TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 557

CONSIDERATIONS OF THE TAKE-OFF PROBLEM

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Washington
February 1936

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SUMMARY

Four considerations of the general take-off problem are presented in this paper: (a) A brief analysis of the importance of static propeller thrust showing that large variations in the initial propeller thrust have but a very small effect on take-off distance. (b) A comparison of the take-off characteristics of an airplane equipped with a propeller having a Clark Y section and equipped with a propeller having an R.A.F. 6 section, which shows the error that might occur from using R.A.F. 6 section propeller data to obtain the take-off performance of a Clark Y section propeller. The R.A.F. 6 section propeller is shown to be superior in take-off to the Clark Y section propeller of the same diameter. (c) An analytical determination of the attitude of an airplane during take-off that will give the minimum air and ground resistance and the development of the equation for the minimum air and ground resistance. (d) A development of a short, accurate method of calculating take-off distance based on the assumption that the reciprocal of the acceleration during take-off varies as the velocity squared.

INTRODUCTION

Many technical papers on the various phases of airplane take-off have been published. Frequently, however, there appear new ideas which affect only particular scattered phases of the subject and which do not receive individual publication. It is the purpose of this paper to present several ideas of this nature which may be of considerable aid in calculating take-off performance and one idea which should correct what appears to be a popular misconception of the importance of static propeller thrust. Although these considerations all concern the general problem of take-off, they are not directly related and therefore will be treated in individual sections.

EFFECT OF STATIC THRUST ON TAKE-OFF PERFORMANCE

That many aeronautical engineers consider static propeller thrust an important factor in take-off is indicated by the fact that the Committee has received numerous letters from men in the industry requesting specific data on static propeller thrust. The stated need for these data is for the purpose of calculating take-off performance more accurately than is possible with the propeller data now available.

Up to the present the Committee has made no static-thrust measurements, partly because of the difficulty of obtaining zero V/nD conditions in wind tunnels and partly because of the fact that the short extrapolations of the curves from the regular propeller tests have been considered quite satisfactory. If, however, the static propeller thrust is an important factor in take-off calculations, as it is so frequently considered to be, tests for its accurate determination should be made.

Figures 1 and 2 are intended to show the true importance of the initial take-off thrust in take-off calculations. Figure 1 shows take-off calculations for a bimotored transport airplane of about 17,500 pounds gross weight. The excess thrust, the difference between the propeller thrust and the thrust required to overcome air and ground resistance, accelerates the airplane. The acceleration at any velocity may be calculated from the relation $a = \frac{g}{W} T_e$ where W is the weight of the airplane and T_e the excess thrust. If this acceleration is divided into the velocity and the result is plotted against the velocity, a curve is developed, the area under which is directly proportional to the take-off distance.

Two arbitrary modifications of the normal thrust curve have been made to show the effect of varying the initial take-off thrust. The first modification was drawn in from a point representing a V/nD of 0.2, the point where the last test point usually occurs in the regular propeller tests. This modification, which represents, perhaps, the maximum error likely to occur from extrapolating the regular propeller curves, reduces the normal static thrust by 25 percent yet increases the take-off distance by an amount almost too small to be measured.

The second modification, taken as an absurd example for the purpose of emphasis, reduces the static thrust to one-third its normal value and yet increases the take-off distance by only 5 percent. It is apparent from this result that precise values of the initial take-off thrust are not necessary for accurate take-off calculations.

It may be seen from this reasoning that building a short incline at the beginning of the runway, as has been done in certain instances, is a very ineffective way of shortening the take-off run as is the method of turning the engine up with brakes locked to obtain a more rapid acceleration at the start. The resulting conclusion is that for a short take-off run one should have the greatest acceleration in the high-speed part of the take-off run where the ground is being rapidly covered.

Figure 2 represents the take-off of an 8,000-pound flying boat. In this example, according to marine practice, the time of take-off as well as the distance has been determined. The time of take-off is proportional to the area under the curve of $1/a$ plotted against V. An extreme modification in the initial propeller thrust leaves only a very small accelerating thrust during the early stages of the take-off and yet increases the take-off distance by only 8 percent. The time to take off has, however, been increased by 25 percent.

The emphasis laid on the take-off time in marine practice may well be analyzed. The use of the criterion "take-off time" probably originated because it was easily measured and because it was thought to be indicative of the distance. Actually, of course, the take-off time is of little importance as long as it is reasonably short. There is some evidence to show that it may be an unreliable criterion if it is assumed to be directly proportional to the take-off distance. The criterion "take-off time" seems to have graduated from a mere convenience into an important design factor with limits specified by the Department of Commerce. The shortest take-off time for a flying boat occurs when the average acceleration during take-off is a maximum, whereas the shortest take-off distance requires a much greater acceleration in the latter part of the run than in the first part. It may be seen then that there is a certain danger in the use of take-off time as a design factor because a boat designed to take off in the shortest time will probably not take off in the shortest distance and take-off distance is, of course, the important criterion even in the case of flying boats.

PROPELLER-THRUST CALCULATIONS IN THE TAKE-OFF RANGE

The method of calculating take-off performance shown in figures 1 and 2 has been used for some years by the N.A.C.A. for computing the performance of flying boats but, in general, it has been considered too lengthy for land-plane computations for which several short methods are available. The use of this method in a simplified form, which will be given later, may, however, be recommended.

Perhaps the most important process in computing take-off distance is the accurate determination of the propeller thrust in the latter part of the take-off run. The data in reference 1 may be used for such computations. It is thought that a common mistake is to use these data, which were obtained from tests of a propeller with an R.A.F. 6 section, to compute the performance of a propeller with a Clark Y section. The difference in the aerodynamic characteristics of the two sections is due to the manner in which they stall and will therefore have an effect on the take-off. Figure 3 shows the take-off characteristics of a pursuit airplane when equipped with a Clark Y section propeller and with an R.A.F. 6 section propeller of the same diameter, the propeller data being obtained from reference 2. These propellers were of fixed pitch and had the same plan form and thickness. The Clark Y section propeller stalls at a higher speed than the R.A.F. 6 section propeller, thus giving the latter propeller a much higher thrust in a fairly important part of the take-off run. This comparison, based on propellers of the same diameter, is not entirely fair to the Clark Y section propeller for the optimum design will give the Clark Y section propeller a considerably larger diameter than the R.A.F. 6 section propeller, and the difference in take-off performance will be less. This comparison does, however, give a better idea of the error involved in using the data of reference 1 to compute the performance of a Clark Y section propeller than any other that might be given.

Figure 3 shows that the Clark Y section propeller gives a 39 percent longer run than the R.A.F. 6 section propeller. Of course, it must also be pointed out that as the blade angle is reduced this difference becomes smaller.

EQUATION FOR MINIMUM AIR AND GROUND RESISTANCE DURING TAKE-OFF

When making take-off calculations by the method used in figures 1 and 3, the problem of computing the air and ground resistance immediately arises and it is necessary to know the attitude of the airplane during the take-off. The angle of attack that will give the minimum air and ground resistance is of considerable interest and may be determined as a problem in maxima and minima. The general equation of the resistance of an airplane during take-off may be written

$$R = \mu W - \mu C_L q S + q f + C_L^2 q S/\pi A$$

where μ is the coefficient of ground friction

W, the weight of the airplane in pounds

C_L, the lift coefficient defined by $C_L = \frac{\text{lift}}{qS}$

q, the dynamic pressure $\frac{1}{2} \rho V^2$ in pounds per square foot

S, the wing area in square feet

A, the effective aspect ratio including ground effect

f, the equivalent parasite area in square feet defined by:

$$\text{parasite drag} = qf$$

If the first partial derivative with respect to C_L is equated to zero,

$$\frac{\partial R}{\partial C_L} = - \mu q S + 2 C_L q S/\pi A = 0$$

we obtain, after transposition, $C_L = \frac{1}{2} \mu \pi A$. This value of C_L gives the minimum air and ground resistance during take-off and defines a particular attitude for any given airplane and field. It should be noticed that the value of C_L for minimum resistance does not vary with the velocity; it is constant throughout the run. A chart giving the optimum value of C_L for a considerable range of the variables μ and A is shown in figure 4. It appears from figure 4 that in a sticky field where μ is high the pilot should take off with tail low, a conclusion agreeing with common experience.

If this optimum value of C_L is substituted in the general resistance equation, the equation for the minimum resistance during take-off will be obtained,

$$R_{min} = \mu W + V^2 \left(\frac{\rho}{2} f - \frac{\pi}{8} \rho \mu^2 b^2\right)$$

or for sea level

$$R_{min} = \mu W + V^2 (0.00119f - 0.000934 \mu^2 b^2)$$

which for any given airplane and field becomes

$$R_{min} = K + K_1 V^2$$

where K and K_1 are the obvious constants

V, velocity in feet per second

b, effective span including ground effect in feet

f, equivalent parasite area in square feet

The factor f may be obtained accurately enough for any take-off calculations from the approximate relation

$$f = \frac{(b.hp.)_o \times \eta_{max} \times 1000}{\rho V_m^3}$$

where $(b.hp.)_o$ is the rated engine power

η_{max}, the maximum propeller efficiency

V_m, the top air speed in feet per second.

This approximation of f is based on the assumption that the induced drag is one-tenth the total drag at top speed.

Figure 5 shows how the minimum ground and air resistance curves for a particular example vary with the ground-friction coefficient μ. It will be noted that the curves always come tangent to the drag curve of the airplane in flight which is, of course, a minimum in the flight condition.

Values of the coefficient of ground friction μ, as given in reference 3, are as follows:

Smooth deck or hard surface	0.02
Good field, hard turf	.04
Average field, short grass	.05
Average field, long grass	.10
Soft ground, gravel or sand	.10 to 0.30

A SHORT METHOD OF COMPUTING THE TAKE-OFF DISTANCE OF LANDPLANES

The foregoing sections have described the method of computing the take-off of landplanes as illustrated in figures 1 and 3. The steps are as follows:

1. Obtain the propeller thrust from reference 1 or any other source.

2. Compute the ground and air resistance from the minimum-resistance equation given in the preceding section.

3. Compute the acceleration from the excess thrust, plot V/a against V and measure the area to obtain the take-off distance.

This method is not very long and is the most accurate available. A study of the variables involved reveals, however, a much shorter method very nearly as accurate. In explanation of the short method it may be said, in brief, that for any airplane there is one particular velocity in its take-off run at which the acceleration, if calculated and substituted in the standard $V^2 = 2as$ formula, will give the exact take-off distance.

The success of the short method depends on the fact that this velocity is, for all airplanes, exceedingly close to the same percentage of the take-off velocity. The reason for this agreement is that in all cases the reciprocal of the acceleration is very nearly a linear function of the velocity squared. The areas under the 1/2a curves given in figure 6 are proportional to the take-off distances and the deviation of these curves from straight lines indicates the degree of inadequacy of the short method. The acceleration at a velocity corresponding to $V_T^2/2$, where V_T is the air speed at take-off in feet per second, will therefore be the value representing the entire take-off run. This velocity is $\sqrt{0.5}\ V_T$ which may be called $0.7\ V_T$.

The short-method equation may then be written,

$$s = \frac{V_T^2}{2a} = \frac{V_T^2 W}{64 T_e}$$

where s is the distance in feet, W the weight in pounds, and T_e the excess thrust calculated for only one air speed which is, for take-off with no wind, $\sqrt{0.5}\ V_T$ or $0.7\ V_T$.

The effect of an inclined runway may easily be included,

$$s = \frac{V_T^2 W}{64\ (T_e \pm W \sin \alpha)}$$

where α is the angle of inclination and the sign depends on whether the airplane is taking off down the slope (+) or up the slope (-).

The effect of wind may also be included without trouble. In this case V_T must be reduced by the value of wind velocity V_W,

$$s = \frac{(V_T - V_W)^2\ W}{64\ (T_e \pm W \sin \alpha)}$$

and T_e must be calculated for a different air speed than in the case of no wind. The air speed is

$$V = 0.7\ V_T + 0.3\ V_W$$

In six examples representing widely different types of airplane the take-off distances calculated by the short method averaged 98 percent of the distances calculated by the long method with one extreme example giving 96 percent. The short method is evidently on the nonconservative side by about 2 percent; if better accuracy is desired, a multiplying factor of 1.02 or 1.03 should be applied to the distance as calculated by the short method.

It has been found that a close approximation to the take-off time for landplanes may be obtained from the relation

$$t = \frac{1.95\ s}{V_T}$$

where t is the time in seconds.

The short method may thus be summarized:

1. Calculate the propeller thrust for the one representative air speed.

2. Calculate the minimum ground and air resistance for the one representative air speed from the minimum resistance equation given in the preceding section.

3. Substitute the difference of these two values in the equations for T_e and solve for distance.

The propeller thrust is best obtained from reference 1; for convenience, however, the general thrust-horsepower curves in figure 7 may be used with some loss in accuracy. The use of these curves should give fairly accurate results for present-day airplanes but, since controllable propellers offer such a broad range of selection, the controllable propeller curve in figure 7 may in certain cases be considerably in error.

Inasmuch as the maximum speed is known, the ratio of V/V_m for the representative air speed may be computed and the ratio of the thrust horsepowers may be obtained from figure 7. Since the maximum efficiency may be easily obtained, the maximum thrust horsepower and the thrust horsepower at the representative air speed may be quickly calculated. The thrust will then be, $T = \frac{t.hp. \times 550}{V}$, where V is in feet per second or $T = \frac{t.hp. \times 375}{V}$ where V is in miles per hour.

CONCLUSIONS

1. The propeller thrust in the early stages of take-off has but a very small effect on take-off distance.

2. A considerable error may result from using R.A.F. 6 propeller data to compute the take-off performance of a propeller with a Clark Y section. A comparison of the take-off performance of two propellers on a particular airplane shows that the propeller of R.A.F. 6 section gives a shorter take-off than the propeller of equal diameter having a Clark Y section.

3. The attitude of an airplane that will give the shortest take-off run does not vary with speed and is represented by the relation $C_L = \frac{1}{2} \mu \pi A$, where C_L is the lift coefficient corresponding to the optimum attitude, μ is the coefficient of ground friction, and A is the effective aspect ratio.

4. A short and reasonably accurate method of calculating take-off results from the fact that the reciprocal of the acceleration is very nearly a linear function of the velocity squared.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., January 27, 1936.

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2. Freeman, Hugh B.: Comparison of Full-Scale Propellers Having R.A.F.-6 and Clark Y Airfoil Sections. T.R. No. 378, N.A.C.A., 1931.

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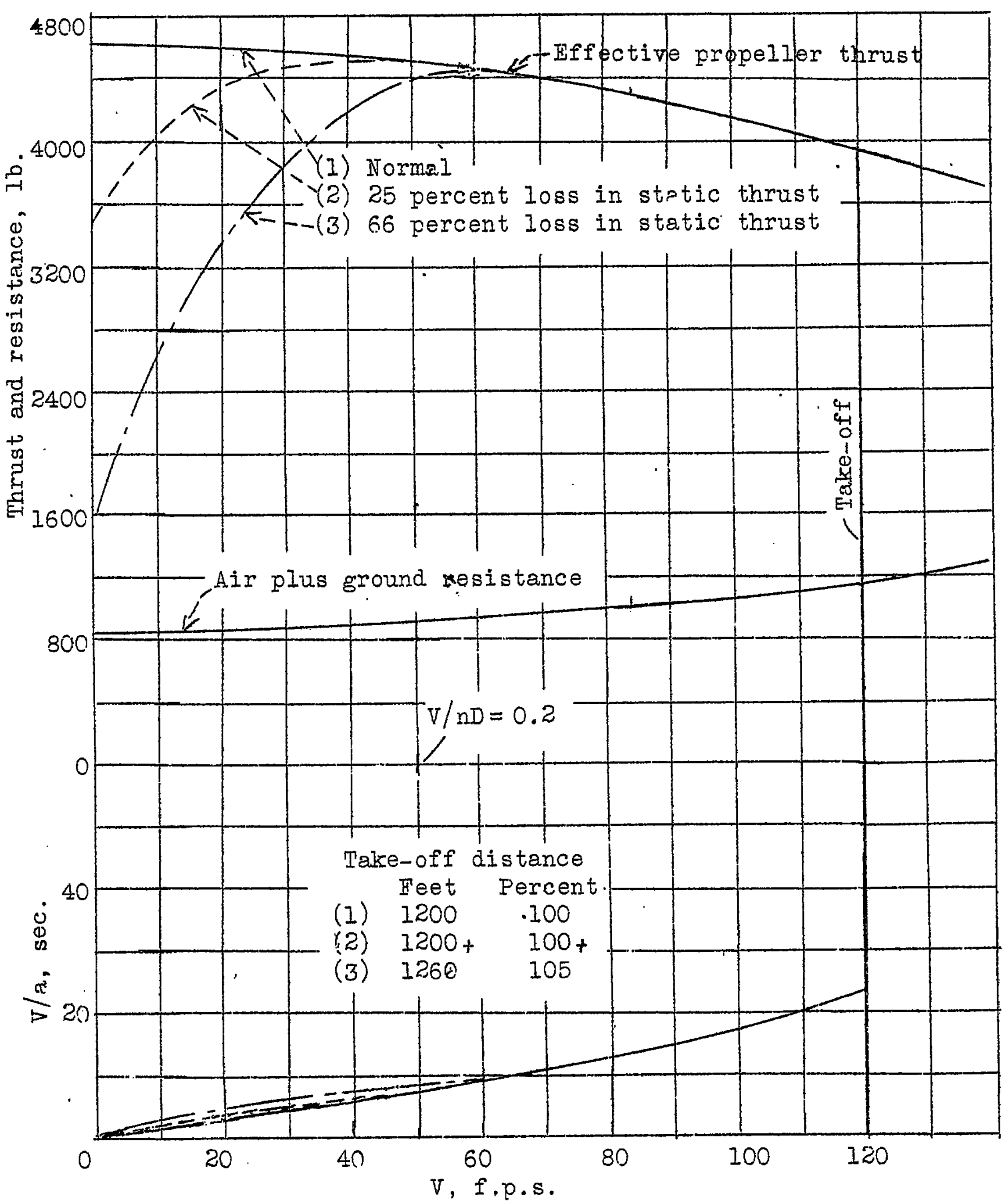
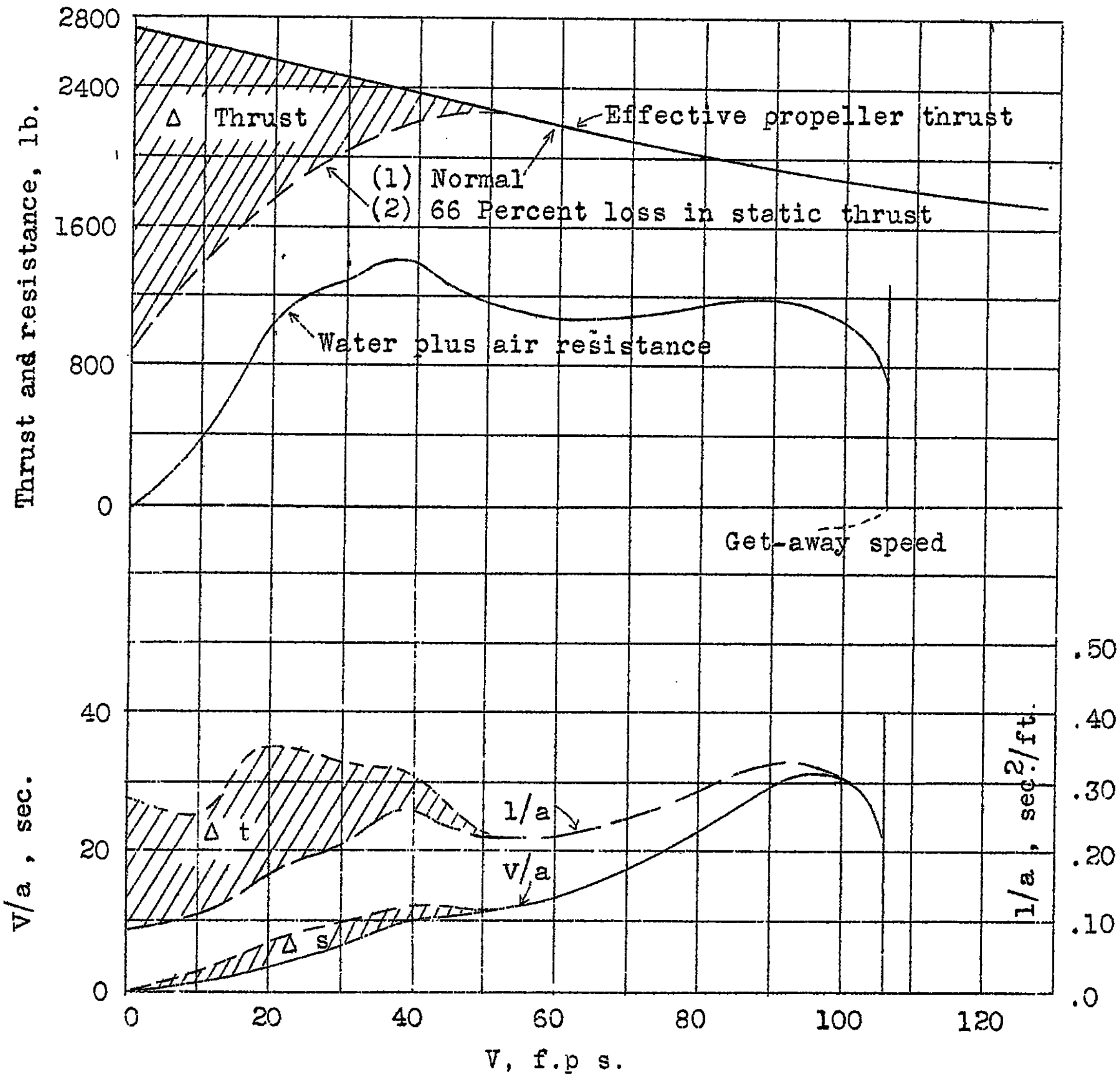


Figure 1.- Effect of variation of initial take-off thrust on the take-off distance of a transport airplane.



Take-off distance, s		
	Feet	Percent
(1)	1480	100
(2)	1600	108

Take-off time, t		
	Seconds	Percent
(1)	24	100
(2)	30	125

Figure 2.- Effect of variation of initial take-off thrust on the take-off distance and take-off time of a flying boat.

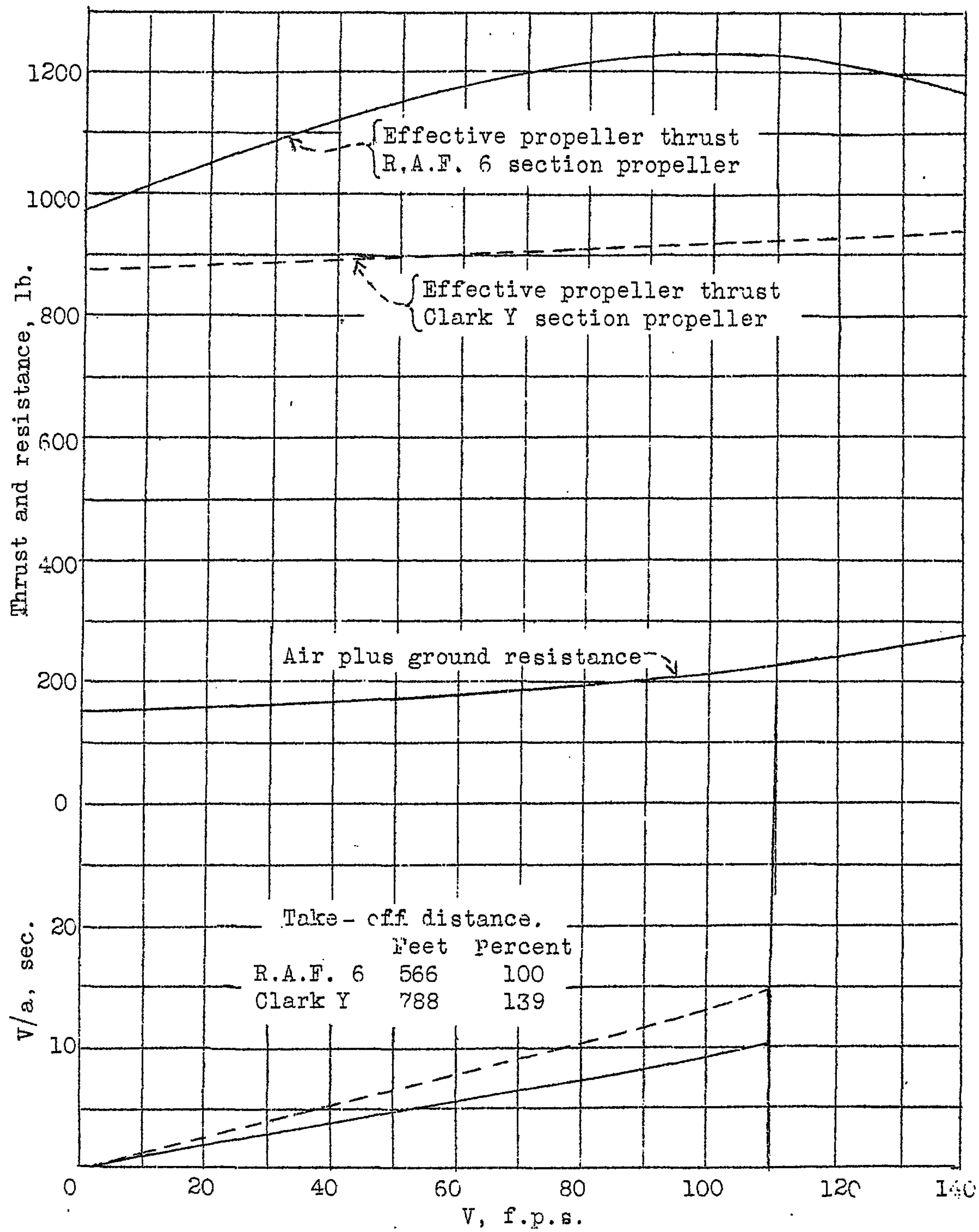


Figure 3.- Comparison of the take-off of a pursuit plane when equipped with fixed-pitch propellers having different blade sections.

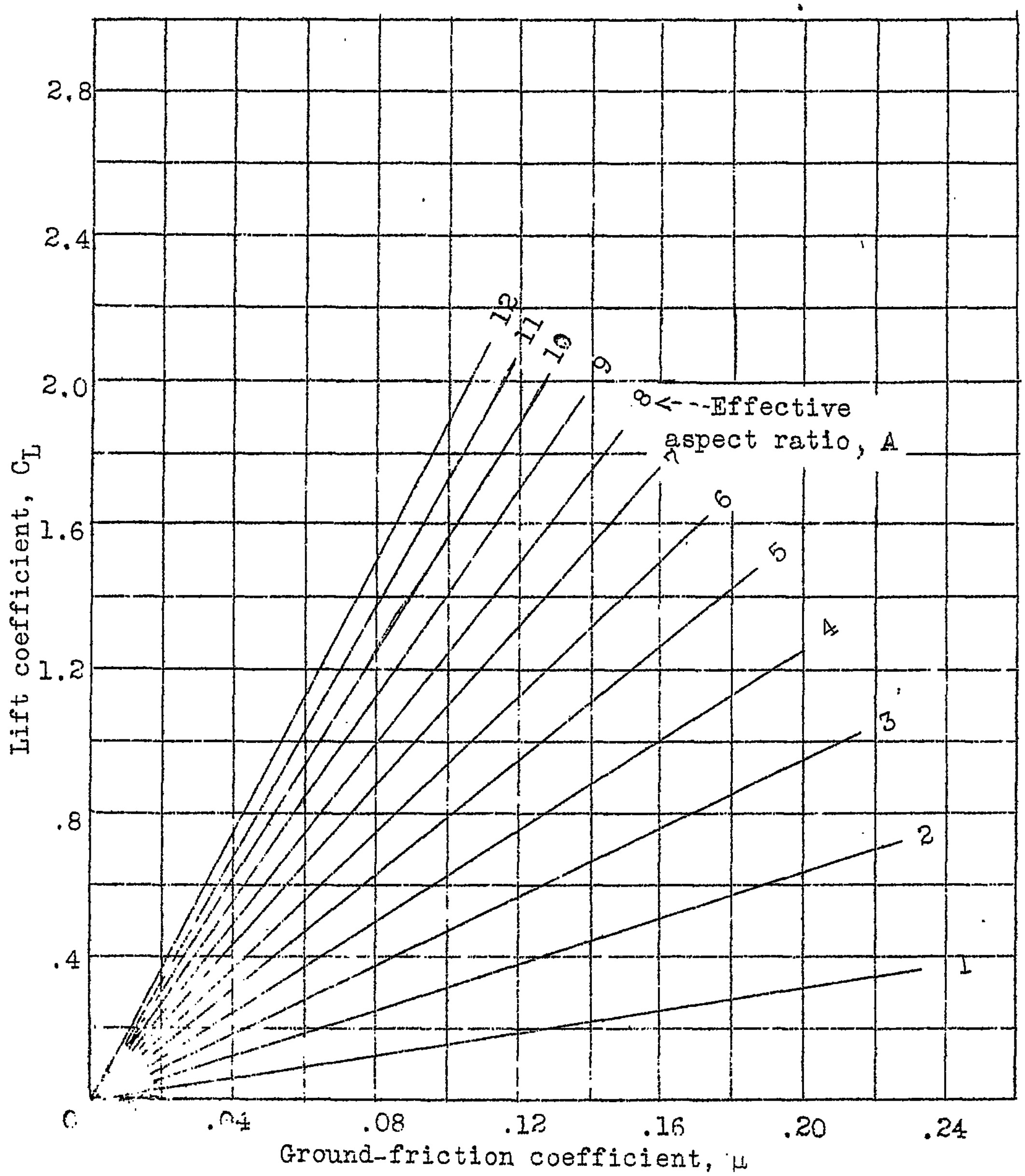


Figure 4.- C_L for mimimum take-off resistance, $1/2\ \mu\pi A$.

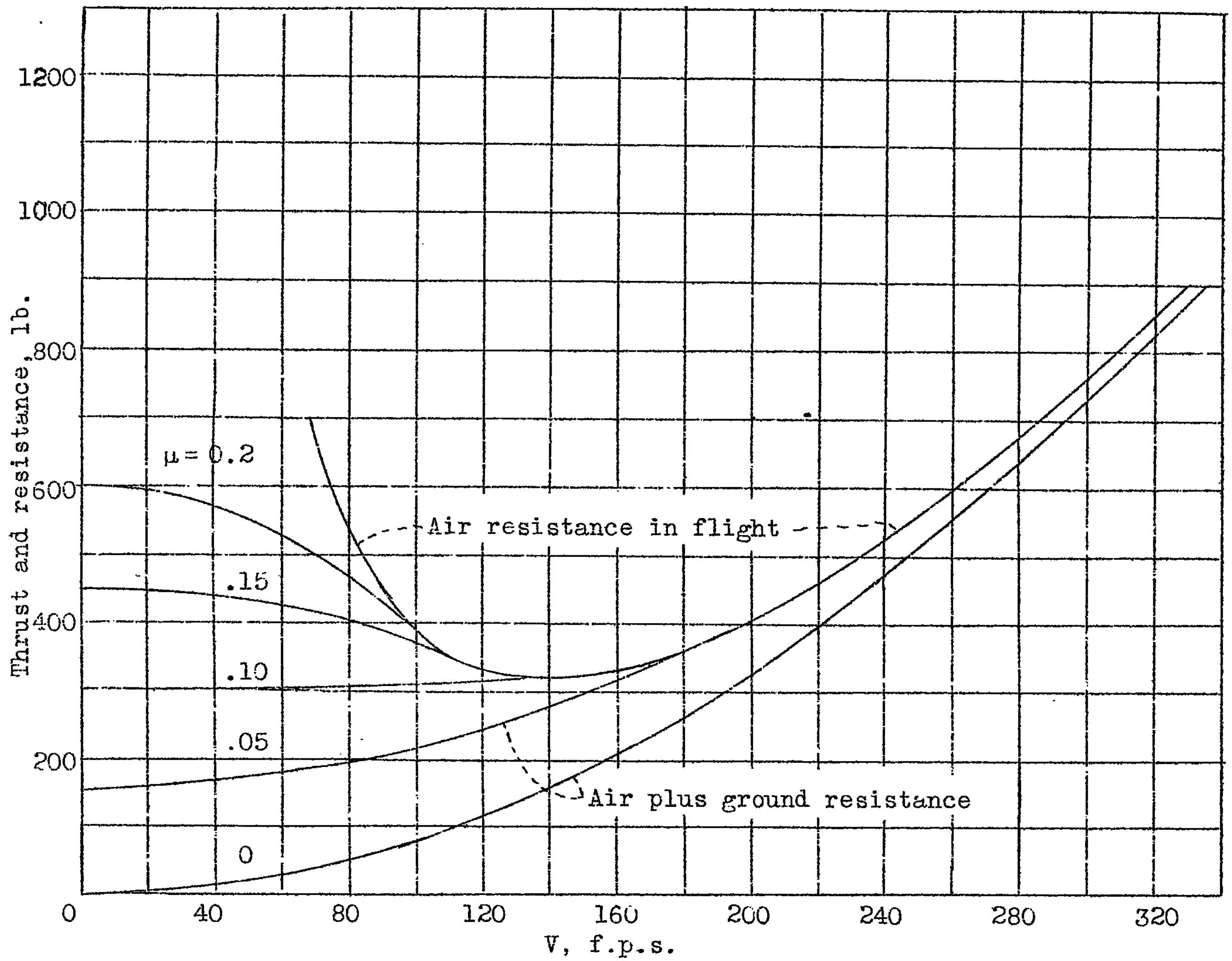


Figure 5.- Curves of minimum resistance during take-off for different values of ground-friction coefficient.

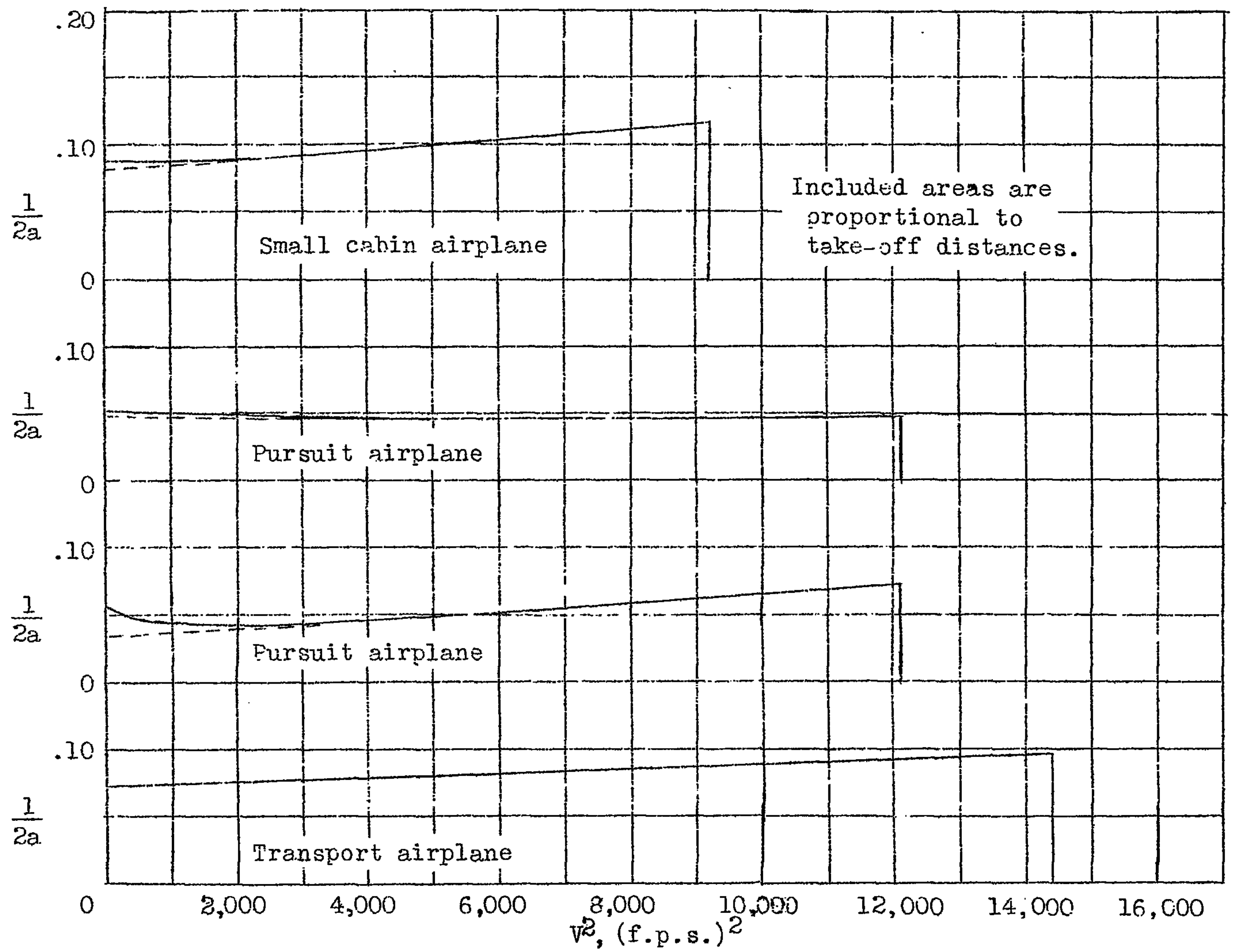


Figure 6.- Curves showing that the reciprocal of the acceleration is very nearly a linear function of velocity squared.

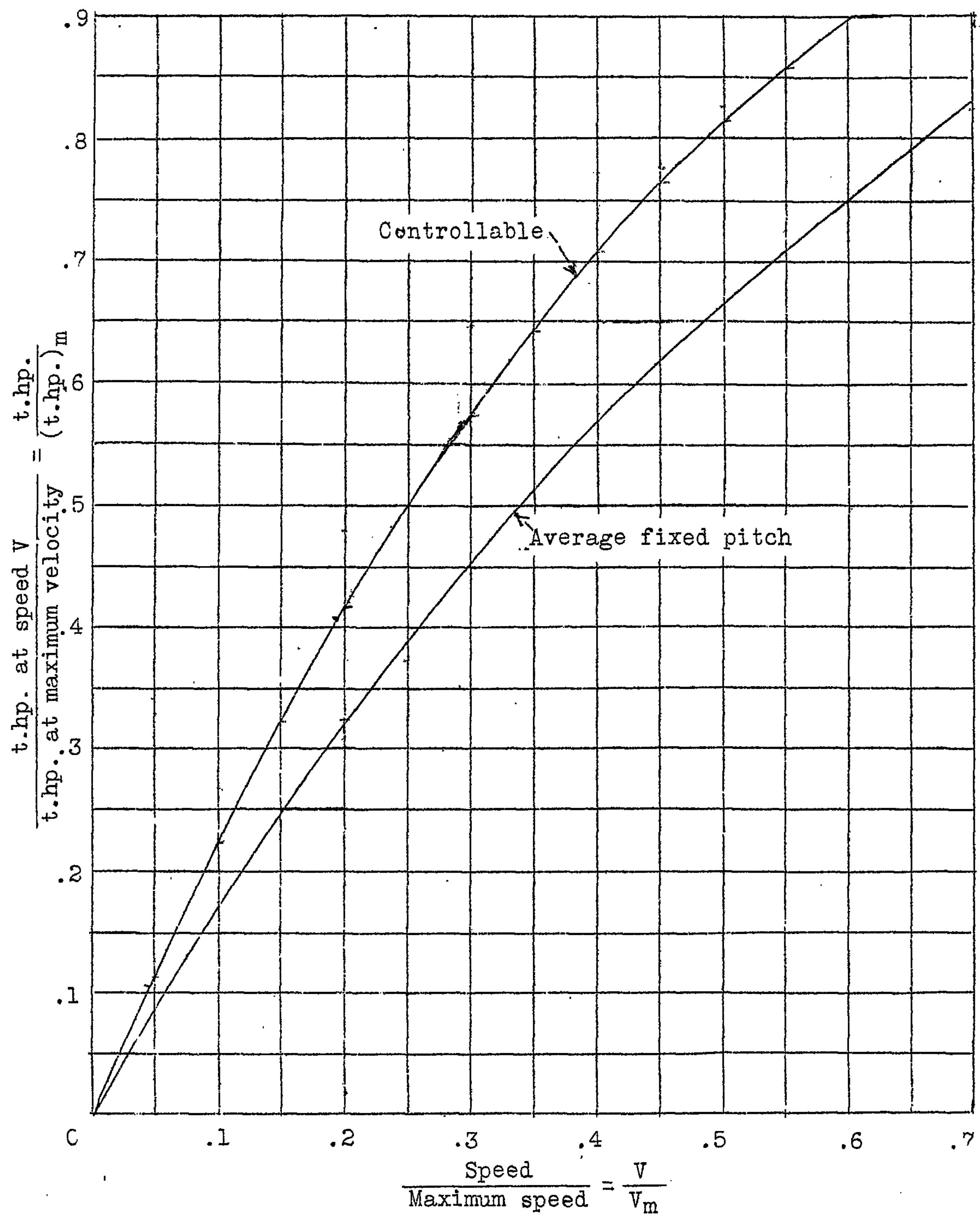


Figure 7.- General full-throttle thrust-horsepower curves.