CASE FILE
COPY

TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

————————

No. 639

————————

A PRELIMINARY INVESTIGATION OF BOUNDARY-LAYER

TRANSITION ALONG A FLAT PLATE WITH

ADVERSE PRESSURE GRADIENT

By Albert E. von Doenhoff
Langley Memorial Aeronautical Laboratory

————————

Washington
March 1938

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 639

A PRELIMINARY INVESTIGATION OF BOUNDARY-LAYER

TRANSITION ALONG A FLAT PLATE WITH

ADVERSE PRESSURE GRADIENT

By Albert E. von Doenhoff

SUMMARY

Boundary-layer surveys were made throughout the transition region along a smooth flat plate placed in an air stream of practically zero turbulence and with an adverse pressure gradient. The boundary-layer Reynolds Number at the laminar separation point was varied from 1,800 to 2,600.

The test data, when considered in the light of certain theoretical deductions, indicated that transition probably began with separation of the laminar boundary layer. The extent of the transition region, defined as the distance from a calculated laminar separation point to the position of the first fully developed turbulent boundary-layer profile, could be expressed as a constant Reynolds Number run of approximately 70,000. Some speculations are presented concerning the application of the foregoing concepts, after certain assumptions have been made, to the problem of the connection between transition on the upper surface of an airfoil at high angles of attack and the maximum lift.

INTRODUCTION

The effects of the transition from laminar to turbulent boundary-layer flow on the aerodynamic characteristics of bodies are known to be large. Reynolds' classic experiments concerning the flow in pipes showed that flow disturbances and surface irregularities make for early transition. Numerous investigators have observed similar effects of such disturbances on the transition from laminar to turbulent boundary-layer flow.

Some steady factors have also been found to affect the occurrence of transition. Investigations of the boundary layer along airfoil sections and airship forms have indicated that an adverse pressure gradient along the surface tends to cause early transition (reference 1). Tollmien (reference 2) has shown that, for a perfect fluid, S-shape velocity profiles such as exist in the presence of an adverse pressure gradient are unstable.

The purpose of the present investigation was to study the nature of boundary-layer transition in the presence of an adverse pressure gradient with disturbing influences, such as stream turbulence and surface roughness, reduced to negligible proportions.

The investigation was carried out with a smooth flat plate in the presence of an adverse pressure gradient with practically zero air-stream turbulence. The range of the tests was limited by the available equipment to comparatively low Reynolds Numbers. The range of Reynolds Number and of pressure gradient was, however, sufficient to include the conditions that exist near the nose of an airfoil at high angles of attack, extending into the full-scale range. As an example of the significance of the results, they are analyzed in relation to the variation with Reynolds Number of the maximum lift coefficient of the N.A.C.A. 2412 airfoil section.

Boundary-layer surveys were made throughout the transition region. The Reynolds Number, based on the distance from the leading edge of the plate to the calculated separation point, was varied from 74,000 to 145,000. The corresponding range of boundary-layer Reynolds Numbers at the separation point is from $R_\delta = 1,800$ to $R_\delta = 2,600$. The investigation was carried out in the N.A.C.A. smoke tunnel because of the low turbulence in the air stream.

APPARATUS AND METHODS

A general view of the installation of the apparatus in the tunnel is given in figure 1. The flat plate was of monel metal and was 16 inches wide, 60 inches long, and 3/16 inch thick. The upper surface of the plate was polished and the leading edge was rounded for smooth flow entry. An adverse pressure gradient was obtained by placing the plate in a diverging channel.

The entrance of the rectangular channel in which the plate was placed was faired from the original smoke-tunnel walls. The width of the channel was 16 inches for the entire length of the plate and the height varied from 6 inches at the leading edge to 10 inches at the trailing edge of the plate. In order to make the structure rigid, the top and bottom of the channel were made up of 3/16-inch aluminum plates supported from 2 by 2 by 1/4 inch duralumin angles. The sides of the channel were of plate glass to make smoke-flow studies possible.

Slots, 1/32 inch wide, for boundary-layer control were located at 1-foot intervals along the channel to prevent excessive thickening and separation of the boundary layer on the sides, top, and bottom of the channel walls. A screen covering the downstream end caused higher pressures to occur inside the channel than outside, thus forcing part of the boundary-layer air to flow through the slots.

The essential element of the hot-wire anemometer used in making the boundary-layer surveys was a fine platinum wire 2 inches long and 0.002 inch in diameter, silver-soldered to two flexible brass supporting prongs of a forked holder that kept the wire in tension. The holder extended through a hole in the top of the channel and was clamped in a vertical micrometer.

Figure 2 shows the electric circuit used. The current through the wire was maintained constant to within 1/4 percent by a current balance that held the current at a constant value independent, within certain limits, of the resistance of the wire and the value of the line voltage. The current was adjusted to the desired value by means of the resistance R_1. Velocities were determined from variations in the resistance of the wire.

A pitot tube was used as a standard for calibrating the hot-wire anemometer and also for maintaining a constant tunnel air speed. The tube was of the standard N.P.L. type and was made according to the drawings in figure 5(a) of reference 3. Because of the low tunnel speeds, a pressure multiplier having a ratio of approximately 100:1 was used. The multiplied pressure was read on a water manometer.

Calibrations of the hot wire were plotted in the usual form:

$$\frac{i^2 \ R \ R_o \ \alpha}{R - R_o} = a + b \sqrt{V}$$

where   i   is the current through the hot wire.

          R,   resistance of wire at air speed V.

          R_o,   resistance of wire at ambient temperature.

          α,   temperature coefficient of resistance of
                hot wire.

          V,   air speed.

    a and b,   constants.

    In order to avoid errors due to changes in the cali-
bration, the wire was calibrated immediately upon the com-
pletion of every run.

                     TESTS AND RESULTS

    The velocity distribution over the plate outside the
boundary layer was determined from observations at the
following five positions along the plate:

    Station - - - - - - - - -   A   B   C   D   E

    Distance back of leading
       edge, in. - - - - - - - 13-1/4   21   27   33   39

    Figure 3 shows the velocity distribution over the
plate. The velocity just outside the boundary layer at
any point along the plate U is given in terms of the
corresponding velocity U_o at station A.

    Boundary-layer surveys were made at stations A, B,
C, and D at each of three tunnel speeds.

    The velocity inside the boundary layer u, is plot-
ted against the distance from the plate y, in figures
4, 5, and 6.

    Although the tests were made for only one value of
the pressure gradient, it can be shown by Reynolds' law

of similarity that running the tests throughout a speed range is equivalent to running them throughout a range of pressure gradients.

PRECISION

The relative accuracy of individual observations is dependent mainly upon the sensitivity of the measuring instruments. It is estimated that the relative accuracy of the hot-wire measurements of velocity is ±2 percent. Because the calibration of the hot wire varied considerably with time, the errors in the absolute values are somewhat greater. As is stated above, this source of error was reduced to a minimum by calibrating the wire immediately upon the completion of every run.

Calibrations of the pitot head given in reference 3 show that it reads the correct dynamic pressure to within 1 percent over the range used. The pressures indicated by it could be determined with an accuracy of ±0.0003 inch of water.

Measurements of distance from the surface are accurate to within ±0.002 inch.

DISCUSSION

The purpose of the first phase of this discussion is to present certain inferences concerning the nature of transition from the relations observed in these tests. Although it is realized that the experimental data at hand are insufficient to justify definite conclusions, they do give an insight into the nature of transition when considered in the light of certain theoretical deductions.

Laminar region.- In order to determine the position on the plate where the laminar boundary layer might be expected to separate, the laminar boundary-layer characteristics were calculated by the von Kármán-Millikan method (reference 4) using the experimentally obtained velocity distribution over the plate. The calculations show that separation occurs when the velocity decreases to 0.898 of its value at the leading edge of the plate. This value corresponds to a distance of 17.5 inches from the leading edge. It is to be noted that the position of the calcu-

lated separation point is independent of the Reynolds Number.

A comparison of the experimental and calculated velocity profiles at station A is given in figure 7. The results are plotted in the form u/U against $\dfrac{y\sqrt{R}}{L}$ where

> L   is the distance from leading edge of plate to laminar separation point.
>
> R,   Reynolds Number UL/v.
>
> v,   the kinematic viscosity.

The experimental data for all speeds when plotted in this form define a single curve, which shows definitely that the boundary layer at this point is laminar. The agreement between the calculated and experimental profiles is similar to that found in reference 5. The results of reference 5 show good agreement between the calculated and experimental separation points; it is therefore reasonable to suppose that the boundary layer on the plate will separate near the calculated position.

Transition region.- Large intermittent fluctuations in velocity were observed at station B for all tunnel speeds. Similar fluctuations were observed at the lower tunnel speeds at station C. It was found that the boundary-layer thickness δ increased rapidly between stations A and C. These results show that transition must begin between stations A and B near the point where laminar separation is expected.

Observations of the smoke flow over the plate showed that the boundary-layer flow was smooth at least as far back as the calculated separation point. As the Reynolds Number was changed, no large movement of the point where transition started was apparent. Boundary-layer velocity profiles at the lower speeds show a region of relatively quiet air next to the plate. These velocity profiles and the direct smoke-flow observations indicate that separation of the laminar boundary layer actually takes place locally near the calculated point and that transition is probably caused by the instability of the subsequent flow.

Examination of the data shows that the position along the surface where the fully developed turbulent boundary

layer first occurs varies with the Reynolds Number. For
the highest speed, the boundary-layer profile at station
C is of the fully developed turbulent type. (See figs 4
and 8.) At the lowest speed, a fully developed turbulent
profile occurs a short distance downstream from station D.
(See figs. 6 and 9.)

The direct smoke-flow observations having indicated
that the transition was closely associated with and fol-
lowed laminar separation, and the measured profiles having
shown that the distance required to reach the fully devel-
oped turbulent layer was affected by the Reynolds Number,
a quantitative relation between separation and transition
was sought. In other words, an estimate was attempted of
the extent of the transition region, which was considered
to lie between the laminar separation point and the posi-
tion where fully developed turbulent profiles were ob-
served. The approximate position of the fully developed
turbulent profile could be determined from these prelimi-
nary experiments but, unfortunately, these experiments
were not sufficiently extensive to permit the direct loca-
tion of the separation point. It was considered justifi-
able, however, to employ a calculated separation point
because the experimental evidence did not conflict with
this location and because the theory had previously been
experimentally verified. An estimate of the extent of the
transition region could thus be made and its variation
with Reynolds Number studied.

The result suggested that the length of the transi-
tion region, defined as the distance from the laminar sep-
aration point to the position of the first fully developed
turbulent boundary-layer profile, can be expressed as a
constant Reynolds Number of approximately 70,000 through-
out the range of the tests.

The development of the turbulent boundary layer can
be seen from a comparison of the velocity profiles for the
different speeds at station C. (See figs. 4, 5, 6, and
8.) At the lowest speed, the velocity profile shows a re-
gion of relatively quiet air next to the plate. As the
speed is increased, this region disappears. The velocity
profile at this point is of the fully developed turbulent
type at the highest speed.

When plotted on logarithmic paper (figs. 8 and 9),
the experimental points for the outer nine-tenths of the

fully developed turbulent velocity profiles fall on a
straight line with a slope of 1/6.4. The deviation of the
points for the inner one-tenth of the boundary-layer thick-
ness is an indication of the extent of the laminar sub-
layer.

From a summarization of the results of the preceding
discussion, it seems likely that transition occurred in
the following manner. The boundary-layer flow was lami-
nar up to the laminar separation point. Downstream from
the separation point, a region of relatively quiet air ex-
isted next to the plate over which the outside air con-
tinued to flow. Because of the instability of such a sys-
tem, turbulence started, finally resulting in the devel-
opment of a turbulent boundary layer. The extent of the
entire transition region, defined as the distance from
the laminar separation point to the position of the first
fully developed turbulent boundary-layer profile, appears
to be approximately expressed as a constant Reynolds Num-
ber

$$\frac{U_s x}{\nu} = 70,000$$

where U_s is the velocity outside the boundary layer
                       at the laminar separation point.

               x,   the extent of the transition region.

In the second phase of the discussion, some specula-
tions are presented concerning the application of the
foregoing deductions to a practical problem. The type of
transition described has been observed near the leading
edge on the upper surface of airfoils just before the
stall (reference 6). In the application of these concepts
to the flow on the upper surface of an airfoil, an attempt
has been made to calculate the variation of maximum lift
coefficient with Reynolds Number.

Speculations on the application to an airfoil section.—
Before the concepts developed from the analysis of the
transition region along the plate can be applied to other
bodies, the flow conditions in the boundary layer along
the body must be examined to see what relation they bear
to the conditions along the plate. Several boundary-layer
calculations by the von Kármán-Millikan method have shown
that, in most cases encountered in practice, laminar sep-
aration-point profiles are nearly similar in shape. Con-

sideration of the small region in the neighborhood of a separation point shows that the only reference length available for forming a nondimensional pressure gradient is the boundary-layer thickness δ. The nondimensional pressure gradient is therefore written in the form $\dfrac{\delta}{q}\dfrac{dp}{ds}$

where q is the dynamic pressure outside the bound-
              ary layer.

         p,   the static pressure.

         s,   distance along the surface.

It can be shown that this pressure gradient and the boundary-layer Reynolds Number R_δ at a laminar separation point are connected by the approximate relation

$$R_\delta \frac{\delta}{q}\frac{dp}{ds} = \text{constant}$$

This relation shows that, if the value of R_δ at the laminar separation point on the airfoil and on the plate is the same, the value of $\dfrac{\delta}{q}\dfrac{dp}{ds}$ will also be approximately the same. Under these conditions, the plate experiments may be considered applicable to the airfoil case.

In addition to the foregoing considerations, some method must be found for taking into account the effects of curvature. The following assumptions, which are necessary to make the calculations possible, are not regarded as quantitatively accurate. They are introduced merely to complete qualitatively the picture of transition as it occurs on a curved surface in the presence of an adverse pressure gradient with practically zero stream turbulence.

In the case of the flat plate, the region of quiet air after the separation point is necessarily thin because the direction of the outside flow is nearly parallel to the plate. Smoke-flow studies of various bodies indicate that, when separation takes place on a curved surface, the flow leaves in a direction approximately tangent to the surface at the separation point. In the calculations it will therefore be assumed that the flow leaves the surface in a direction tangent to the surface at the laminar separation point.

On the flat plate, the entire length of the transition region was expressed as a constant Reynolds Number run of 70,000. This value, however, represents the length corresponding to the attainment of a fully developed turbulent boundary-layer profile, whereas the turbulence must begin farther upstream; at least the turbulence may be considered sufficiently developed to provide the necessary scouring action required to prevent separation at a position represented by a Reynolds Number run less than 70,000. For this reason, the distance from the laminar separation point to the point from which the turbulence begins effectively to spread will be assumed to correspond to a Reynolds Number run of, say, 50,000.

Both the theory of spreading turbulent jets and experiment show that turbulence tends to spread in a linear manner (reference 7). The angle of spread depends upon the particular conditions. The case of flow closing in after laminar separation seems to be analogous to the spreading of a jet. For the present it will be assumed that the branch of the envelope of the turbulent flow inclined toward the surface makes an angle of 15° with the line of separation. This angle is within the range of those observed for spreading jets of different types. If the spreading turbulence does not reach the body, the boundary layer cannot re-establish itself and permanent separation may be considered to have taken place.

The analysis was made in the following manner. The position of the laminar separation point was calculated according to the method of reference 4 after finding the pressure distribution by the method of reference 8 for several lift coefficients. A straight line was drawn tangent to the profile at the separation point. Another line, making an angle of 15° with the line of separation, was drawn tangent to the airfoil surface. The distance from the separation point to the point of intersection of the two lines is considered to represent a Reynolds Number

$$R_T = \frac{U_S x_T}{\nu} , \quad \text{of } 50,000$$

where U_S is the velocity outside the boundary layer at the separation point on the airfoil.

x_T is the length of the line from the separation point to the point of intersection.

The Reynolds Number of the airfoil corresponding to maximum lift for the lift coefficient chosen was then computed according to the relation

$$R = 50,000 \frac{U_o}{U_S} \frac{c}{x_T}$$

where U_o is the reference velocity for the airfoil.

         c,   the airfoil chord.

Figure 10 shows the graphical construction, for one lift coefficient, on the N.A.C.A. 2412 airfoil section.

Figure 11 gives a comparison between the experimental variation of the maximum lift coefficient with Reynolds Number (reference 9) and the calculated values. By the Reynolds Number range indicated as the "range of present experiments" is meant the range of Reynolds Numbers throughout which R_δ at the separation point falls within the range of values obtained on the flat plate. The qualitative agreement of the two curves, especially throughout the range of the present tests, suggests that, at moderate Reynolds Numbers, the general nature of transition on the upper surface near the leading edge of an airfoil at high angles of attack probably does not differ essentially in character from that discussed in this report.

Much more experimental data through a wider range of Reynolds Number both on a flat plate and on other bodies are needed to extend and verify the tentative theory of transition set forth in the present report. It seems doubtful that the apparent mechanism of transition observed in the range of these tests would continue operative with indefinite increase of the Reynolds Number. As this increase occurs, the transition region becomes continually shorter in length, approaching the order of magnitude of δ at the laminar separation point. This condition for the N.A.C.A. 2412 airfoil, however, does not occur until R is above 7,000,000, when the run is still approximately 10 δ. A comparison of the experimental and calculated scale-effect curves indicates that turbulent-layer separation effects become of major importance before this Reynolds Number is reached.

Langley Memorial Aeronautical Laboratory,
    National Advisory Committee for Aeronautics,
        Langley Field, Va., February 11, 1938.

REFERENCES

1. Squire, H. B.: Note on Boundary Layer Flow. R. & M.
        No. 1664, British A.R.C., 1935.

2. Tollmien, W.: General Instability Criterion of Lam-
        inar Velocity Distributions. T.M. No. 792, N.A.C.A.,
        1936.

3. Ower, E., and Johansen, F. C.: On a Determination of
        the Pitot-Static Tube Factor at Low Reynolds Num-
        bers, with Special Reference to the Measurement of
        Low Air Speeds. R. & M. No. 1437, British A.R.C.,
        1932.

4. von Kármán, Th., and Millikan, C. B.: On the Theory
        of Laminar Boundary Layers Involving Separation.
        T.R. No. 504, N.A.C.A., 1934.

5. von Doenhoff, Albert E.: An Application of the von
        Kármán-Millikan Laminar Boundary-Layer Theory and
        Comparison with Experiment. T.N. No. 544, N.A.C.A.,
        1935.

6. Jones, B. Melvill: Stalling. R.A.S. Jour., vol.
        XXXVIII, no. 285, Sept. 1934, pp. 753-769.

7. Prandtl, L.: The Mechanics of Viscous Fluids. Sec.
        25, Spread of Turbulence. Vol. III, div. G of
        Aerodynamic Theory. W. F. Durand, editor. Julius
        Springer (Berlin), 1935, pp. 162-178.

8. Pinkerton, Robert M.: Calculated and Measured Pres-
        sure Distributions over the Midspan Section of the
        N.A.C.A. 4412 Airfoil. T.R. No. 563, N.A.C.A.,
        1936.

9. Jacobs, Eastman N., and Sherman, Albert: Airfoil Sec-
        tion Characteristics as Affected by Variations of
        the Reynolds Number. T.R. No. 586, N.A.C.A., 1937.

Fig. 1
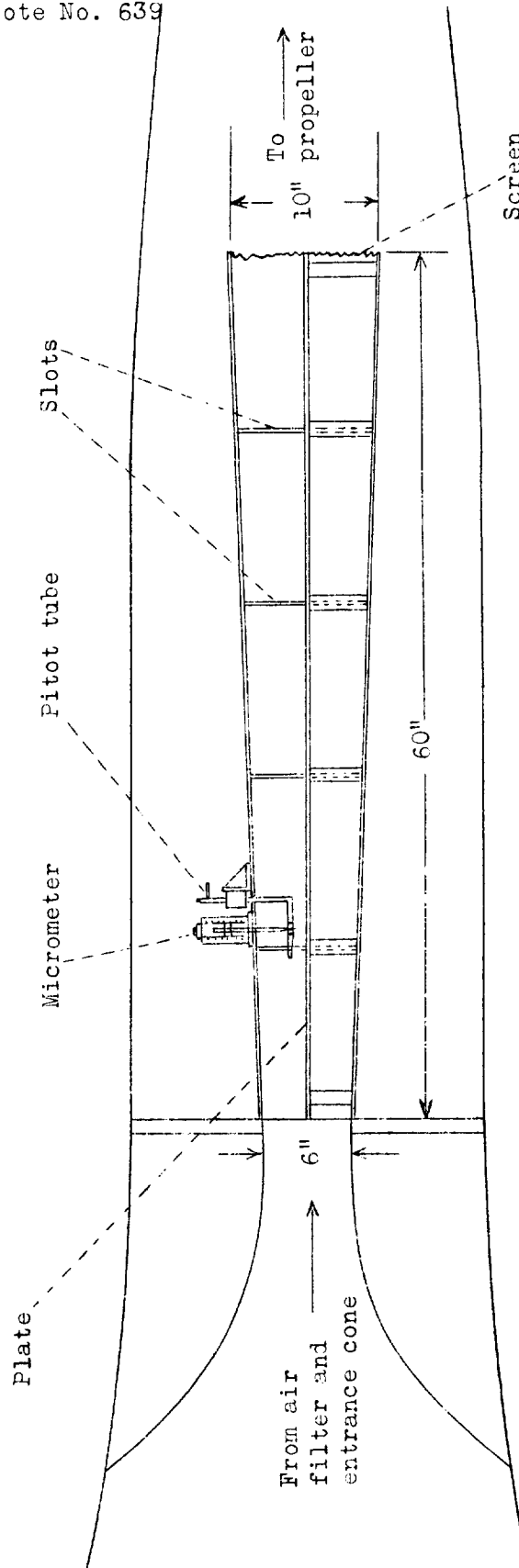
Figure 1.- Installation of plate and instruments in tunnel.

Fig. 2
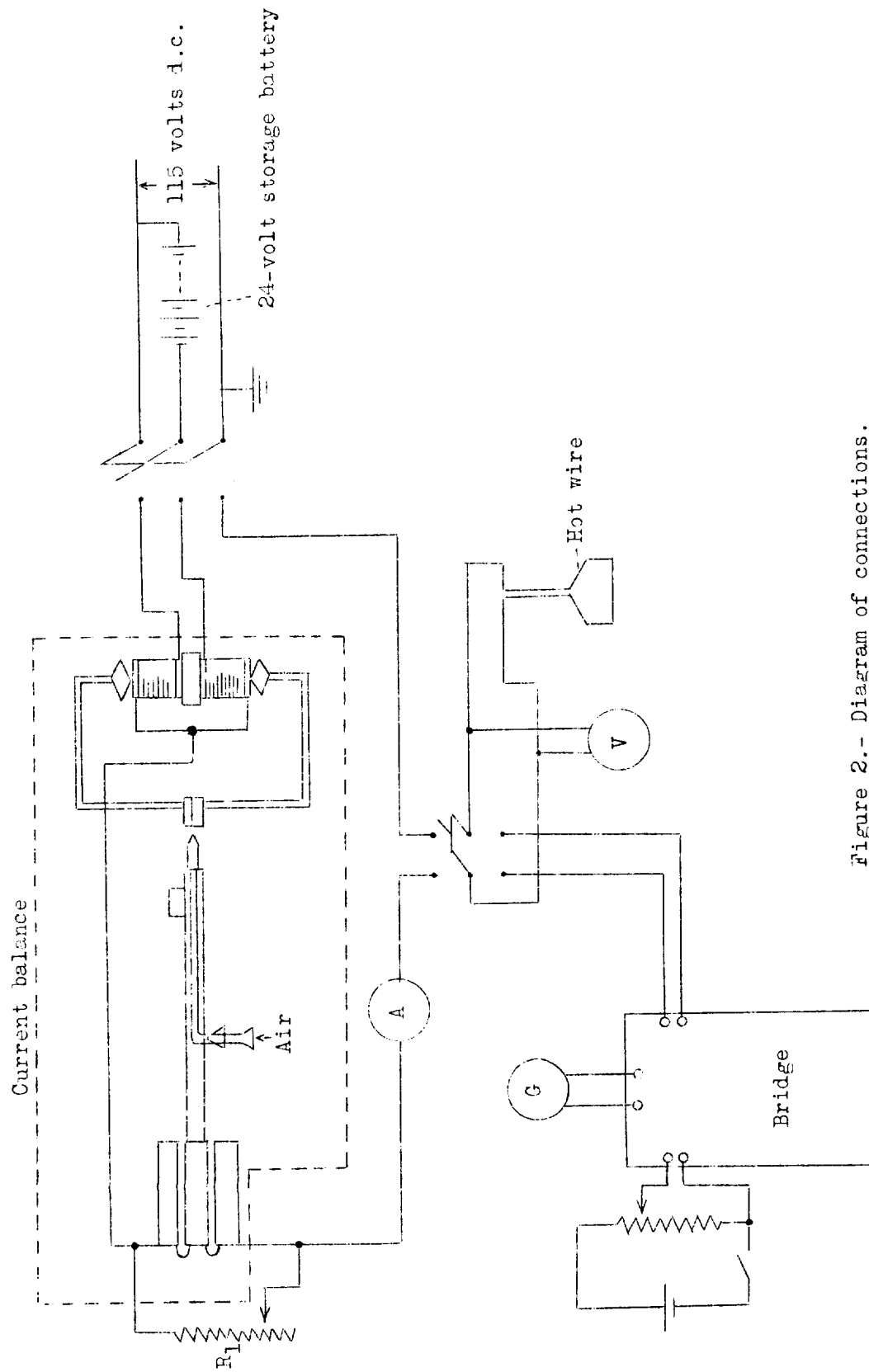
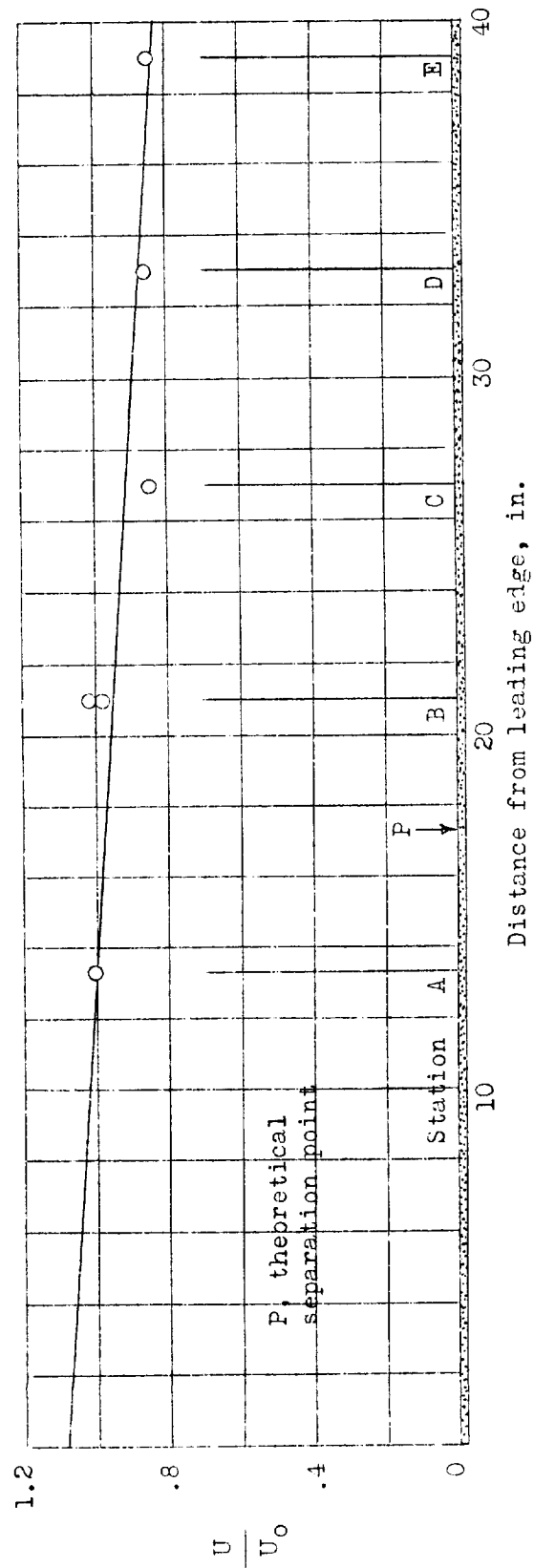Figure 2.- Diagram of connections.

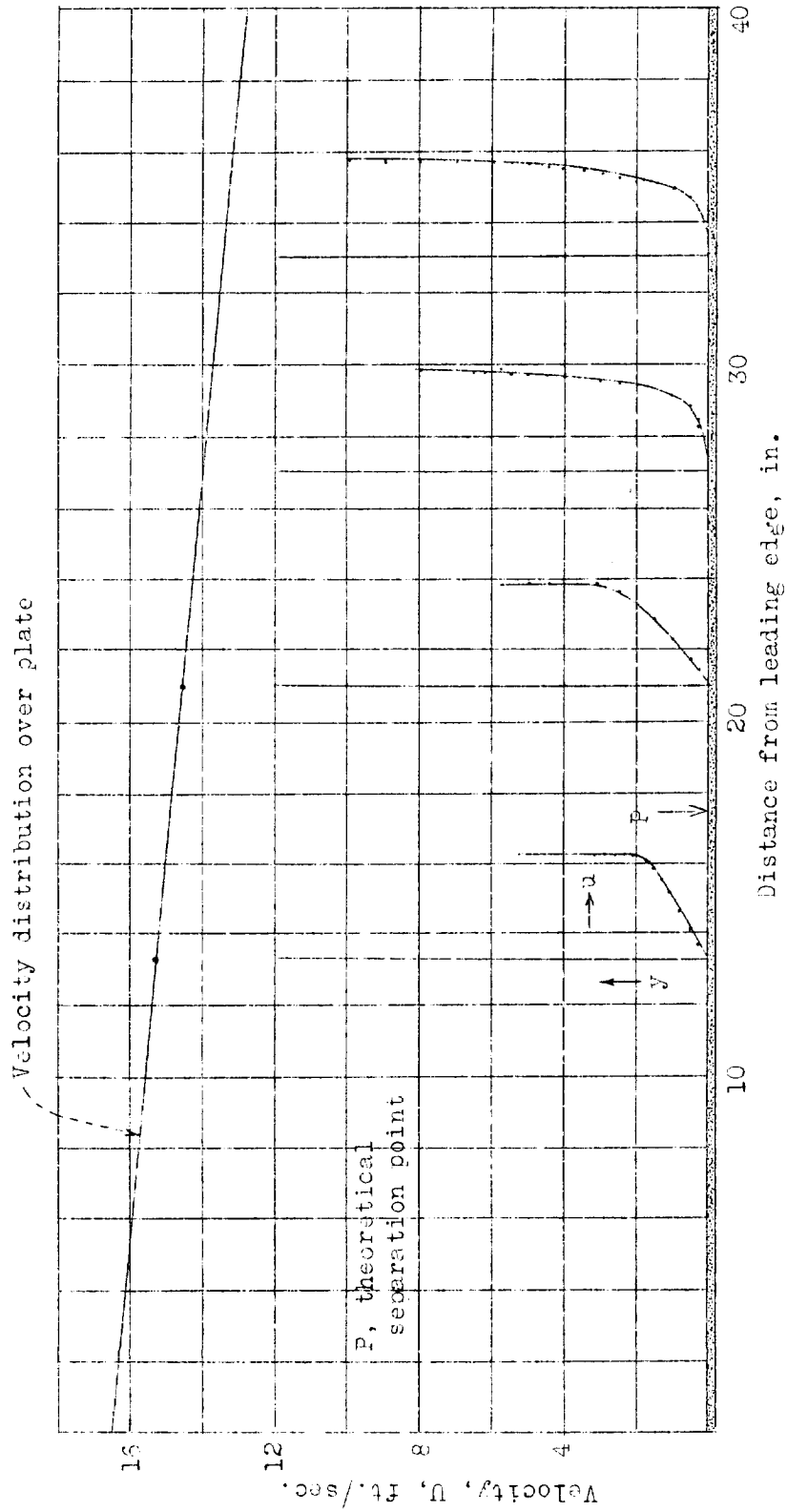Figure 3.- Velocity distribution over plate.

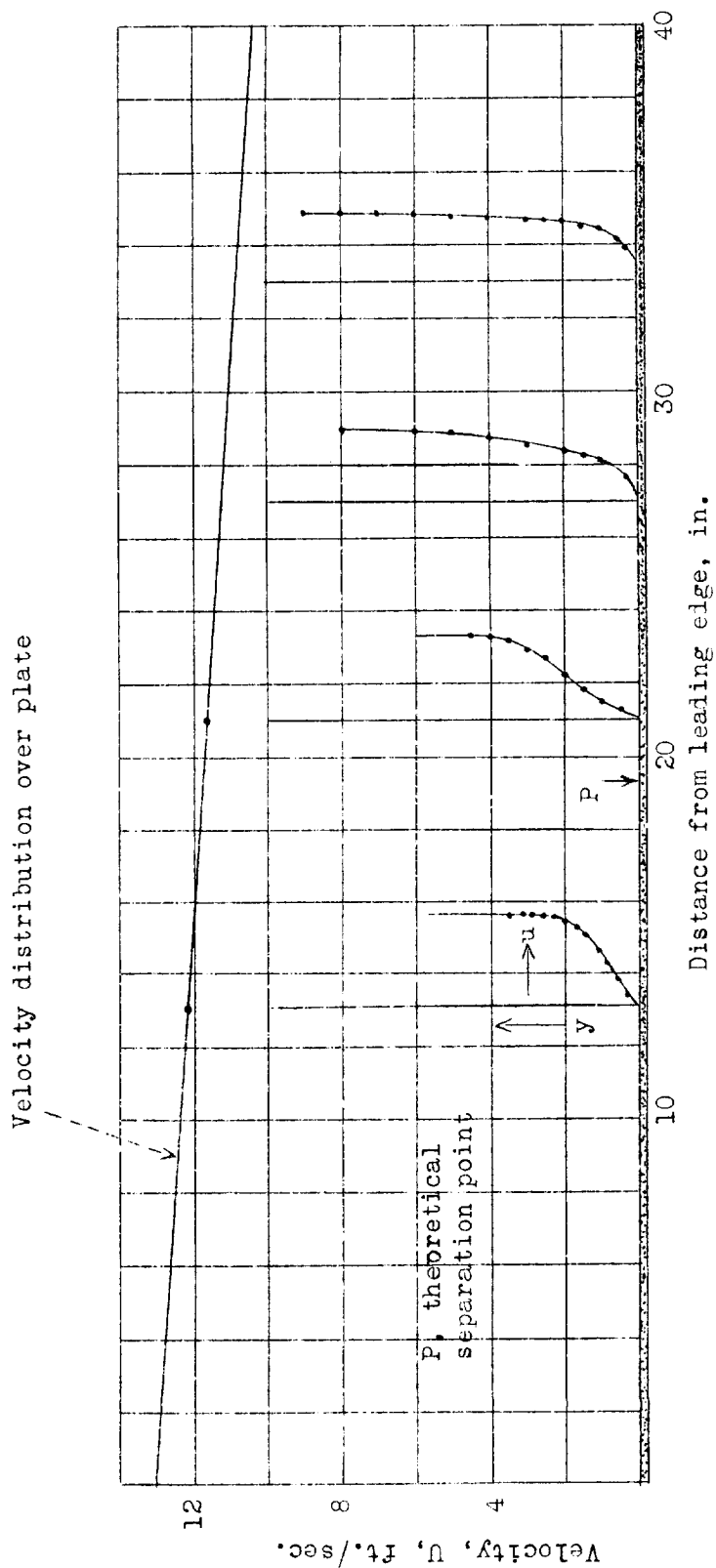Figure 4.- Boundary-layer profiles. $U_o = 15.3$ ft./sec.

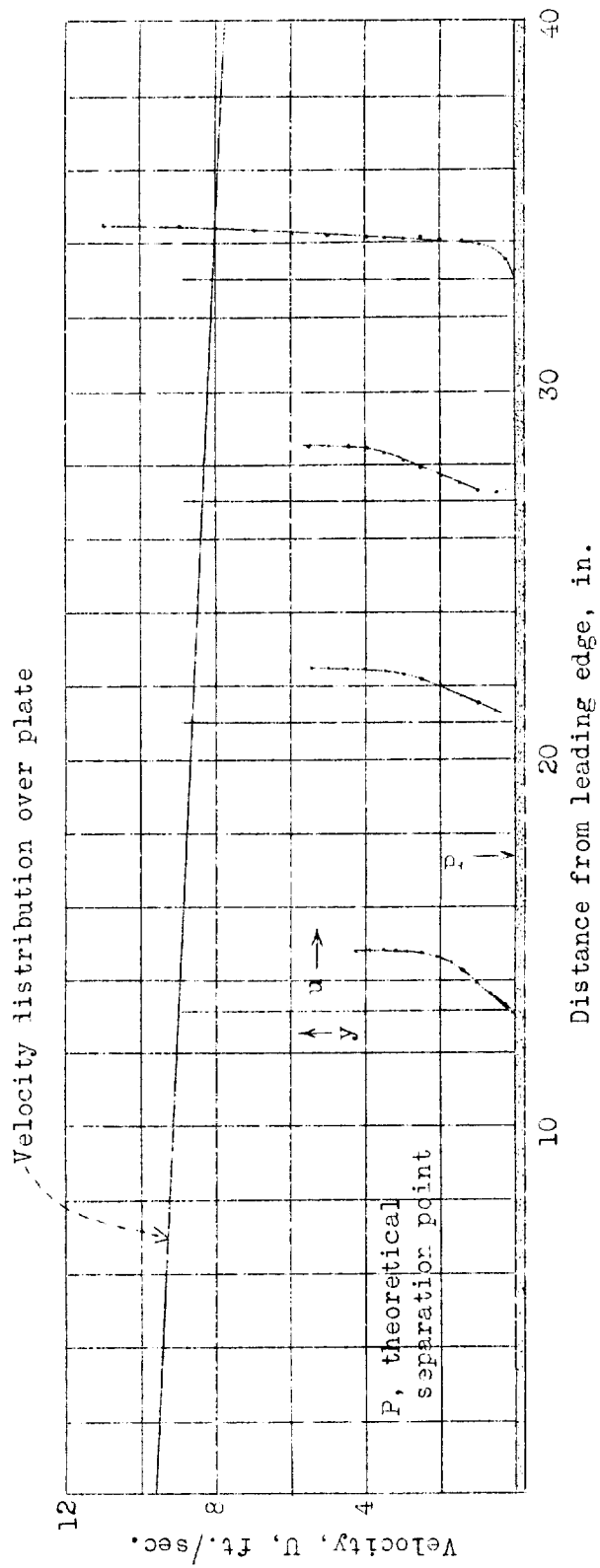Figure 5.- Boundary-layer profiles. U_o = 12.2 ft./sec.

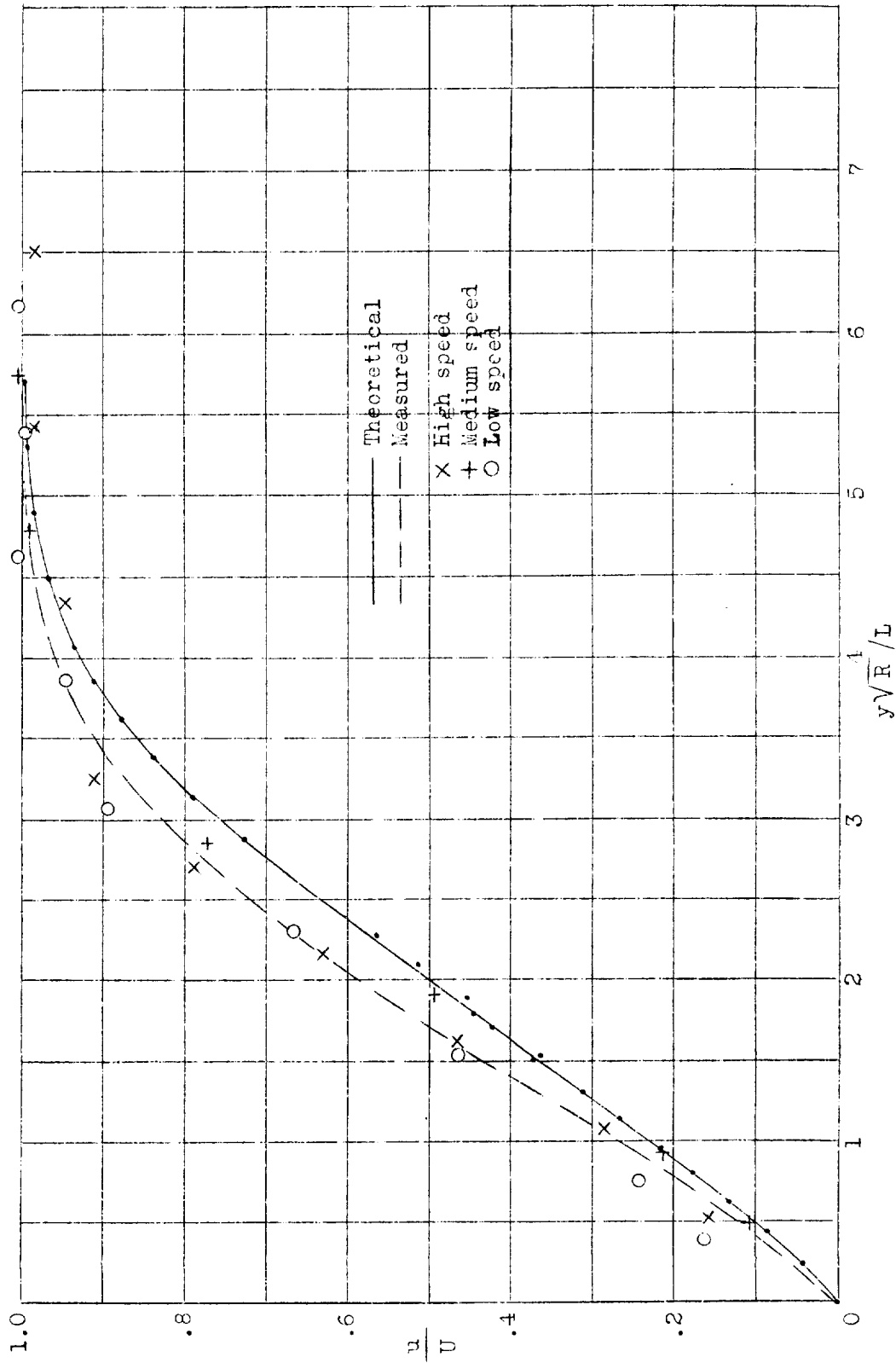Figure 6.- Boundary-layer profiles. U_o = 8.0 ft./sec.

Figure 7.- Theoretical and measured boundary-layer velocity distributions. Station A.
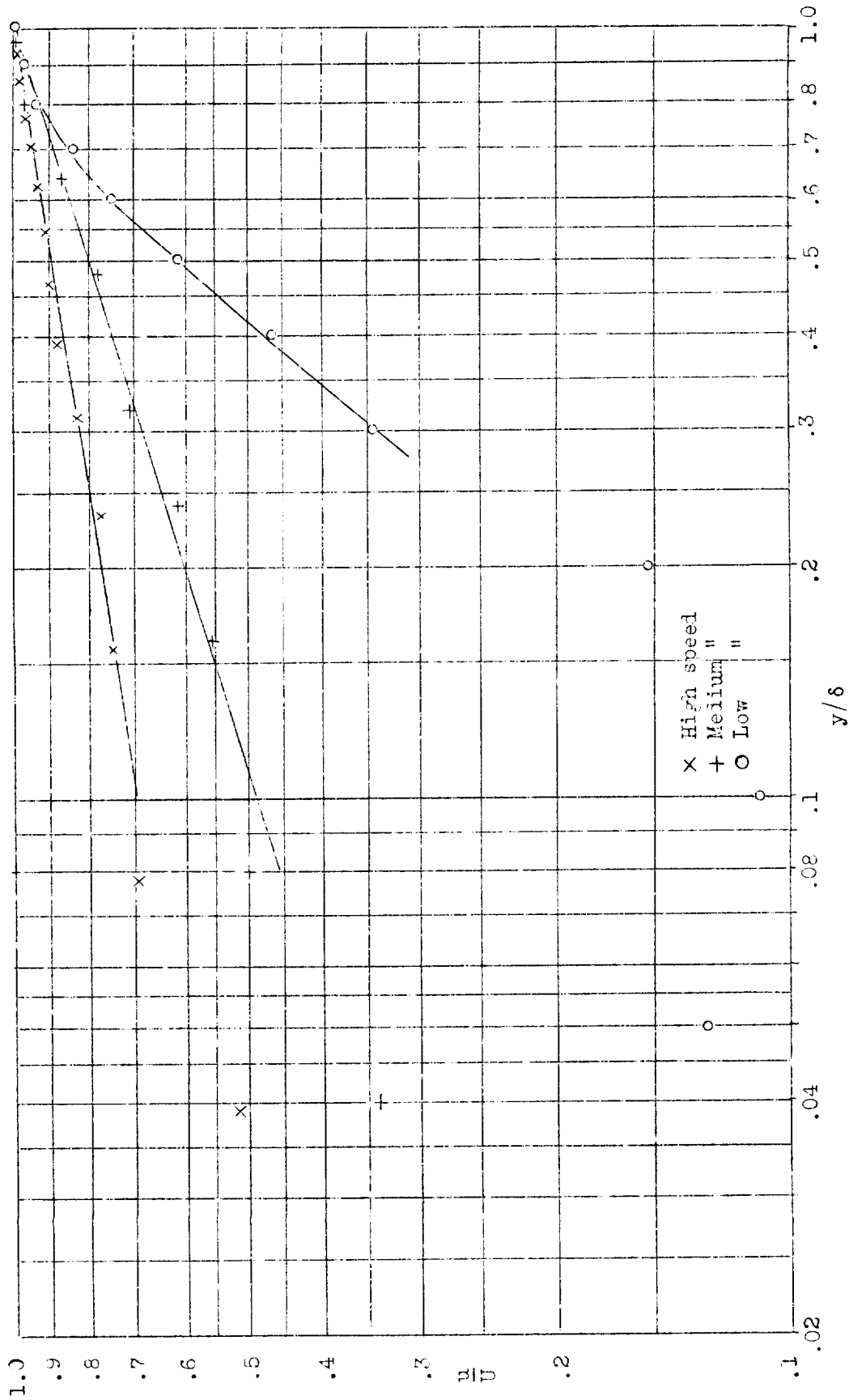
Figure 8.- Logarithmic plot of boundary-layer velocity distributions. Station C.
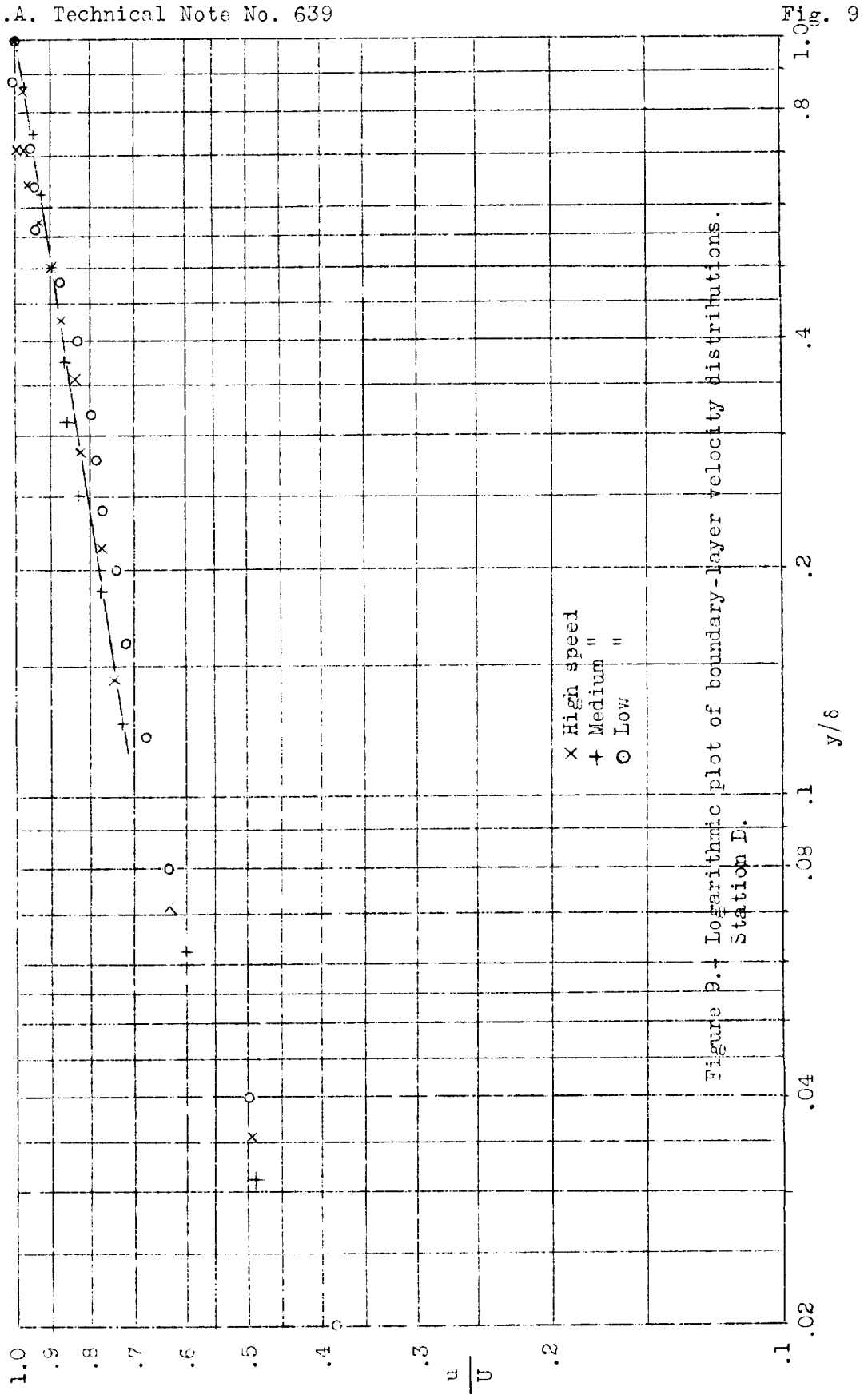
Fig. 9

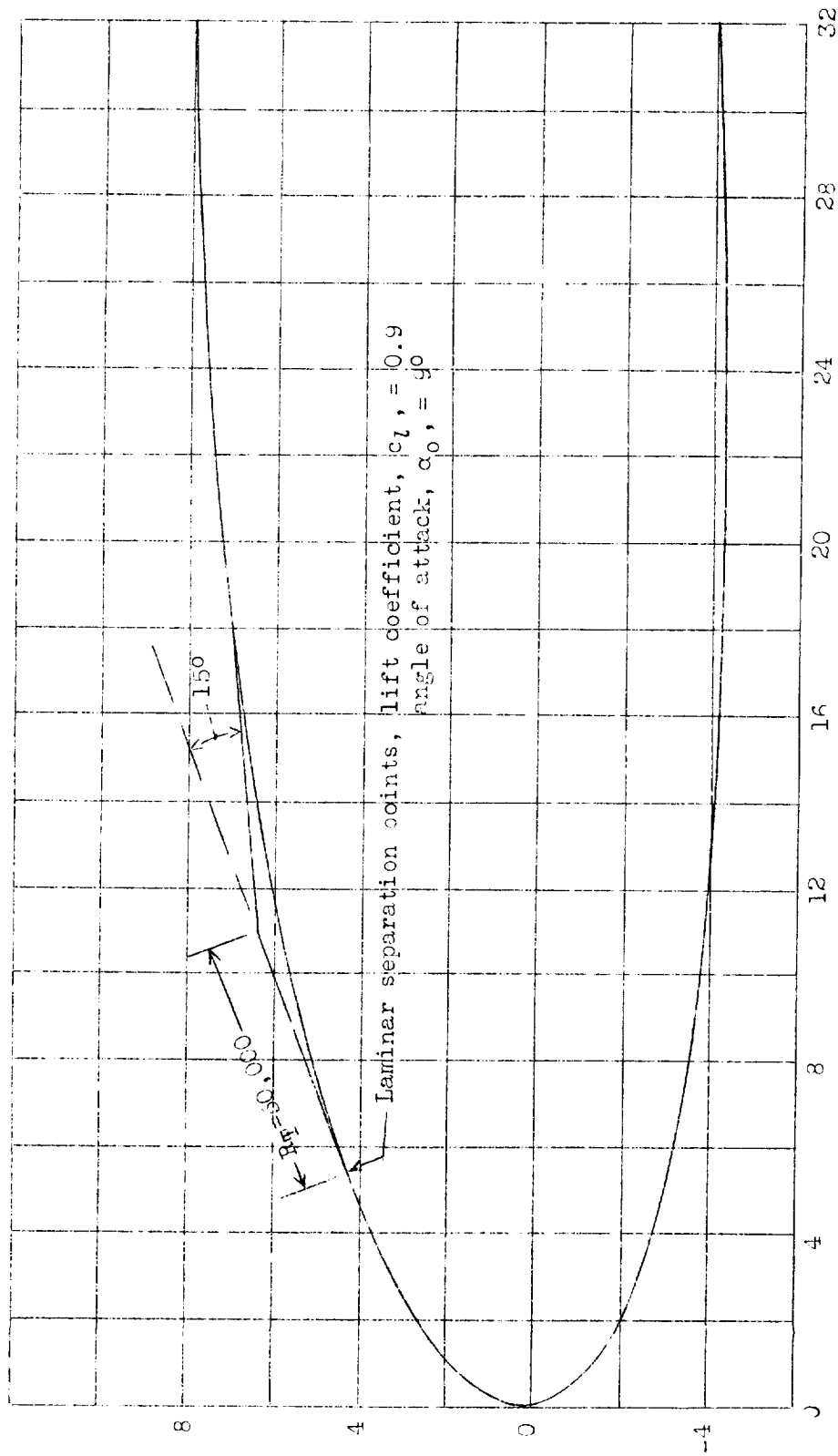Figure 9.- Logarithmic plot of boundary-layer velocity distributions. Station D.

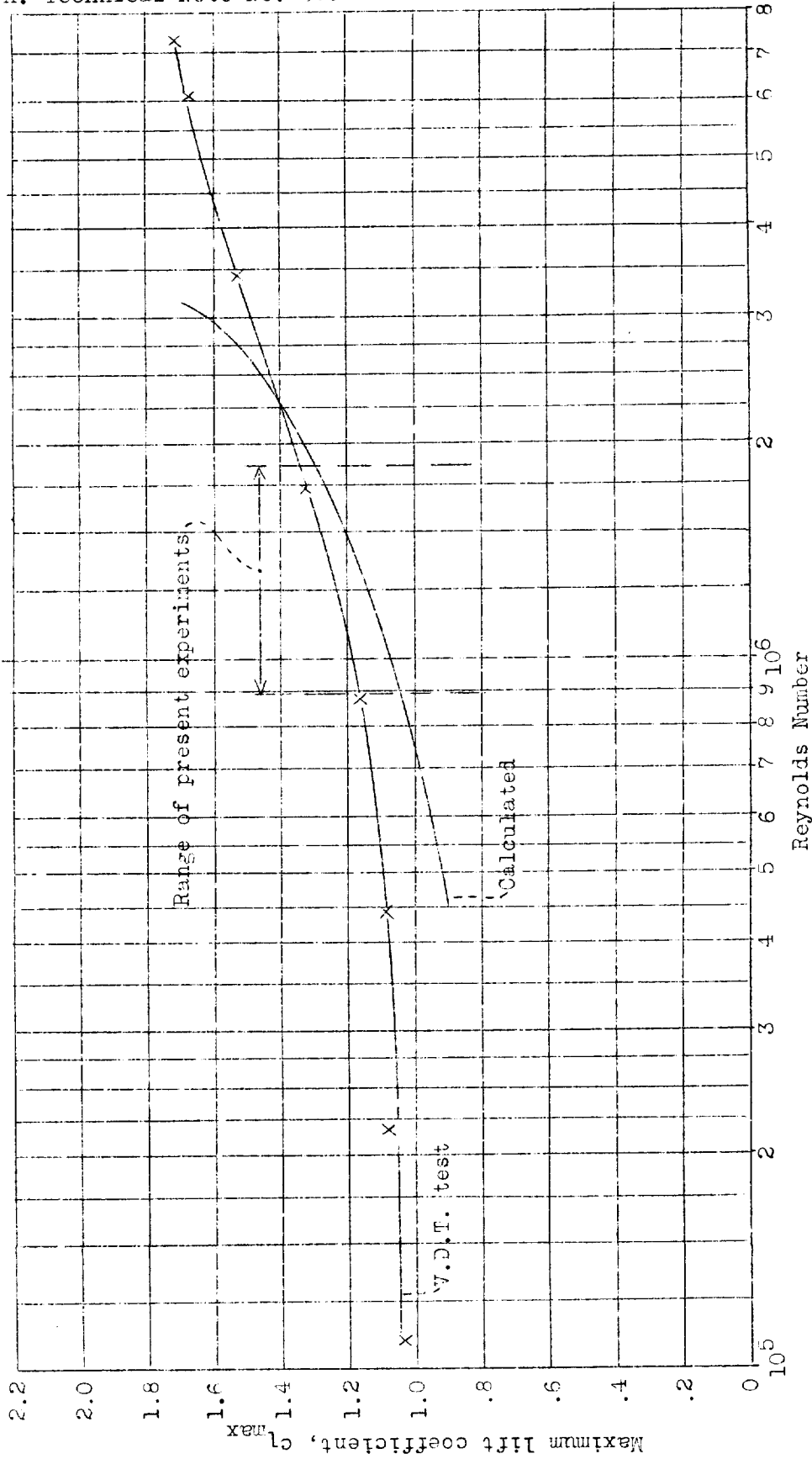Figure 10.- Transition conditions for $c_{l\,max} = 0.9$, $R = 445,000$. N.A.C.A. 2412 airfoil section.

Figure 11.- Calculated and experimental curves of scale effect on maximum lift for the N.A.C.A. 2412 airfoil section.