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WING RESISTANCE NEAR THE GROUND

By C. Wieselsberger.

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Changes in wing resistance near the ground are important for the more accurate determination of the conditions in the taking off and landing of an airplane. It has been found** that the wing resistance diminishes on approaching the ground, while the lift increases somewhat, thereby making the lift-drag ratio more favorable. In the present treatise, a convenient method will be indicated, which makes it possible to determine the polar curve of an airplane at short distances from the ground by a simple short calculation, when the polar curve is known for flight in unlimited space. The satisfactory agreement between experiment and calculation is demonstrated by the results of two experiments with models.

The polar curve for unlimited space is converted in the present case, with the aid of L. Prandtl's wing theory and the multiplane theory.*** According to this theory, the air flow about the

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** Compare A. Betz, "Lift and Drag of a Wing near a Horizontal Surface," (the ground) "Zeitschrift für Flugtechnik und Motorluftschiffahrt," 1912, p.217.

*** L. Prandtl, "Wing Theory," 1st and 2nd papers, Bulletin of the Göttingen Scientific Society, 1919. Mathematics-physics class, further, L. Prandtl, "The Induced Drag of Multiplanes," Technische Berichte, Vol. III, No. 7. Figs. 1 and 3 are taken from the latter.

wing can be calculated on the assumption that the lift is distributed over the wing span in the form of half an ellipse, which is accurate enough for most practical cases. In this connection, we will utilize the theoretical consideration that a vortex band goes out from the trailing edge of each wing. The axes of the elementary vortices of this band are nearly parallel to the direction of flight and the width of the band is equal to the wing span. The added disturbing velocity resulting from this vortex band at any point is the integral of the disturbing velocities produced by the individual elementary vortices, whereby the former are calculated according to the Biot-Savart law. If a wing is in an air flow which is disturbed by a second wing, only the vertical component of the disturbing velocity comes into consideration for the induced drag of the first wing, since the inflow direction and therewith the induced drag are changed by the vertical components of the disturbing velocity at the place of the supporting line. The vertical velocities, in the vertical plane passing through the middle of the chord, were calculated and graphically represented in Fig. 1, for a series of distances from the supporting line, by K. Pohlhausen, at the suggestion of L. Prandtl, on the assumption that the lift is distributed in half an ellipse over the wing span. The ratio of the distance of the point considered to the wing span is represented by h/b. The vertical velocity of the wing itself $(h/b = 0)$, which is constant in the elliptical distribution of the lift over the entire width of the span, is here made 1, and the span 2. The actual vertical velocity w_{11}' on the place of wing 1, due to wing 1', is therefore

$$w_{11}' \mp \frac{2A}{\pi \rho v b^2} \times z \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (1)$$

in which A is the lift of wing 1', v the flight velocity and ρ the density of the air. z can be obtained from Fig. 1 for the corresponding h/b.

In order to investigate the change in the resistance near the ground, we utilize the principle of reflection. We replace the surface of the ground by wing 1' reflected by the ground (Fig. 2) and calculate (by a method analogous to that for calculating the drag of a multiplane from the drag of a monoplane) in what manner the air flow about wing 1 will be affected by its image. We denote its distance from the ground by h/2. Wing 1 is now on the pressure side of wing 1'. We already recognize qualitatively that the disturbing velocity due to 1' on the place of wing 1 is directed upward. The resulting direction of flow on wing 1, which is found by the geometric addition of the original direction of the velocity v and the vertical velocity w_{11}' due to wing 1', and whose direction is indicated by v', is therefore as we see, deflected downward somewhat less than in the undisturbed condition. The induced drag near the ground must therefore be smaller than at a higher altitude, since with decreasing distance between wing 1 and its image 1', the disturbing velocity increases from 0 to a maximum, as shown in Fig. 1.

For the quantitative relations, it is to be noted that the vertical disturbing velocity, as shown by Fig. 1, varies also along the span of wing 1. We obtain the change in the induced drag by multiplying the lift component of each wing element by the sine of the angle of attack of the air stream at the given point and integrating over the whole span. The small angle of attack has the value $\frac{w_{11}'}{V}$ and consequently the change in drag is

$$W' = -\int_{-b/2}^{+b/2} \frac{w_{11}'}{V} \, dA \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

in which the minus sign, in accordance with the above consideration, indicates that the wing resistance is diminished. The value of the integral was determined planimetrically for different values of h/b and the result expressed by the influence coefficient σ, which is defined by the equation

$$W' = -\sigma \frac{A^2}{\pi \, q \, b^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

in which q = dynamic pressure. In Fig. 3, the values of σ are plotted against the ratio $h : \frac{b_1 + b_2}{2}$. This method was chosen with reference to the ratios yet to be considered, which arise in biplanes with unequal wing spans (b_1 and b_2). In the case now under consideration, in which the span of the real wing is like that of the reflected one, the ratio $h : \frac{b_1 + b_2}{2}$ is identical with h/b and the corresponding values of σ are given by the curve $\mu = 1$. If we desire a more accurate value than the graphic diagram gives, we may employ the following approximation formula of

Prandtl, which holds good from $h/b = 1/15$ to $h/b = 1/2$

$$\sigma = \frac{1 - 0.66\ h/b}{1.05 + 3.7\ h/b} \quad . . . . (4)$$

The change in drag may therefore be calculated according to equation (3) in a very simple manner. We only have to determine the influence coefficient σ for the value h/b from Fig. 3, or with the aid of equation (4). The other quantities in equation (3) are to be considered as given. We hereby assume that the lift near the ground is the same as higher up. With the introduction of the dimensionless coefficients, equation (3) may also be written in the form

$$c_w' = - \sigma \frac{c_a^2}{\pi} \times \frac{F}{b^2} \quad . . . . (5)$$

in which c_w' represents the change in the drag and F the surface area of the wing.

The present calculations have to do with a monoplane near the ground. If it is deisred to investigate the relations for a biplane, it is necessary to know the lift components of both wings. This is generally known for a finished biplane. Otherwise, the lift components may be subsequently determined from the quantities upon which they depend, especially from the surface area of the wings and the angle of attack. Here it is well to remember that (according to the results of the first approximation of the multiplane theory with a given total lift, span and vertical distance between the two wings) the induced drag acquires a minimum value with a definite distribution of the total lift over both wings of the biplane. In practice, we must always endeavor to produce this condition.

The vertical disturbing velocities resulting from the reflected wings 1' and 2', are represented diagrammatically in Fig. 4. The total amount, about which the drag diminishes near the ground and which is given by equation (3) for the monoplane, is made up of four members in the case of the biplane. If W_{11}' designates the drag decrease of wing 1 through the influence of wing 1', and if the other components of the drag variations are designated in a similar manner, the total drag decrease is

$$W' = W_{12}' + W_{11}' + W_{22}' + W_{21}' \quad . \; . \; . \; . \; . \quad (6)$$

According to the multiplane theory, however, $W_{12}' = W_{21}'$ and the latter, on account of the mirror symmetry $= W_{21}'$, so that the resulting expression for the drag decrease is

$$W' = 3\, W_{12}' + W_{11}' + W_{22}' \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

The influence coefficient σ, which is necessary for calculating the above summands, is, in the case of a biplane with unequal wing spans, a function of both wing spans, b_1 and b_2. If we designate the ratio $b_2 : b_1$ by μ, the value of σ, for the value of μ in question, can be obtained from Fig. 5, in which the values of σ, for $\mu = 0.8$ and 0.6, are plotted against the ratio $h : \frac{b_1 + b_2}{2}$. For other values of μ, σ can be interpolated with sufficient accuracy. $\mu = 1$ comes into consideration for equal spans of the upper and lower wings. For $\mu = b_2/b_1 > 1$, the same values are to be taken as for b_1/b_2. h always denotes the distance of the wing under consideration from the wing causing the disturbance.

Some time ago two experiments were carried out in the Göttingen laboratory on a monoplane model, of 134 cm span, with fuselage and elevator, whereby the air forces were measured once in unlimited space and once near the ground. The surface of the wing, to which the coefficients were applied, was 1675 cm². Consequently, F/b^2 had the value 0.11. The distance of the wing from the ground varied a little for the individual angles of attack. The mean value of h/2 was 15 cm. The polar curves, determined experimentally for both these cases, are given in Fig. 5. The change c_w' of the drag coefficient was now determined in accordance with the above instructions. For the influence coefficient, we have $\sigma = 0.432$, since $h/b = 0.242$. If we substitute this value and the value of F/b^2 in equation (5), we obtain

$$c_w' = - 0.015 \, c_a^2.$$

If we carry the values of c_w', calculated for different lift coefficients toward the left, from the polar curves, measured in the free air flow, we obtain the curve indicated by dashes. It is evident that this curve fully agrees with the measured values of the lift coefficients up to about $c_a = 1$. For very large lift values, we obtain deviations for which no satisfactory explanation can yet be given.

Translated by the National Advisory Committee for Aeronautics.

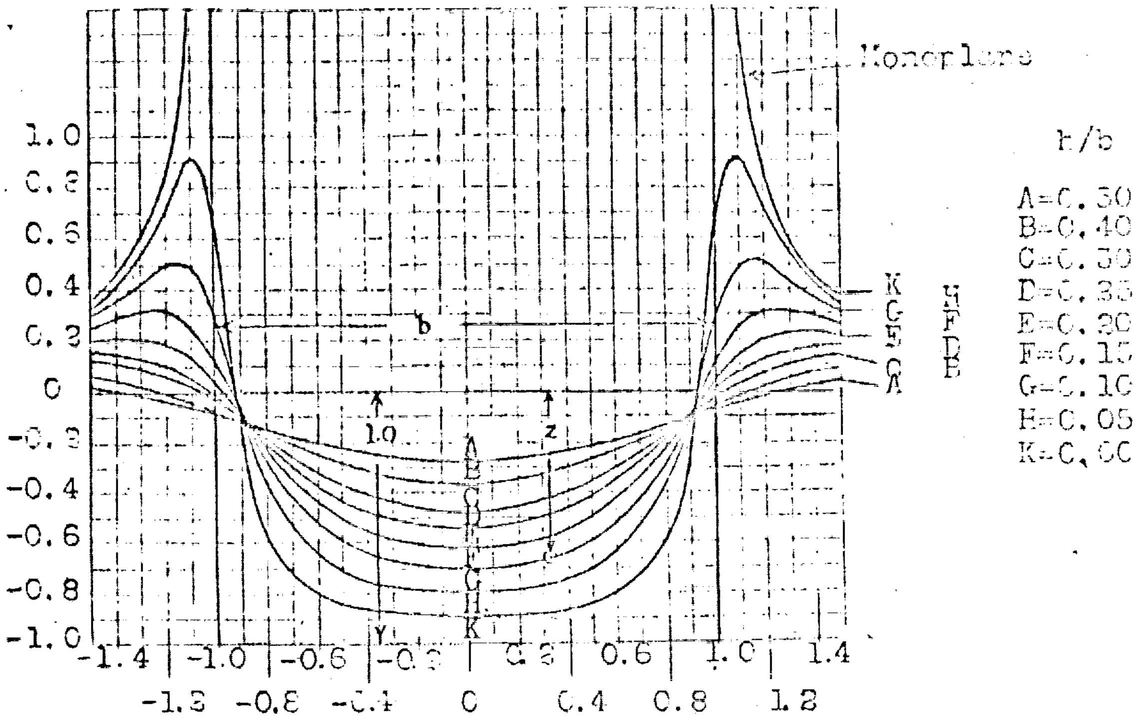

Fig. 1 - Vertical component of the disturbing velocity plotted against the longitudinal and vertical distance from the wing.

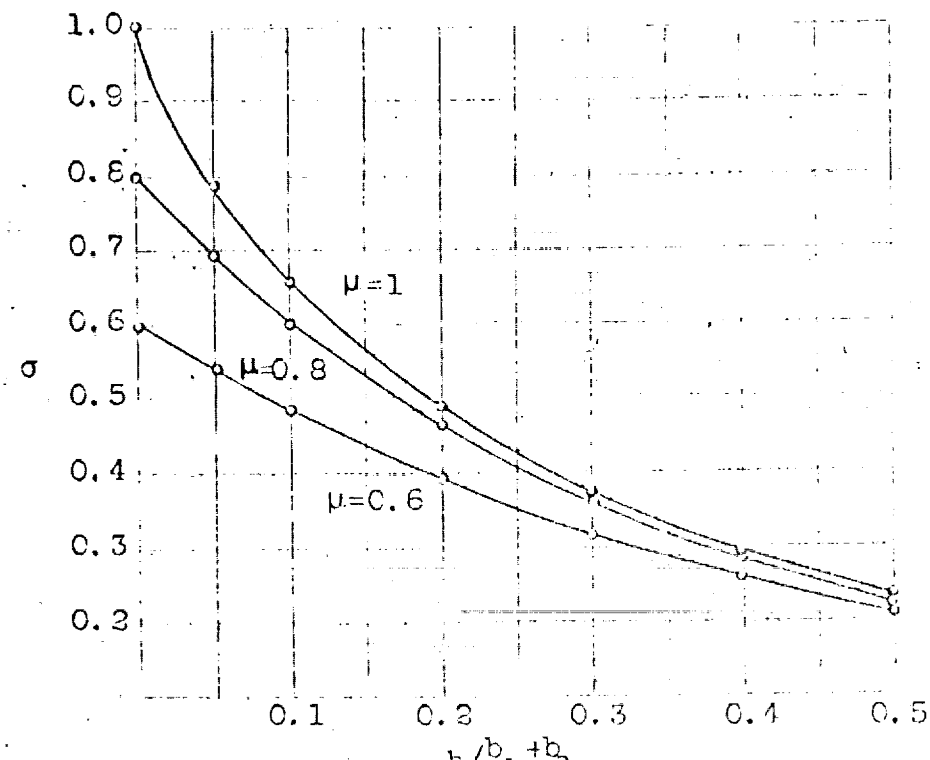

Fig. 3 - Interference coefficient σ plotted against $h/\frac{b_1+b_2}{2}$ and $\mu = b_2/b_1$

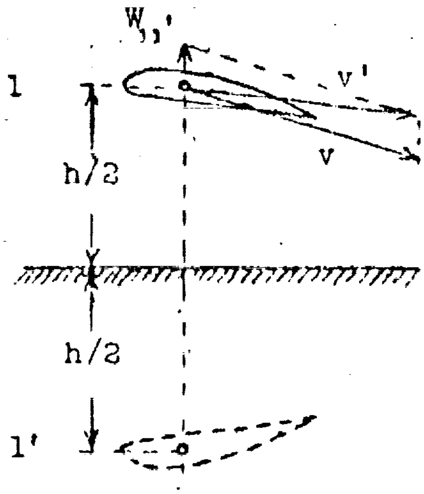

Fig. 2.

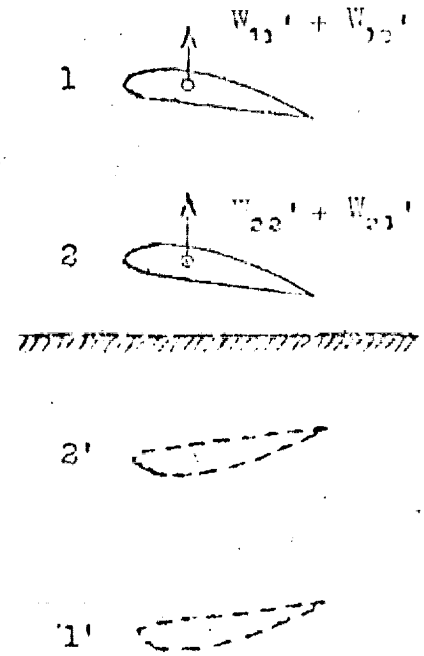

Fig. 4.

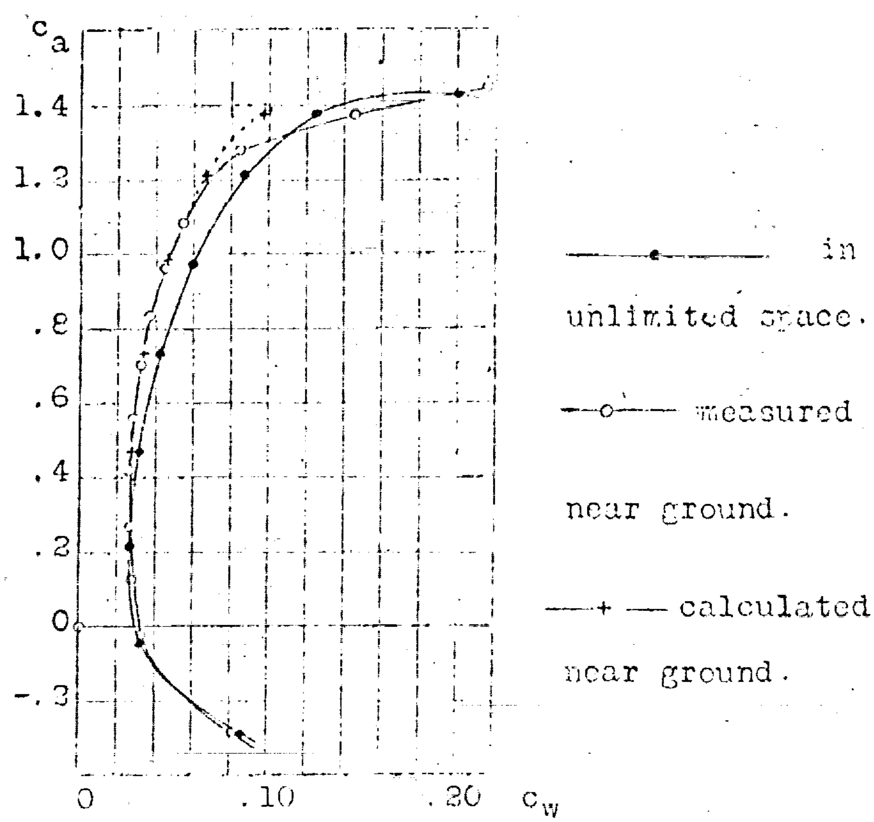

Fig. 5 - Measured and calculated polar curves near the ground.