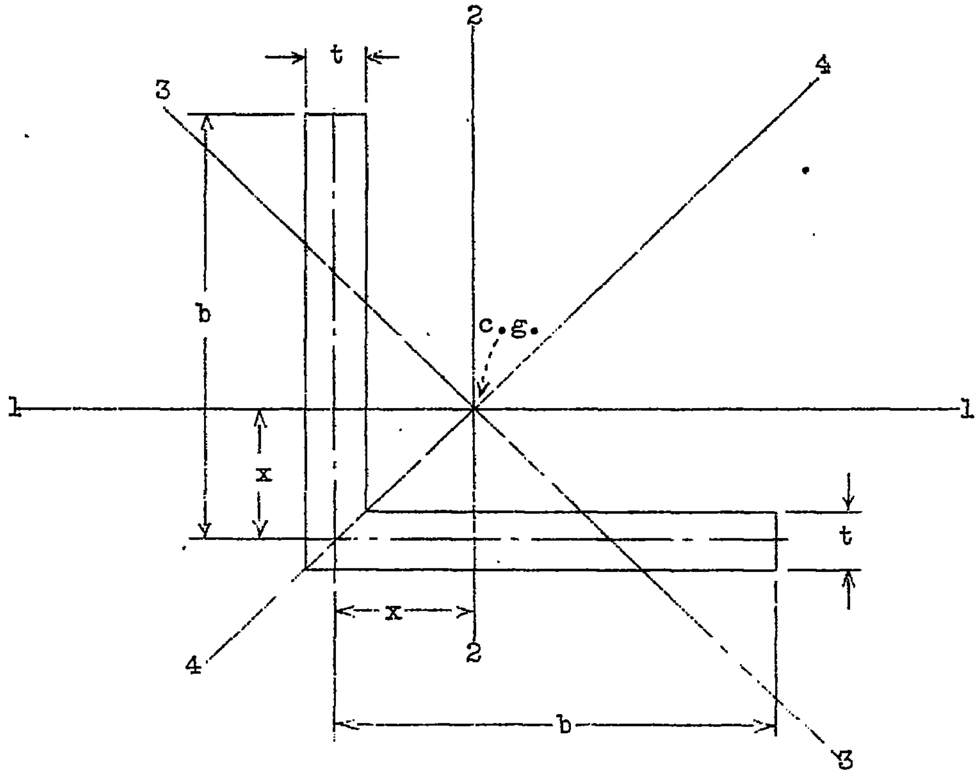


Fig.1 Equal angle section.

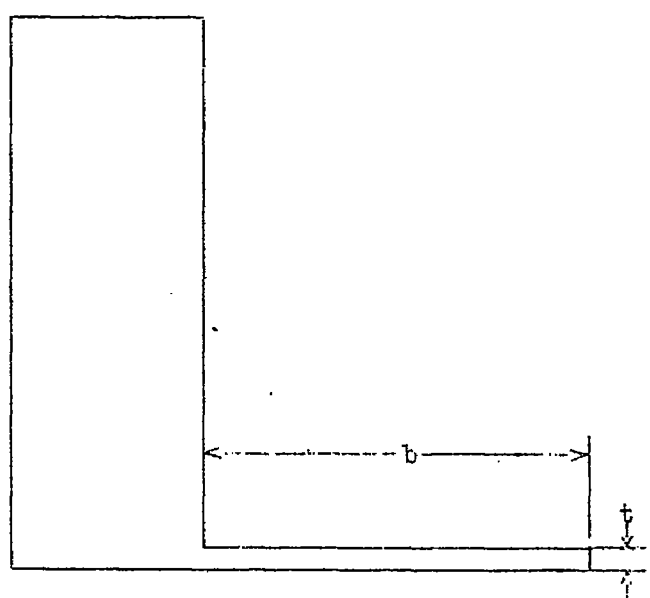


Fig.2 Angle of odd proportions in which local wrinkling might occur.

Ref.	Column		A	b	t	b/t
▽	2	H	0.219	0.931	0.118	7.9
+	2	O	.0475	.475	.050	9.5
○	2	P	.309	1.238	.125	9.9
△	3	S-3	.085	.760	.0552	13.8
◁	2	L	.0544	.680	.040	17.0
×	2	N	.098	.975	.050	19.5
□	3	S-1	.061	.768	.0394	19.5
▷	4	A-1, A-7	.0426	.735	.029	25.4

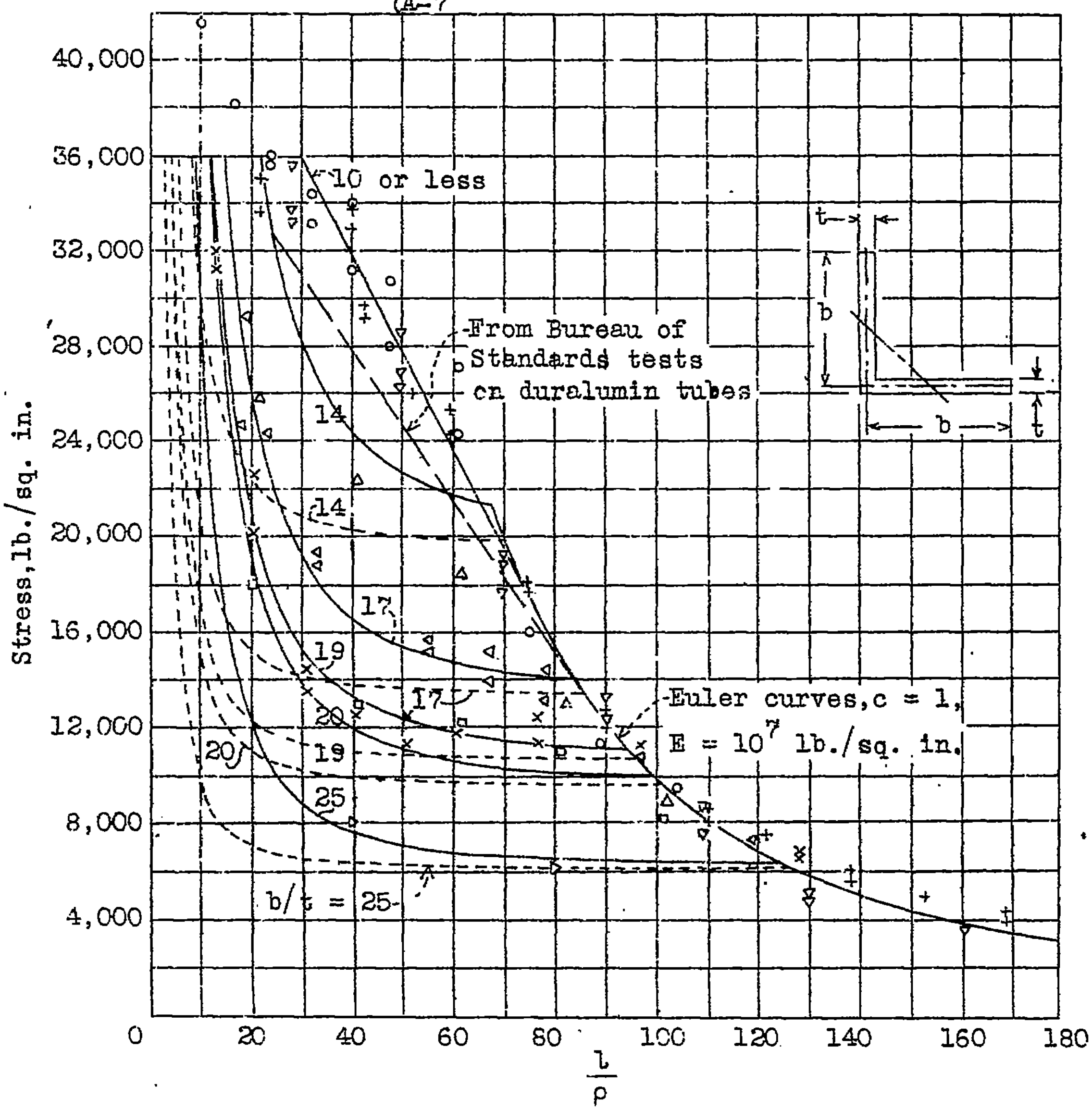


Fig.3 Column chart summarizing test data.

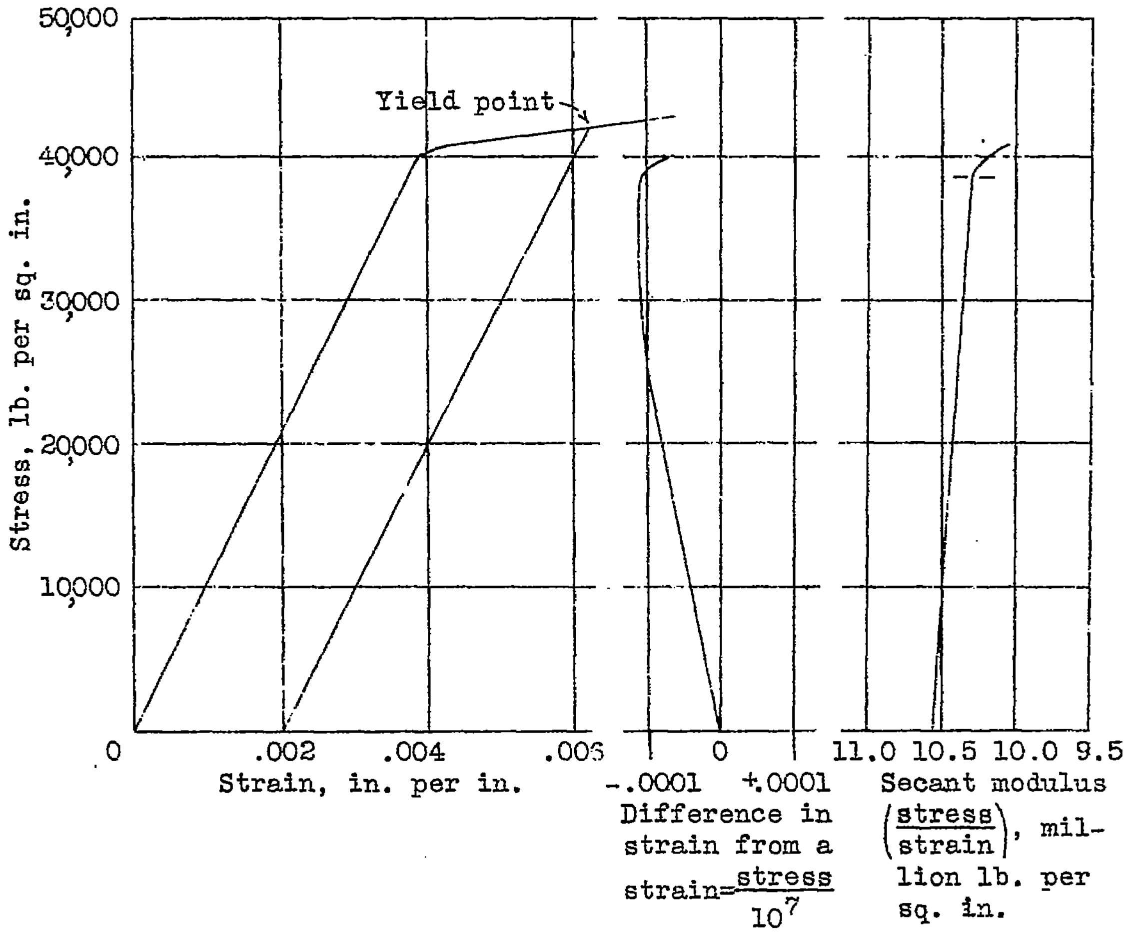


Fig. 4 Stress - strain curve for sheet duralumin 0.011 inch thick parallel to the direction of rolling.

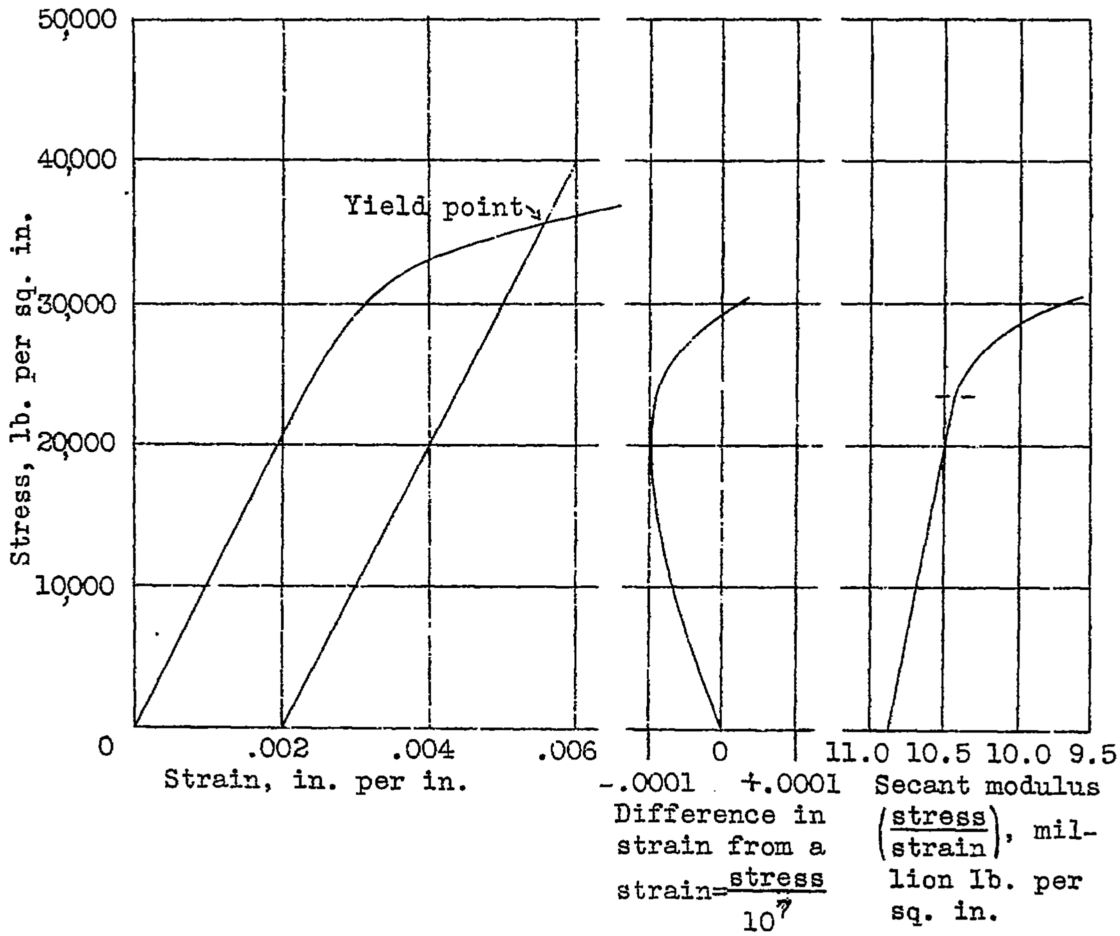
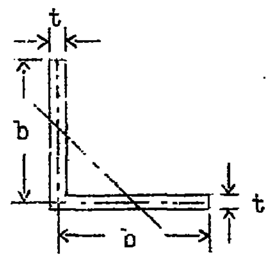


Fig. 5 Stress - strain curve for sheet duralumin 0.011 inch thick normal to direction of rolling.

Note:

This column chart is for material which conforms to the strength proporties of:
Army specification 57-187-1-type B
Navy specification 44T21-type B
Navy specification 47A3a-type 3



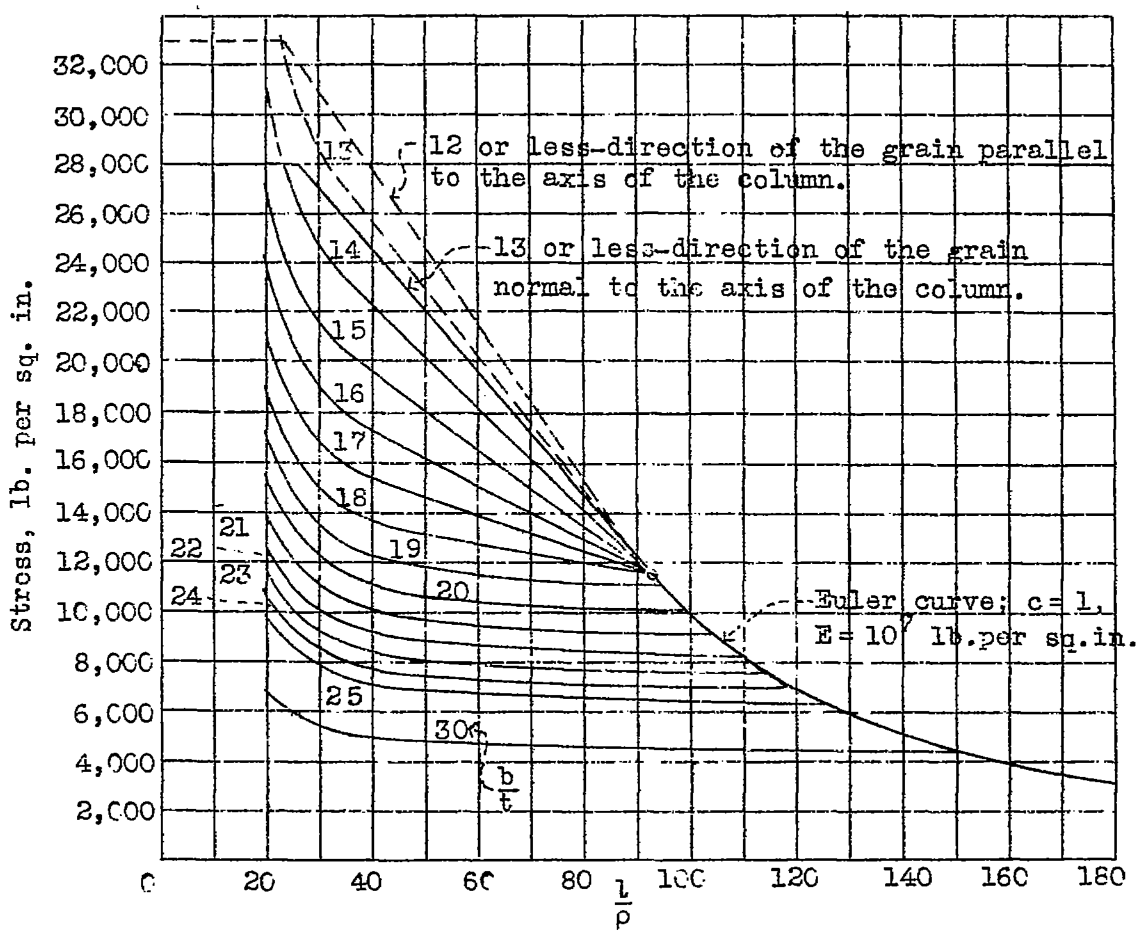
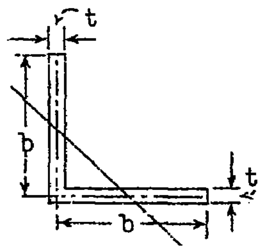


Fig. 6 Column chart for equal duralumin angles, c = 1.

Note: This column chart is for material which conforms to the strength properties of:
Army specification 57-187-1-Type B
Navy specification 44T21-Type B
Navy specification 47A3a-Type 3



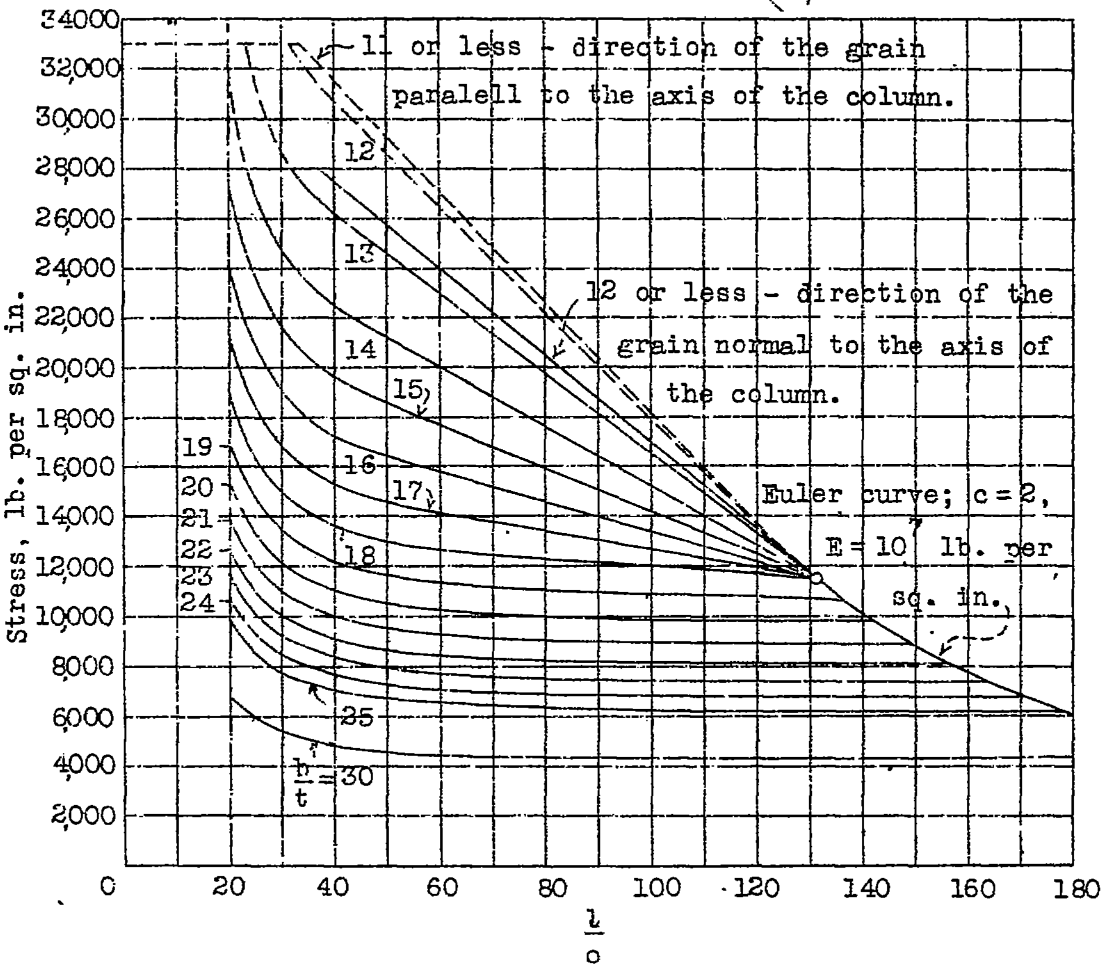


Fig. 7 Column chart for equal duralumin angles, c = 2