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SUMMARY

Torsion tests were made on 51ST aluminum-alloy seamless tubes having diameter-to-thickness ratios of from 77 to 139 and length-to-diameter ratios of from 1 to 60. The torsional strengths developed in the tubes which failed elastically (all tubes having lengths greater than 2 to 6 times the diameter) were in most cases within 10 percent of the value indicated by the theories of Donnell, Timoshenko, and Sturm, assuming a condition of simply supported ends.

INTRODUCTION

In the design of aircraft, lightweight trains, tanks, and pipe lines, problems involving the strength of thin curvilinear sections subjected to shear are frequently encountered. The strength in such cases is more often dependent upon the stability of the section than upon the strength of the material of which it is composed, and solutions are necessarily based upon the results of both tests and theoretical analyses. A study of the torsional strength of thin-wall cylindrical sections covers the simplest case of the general problem and, for that reason, this type of section has been the field for numerous investigations. It is the purpose of this report to present additional experimental data, obtained from aluminum-alloy seamless tubes, and to compare the test results with several of the existing theories of torsional stability.

In previous investigations of the torsional strength of round tubing, emphasis was placed upon the determination of:

1. The shearing properties of wrought aluminum alloys (reference 1), and

2. The effect of the ratio of diameter to wall thickness (D/t) upon the torsional strength of tubes of approximately the same length (reference 2).

Although the length of tubing is usually not considered as a variable factor in tests to determine shearing properties (reference 3), it does have an important bearing, within limits, upon the torsional strength of tubing that fails because of elastic instability.

In 1933, L. H. Donnell (reference 4) presented a theoretical solution of the torsional-stability problem, including the length factor, and gave numerous experimental data in support of his conclusions. The tests reported by him were all made on fabricated specimens having longitudinal seams, either lapped or spliced. It seemed desirable to obtain some experimental data on seamless tubing, particularly as the Aluminum Company has made no previous investigations of this kind. Added interest has been attached to these tests in view of the recent theoretical analysis made by R. G. Sturm (reference 5), in which one general expression is given for the critical shear stress for all lengths of tubing, whereas Donnell's theory necessitates the use of two formulas, one for short and medium tubes and the other for long slender tubes. The theoretical solutions of Timoshenko (reference 6) and of Schwerin (reference 7) apply only to long slender tubes. Torsion tests of a number of steel and aluminum-alloy tubes of various sizes and lengths were made at the National Bureau of Standards (reference 8).

The objects of this investigation were:

1. To determine the influence of diameter thickness (D/t) and length-diameter (L/D) ratios upon the torsional strength of thin-wall aluminum-alloy tubing.

2. To compare the results of the tests with existing theories of torsional stability.

DESCRIPTION OF SPECIMENS AND PROCEDURE

The following sizes* of 51ST seamless round tubing were tested in duplicate:

1. 1.003 in. O.D. × 0.977 in. I.D., having a D/t ratio of 77, in lengths** of 1, 2, 4, 8, 16, 28, and 40 times the diameter.

2. 1.878 in. O.D. × 1.842 in. I.D., having a D/t ratio of 104, in lengths of 1, 2, 4, 8, 16, 22, 40, and 60 times the diameter.

3. 2.500 in. O.D. × 2.464 in. I.D., having a D/t ratio of 139, in lengths of 1, 2, 4, 8, 16, 32, and 45 times the diameter.

Aluminum alloy 51ST was selected because it provided the highest yield strengths available in the foregoing sizes of commercial tubing. Table I gives a summary of the tensile properties. The moduli of elasticity, shown on the tensile and compressive stress-strain curves (figs. 1, 2, and 3) averaged about 9,600,000 pounds per square inch. Although these moduli are somewhat below the value usually found for the strong aluminum alloys, they are not seriously out of line with previous determinations for this particular alloy.

The torsion tests on all tubes having a length less than 44 inches were made in the 1,200 foot-pounds capacity Amsler torsion machine, using the 240 and 400 foot-pounds capacity ranges. The tubes longer than 44 inches were tested in the large lathe in the machine shop, using the set-up shown in figure 4. One end of the tubing was gripped in the chuck of the lathe, which was locked in a stationary position, and the other was mounted on a ball-bearing center in the tail stock. Torque was applied by dead weights suspended from a horizontal lever arm clamped to the end of the tubing as shown. Close-fitting steel plugs, approximately 4 inches long and having a generous radius on the leading edge, were used in all tubes to provide support for clamping during the tests.

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*A third specimen was used for a check test in some cases.

**Exclusive of 8-inch length provided in all tubes for grips of testing machine.

RESULTS AND DISCUSSION

Tables II, III, and IV give the results of the tests on all tubes. Figure 5 shows the relation between the average shearing stresses developed at failure and the D/t and L/D ratios of the tubing. The shearing stresses corresponding to the maximum applied torques were computed from the relation

$$s = \frac{T}{2\pi r^2 t} \tag{1}$$

where T is the torque producing failure, in.-lb.

r, mean radius, in.

t, wall thickness, in.

s, shear stress, lb. per sq. in.

The influence of the proportions of the specimens is clearly indicated by the fact that the stresses ranged from a maximum of 21,800 pounds per square inch, obtained on the shortest length of tubing having a D/t ratio of 77, to a minimum of 4,600 pounds per square inch, obtained on the longest specimen having a D/t ratio of 139. The highest values were in the vicinity of the shearing yield strength of the material, while the lowest were in the range where failure was obviously due to elastic instability and the modulus of elasticity and Poisson's ratio were the only properties of the material involved.

Figures 6, 7, and 8 show the failures obtained in the specimens tested in the Amsler torsion machine. All may be classed as instability failures, although it appears from the shape of the curves in figure 5 that the action of the shorter specimens, having D/t ratios of 77 and 104, was not entirely elastic. The reversed curvature shown for the range of low L/D ratios on these tubes is typical of that found in column curves where failures result from a combination of elastic and plastic action. Figure 4 shows one of the thinnest walled tubes (D/t = 139) photographed just before failure. Although the buckling of the tube walls was quite severe, the action in this case was apparently elastic, as the deflections disappeared when the load was relieved.

Figures 9, 10, and 11 show a comparison between the torsional strengths developed in the tests and the corresponding theoretical values. The theoretical curves for critical shear stress attributed to Donnell were computed from the following relations:

1. For short and moderately long tubes with simply supported ends, where the quantity

$$\frac{1}{\sqrt{1-\mu^2}} \times \frac{L^2 t}{D^3} \text{ is less than } 5.5,$$

$$s = \frac{Et^2}{(1-\mu^2)L^2}\left[2.8 + \sqrt{2.6 + 1.40\left(\sqrt{1-\mu^2}\,\frac{L^2}{tD}\right)^{3/2}}\right] \quad (2)$$

where L, t, and D are length, wall thickness, and mean diameter of tube, respectively, in.

s, critical shear stress, lb. per sq. in.

E, modulus of elasticity (9,600,000 lb. per sq. in. for the 51ST tubes tested).

μ, Poisson's ratio (0.33).

2. For long slender tubes where the quantity

$$\frac{1}{\sqrt{1-\mu^2}} \times \frac{L^2 t}{D^3} \text{ is greater than } 5.5,$$

$$s = 0.77\,E\sqrt{\frac{1}{(1-\mu^2)^{3/2}}\left(\frac{t}{D}\right)^3} \quad (3)$$

The critical shear stresses attributed to Timoshenko in the so-called "long-tube" range were computed from the relation

$$s = \frac{E}{3\sqrt{2}\,(1-\mu^2)^{3/4}}\left(\frac{2t}{D}\right)^{3/2} \quad (4)$$

The theoretical curves attributed to Sturm were obtained by means of the relation,

$$s = KE\left(\frac{t}{D}\right)^2 \qquad (5)$$

where values of K are shown in figure 12.

In the computation of the theoretical values of torsional strength, a condition of simply supported ends was assumed. For the sizes of tubing considered the difference between clamped and simply supported ends, according to Donnell, is only about 10 percent in the short-tube range while the end condition factor is omitted entirely in the long-tube range. Sturm's theory indicates a maximum difference between clamped and simply supported ends of about 10 percent with smaller differences for increasing lengths of tubing.

As far as the results of these particular tests are concerned, there appears to be little difference between the applicability of the torsion theories considered. Within the range of elastic instability failures, which apparently included all specimens having lengths greater than two to six times the diameter, the observed torsional strengths in most cases were within 10 percent of the theoretical values as computed by any of the equations given. As shown in figures 6, 7, and 8, the theoretical curves computed by means of Sturm's equation (5) were below those obtained by means of Donnell's equations (2) and (3), while Timoshenko's equation (4) gave results in almost exact agreement with equation (5) in the long-tube range. The experimental values shown for the tubes that failed elastically fell for the most part between the theoretical curves of Donnell, Sturm, and Timoshenko in the long-tube range and coincident or slightly above Donnell's curve in the short-tube range. It might be supposed that, since all the tests were made on specimens having at least partly fixed ends, the experimental values should lie above the theoretical curves for simply supported ends. The fact that the difference in strength for the two end conditions is relatively small, however, and that any out-of-roundness or nonuniformity in wall thickness tends to compensate for the effect of fixity at the ends, makes it difficult to formulate any definite conclusions regarding the lack of agreement between the experimental results and the theories. Sturm's solution is somewhat easier to apply than Donnell's in that one general expression covers all sizes of tubing and it is not necessary to make a length classification, although it does have the disadvantage that interpolations must be made for K values in figure 12.

It is of interest to point out that the shearing strengths obtained on the longest tubes were in very close agreement with the critical shear buckling stresses for curved plates having the same ratios of R/t, given in table 18 of the Structural Aluminum Handbook (1938). The stresses given in the handbook were obtained by a formula that is substantially the same as Timoshenko's formula for long tubes, previously referred to, and, of course, are applicable only to extremely long lengths of curved plate. For short lengths of curved plate, the values given in the handbook are ultraconservative.

CONCLUSIONS

The results of these torsion tests on several different sizes of 51ST seamless round tubing may be summarized as follows:

1. The maximum shearing stresses developed in the tubes having D/t ratios of 77, 104, and 139, for lengths equal to the diameter, were computed by means of equation (1) to be 21,800, 19,200, and 18,400 pounds per square inch, respectively. For lengths of 40 times the diameter in the same size of tubing, the corresponding maximum shearing stresses were 10,400, 7,500, and 4,800 pounds per square inch, respectively.

2. Elastic-instability failures were apparently obtained in all the tubes tested having lengths greater than two to six times the diameter. For shorter lengths, failures resulted from a combination of yielding of the material in shear and buckling.

3. The torsional strengths developed in the tubes that failed elastically were, in most cases, within 10 percent of the values indicated by the theories of Donnell, Timoshenko, and Sturm, assuming a condition of simply supported ends.

4. Sturm's theory indicated critical shearing strengths below those of Donnell in all cases but in close agreement with those of Timoshenko in the long-tube range. The test values were found to lie for the most part within the limits indicated by the different theories.

5. Although some end fixity was undoubtedly obtained in the tests, the unknown degree to which this effect was compensated for by out-of-roundness in the tubes and eccentricities of loading makes it difficult to differentiate between the accuracy of the different theories.

Aluminum Company of America,
Aluminum Research Laboratories,
New Kensington, Penna., Dec. 1, 1938.

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TABLE I

Tensile Properties of 51ST Tubing

(P.T. No. 051037-C)

Tube size	Tensile strength (lb./sq.in.)	Yield strength (0.2% set) (lb./sq.in.)	Elongation in 2 inches (percent)
1.003" O.D. x 0.977" I.D.	46,300	43,000	5.5
1.878" O.D. x 1.842" I.D.	46,600	43,500	6.0
2.500" O.D x 2.464" I.D.	46,500	42,500	7.0

TABLE II

Torsional Strength of 1.003" O.D. × 0.977" I.D. 51ST Tubing
D/t = 77

Specimen	Length / Diameter	Maximum torque (ft.-lb.)		Corresponding maximum shear stress* (lb./sq. in.)
1	1	36.0		21,600
2	1	36.5		21,900
			Average	21,750
3	2	35.7		21,400
4	2	36.5		21,900
			Average	21,650
5	4	35.3		21,200
6	4	35.5		21,300
			Average	21,250
7	8	33.5		20,100
8	8	32.0		19,200
			Average	19,650
9	16	22.5		13,500
10	16	20.8		12,500
10a	16	22.3		13,400
			Average	13,100
11	28	19.4		11,600
12	28	18.0		10,800
12a	28	17.8		10,700
			Average	11,000
13	40	18.0		10,800
14	40	16.5		9,900
			Average	10,350

*Computed for mean fiber (see equation (1))

Note: All tests made in Amsler torsion machine.

TABLE III

Torsional strength of 1.878" O.D. × 1.842" I.D. 51ST Tubing
D/t = 104

Specimen	Length / Diameter	Maximum torque (ft.-lb.)		Corresponding maximum shear stress* (lb./sq. in.)
1	1	154.0		18,900
2	1	159.5		19,500
			Average	19,200
3	2	153.5		18,800
4	2	161.0		19,700
			Average	19,250
5	4	144.5		17,700
6	4	151.5		18,600
			Average	18,150
7	8	113.0		13,900
8	8	114.0		14,000
			Average	13,950
9**	15.5	79.6		9,800
10	16	79.5		9,800
11	16	78.5		9,600
			Average	9,700
12	22	67.0		8,200
13	22	69.0		8,400
			Average	8,300
14	40	61.1		7,500
15	40	61.8		7,600
			Average	7,550
16	60	60.2		7,400
17	60	59.6		7,300
			Average	7,350

*Computed for mean fiber (see equation (1))

**Specimens 9, and 14 to 17, inclusive, were tested in the lathe in the machine shop (see fig. 1). All others were tested in the Amsler torsion machine.

TABLE IV

Torsional Strength of 2.500" O.D. x 2.464" I.D. 51ST Tubing
D/t = 139

Specimen	Length / Diameter	Maximum torque (ft.-lb.)		Corresponding maximum shear stress* (lb./sq. in.)
1	1	268		18,400
2	1	268		18,400
			Average	18,400
3	2	247		17,000
4	2	240		16,500
			Average	16,750
5	4	184		12,600
6	4	180		12,400
			Average	12,500
7	8	134		9,200
8	8	134		9,200
			Average	9,200
9	16	94		6,500
10	16	94		6,500
			Average	6,500
11**	32	70.8		4,900
12	32	71.3		4,900
			Average	4,900
13	44.8	67.5		4,600
14	44.8	67.3		4,600
			Average	4,600

*Computed for mean fiber (see equation (1)).

**Specimens 11 to 14, inclusive, were tested in the lathe in the machine shop (see fig. 1). All others were tested in the Amsler torsion machine.

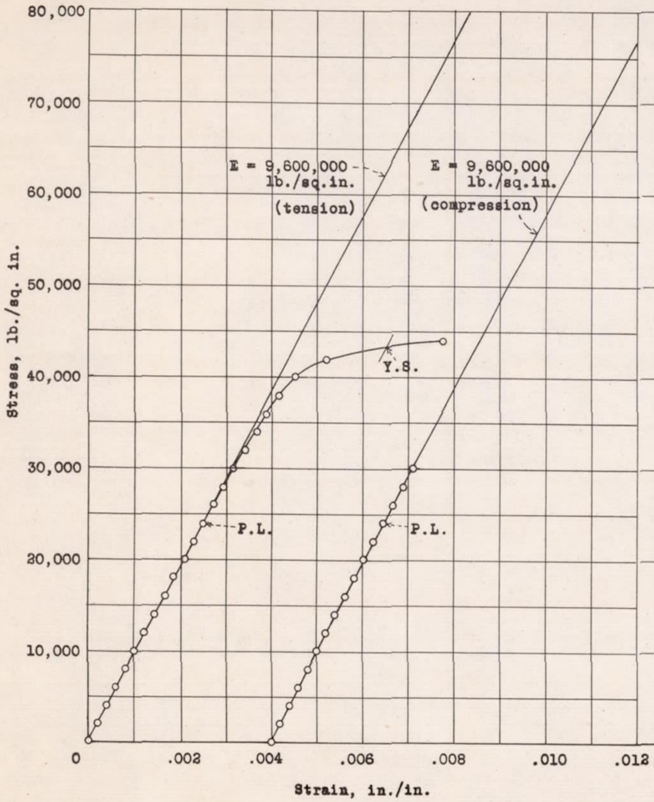


Figure 1.- Stress-strain curves for 51ST aluminum-alloy tubing 1.003 in. O.D. x 0.977 in. I.D.

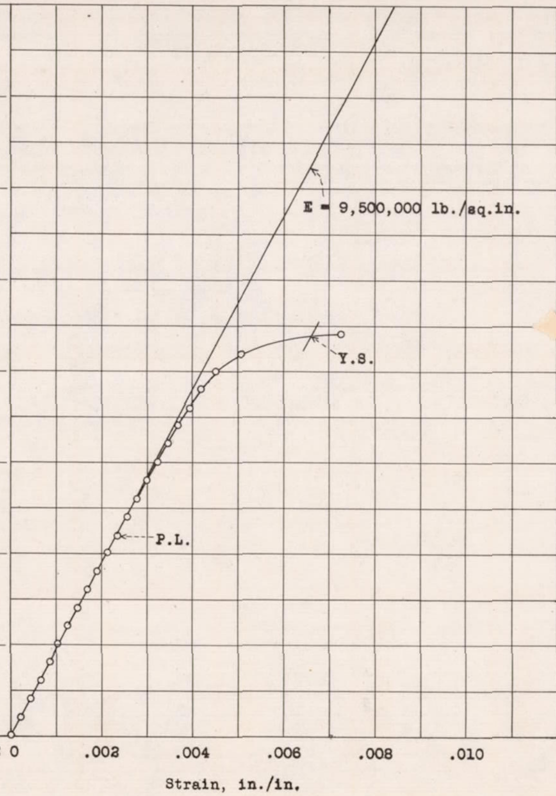


Figure 2.- Stress-strain curve for 51ST aluminum-alloy tubing 1.878 in.O.D. x 1.842. in. I.D.

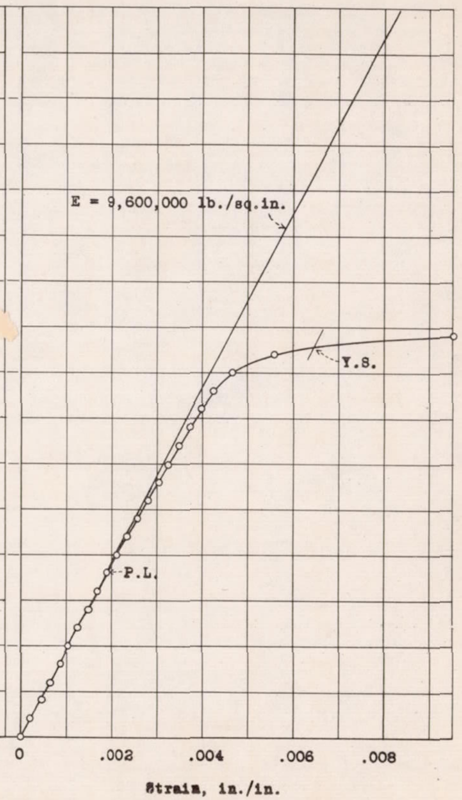


Figure 3.- Stress-strain curve for 51ST aluminum-alloy tubing 2,500 in. O.D. x 2.464 in. I.D.

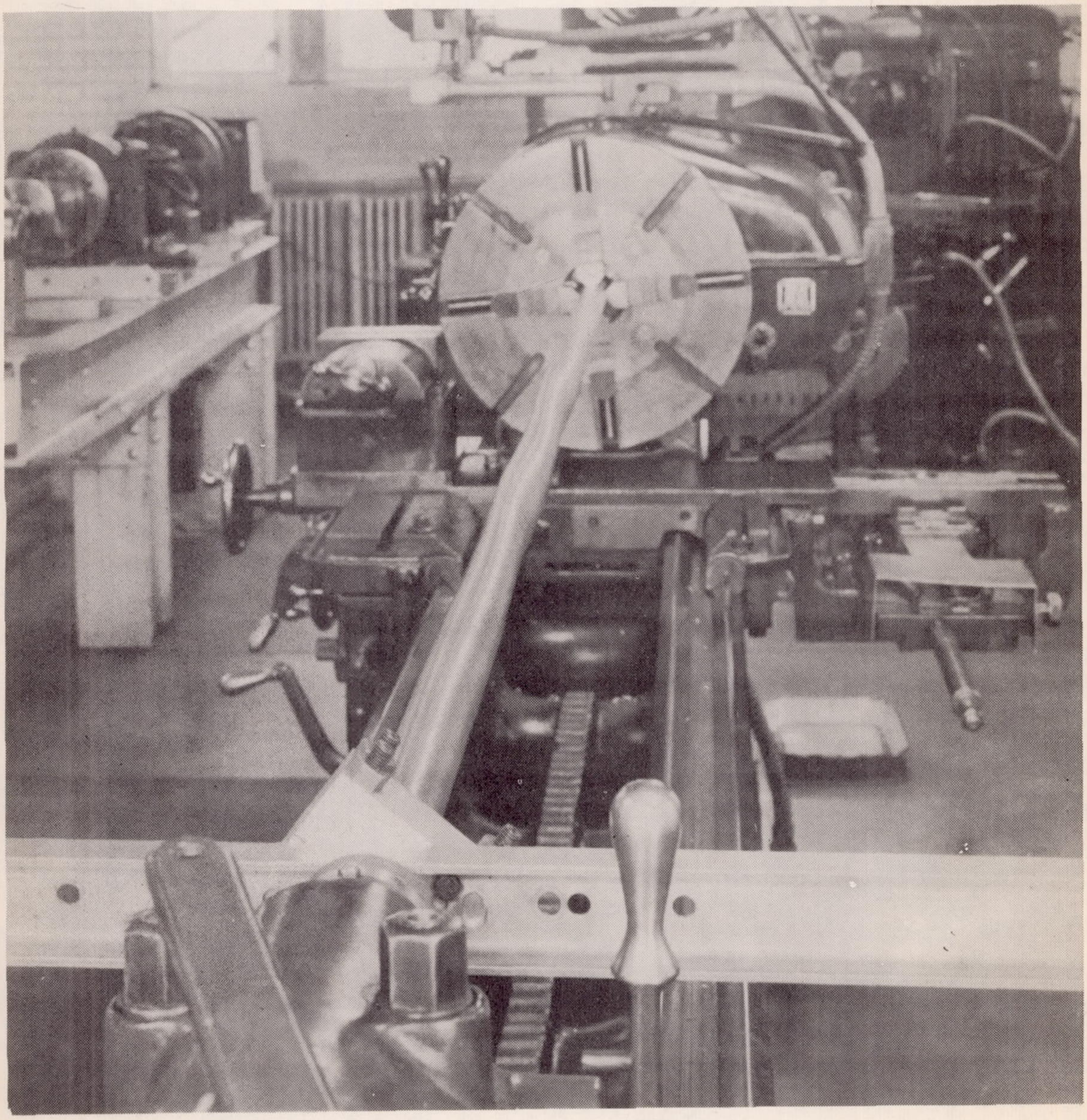


Figure 4.- Torsion test set-up in lathe. Tube D/d - 139.

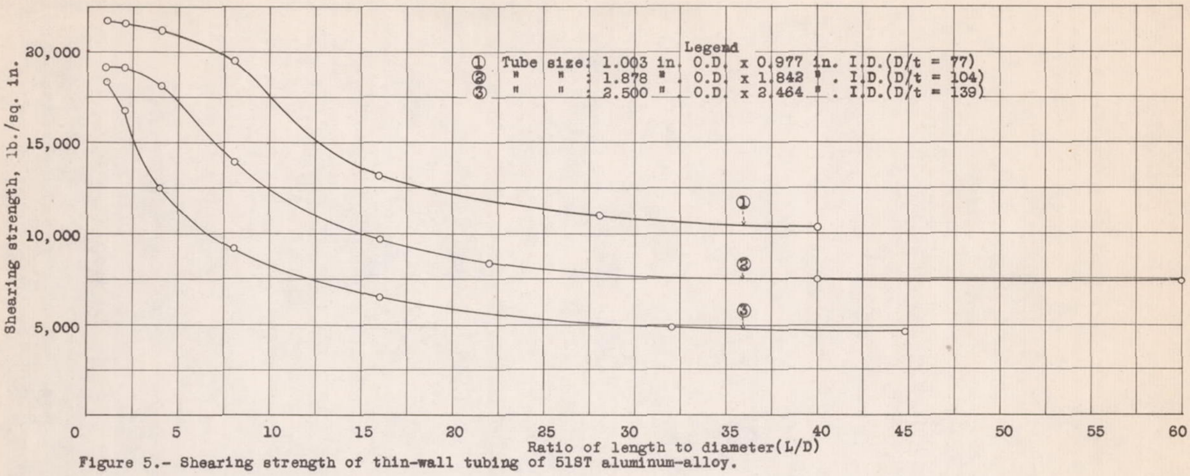


Figure 5.- Shearing strength of thin-wall tubing of 51ST aluminum-alloy.

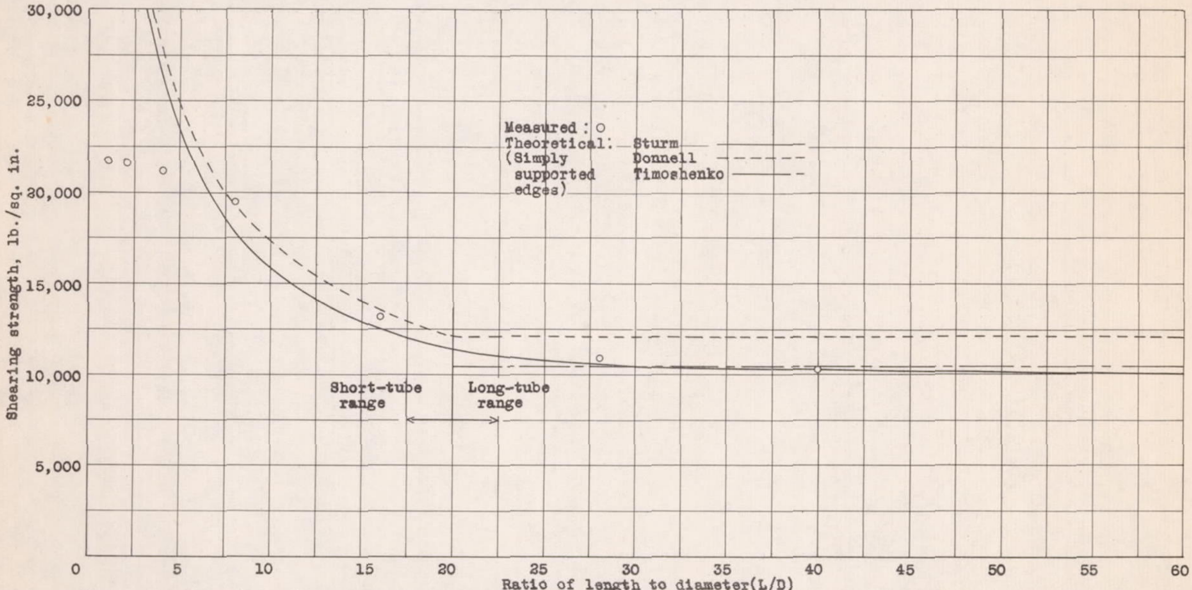


Figure 9.- Comparison of measured and theoretical values of shearing strength in torsion for 51ST aluminum-alloy tubing of 1.003 in. O.D. x 0.97 in. I.D.(D/t = 77).

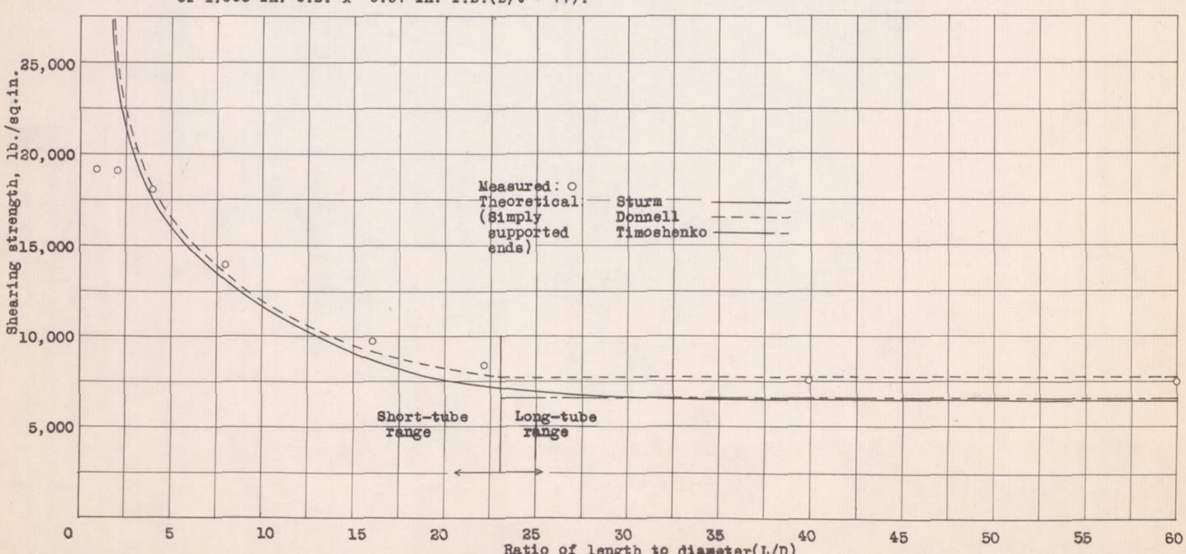
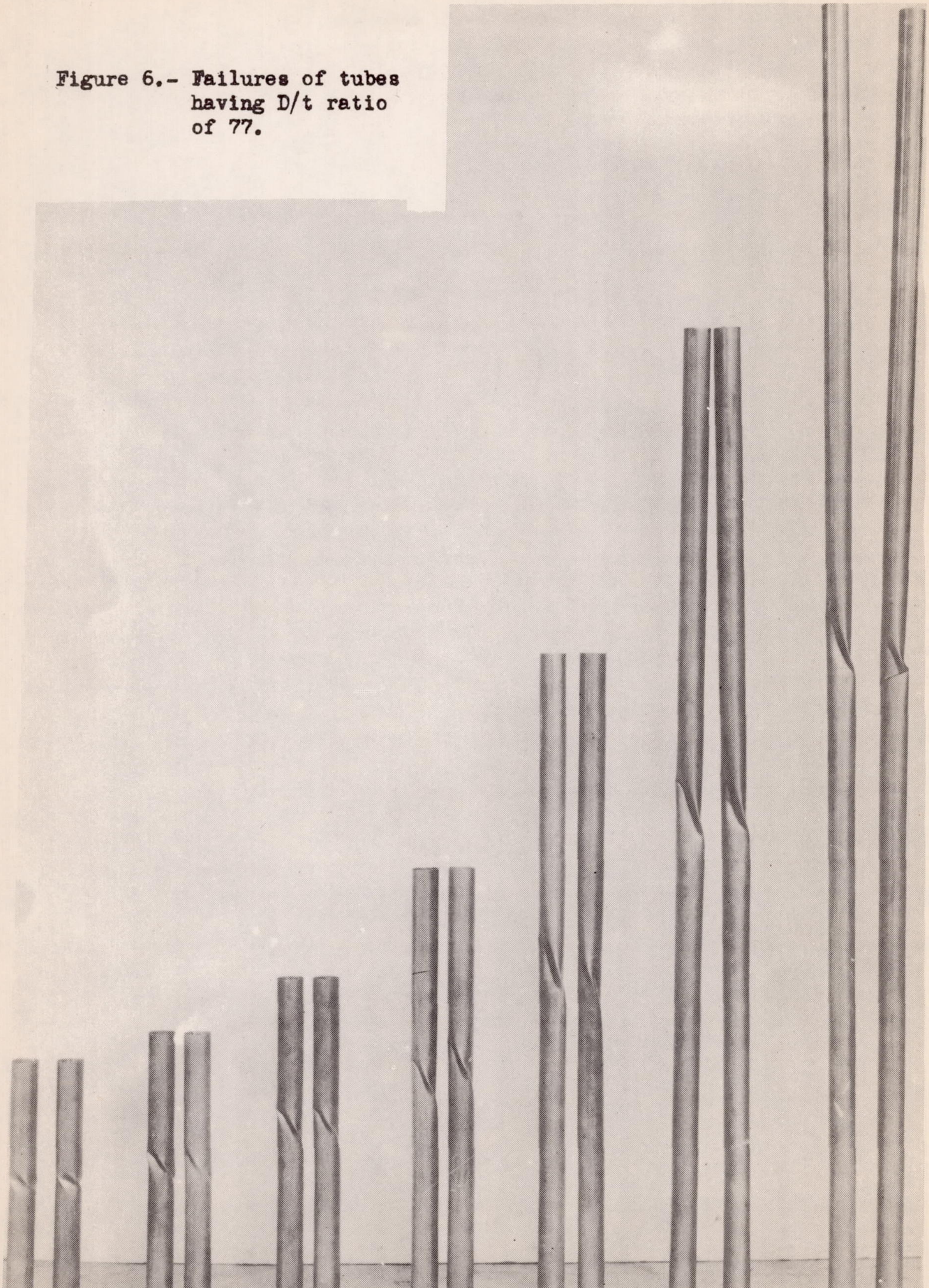


Figure 10.- Comparison of measured and theoretical values of shearing strength in torsion for 51ST aluminum-alloy tubing of 1.878 in. O.D. x 1.842 in. I.D.(D/t = 104) .

Figure 6.- Failures of tubes having D/t ratio of 77.



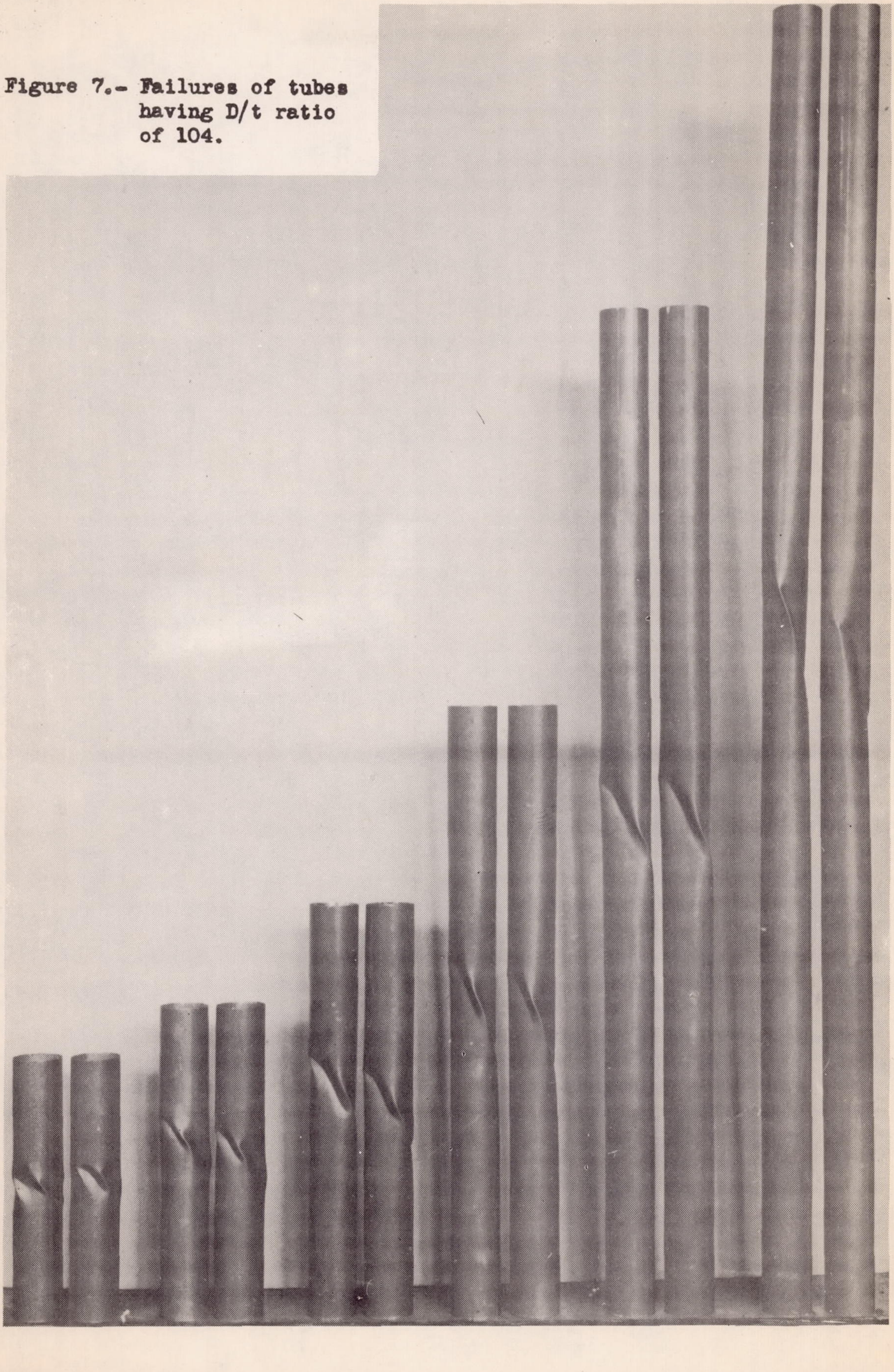


Figure 7.- Failures of tubes having D/t ratio of 104.

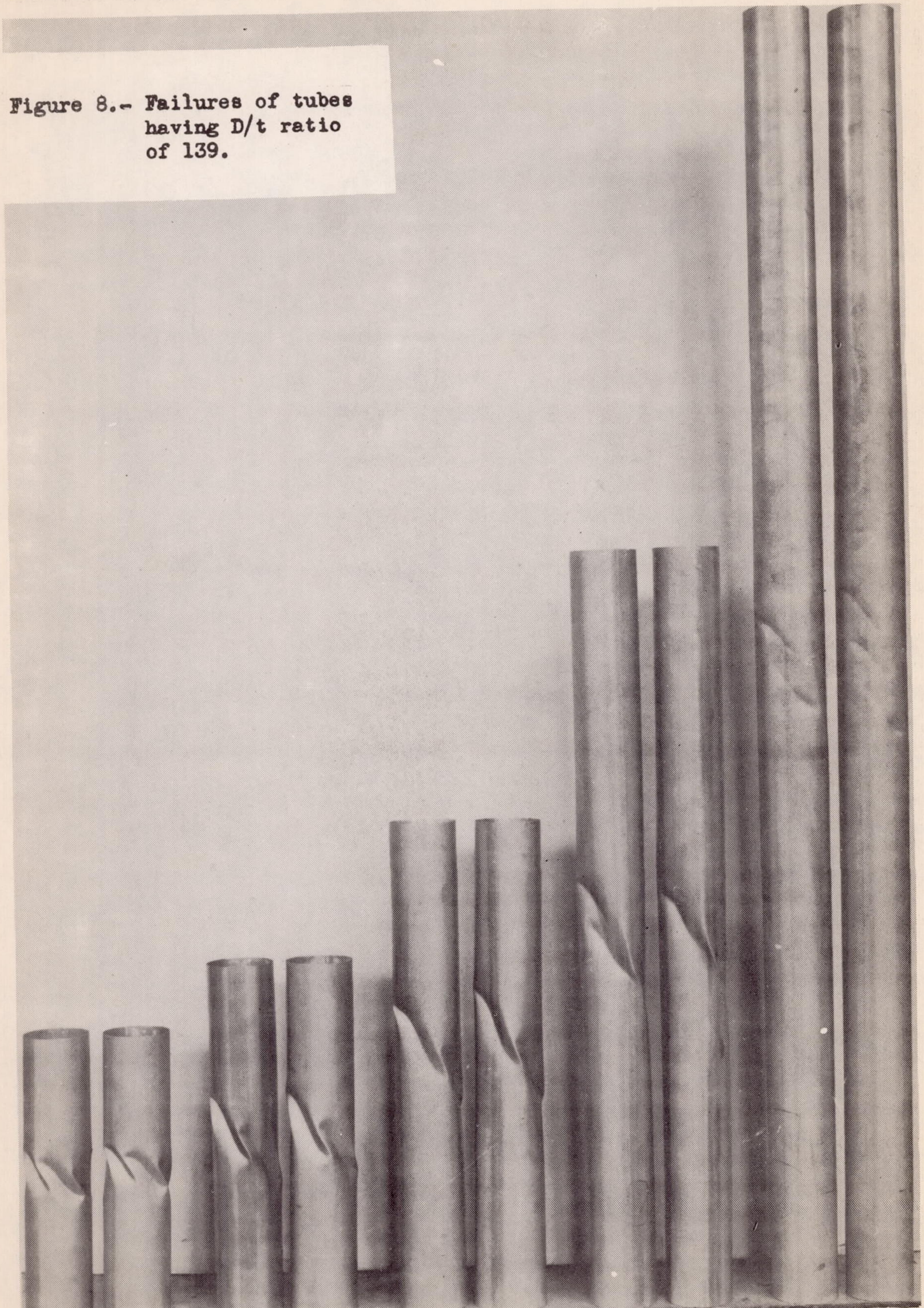


Figure 8.- Failures of tubes having D/t ratio of 139.

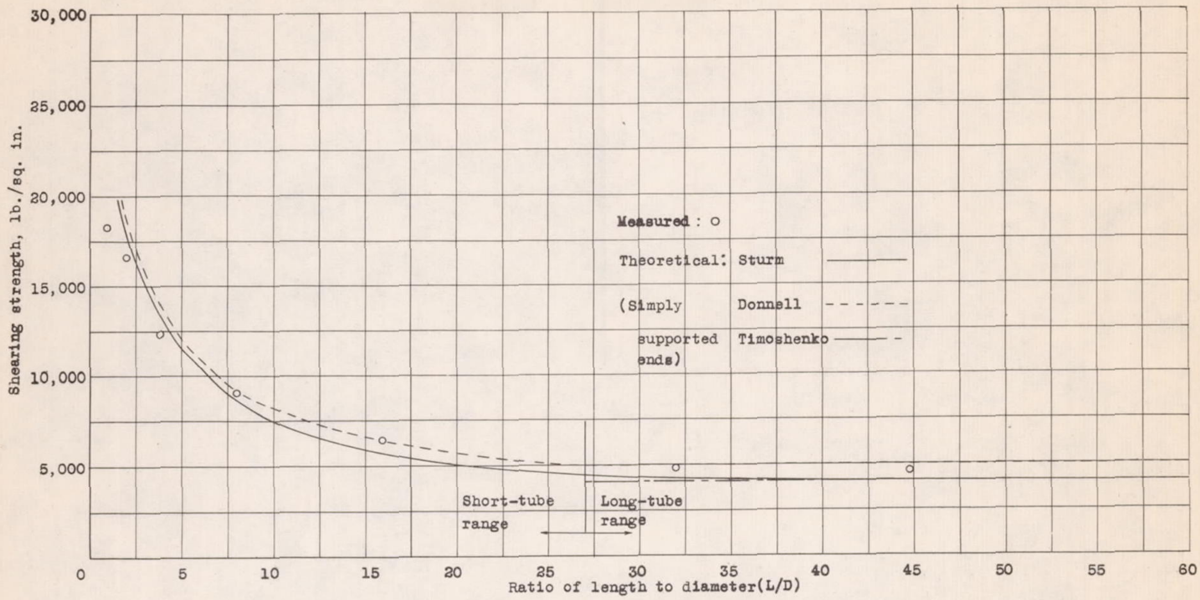


Figure 11.- Comparison of measured and theoretical values of shearing strength in torsion for 51ST aluminum-alloy tubing of 2.500 in. O.D. x 2.464 in. I. D. (D/t = 139)

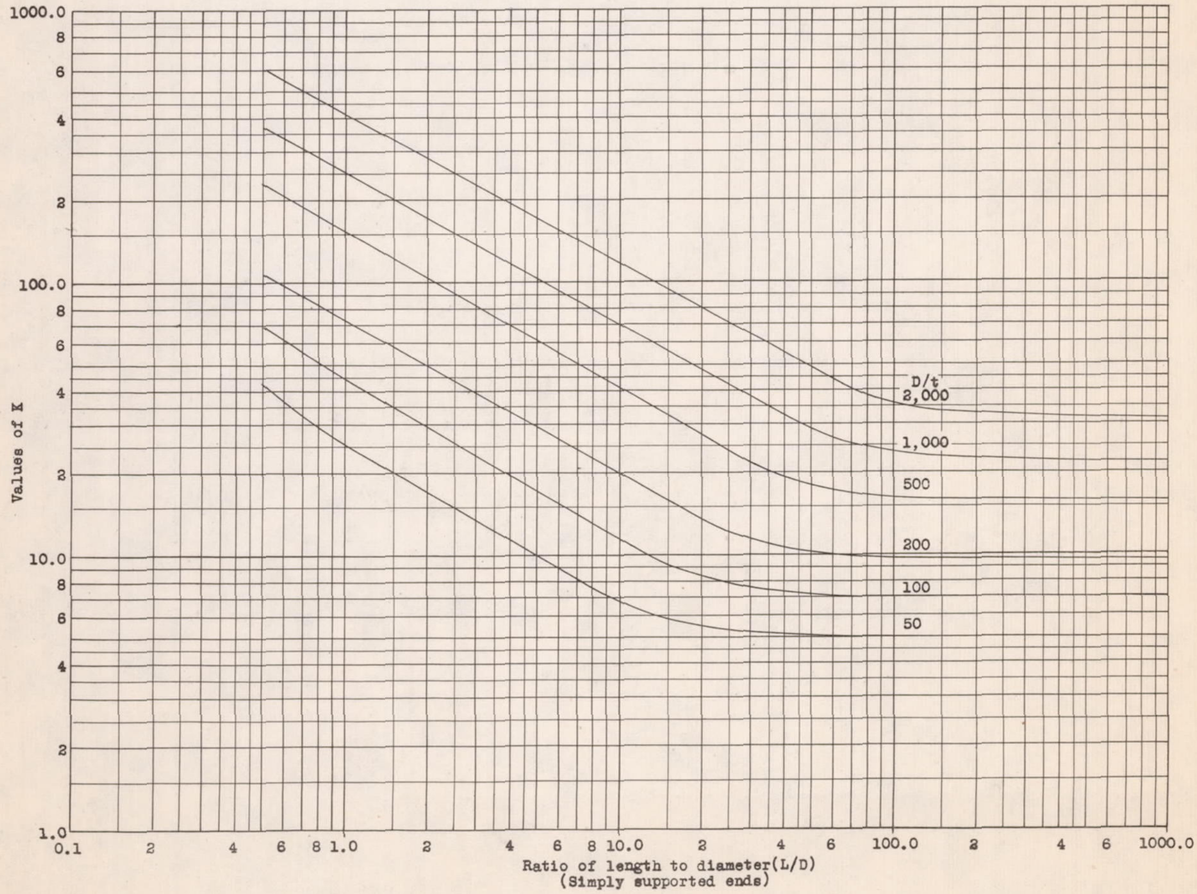


Figure 12.- Values of K for equation (5).