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ELASTIC PROPERTIES OF CHANNELS WITH UNFLANGED LIGHTENING HOLES

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SUMMARY

Fifty-eight lightened and five unlightened aluminum-alloy channels were tested as simply supported beams in pure and/or simple bending produced by loads parallel to the plane of symmetry, and fifty-three lightened and four unlightened aluminum-alloy channels were similarly tested under loads parallel to the back, in order to develop empirical formulas for the effect of unflanged lightening holes in the back on the position of the effective centroid and on the magnitude of the effective moment of inertia of the section. Forty lightened and four unlightened aluminum-alloy channels were tested as pin-ended columns to determine the effect of unflanged lightening holes in the back on the position of the effective centroid and the column stiffness. Reasonable empirical formulas for these effects were developed from the test data. An empirical formula was also developed for estimating the effect of unflanged lightening holes on the deflection of a channel due to shear deformation.

Fifty-six lightened and seven unlightened aluminum-alloy channels were tested as cantilever beams to determine the effect of unflanged lightening holes in the back on the location of the shear center, and a reasonable empirical rule to allow for this effect was developed. Additional data from these tests were studied in an unsuccessful attempt to develop a reasonable formula for the effect of unflanged lightening holes on the torsional stiffness of a channel.

INTRODUCTION

In 1934-35 an initial study of the effect of lightening holes on the elastic properties of channels was made by Mr. C. Glasgow, who tested 20 aluminum-alloy specimens in compression and in bending due to loads in the plane of symmetry. The chief result of his work was to indicate that, until the effects of unflanged holes had been more definitely determined, further study should be restricted to channels with holes of that type.

Studies of the effect of unflanged or "plain" holes were carried out by Messrs. F. C. Allen and J. C. Silliman in 1936-37, Messrs. A. J. Carah and J. W. Park in 1937-38, Mr. J. W. Scarbrough, Jr., in 1939-40, and by Mr. R. J. Wellman in 1940-41. The present report covers the work of these later investigators, as combined and analyzed by the writer.

The objectives of the tests under consideration were to determine the influence of plain round holes in the web of a channel on:

1. Stiffness against bending produced by forces parallel to the principal axes of the cross sections

2. Stiffness against torsional deformation

3. The location of the resultant axial compression compatible with zero transverse deflection

4. The position of the shear center of the cross section

These influences were not determined for all the specimens tested, but each was determined from enough specimens to permit the development of some empirical design rules.

Prosecution of the project covered by this report was made possible by the gift of test specimens from the Boeing Airplane Co. and the former Northrop Aircraft, Inc., now the El Segundo Division of the Douglas Aircraft Co., Inc., and financial support from the National Advisory Committee for Aeronautics. The writer of the present report wishes to acknowledge also his debt to the students who carried out the tests reported upon: Messrs. F. C. Allen; A. J. Carah; C. Glasgow; J. W. Park; J. W. Scarbrough, Jr.; J. C. Silliman, Jr.; and R. J. Wellman. Thanks are also due to Messrs. R. Jackson for assisting in the tests; H. Ponsford and A. E. Anderson for preparing diagrams and similar work; F. D. Banham, W. H. Cadwell, and T. J. Palmateer for constructing test equipment; and Professors M. S. Hugo, C. Moser, and S. Timoshenko for helpful advice to both the writer and the students who did the actual testing.

TEST MATERIAL

The tests covered in this report were made on a group of 17S-T aluminum-alloy channels donated to Allen and Silliman by the Boeing Aircraft Co. and a group of 24S-T aluminum-alloy channels donated to Allen and Silliman by the former Northrop Aircraft, Inc., now the El Segundo Division of the Douglas Aircraft Co., Inc.

The major dimensions of the specimens shown in figure 1 are listed in table 1. In this table and in the remainder of the report specimens furnished by Boeing are indicated by a plus sign and those furnished by Northrop, by a minus sign preceding the specimen number. In table 1 the over-all width of back B, the over-all width of side S, the lightening-hole diameter D, and the lightening-hole pitch P are nominal dimensions. The thicknesses t were obtained by weighing the specimens and computing the thickness from the weight and the developed area was obtained by assuming a density of 0.1011 pound per cubic inch. Numerous check measurements of the thickness were made with micrometer calipers; but, as there was considerable variation in the observed thicknesses of individual specimens, the values computed from the weights are considered more reliable. Although these thicknesses are recorded to three significant figures, the third figure is not reliable.

In the "lightening parameter" D^2/Pb, b is the distance $B-t$ between the midlines of the channel legs. This parameter multiplied by 25π is the percentage of the area of the back occupied by the holes.

Since the investigation was limited to the influence of holes on the <u>elastic</u> properties of the specimens, the only material properties of interest were Young's modulus E and the shearing modulus G. Tests were made to check the Young's modulus of a few of the specimens and the results varied little from the standard values used in the analysis of test results. More such tests might have been made, had it been considered that the results would have justified the trouble. The elastic properties, however, are not subject to such wide variations as properties like the yield and ultimate stresses and the objective of the study was to obtain empirical formulas that could be applied to "run of the mill" material rather than to validate a refined theoretical analysis. Furthermore, in a consultation with engineers of the National Bureau of Standards no practicable method of checking the shearing modulus of the thin flat sheets used in the specimens was suggested. It was therefore decided to make all computations on the basis of the standard values $E = 10,300,000$ pounds per square inch and $G = 3,850,000$ pounds per square inch.

APPARATUS AND TEST PROCEDURE

Types of Test

The tests were of four types:

1. Tests with the channel simply supported near each end and subjected to transverse loads acting in the plane of symmetry of the specimen

2. Tests with the channel simply supported near each end and subjected to transverse loads acting in a plane, parallel to the web, which passed through the experimentally determined shear centers of the specimen cross sections

3. Tests with the channel supported as a cantilever and subjected to a concentrated load, at the free end, acting parallel to the web

4. Tests with the channel supported between knife edges, or their equivalent, parallel to the web, and loaded as a pin-ended column

The first two types of test were used to determine the apparent stiffness EI in bending. Most of these tests were made with two concentrated loads so proportioned that the portion of the span between those loads would be subjected to "pure" bending. The remainder were made with a single concentrated load at midspan. In this report the resulting combination of shear and bending is termed "simple bending."

The third type of test was made to determine experimentally the location of the shear center, to determine the torsional stiffness of the member GJ, and to obtain an additional value of the apparent stiffness EI in bending. The combination of transverse shear and bending used in this type of test might also be called simple bending, but is termed in this report "cantilever bending" to distinguish it from the conditions existing in the first two types of test.

The fourth type of test was made to determine: (1) the apparent stiffness EI from the action of the specimen as a long pin-ended column, and (2) the position necessary for the resultant axial load if no lateral bending were to result from its application. For convenience this position is termed the "effective centroid" of the section, and the line parallel to the web of the specimen which passes through the effective centroid is called the "effective neutral axis" of the specimen.

Tests of Simply Supported Specimens

Load in plane of symmetry.- A general view of the apparatus constructed and used by Allen and Silliman for their tests of the first type is given in figure 2. The method of applying the load is shown diagrammatically in figure 3. The weight of the shot bags placed on the load pan W was transmitted by wires, the horizontal loading bar H, and the cross arms C to the loading rods A, which rested directly on the specimen. The specimen was, in turn, supported through the reaction rollers B, which rested in V-shape grooves in the cast-iron blocks indicated at K in figure 4. Near one end the specimen rested directly on the reaction roller, as shown in figure 5. Near the other end it was separated from the reaction roller by the roller pad (R in fig. 4) shown in figure 6, which allowed that end of the specimen to move horizontally without restraint. The loading rods and reaction rollers were held in the desired locations by the loading templet T. This is composed of two parallel steel plates, slotted to receive the rods and rollers, reinforced longitudinally by angle irons, and held apart by four steel spacing plates. All the tests of Allen and Silliman were made with the central portion of the specimen subjected to pure bending; so the loading rods were placed in the two slots located 4 inches from those for the reaction rollers, as shown in figures 2 and 4. The distance between the slots for the reaction rollers was 32 inches. Deflections of the specimen at or near the supports, loading points, and midspan were made by means of Ames dial gages supported from a wood plank, which was supported in turn from the main I-beam.

The testing procedure was simple. The specimen was usually so located that the lightening holes were symmetrical about midspan, the loading rods and reaction rollers located by means of the templet assembly, and the dial gages put in position to measure the vertical movements of the four rods and the web of the specimen at midspan. After a tare load of 5 or 10 pounds had been placed in the load pan to take up any "slack," each dial gage was set to read zero. Loads were added in 5-, 10-, or 25-pound increments, according to the size of the specimen, until it was estimated that a maximum stress about equal to one-half the yield point of the material had been reached. Since there seemed to be a slight amount of friction between the loading rods and the edges of the templet slots, those rods were lightly tapped after each load increment before the dial-gage readings were recorded. It was found that this tapping made it possible to obtain much straighter load-deflection diagrams from the recorded data. After the maximum desired load had been reached, the specimen was unloaded in equal steps and the dial-gage readings were recorded. In this manner the deflection readings for each load were checked. A sample data sheet is shown in table A1.

In order to compute the stiffness EI from the observed deflections, the readings of dials 2 and 4 at the loading points were subtracted from those of dial 3 at midspan. The average of these differences was then taken as the deflection of the point at midspan from a straight line joining the points of load application. These deflections were next plotted against the loads producing them, and the slope of the straight line determined by them was computed. The value of δ/W thus obtained was then inserted in the appropriate beam deflection formula, which then was solved for EI. The resulting values of EI_{xx} are recorded in tables 2 and 3.

Essentially the same testing apparatus and procedure were employed by Wellman in tests with the plane of loading normal to the back of the channel. A somewhat different procedure, however, was used for computing the "observed EI" from the observed deflections. Instead of plotting the differences between the midpoint deflection and the average of the load-point deflections, a separate curve was plotted for the deflections of each of those points and also those of the reaction points under dials 1 and 5. The value of δ/W inserted in the beam deflection formula was then the difference between the slope of the line plotted from the deflections at dial 3 and the average of the δ/W values obtained from the lines representing the deflections at dials 2 and 4. The stiffnesses EI obtained in this manner are termed "pure bending stiffnesses." In addition, the value of δ/W for the midspan was subtracted from the average of the δ/W values for the support points and the result inserted in the appropriate beam deflection formula, which was solved to obtain the value of EI termed the "two-load bending stiffness." Both these values are listed in tables 2 and 4.

Load parallel to web.- For the tests on simply supported beams with the plane of loading parallel to the web, it was necessary to make some minor changes in the test apparatus and procedure. Had the loading rods and reaction rollers rested directly against the specimen, the plane of loading would not have passed through the shear centers of the cross sections and the specimen would have been subjected to torsion as well as bending. This was circumvented by the use of the loading frames shown in figure 7. These frames were made of square-section steel bars held together with machine screws. Hardened knife edges were inset in the upper and lower members in such position that the specimen could be located with its shear center on the line joining the knife edges. The specimen was held in the desired position with reference to the frame by machine screws and blocks of synthetic resin, as shown in figure 7. In the tests the loading rods rested on the upper knife edges of the frames at the loading points. At the reactions the lower knife edge of one frame rested on the reaction roller, while that at the other end rested on the top of the roller pad. When the specimen was loaded and supported in this manner, it was unstable with respect to rotation about a longitudinal axis. To prevent it from rolling over, the vertical guides shown in figure 8 were clamped to the I-beam near each end of the specimen.

Except for the use of the loading frames and end guides, the apparatus and procedure of Carah and Park for tests in pure bending with the plane of loading parallel to the web were essentially the same as those of Allen and Silliman for tests with the loads normal to the web. One minor change also was made by Carah and Park in the determination of the effective EI from the test data. Allen and Silliman measured the deflection of the points of load application by dial gages measuring the vertical movements of the loading rods. Carah and Park attached small synthetic-resin blocks to the channel web with their upper surfaces at midheight of the specimen. Since the loading frames made it impossible to place these blocks exactly at the loading points, they were placed a little closer to the midspan, so that the dial-gage spindles would clear the loading frames. The formula for computing EI from the deflections was suitably modified to allow for the actual distance between the points at which the deflections were measured. Scarbrough and Wellman made their tests in pure bending with the loads parallel to the web in essentially the same manner as Carah and Park. Their chief modification was to omit the use of blocks attached to the web for measuring deflections and to measure the deflection of points on the upper flange as close as possible to the web.

In addition to the tests in pure bending, Carah and Park, Scarbrough, and Wellman made tests in simple bending with the loads parallel to the web. In these tests the horizontal loading bar was dispensed with and the load pan hung from a single loading rod placed in the slots at the center of the templet. In these tests it was necessary to measure the deflections at but three points, at each reaction, and near the loading point at midspan. The method of determining apparent EI from the test data was essentially the same as in the pure bending tests; the pertinent dimensions and the observed values of δ/W were inserted in the appropriate beam deflection formula, which was solved for EI.

The values of apparent EI obtained in both pure and simple bending tests are recorded in table 5. Wellman also used data from his tests, made primarily to obtain the stiffness in pure bending, to obtain stiffnesses in "two-load bending" similar to those he obtained from the tests with the loads normal to the back of the specimen. These results also are recorded in table 5.

Cantilever-Beam Tests

In the third type of test one end of the specimen was clamped to a heavy vertical steel column in such a manner as to minimize possible rotation at that end. At the free end a T-shape fitting had its "vertical" member bolted to the web at midheight, the cross bar forming a horizontal platform on which the load could be applied at varying distances from the web of the specimen. The load pan was suspended from a steel loading bar which rested on the cross bar of the T-shape fitting. In order to have single-point contact between these two members a small hole was drilled in the loading bar, into which a bearing ball was forced. The position of the bearing ball with respect to the web of the specimen was measured by a micrometer screw attached to the loading bar. In the first group of these tests, those made by Carah and Park, the deflection of the free end of the specimen was measured by an Ames dial gage supported from a platform resting on the floor of the laboratory. In the same tests the rotation of the specimen was determined from the vertical movement of the ends of a steel rod passing through, and normal to, the web a short distance from the free end. These deflections were measured by dial gages supported from the same platform as the gage measuring the deflection of the end of the specimen. The arrangement of these gages and other apparatus at the free end of the specimen is shown in figure 9.

The specimen was first clamped to the vertical support with its web in a vertical plane and its longitudinal axis horizontal. The dial gages were then set to zero and load applied in small increments. After each increment of load the channel was tapped lightly to eliminate friction effects, and the load bar was shifted in position by the micrometer screw until the deflections of the ends of the transverse rod were equal. When the twist of the specimen had been thus eliminated, it was considered that the point of load application was coincident with the shear center of the cross section. The position of the point of load application and the dial-gage readings were then recorded, as shown in table A2. A load-deflection curve was plotted as the test proceeded and care was taken to keep the imposed load below the magnitudes that might cause yielding of the material or buckling of the member. In these tests it was noticed that the position of the shear center seemed to change slightly under low loads but eventually reached a stable position, as shown by the curve of figure A1. The shear-center distances recorded in table 5 are based upon the locations at which the larger loads produced torsionless bending. The apparent EI in cantilever bending was obtained by inserting the slope of the load-deflection curve in the appropriate beam-deflection formula. The results of these computations are also included in table 5.

A few tests were made by Carah and Park to determine the critical load under cantilever loading. With the channel firmly clamped in place as a cantilever beam, the loading device was fixed in position. As in the shear-center tests the web was adjusted to a vertical position and the channel leveled. The two side gages were then placed under the cross bar and set at zero with the free end of the specimen under no load. Since the shear center tended to shift slightly under low loads, increments of weight were added and the load position adjusted each time until no further shift was necessary, as determined by equal deflections of the two side gages. These gages were then removed and the load increased until the channel buckled. Deflections were not measured in these tests.

In order that the specimens used in these tests would not be permanently damaged, a platform of shot bags was built up to about one-half inch of the lower surface of the specimen. This caught the member after buckling took place and prevented permanent deformation of the aluminum-alloy specimens. This precaution appeared to be effective since the members were not damaged by the buckling, but, upon releasing the load and applying it a second time, the critical load was found to be practically unchanged.

In parallel tests to determine the critical load, the weight was applied at the centroid of the section as determined by computation. Otherwise the tests were carried out in the same manner as those with the load applied at the shear center. The critical loads found in these two groups of tests were observed and are listed in table 6.

Scarbrough and Wellman's cantilever-beam tests were made with the same apparatus, except for the deflection and rotation measuring systems, as those of Carah and Park. The angle of torsional rotation was measured by the use of a telescope and vertical meter stick attached to a steel tripod located several feet from the loaded end of the specimen and a mirror glued to the loaded end of the specimen opposite the scale. The scale reading was reflected from the mirror to the telescope as shown in figures 10 and 11.

In order to preclude errors due to rotation of the specimen at the supported end, the deflections of the free end were measured from a reference bar attached to the web of the specimen a small distance from the supported end, as shown in figure 12. When the member deflected under load, the reference bar remained parallel to the line tangent to the elastic curve at the connection point. To obtain sufficient rigidity the reference bar and the vertical member connecting it to the web of the specimen were braced by a diagonal member, as can be seen from figure 10. In order to measure the deflections of the free end of the specimen with respect to the reference bar, a standard micrometer screw was set vertically in a steel block bolted to the end of that bar. Contact between the screw and the specimen was indicated by the closing of a $1\frac{1}{2}$-volt electrical circuit, as shown in figure 13. A pointed cap mounted on the end of the micrometer screw contacted mercury in a small basin attached to the top flange of the test specimen so as to complete the circuit and light a flashlight bulb.

In starting a test the point of application of the load was set near the expected location of the shear center, and the distance from the back of the channel was measured with the micrometer screw and recorded. Load increments of from 2 to 5 pounds were applied, readings on the scale were made through the telescope and recorded, and a "load-rotation" (actually a load against scale reading) diagram was plotted as the test proceeded. The load was carried only to values which would cause no buckling of the flanges. The test was then repeated with the load application point reset, preferably on the opposite side of the shear center (so as to cause rotation in the opposite direction). Typical load-rotation diagrams are shown in figure A2. In order to find the position of the shear center, the slopes dR/dW of the lines plotted from these tests were plotted against the distance from the back of the channel to the point of load application. Since the torsional rigidity of each particular channel is a constant, a straight line drawn between the two points should intersect the axis (zero slope) at the shear-center distance, as shown in figure A3. The point of load application was therefore set at the location determined in this manner and the test repeated. If any rotation occurred, and it usually did, the point of load application was again moved slightly and the test repeated until any rotation indicated by the scale readings was negligible. This system of measuring rotations not only was more sensitive than that of Carah and Park but also lent itself to a determination of the apparent torsional stiffness of the channel.

The observed torsional stiffness of the specimens is reported in table 5 in the form of values of M_t/θ, in which θ is the rotation in radians of the mirror near the free end of the specimen produced by a constant torsional moment M_t. In computing these figures M_t was taken as the product of a convenient load increment ΔW and the distance d' from the experimentally located shear center to the actual point of load application. As can be seen from figure 11, for the small rotations encountered, 0.1 radian or less, θ could be obtained from the relation $\theta = \Delta R/2L_r$ where ΔR is the change in meter-stick readings produced by the imposition of ΔW, and L_r is the distance from the meter stick to the mirror attached to the specimen. Use of these expressions produces the relation

$$\frac{M_t}{\theta} = 2\, L_r d' \frac{\Delta W}{\Delta R} \qquad (1)$$

in which L_r must be measured in the same units as ΔR. Since the separate tests on a single specimen did not always give the same value for $d'\Delta W/\Delta R$, this quantity was computed for each position of the load and the average used in the expression for M_t/θ to get the value given in table 5. Where there was considerable spread in the magnitudes of $d'\Delta W/\Delta R$, those deviating excessively from the mean were neglected in computing that value. Usually this meant neglecting values obtained from tests in which d' was relatively large.

Column Tests

The column tests of Allen and Silliman were made in a hand-operated 20,000-pound-capacity Olsen testing machine, with the specimen located between knife edges parallel to the plane of its web. In order to permit controlled changes in the distance between the plane of the knife edges and the plane of the channel web, the end fittings shown in figure 14 were used. In this fitting the lower plate rests directly on the knife edge. The upper plate slides on the lower; its movement normal to the knife edge is effected by the screw, the head of which appears in the figure at the right-hand side of the fitting. The horizontal distance from the back of the channel to the plane of the knife edges is measured by the micrometer supported from the lower plate. The deflection of the specimen at midspan was measured by the dial-gage arrangement shown in figure 15. The gages were mounted on the operating screws of the testing machine in such a manner that the tips of the spindles opposed each other. This practically eliminated the undesirable unbalanced side load which the spring in a single dial gage would have produced. When the effective neutral axis had been approximately located, however, these gages were removed so the ultimate load would be unaffected by uncertain midspan conditions. The ends of the specimens were embedded in type-metal pads to prevent local failure of the thin sections being tested. It was found later that such pads were unnecessary and they were omitted in Wellman's tests.

In testing, the specimen was carefully located in a vertical position with its web parallel to the knife edges and subjected to a tare load of about 70 pounds. The channel was then moved by manipulation of the end fittings until the knife edges were in line with the estimated position of the effective neutral axis. Additional load was then imposed and the amount and the direction of the midspan deflection were noted. The load was then reduced to the tare value and the position of the specimen with respect to the knife edges changed so as to reduce the eccentricity loading. This procedure was repeated with the specimen subjected to larger and larger loads as the eccentricity was reduced until the position at which there would be no transverse deflection prior to buckling had been bracketed within a range of 0.010 inch or less. At this stage the midspan dials were removed and the specimen tested to buckling failure.

Wellman, in his column tests, used different apparatus from that employed by Allen and Silliman. His specimens were strained by the 200,000-pound Riehle testing machine and the axial loads were measured by the combination of the 20,000-pound-capacity Emery hydraulic capsule and Bourdon tube gage employed in the tests reported in reference 1. A general view of this arrangement is shown in figure 16. He also constructed a new pair of end fittings.

In the Allen and Silliman tests the "pin-ended length" of the specimen, measured between the knife edges, exceeded the actual length of the specimen by about $1\frac{1}{4}$ inches. In order to eliminate the uncertain effect of the stiffness of the end fittings on the Euler load of the specimen, Wellman used the fittings, similar to those described by H. Barlow in reference 2, which are shown in figure 17. Each fitting consisted of a round steel loading bar mounted between two ball-bearing rings, which in turn were held by a steel U-frame. The loading bar was notched to a depth 3/16 inch greater than one-half the diameter of the loading bar. This allowed the top surface of a movable platform made of hardened steel, 3/16 inch thick, to rest at a depth of exactly one-half the diameter of the loading bar, so as to permit the location of the neutral axis of the specimen on the center line of rotation. The actual movement of the platform was effected by means of the adjusting screw shown just below the micrometer in figure 17. On one side of this movable platform was a raised edge 1/8 inch high, against which the back of the specimen rested. The position of the channel with respect to the center of rotation of the cylinder was measured by means of a micrometer shown in figure 17, just above the platform adjusting screw. Arms to carry a weight to counterbalance any initial moment set up by the micrometers were attached to the ends of the movable cylinders. In using this fitting, the effective pin-ended length of the column was the distance between loading platforms and therefore just equal to the length of the specimen.

Aside from the use of the new end fittings and the omission of the type-metal pads on the ends of the specimens, Wellman's procedure was the same as that of Allen and Silliman. In both sets of column tests, the major recorded quantities were the critical load in pounds and the distances from the reference point to the back surface of the specimen near its ends. The former was inserted in the Euler formula for pin-ended columns

$$EI_e = P L^2/\pi^2 \qquad (2)$$

to determine the "column stiffness." From the latter the distance from the surface of the back of the specimen to the effective neutral axis Y_o was readily obtained. The observed values of EI_e and Y_o are listed in table 2.

TEST RESULTS

Properties in Bending about X-X Axis

Values of EI_{xx} obtained from tests in bending with the plane of loading perpendicular to the back of the specimen are listed in tables 2, 3, and 4. Most of these tests were made with the downward loads applied at the free edges of the flanges and the point midway between those loads also midway between the centers of adjacent holes. The results of the tests with the specimen in this "back down pitch centered" position are listed in table 2. In this table the values obtained by Allen and Silliman for EI from their tests in pure bending are shown in column 2. Columns 3 and 4 give the values of EI obtained from Wellman's tests. Those in column 3 are his two-load bending, while those in column 4 are his pure-bending values.

In addition to the tests with the specimen in the back-down pitch-centered position, Wellman tested five specimens in three other positions: (1) back down, holes centered --that is, with the midpoint between the down loads opposite the center of a lightening hole; (2) flanges down, pitch centered; and (3) flanges down, holes centered. The values of EI obtained from all four tests of each of these specimens are shown in table 4. Allen and Silliman ran a few tests with the specimens in the back-down hole-centered position. They did not report the numerical results, but stated that the stiffness in the hole-centered position was less than the stiffness in the pitch-centered position and that the difference was of the order of 3 percent.

In order to investigate the possibility that appreciable error in the observed values of EI might result from local deformation of the channel flanges under the load rollers, Allen and Silliman made several tests in which the deflections were measured at midspan and at points 2 inches inboard from the load rollers. The values of EI obtained from these tests and the corresponding tests by their standard method are listed in table 3.

The values of stiffness EI_e and distance from the surface of the back to the effective neutral axis Y_0 as obtained from the column tests are also shown in table 2. The results of Allen and Silliman's tests are listed in columns 5 and 7 and those of Wellman, in columns 6 and 8.

Properties Determined from Loads Parallel to Web

The major results of the tests with the loads acting parallel to the specimen webs are listed in table 5. In this table the results obtained by the different experimenters are placed on separate lines instead of in separate columns. The source of the data is given in column 1 and the footnotes to the table. The observed values of EI_{yy} listed in columns 2, 3, 4, and 5 are those obtained from tests in pure bending, simple bending, two-load bending, and cantilever bending, respectively, by the methods described in the section on Test Apparatus and Procedure. Since no allowance was made for deflection due to shear deformation in computing these quantities, only those obtained from the tests in pure bending should be taken to represent the true stiffnesses EI of the channels. The remainder are "apparent" values, which are useful as measures of the variation of deflection with load for the specific loading patterns employed. Columns 6 and 7 show the observed and computed distances from the midline of the back to the shear center and column 8 the differences between those distances. The computed shear-center distances of column 7 were obtained from equation 6:17 (on p. 162 of reference 3), the existence of the lightening holes being neglected. In column 9 are the torsional stiffnesses M_t/θ obtained by the methods previously described.

In table 6 are listed the critical loads obtained by Carah and Park in their cantilever tests and some simple ratios of these critical loads which indicate the effect on the strength of the specimen of changing the position of load application.

PRECISION

Each experimenter analyzed his test procedures, estimating the degree of precision of and the probable error in each of the readings taken, and determining from those data the probable precision of the end products, apparent values of EI, shear-center distances, and so forth. All these studies indicated that the values of apparent stiffness obtained from the tests would be correct to within 5 percent or less and the distances correct to within 0.01 inch or less. Comparison of the results obtained by different observers or in different tests of the same specimen and the studies made to develop empirical formulas for the properties under consideration, however, show that the precision of the experimental work was not uniformly that good.

Since the analyses of probable precision first mentioned proved to be over-optimistic, they are not included in this report. The following discussion is therefore devoted primarily to a study of the divergences found between the work of the different experimenters. From this discussion it will appear that, while enough work of satisfactory precision was done to permit the formulation of better rules for practical design than now exist, the basis of these rules is not so sound as would be desirable. The lack of precision of many of the tests also made it impracticable to obtain reliable information on the effects of some of the variables that it would be desirable to study in more detail.

Bending Tests - Load Normal to Plane of Channel Web

Comparisons of bending tests made of the same specimen by Allen and Silliman and by Wellman show considerable differences in apparent EI. Such comparison was possible on 15 specimens and the values of EI obtained from the Wellman tests ranged from 2.5 percent below to 8.2 percent above Allen and Silliman's figures. The arithmetic mean difference was 3.48 percent and the algebraic mean difference 3.15 percent. (In this report the adjective "arithmetic" is applied to mean and median values computed from the absolute magnitudes of a group of quantities. Use of the term "algebraic" indicates that the signs of the individual quantities were considered in the computation of a mean or median.) Both arithmetic and algebraic median differences were 3.2 percent. It should be noted that for only one specimen (-19) was the stiffness obtained from Wellman's test less than that reported by Allen and Silliman.

Allen and Silliman reported that they made a number of check runs of individual specimens and found that the results did not deviate from a mean by more than 1.75 percent. Wellman did not report the results of check runs under the same conditions but he did report the results shown in table 4 of tests with the specimens in four different positions in the jig. These showed variations of EI as large as 14 percent without any discernible relation between change in apparent EI and specimen position. Allen and Silliman, on the other hand, reported that, when the specimen was tested with a hole at midspan, the apparent EI was about 3 percent less than when the midspan point was midway between the centers of adjacent holes. These facts make it appear that the Wellman results were not as precise as those of Allen and Silliman, and they are not given as much weight in the following discussion.

In his first computations of EI from the test logs Wellman used the same method of computing W/δ as had been employed by Allen and Silliman; the deflection of the point at midspan was subtracted from the average of the deflections at the loading points. As this did not give satisfactory results, he modified his procedure by finding the slope of the load-deflection curve for each point at which deflections were measured and combining those slopes to get the value of W/δ to insert in the beam deflection formula. It was hoped that in this way the effects of individual poor readings would be minimized.

Another attempt to meet the situation was the computation of the two-load bending stiffnesses from the slopes of the load-deflection curves of midspan and the reaction points. In this manner the differences used were approximately doubled and the results made more consistent and probably more reliable. Comparison of the pure bending and two-load bending values of EI obtained from the Wellman tests is of interest. In 48 tests the two-load bending EI ranged from 0.920 to 1.082 times the pure-bending value, the average ratio being 0.9928, with almost equal division between values above and below 1.00. Since there would be shear deformation of the channel legs in the end segments, the observed two-load bending EI should be less than that for pure bending; but, since the ratio of span to depth of section was large, the difference should be small. Insofar as the average ratio is concerned, the difference of less than 1 percent is very reasonable; but the larger differences for the individual specimens, particularly those in which the two-load-bending figure was the larger, can be best explained as due to a lack in precision of technique. It was therefore concluded that for these tests the effect of shear deformation should be neglected and the observed values of two-load bending stiffness used in the more detailed analysis of the data rather than the pure-bending figures. It was these two-load bending stiffnesses that were compared with the Allen and Silliman results in the preceding discussion. Had the pure-bending figures been used, the differences would have ranged from -4.1 to 9.1 percent and the mean figures would have increased from 3.48 to 4.57 percent and from 3.15 to 3.91 percent and the median from 3.2 to 4.1 percent or 3.7 percent, according to whether arithmetic or algebraic averages are obtained.

The tests made by Allen and Silliman to determine the possible effect of local deformation of the specimen under the loading rods resulted in the observed EI values of table 3. Since the differences between comparable pairs of these values were all less than 2 percent and were not consistently in the same direction, it is reasonable to assume that the effect of such local deformation was negligible.

Column Tests

Seven of the channels tested by Allen and Silliman were retested by Wellman, and the indicated values of EI in the retests ranged from 2.6 to 22.5 percent in excess of those in the first tests, the average increase being 10.81 percent and the median, 7.6 percent.

One possible cause of the differences between the results from tests of the same specimen is a difference in the method of determining the critical load. In some of Allen and Silliman's tests, it was noticed that after the peak load was reached the channel bowed suddenly, with a resultant decrease in the equilibrium load of the specimen. Figure 18 illustrates diagrammatically the implied load-deflection relationship. The conclusion was reached that the load registered before the sudden deflection was somewhat higher than the actual Euler load. The existence of this peak load was attributed to two factors: (1) While the knife edges were assumed to be theoretically perfect, they offered a small restraining moment on the ends of the specimen and thus made the restraint coefficient somewhat greater than unity. As the channel deflected, however, this restraint was eliminated and a decrease in the load required to hold the specimen in equilibrium in its bent configuration resulted. (2) Before failure the major axis of the channel may have been misalined with respect to the knife edges. Bending would then occur around an oblique axis about which the moment of inertia would be relatively large. In the process of deflecting, the channel would rotate slightly with the result that the bending would finally be about the major axis and the load required for equilibrium in the bent configuration would be correspondingly reduced. In Allen and Silliman's tests, the ultimate load recorded, therefore, was that maintained by the specimen after it had begun to exhibit definite lateral deflection. Wellman did not follow this practice but considered the maximum load carried prior to buckling as the Euler load for each specimen. This may well account, at least in part, for his obtaining higher values of critical load than Allen and Silliman.

While this difference in the determination of the critical load might account for the smaller discrepancies between Wellman's and Allen and Silliman's figures, it appears unlikely that it is sufficient to explain completely the larger discrepancies. If, however, in some of Wellman's tests there was serious lack of parallelism between the axes of rotation of the end fittings, that might account for his high values of critical load. That it was probably Wellman's rather than Allen and Silliman's test results that were most in error is indicated by the fact that the results of Allen and Silliman are easier to correlate with theory. In the development of rules for predicting EI, Allen and Silliman's data are therefore given more weight than those of Wellman.

Allen and Silliman noted that the magnitude of the apparent critical load for a given channel was dependent on whether the direction of buckling caused an increase or a decrease in the compressive stresses at the free edges of the section. In general, the critical load was larger if the buckling caused a reduction in the compressive stresses at the free edges, there being but a single exception to this rule. The average difference was about 3 percent and the maximum difference, 7 percent. In determining the value of EI from Allen and Silliman's column tests, the critical loads used were those obtained from the tests in which the buckling caused a decrease in the compressive stresses in the free-edge fibers. Wellman made no comparable study of the variation of critical load with direction of buckling, but his records show that, in tests of 26 specimens, in 20 cases buckling under maximum load reduced the compressive stress at the free edges, in five it increased that stress, and in one column failure under the maximum load caused an increase in one test and a decrease of compressive stress at the edge fibers in the other.

Discrepancies also exist between the observed distances from the back of the specimen to the effective neutral axis. In both series of tests the distances from the fixed reference lines to the specimen were bracketed to within 0.003 or 0.004 inch. Therefore, in spite of possible errors in obtaining the true positions of the reference lines to the resultant loads, the effective neutral axis positions were believed to be correct to less than 0.010 inch. Comparison of Allen and Silliman's with Wellman's results, however, showed Wellman's to be consistently in excess of Allen and Silliman's. According to Wellman, the distance Y_0 was from 0.005 to 0.025 inch in excess of the distances determined by Allen and Silliman, the average excess being 0.0167 inch and the median, 0.016 inch. When all the circumstances are taken into consideration, it is believed that the Allen and Silliman results are the more reliable.

Simply Supported Beam Tests - Loads Parallel to Web

In the tests of the channels as simply supported beams with the loads parallel to the plane of the web, the same sources of error existed as in the pure-bending tests with the loads in the plane of symmetry. Most of these sources might be expected to produce approximately the same percentage errors in the observed values of EI. Owing to the fact that the deflections measured in the tests with the loads parallel to the web were smaller than those obtained with the loads in the plane of symmetry, the probable error due to lack of precision in determining W/δ was greater with the loads parallel to the web than with the loads in the plane of symmetry. This is particularly true with respect to the pure-bending tests. In the simple and two-load bending tests the observed deflection differences were larger than in pure bending, and the resulting EI values were correspondingly more precise.

Here again a better idea of the actual reliability of the observed EI values can be obtained from a comparison of the results of different experimenters than from a theoretical analysis of the possible causes of error. For the four specimens tested in pure bending both by Scarbrough and by Carah and Park, the stiffnesses EI obtained by Scarbrough were 0.94, 0.94, 0.99, and 1.03 times those reported by Carah and Park. This can be considered very good. When Wellman retested three of Carah and Park's specimens, however, he got values of EI 0.97, 1.18, and 1.28 times the values from the first tests. His deviations from Scarbrough's figures were even greater, the ratios being 1.24, 1.56, and 2.25. As will be shown in the section on Discussion of Test Results, Scarbrough's values of EI are those most nearly in accord with the computed theoretical values, and Carah and Park's values are nearly as good. For many of his specimens, however, Wellman obtained values of EI in pure bending that are too far from any reasonable theoretical figures to be believed. For some reason he appears to have been unable to obtain reliable figures for W/δ at the loading points, but it has not been possible to determine the exact source of his difficulty.

For most of the Carah and Park and Scarbrough tests in pure bending it would appear that the results are correct to within 5 percent. They reported, however, that specimens –0, –17, and +12 were initially badly twisted and specimen –29 was damaged in an early test. The results for those members are therefore unreliable. They also reported that some of the other channels for which the observed values differed considerably from what was expected were probably eccentrically loaded but gave no supporting evidence.

The values of apparent EI obtained from the simple-bending tests should be somewhat more precise than those from the pure-bending tests on account of the greater differences between the values of W/δ from which they were computed. This judgment appears reasonable in the light of the few tests made on the same specimen by more than one investigator. Thus for two specimens Scarbrough obtained values of 1.04 and 1.10 times those of Carah and Park. Wellman, however, obtained values 1.13, 0.88, and 0.86 times those resulting from Carah and Park's tests of the same specimen and 1.59, 0.85, and 0.83 times the corresponding figures of Scarbrough. Although the maximum deviation of Scarbrough's from Carah and Park's figures rises to 10 percent, that was found in but one test. Wellman's figures for simple bending, however, are much more nearly in agreement with those of the other experimenters than his pure-bending figures. It is therefore believed reasonable to assume that most of Carah and Park's and Scarbrough's figures for simple bending are correct to within 5 percent and the remainder to within 10 percent but that, while Wellman's figures for simple and two-load bending are better than those for pure bending, many of them include appreciable errors and cannot be depended upon when they indicate conclusions at variance with those deduced from the other tests.

When the simple-bending tests were made with the load parallel to the plane of the web, it was suggested that error might result from slipping of the specimen in the loading clamps or from lateral bowing. Slipping of the specimen in the clamps was unlikely since it was very rigidly blocked in and the blocks were fixed in place by tightening a series of screws. Sufficient bearing area was allowed that the blocks of plastic resin did not crush, a resin with a bearing strength of 23,000 pounds per square inch being used. One channel was bowed laterally by pushing at its midpoint in order to determine qualitatively the effect of curvature. It was pushed sideways several times as far as any bowing noted in the tests, and the error in deflection amounted to only 0.0002 or 0.0003 inch. It was therefore considered that the very small lateral deflections noticed in the actual tests had a negligible effect on the precision of the test results.

Cantilever-Beam Tests

The precision of the results of the cantilever-beam tests is difficult to estimate. All the experimenters found that a change of but a few thousandths of an inch in the position of the load from that associated with torsionless bending would produce appreciable twist. In only one case, however, did two experimenters find the same shear-center position for a given specimen. With one specimen the difference in observed shear-center positions was nearly 1/8 inch. These differences make it impossible to use the test data of this report to develop a reliable quantitative expression for the effect of holes on the shear-center position, though the differences do not prevent the obtaining of valuable qualitative information on that point.

The values of apparent EI obtained by Carah and Park from their cantilever-beam tests were so much lower than the results of their pure- and simple-bending tests that they concluded that a large part of the total deflection of a lightened channel was due to shear. It was suspected, however, that part of the difference may have been the result of rotation of the specimen at the point of support. Scarbrough and Wellman therefore measured their deflections from an arm supported by the specimen in such a manner that the results would not be affected by rotation of the specimen as a whole. Their results show pretty plainly that Carah and Park's low values of apparent EI were due primarily to such rotation and are not to be relied upon. Study of their figures, however, indicates that Scarbrough and Wellman's cantilever-bending figures are for the most part probably correct to within ±10 percent or better.

Carah and Park did not attempt to find M_t/θ, the torsional stiffness, in their tests. Scarbrough made tests for this quantity and his results appear to be reasonably consistent. Wellman's figures agree fairly well with Scarbrough's but their consistency is not as good. Both men used average values of rotation per unit torsional moment obtained from tests with different points of load application. Since there was less spread between the figures averaged by Scarbrough than between those averaged by Wellman, it is not surprising that Scarbrough's results are more consistent. Nevertheless, it has been found impossible to reconcile Scarbrough's figures with theory or to obtain a reasonable estimate of their precision. This problem is gone into in more detail in the section on Discussion of Results.

DISCUSSION

Properties about Centroidal Axis Parallel to Back

The column tests and the bending tests with the loads normal to the back of the specimen were made primarily to develop a method of predicting the stiffness EI_{xx} against bending about the centroidal axis parallel to the back of the specimen, and the distance Y_o of that axis from the outside surface of the back. The more important test data and computed quantities used for that purpose in this study are given in tables 7 and 8. Table 7 contains the values of I/t and Y_0 obtained from the individual tests and the corresponding values computed on the basis of three alternative assumptions regarding the effect of the lightening holes. In this table the results of tests by Allen and Silliman and of tests by Wellman are shown on separate lines. The values of observed I/t shown were obtained by dividing the EI values of table 2 by 10,300 times the thicknesses t recorded in table 1. This was done because it was found more convenient to compare values of I/t than values of EI. The values of Y_0 in table 7 are taken directly from table 2. In table 8 are shown maximum, minimum, mean, and median percentage difference between various comparable observed and computed values of I/t and Y_0. The methods by which these values were obtained and what appears to be their significance are discussed below.

The values of I/t obtained in the two types of test, bending and axial compression, are directly comparable and, unless the character of the effect of the lightening holes should be a function of the type of test, should be the same for any specific specimen. From table 8 it can be seen that for the specimens tested both ways by Allen and Silliman the agreement of I/t values is quite good. It is really better than is suggested by the extreme percentage difference of -11.6 percent since, if the values for channels -22 and -26 are neglected, the spread for the remaining 23 specimens of this group is only from -3.8 to 5.3 percent.

Wellman obtained EI from both column and beam tests of 14 specimens, only two of which were tested both ways by Allen and Silliman. For those two specimens (-7 and -15) the column stiffnesses deviated from the beam stiffnesses by only -0.6 and 1.2 percent; whereas the deviations of the Allen and Silliman values were -1.2 and 3.4 percent, respectively. Although Wellman obtained better agreement between the two EI values for these specimens than did Allen and Silliman, he did not get such good agreement for the others, as can be seen from the second line of table 8. In this group of tests only four of the specimens showed differences above 10 percent, but for each of these four the difference exceeded 25 percent.

The third line of table 8 summarizes the results of comparing the results of column tests by Wellman with beam tests by Allen and Silliman. Since the Wellman values of I/t in bending are consistently higher than those of Allen and Silliman, the percentage deviations of the column-test I/t values from the bending-test values are higher for this basis of computation than when the Wellman bending values are used. This condition is intensified by the inclusion of four specimens for which no bending tests were made by Wellman, and the deviations of the Wellman column-test results from the beam-test results of Allen and Silliman range from 4 to 21 percent.

The relatively close agreement between the Allen and Silliman values of I/t obtained from the two types of test indicates that the influence of the lightening holes on the stiffness in bending is the same as on the stiffness in column action. It is the opinion of the writer that the greater spread of the Wellman results indicates primarily that in the Wellman column tests there was undesired end restraint which caused the observed values of I/t to be fictitiously high and that the results of these tests should not be taken to invalidate the afore-mentioned conclusion.

The most important information sought from the column tests and the tests in bending with the load normal to the channel back was a method for estimating the location of the effective neutral axis and the effective moment of inertia about that axis. The obvious method of allowing for the effect of the lightening holes on these quantities is to compute them for a channel with the same values of width of side S and thickness t as the actual member but to reduce the width of back B by an amount which would depend on the diameter of the lightening holes and possibly their pitch and other dimensions of the specimen. Thus the problem reduces to that of developing a method for computing what may be termed the effective diameter of the lightening holes D_e.

The simplest assumption is that D_e, the effective diameter, and D, the actual hole diameter, are identical. This implies that the entire strip of material between the lightening holes contributes nothing to the stiffness of the member and seems to be the most conservative assumption that would be reasonable. Values of I/t and Y_o based on this assumption are listed in columns 4 and 8 of table 7.

In computing these figures and all other values of I/t and Y_0, the sectional area was assumed concentrated along the section midline. The values of Y_0 and the radius of gyration were first computed, neglecting any effect of the fillets. The values of I/t were then computed as the product of the radius of gyration squared and the length of the midline, the effect of the fillet being taken into account in computing the length of the midline.

The results of comparing the observed values of I/t with those computed on the assumption that $D_e = D$ are summarized in table 8. In every case the value of I/t obtained from a bending test is in excess of that found by computation, the average difference being nearly 15 percent; and there is considerable scatter in the ratios of the two values. All but three of the column tests gave higher indicated than computed values of I/t. For one of these tests the difference was only –1.0 percent, while for the other two it was –11.0 and –14.8 percent. Unfortunately, neither of the two latter members (+34 and +39) was tested in bending, so it is not possible to determine whether their observed I/t values were as excessive as most of those found by Wellman. The results of the other tests, however, suggest that with these two members it may have happened that, instead of being subject to unexpected restraint at the ends in the column tests, they were subjected to eccentric loading.

Study of the individual differences between the observed and computed values of I/t showed that they tended to increase with increase in the pitch of the lightening holes. This appeared reasonable since it does not seem possible that the material between holes makes no contribution to the stiffness of the specimen, and the further the distance between holes the greater should be the effect of this material. Several methods of making a more refined allowance for the effect of the holes than assuming $D_e = D$ were tried. The most satisfactory proved to be the assumption that $D_e = D'$ where

$$D' = (0.2 + 1.5D^2/Pb)D \tag{3}$$

Computations of I/t and Y_0 on the basis of this assumption led to results that are given in tables 7 and 8. The agreement between Allen and Silliman's observed

I/t and these computed values is quite good for both the column and the beam tests. It is really better than is indicated in table 8 since for only one specimen (-23) is the divergence of the beam test from the computed value more than 10 percent. With the Wellman column tests the agreement is not very good, on account of the excessive observed values, and that between Wellman's beam test and the computed values must be considered as only fair.

The chief objection to using $D_e = D'$ as a basis of design is the unfortunate fact that the five specimens without lightening holes included in the bending tests gave indicated values of I/t ranging from 6.8 to 9.7 percent above the computed values. At least part of this apparent error seems to have been due to the use of the nominal values of section dimensions in computing I/t. For four of these specimens Allen and Silliman measured the actual dimensions to the nearest 0.001 inch for use in computing the moment of inertia. The resulting values of I/t average about 7 percent greater than those shown in table 7. About half of this difference can be accounted for by Allen and Silliman's neglect of the effect of the fillets. The remainder is most likely due to the fact that the actual width of leg is in excess of the nominal. For three of these specimens the difference between Allen and Silliman's observed and computed values of I/t is less than one-half of 1 percent and for the other one only 2.8 percent. If it is to be assumed that these effects of manufacturing tolerances existed in all the specimens, it would mean that the observed values of I/t should be reduced something like 10 percent before comparing them with the computed values. Alternatively, the desired expression for D_e would be one that would produce values of I/t about 10 percent less than the observed values. Since Allen and Silliman reported that in only a few specimens was the actual width of leg more than 0.002 inch in excess of the nominal, such a large adjustment of the observed values does not appear necessary.

Nevertheless a study was made to find an expression for D_e that would give computed values of I/t about 10 percent less than the observed values. As a result the values of I/t and Y_o were computed on the assumption that

$$D_e = D'' = (0.7 + D^2/Pb)D \quad \text{(or } D\text{, whichever is the smaller)} \qquad (4)$$

These values are listed in table 7 and their average deviations from the observed values are shown in table 8.

On the whole the tests appear to indicate that reasonably close, though slightly unconservative, estimates of I/t can be obtained by the use of $D_e = D'$ in computation. This is indicated by both the extreme and the average quantities listed in table 8, and also by figure 19 in which the values of I/t obtained in the beam tests are plotted as ordinates and the values computed with $D_e = D'$ are used as abscissas. For more conservatism, a somewhat greater value of D_e may be used, but there appears no reason why a value in excess of D'' should be employed.

It might be thought that the effective hole diameter would be influenced by the thickness of the material, but a study of the results on the specimens thicker and thinner than the average indicated that there was no such effect. Another variable that was considered was the width of the channel flange S. Figures 20 and 21 show the percentage differences between observed and computed I/t grouped according to nominal width of side S. The computed values used in preparing figure 20 are based on $D_e = D$ and those in figure 21, on $D_e = D'$. The observed values are those obtained from bending tests. Those figures show no definite trend associated with variation in S.

In the foregoing study the stiffnesses in bending have been those obtained with the specimen in the back-down pitch-centered position. Allen and Silliman reported, however, that with the specimen in the back-down hole-centered position the observed I/t was reduced about 3 percent. If this finding is relied upon, it would appear desirable, in design, to compute I/t on the basis of an assumed D_e somewhat larger than D'. Also it would suggest that perhaps the criterion for D_e should be based on tests with the specimen in one of the flange-down positions. Wellman, however, tested six channels in all four positions, with the results summarized in table 4. From these results it would appear that there is no consistent trend, and the differences in the results of separate tests on the same specimen may be due to factors other than the position of the specimen in the test jig.

Although the assumption that $D_e = D'$ gave the best correlation between the observed and computed values of I/t, the assumption that $D_e = D$ was considerably better for the distance Y_o. This is shown by table 8 in which are listed extreme, average, and median values of observed Y_o minus computed Y_o in percent of the latter. This is not conclusive evidence that the best figure for Y_o is to be obtained by assuming $D_e = D$ in computations. Where D_e is assumed equal to D', the apparent distance from the back of the web to the effective neutral axis appears to exceed the computed value. If the specimens had been originally straight, this would indicate an error. It was noticed, however, that practically all the specimens were curved in manufacture in such a way that the flanges were in initial tension. It can be seen from figure 22 that, under these conditions, the line of action of the axial load that would cause a minimum of bending would intersect the end cross sections at a greater distance from the back than the actual effective neutral axis. There is insufficient evidence, however, regarding the amount of initial curvature of the specimens to permit a definite conclusion as to just which would be the best assumption for D_e in computing the location of the effective neutral axis. For practically all the channels tested, however, the difference between using $D_e = D$ and $D_e = D'$ would not exceed 0.05 inch, and that would be sufficiently close for most practical design work.

Stiffness EI about Axis of Symmetry

The tests of channels as simply supported beams with the plane of loading parallel to the web were made primarily to develop a method for predicting the deflections that would be produced by such loadings. When dealing with beams with relatively thick webs, the deflections due to shear deformation may usually be neglected. With trusses, on the other hand, the effect of the deformations of the web members is too great to be neglected, and the same is true with respect to the shear deformation of very thin webs of beams. It was expected that the lightening holes in the webs of the channels tested in this investigation would make the channels act like trusses or very thin webbed beams and that the effect of shear deformation of the web would have to be taken into account.

A first step was to obtain values of apparent EI by substituting the slopes δ/W of the observed load-deflection curves in the appropriate beam-deflection formulas and solving for EI. The formulas used for this purpose were the conventional beam deflection formulas (such as those of table 4:1 on p. 94 of reference 3) in which no provision is made for the effect of shearing deformation. The values of apparent EI obtained in this manner are recorded in table 5.

Of these values, only those obtained from the tests in pure bending represent the true values of EI, and it is desirable to see how closely they agree with values computed from the dimensions and material of the specimens. It is to be expected that the observed moment of inertia of a lightened channel should lie somewhere between that of an otherwise identical channel without lightening holes, and the latter quantity minus the moment of inertia of the area removed from the web cross section through the center of a hole. The former quantity may be termed the "full back" and the latter the "full hole" moment of inertia of the cross section. In the present study it was considered simpler to work with values of I/t, the moment of inertia divided by the material thickness, than with the moment of inertia itself. Values of $(I/t)_{FB}$ and $(I/t)_{FH}$ computed for the channels tested in pure bending are listed in columns 2 and 3 of table 9. In computing these values the first step was to compute the square of the radius of gyration of the section midline about its axis of symmetry, no account being taken of the effect of the fillets at the junctions of the web and flanges, the computation being made by use of the formula

$$\rho_{yy}{}^2 = \frac{b^2\,(k + 6)}{12\,(k + 2)} \qquad (5)$$

where b is the width of back (or web), and k the ratio of the width of side (or flange) s to b. This value was multiplied by the developed length of the midline

$$L = 2s + b - 0.8584\,r \qquad (6)$$

where r is the radius of the fillet. This gave values of I/t from 3 to 4 percent lower than would have been obtained if the effect of the fillet had been neglected entirely. After computing $(I/t)_{FB}$ in this manner, $(I/t)_{FH}$ was obtained by subtracting $D^3/12$, where D is the diameter of the lightening hole.

The fourth column of table 9 shows the values of I/t obtained by dividing the observed EI in pure bending of column 2, table 5 by 10,300,000, the standard value of E.

It may be noticed that for most of the specimens the observed I/t was greater than the computed $(I/t)_{FB}$, and the percentage excess for each specimen is listed in table 9, column 5. For those tests in which the observed I/t was less than or only a little greater than $(I/t)_{FB}$, its percentage excess over $(I/t)_{FH}$ is listed in table 9, column 6. Since it is to be expected that the deviation of observed I/t from the computed values would probably be a function of the amount of material removed by the lightening holes, the values of the lightening parameter D^2/Pb are recorded in table 9, column 7.

Study of table 9 shows that nearly all of Scarbrough's observed values of I/t fell either between $(I/t)_{FB}$ and $(I/t)_{FH}$ or very close to those values. Carah and Park's observed values were also close to $(I/t)_{FB}$ but tended to be a little higher rather than a little lower than those values. Wellman's results, however, were widely scattered and ranged from 30 percent below $(I/t)_{FB}$ to about 2.5 times $(I/t)_{FB}$. These facts can be seen even more clearly from figure 23 in which the values of $100[(I/t)_{obs} - (I/t)_{FB}]/(I/t)_{FB}$ are plotted against D^2/Pb. In this and the following figures, the results of tests by Carah and Park are indicated by multiplication signs, those of Scarbrough by circles, and Wellman's by plus signs. Many of Wellman's results are not shown on this figure because the points would have fallen outside of its boundaries.

In the 19 tests made by Carah and Park the algebraic mean percentage excess of $(I/t)_{obs}$ over $(I/t)_{FB}$ was 3.09 and the algebraic median figure, 1.5. The corresponding figures for the 16 tests made by Scarbrough were: mean, −1.65 percent and median, −1.25 percent. Wellman, however, had an average excess of 41.86 percent with a median figure of 30.0. If all tests are considered as forming a single group, the average excess of $(I/t)_{obs}$ over $(I/t)_{FB}$ is 20.17 percent with a median figure of 7.3. It is obvious, however, that owing to some defect in Wellman's technique his results cannot be trusted. If the other two groups are combined, the average excess is only 1.05 percent and the median figure, 0.9 percent.

If only Scarbrough's results were considered, it would be possible to draw a pretty satisfactory empirical curve to show the percentage decrease in $(I/t)_{FB}$ to be expected to result from a given value of D^2/Pb. Unfortunately, however, the number of Scarbrough's tests was too small and covered too limited a range of section proportions to make it advisable to use them as the basis for a curve of this kind for design use.

The combined data of Scarbrough and Carah and Park, however, furnish good evidence that the stiffness of the channels in pure bending was little reduced by the presence of the lightening holes. It is believed that their results are sufficient basis for the recommendation that for most practical work, unless D^2/Pb is relatively large, the effect of the lightening holes may be neglected in computing the moment of inertia of the section; while, if D^2/Pb is large or there is special need for conservatism, there is no need to use a lower value of I/t than $(I/t)_{FH}$. In most cases of this kind it should be sufficient to use the arithmetic mean of $(I/t)_{FB}$ and $(I/t)_{FH}$.

Effect of Shear Deformation

The total deflection of a beam subjected to combined bending and shear is the sum of the deflections due to the two types of stress acting independently. Therefore write

$$\delta_t = \delta_b + \delta_s \tag{7}$$

where δ_t is the total deflection, δ_b is the deflection due to bending given by the usual deflection formulas, such as those in table 4:1 on page 94 of reference 3, and δ_s is the deflection due to shear. The general expression for δ_s is given by equation 12:7 on page 386 of reference 3 as

$$\delta_s = \int \frac{k\ s\ V\ dx}{A\ G} \tag{8}$$

where s is the shear at a section due to a unit load system based on a unit load at the point for which the deflection is desired, V is the total shear on a section, A the cross-sectional area, G the shearing modulus of the material, and k is a constant depending on the shape of the cross section. For a beam of constant section and length L subjected to a concentrated load at midspan, this becomes

$$\delta_s = \frac{k\ W\ L}{4\ A\ G} \tag{9}$$

for the shear deflection at midspan.

With specimens like the lightened channels under consideration, the value of k is unknown and its empirical determination was one of the objectives of the simple-bending tests. In these specimens the vertical deflection due to shear deformation of the flanges is negligible and it is reasonable to replace A by bt, the gross sectional area of the web. Since it would be difficult to separate the effects of the flanges and the holes in the web, it appears best to combine the two effects and write

$$\delta_s = \frac{W\ L}{4\ K\ b\ t\ G} \tag{10}$$

where K is a factor which takes account of the shape of the section, the use of the web area in place of the total area, and the effect of the holes.

In order to obtain K empirically, the first step was to compute $Et\delta_t$ for a load of 100 pounds at midspan from the observed values of apparent EI indicated by the simple-bending tests and recorded in column 3 of table 5. The conventional formulas were then used to compute $Et\delta_b$ using the computed values of $(I/t)_{FB}$ and $(I/t)_{FH}$. Then, by subtraction two values of $Et\delta_s$ were obtained for each simple bending test, one based on each of the alternative values of I/t. These values were next inserted in equation (10) which was solved for K, E being assumed equal to 10,300,000 and G to 3,850,000 pounds per square inch, in these computations.

The values of K obtained from the Carah and Park and the Scarbrough tests in this manner, when plotted against D^2/Pb, formed a fairly definite band. That of the points based on $(I/t)_{FB}$ appeared a little more clearly defined than that based on $(I/t)_{FH}$ though there was not much choice between the two. Both exhibited considerable scatter of the points, but that was to be expected since the values of K were based on small differences between relatively large numbers and also had to absorb all errors of precision in making the tests. The formula obtained in this manner was

$$K = 0.5 - D^2/Pb \qquad (11)$$

The value of $Et\delta_t$ for a 100-pound load was then recomputed, using this value of K and $(I/t)_{FB}$, for comparison with that obtained from the apparent EI in simple bending. These two values and their percentage difference are listed in columns 2, 3, and 4 of table 10. The percentage differences for the Carah and Park and the Scarbrough tests are also plotted against D^2/Pb in figure 24. From this figure and the table it can be seen that, if δ_b is computed from the ordinary bending formula using I_{FB} as the moment of inertia and δ_s is computed from equation (10) using the value from equation (11) for K, the resulting value of $\delta_t = \delta_b + \delta_s$ is sufficiently close to that obtained from the tests for most practical purposes. The mean deviation of the deflection computed in this manner from the corresponding observed deflection of the Carah and Park and Scarbrough tests is -1.02 percent and the median, -0.4 percent. If the results of Wellman's tests are included, the mean deviation rises to 2.54 percent and the median to 0.7 percent. It is to be noted that Wellman's results are much closer to the computed results here than in the pure-bending tests.

Formulas were developed in the same manner for the total deflection under a specified intensity of the cantilever and two-load bending loadings, and the values of

$Et\delta_t$ were similarly computed. These values of $Et\delta_t$ and the corresponding figures from the test data are listed in table 10.

The percentage differences between the observed and computed values of $Et\delta_t$ for cantilever bending are also plotted against D^2/Pb in figure 25. From this figure it can be seen that in practically every test the observed deflection exceeded the computed, but the average excess was only 4.18 percent for the Scarbrough and 6.97 percent for the Wellman tests. The corresponding median deviations were 3.9 and 6.5 percent. On the other hand, the average excess in the Carah and Park tests was 37.31 percent and the median, 35.7 percent. The excessive observed deflections of Carah and Park were at first thought to reflect primarily the effect of the lightening holes in reducing the resistance of the web to shear. The results of Scarbrough and Wellman, however, indicate that they were more likely the result of rotation of the specimen at its point of support. Study of the test apparatus will show that any rotation of the specimen at the support would cause an increase of the measured deflections, while that would not be the case with the arrangement used by Scarbrough and Wellman. The latter may have produced some deformation of the channel web where the reference arm was attached, and the deflections measured may not have been measured exactly from a tangent to the elastic curve at that point. It is believed that these factors may be responsible for the fact that the differences between computed and observed values of $Et\delta_t$ for the Scarbrough and Wellman cantilever bending tests tend to be larger than the differences for the simple bending tests of those experimenters. The results indicate pretty clearly, however, that Scarbrough and Wellman's measurements came much closer to being what they were intended to be than the deflection measurements of Carah and Park.

Only for Wellman's tests were the values of EI in two-load bending determined. These were used to determine observed values of $Et\delta_t$ which were compared with corresponding computed values. Again the observed values tend to exceed the computed, but there is considerable spread. If the three largest differences are neglected, the average arithmetical difference between the two values of $Et\delta_t$ is 9.44 percent and the median 8.0 percent. If the signs of the differences are taken into account, the mean difference is only −0.97 and the mean −0.35 percent.

While D^2/Pb was considered the most probable parameter with which the difference between observed and computed values of $Et\delta_t$ would vary, studies were made of the variation of these differences with width of side s and with the ratio of width of side s to width of back b. No special trend with respect to either of these variables was detected.

On the whole it is considered that the method of computing deflections used in the preparation of table 10 gives sufficiently accurate results to be employed in most design work. Admittedly the precision is not as good as might be desired and the test data are not of as good quality as could be wished. On the other hand, in practical work the deviations of actual from computed deflections resulting from standard tolerances for sheet thicknesses, bend radii, and so forth, are of such magnitude that it would be futile to attempt very great precision. Therefore, until more accurate and extensive tests have been carried out, it is believed that designers will find this method of value.

One obvious weakness of the method is that it appears to result in stiffnesses which tend to exceed those obtained by test. The designer could easily avoid difficulty on this score by using $(I/t)_{FH}$ instead of $(I/t)_{FB}$ in computing the fraction of the deflection due to bending. It seems hardly necessary to make any further correction to the deflection due to shear. It is true that the formula for K probably has a low accuracy, but the term in which it appears normally represents such a small part of the total deflection that a relatively large percentage error in K will cause but a small percentage error in the final result.

Shear-Center Location

For a channel without lightening holes the distance from the midline of the back to the shear center can be obtained from the relation

$$d = \frac{s^2 b^2 t}{4I} \qquad (12)$$

given in article 6:5, pages 161-164 of reference 3. When the back is pierced with lightening holes, more of the normal stress must be carried by the flanges and the shear flow in the flanges is thereby increased. One result is an increase in the moment of the couple produced by the flange shear forces and therefore an increase in the distance from the web to the shear center. Alternatively, it might be reasoned that since the presence of lightening holes would decrease the effective moment of inertia, the result would be to increase the distance d.

The amount by which the shear center would be displaced as the result of employing lightening holes of a given size and spacing would be very difficult to estimate theoretically. In the hope of developing an empirical rule, equation (12) was used to compute the theoretical shear-center distance listed in column 7 of table 5. Comparison of these figures and the test results verifies the expectation that the effect of the holes would be to increase the distance from the channel web to the shear center. With a few specimens the observed shear-center distance is less than the theoretical, but on the average it is 0.0464 inch greater. The relation between the computed and observed shear-center distances is also indicated in figure 26, where the observed distances are plotted as ordinates and the computed values as abscissas.

When the percentage differences between the observed and computed values for the shear-center distance were plotted against D^2/Pb, there was some indication that such differences increased with that lightening parameter. The plotted points were too scattered, however, to use them as the basis for formulating an equation for the relation. There were several groups of specimens which differed only in hole diameter, hole pitch, or D^2/Pb. Study of these groups failed to disclose any clear relationships affecting shear-center distance which would be of value to the designer.

The differences between the observed and computed shear center distances of table 5 range from -6.5 to 17.0 percent of the width of flange, the algebraic mean being slightly under 5.0 percent. For most practical work it would be sufficient to assume that the shear center of a lightened channel would lie between the point indicated by equation (12) and a point 10 percent of the flange width farther from the web.

Torsional Stiffness

Since the channels used in this study were formed from flat sheets with developed widths considerably in excess of 16 times their thicknesses, their torsional stiffnesses GJ were assumed to conform to the relation

$$GJ = G\, w_e\, t^3/3 \qquad (13)$$

in which G is the shearing modulus of elasticity, t is the thickness of the material, and w_e is the effective width of the developed section. For the specimen without lightening holes the effective width should be the same as w, the actual developed width of the section. For the lightened channels the effective width was expected to lie somewhere between w and w – D, where D is the diameter of the lightening holes. It was hoped that by study of the test results it would be possible to obtain an empirical expression for w_e that would be between these two figures.

The quantity GJ of equation (13) is the ratio of applied torsional moment to resulting twist in radians per inch. Since the observed values of M_t/θ in table 5 are ratios of applied torsional moment to total twist, "observed" values of GJ were first obtained by multiplying observed M_t/θ by the distance from the face of the support to the mirror attached to the web, which was 31.125 inches in Scarbrough's tests and 27.876 inches in Wellman's tests. For purposes of comparison two values of GJ were computed from equation (13). One of these values was based upon the assumption that $w_e = w$ and the other, on the assumption that $w_e = w - D$. In both cases G was taken as 3,850,000 pounds per square inch. These three values of GJ are listed in table 11. Figures 27 and 28 show the values of observed GJ plotted against the two computed values of that quantity.

In each of these figures nearly all the points representing Scarbrough's tests (indicated by circles) lie fairly close to a straight line, the deviations from such a line being less in figure 28 than in figure 27. Most of Wellman's tests gave points falling reasonably close to the same lines but exhibited much more scatter. This is believed to be due largely to the fact that Wellman's values of M_t/θ were averages from tests with greater eccentricities of loading than those present in the Scarbrough tests. As a result, Wellman's individual values of M_t/θ deviated more from the means listed in table 5 than did Scarbrough's and his means are therefore considered less reliable.

In spite of the fact that most of Scarbrough's points come quite close to falling on straight lines in figures 27 and 28, these data fail to provide a rule for determining the value of w_e to be used in equation (13). In the first place, the lines in question would not pass through the origin; this fact indicates either that G is not a constant or that J is not directly proportional to $w_e t^3$, as indicated in equation (13), but some constant must be added to that relation. Equally important is the fact that the observed values of GJ are so much larger than the computed ones that some important factor must have been omitted from the computations.

The situation is further complicated by the data from specimens +21 and +34. In both figures the points for specimen +21 fall considerably to one side of the band formed by the points from the other tests. These points may be disregarded, however, since Scarbrough reported that channel +21 was initially twisted and consistently gave test results which lacked conformity with those from the other specimens. Much more important are the points for specimen +34, which was the only one without lightening holes that was tested in torsion and which was tested by both experimenters. In figure 27, with GJ computed on the basis of $w_e = w$, the points for +34 fall within the band defined by the other test data; but in figure 28 with GJ computed on the basis of $w_e = w - D$, the points for this specimen fall considerably to one side of the band. This appears to be additional evidence that some highly important factor was neglected either in computing GJ from equation (13) or in obtaining from the tests the observed quantities to be compared with the computed values.

The most obvious factor that was neglected was the possible restraint at the support against warping of the cross sections. In preparing figures 27 and 28 this was not taken into account since the possible magnitude of its effect was not realized. These figures showed such great differences between the observed and the computed stiffnesses that an investigation was made to estimate the possible effect of complete restraint of the cross section at the support against warping. This was done by applying the formulas of Timoshenko in reference 4.

The first step was to compute effective lengths from the relation

$$L_e = L - a \tanh \frac{L}{a} \qquad (14)$$

where

$$a^2 = \frac{EI_f}{2GJ}\left(1 + \frac{t\ b^3}{4\ I_y}\right) \qquad (15)$$

where

L_e effective length of specimen

L actual length of specimen

E Young's modulus

I_f moment of inertia of one flange about its minor axis of symmetry

I_y moment of inertia of entire cross section about its axis of symmetry

t thickness of material

b distance between flanges

GJ torsion constant obtained from equation (13)

In computing a^2 from equation (15), I_f was taken as equal to $ts^3/12$, where s is the width of a flange. Since $I_y = tb^3/12 + tsb^2/2$, equation (15) reduces to

$$a^2 = \frac{EI_f}{2GJ}\left(1 + \frac{3b}{b + 6s}\right) \qquad (16)$$

Two values of L_e were computed for each specimen, one based on each of the values of computed GJ listed in table 11. These values of L_e are shown in columns 5 and 6 of that table. Finally, "theoretical" values of M_t/θ were obtained for insertion in columns 7 and 8 of table 11 by dividing each computed value of GJ by the corresponding value of L_e.

The observed values of M_t/θ are plotted against the theoretical values of this quantity in figures 29 and 30. From these figures and the data of table 11 it can be seen that the observed stiffnesses are still considerably in excess of the computed values. They average 24.2 percent greater than the stiffnesses computed on the basis of $w_e = w$ and 54.6 percent greater than those based on $w_e = w - D$. The median figures are 22 and 50 percent, respectively.

The percentage differences between observed and computed values of M_t/θ were plotted against the lightening parameter D^2/Pb against the width of side S, and against the absolute computed values of M_t/θ to see if any interesting trends would be revealed. In the following remarks this percentage difference is called the excess stiffness. The excess stiffnesses showed no definite trend of variation with D^2/Pb. They did, however, appear to have a tendency to decrease with increase in the width of leg, though it must be admitted that the plotted points showed too much scatter to permit the formulation of an algebraic expression to represent this tendency. The excess stiffnesses based on $w_e = w$ showed no consistent trend of variation with the absolute magnitude of computed stiffness, but those based on $w_e = w - D$ showed a definite tendency to decrease as computed M_t/θ increased. Here again, however, there was too much scatter of the plotted points to permit expressing the trend by an algebraic relation.

From the data of table 11 it is obvious that the stiffnesses obtained in the tests were due primarily to the shape of the section and the restraint of the supported end against warping. It is equally apparent, however, that these factors do not completely account for the differences between the observed and computed values of GJ. One source of the discrepancy may well be the use of too low a value of the shearing modulus of elasticity G. In this connection it may be recalled that in reference 1 the writer reported on some tests of extruded aluminum-alloy channels and flat strips subjected to torsion, which indicated values of G appreciably larger than the standard value of 3,850,000 pounds per square inch used in this report. In those earlier tests the value of G obtained from tests of flat strips was 4,500,000 pounds per square inch and that from tests on a channel section was 5,000,000 pounds per square inch. Had such values of G been used in the present investigation, the computed stiffnesses would have been much closer to those observed, and it might have been possible to obtain an empirical expression for effective width w_e. The tests of reference 1, however, were very few in number and rather crude in character, and the writer believes that the high values of G obtained from them represent not so much an error in the accepted value for that property as lack of complete applicability of the formulas from which they were computed. Some additional sources for the discrepancy must be looked for.

While it is clear that it is incorrect to neglect the effect of restraint against warping, it is not so clear that the method used to account for that factor is the correct one. In the development of his formulas, Timoshenko assumed complete restraint against warping of the cross section at the supported end. This, however, is an ideal condition which could hardly be attained in a test. On the other hand, deviation from the ideal condition would result in an actual stiffness less instead of greater than that computed. Similarly, the measurement of the length of the specimen from the face of the support would tend to increase the computed stiffnesses. Both these factors would thus cause discrepancies in the opposite direction from those observed.

There is a possibility, however, that in some manner which the writer has been unable to visualize the construction of the support was of such character that the specimen was not subjected to the constant torsional moment assumed in the analysis but to a varying torque of lower average intensity. Another possibility is that the theory expressed in Timoshenko's formulas is incomplete and that a more refined theory would indicate greater stiffnesses.

On the whole it must be admitted that these tests failed to indicate a satisfactory method of computing the effect of lightening holes on the torsional stiffness of channels. On the contrary, they serve mainly to indicate some of the difficulties attendant upon an experimental determination of torsional stiffness and cast some doubt on the validity of present methods for computing the torsional stiffness of unlightened channels. In so doing they emphasize the desirability of additional research in this field.

CONCLUSIONS

1. The position of the centroidal axis parallel to the back of a channel lightened by unflanged holes and the moment of inertia about that axis can be computed by assuming the actual width of back reduced by $\left(0.2 + 1.5\,\frac{D^2}{Pb}\right)D$ where D is the diameter of the lightening holes, P is the pitch of the holes, and b is the distance between the midlines of the flanges.

2. For a more conservative figure the assumed effective reduction in the width of back may be taken as $\left(0.7 + \frac{D^2}{Pb}\right)D$ or D, whichever is the smaller.

3. The moment of inertia obtained from the procedures of conclusions 1 and 2 may be used for the practical estimation of deflections, or of critical loads according to the Euler formula.

4. In computing the effective stiffness about the axis of symmetry of a channel with unflanged lightening holes, the effect of the hole may be disregarded for most purposes. If a more conservative figure is desired, even when D^2/Pb is large, there is no indication that the value of I for the cross section need be reduced by more than $tD^3/12$, where t is the thickness of the material.

5. The deflection due to shear deformation of a channel with unflanged lightening holes subjected to loads parallel to the back may be estimated for practical purposes from the relation.

$$\delta_s = \int \frac{s\ V\ dx}{K\ b\ t\ G}$$

where

δ_s deflection due to shear deformation

s shear on a section due to a unit load at point for which deflection is being computed

V total shear on a section

K $0.5 - D^2/Pb$

b distance between midlines of channel flanges

t thickness of material of channel

G shearing modulus of elasticity

6. The position of the shear center of a channel with unflanged lightening holes is farther from the web than for a similar channel without holes. For design purposes the shear center of the lightened channel may be assumed to lie between its theoretical location for the corresponding unlightened channel and a point 10 percent of the flange width farther from the back.

7. Special precautions are necessary if reliable figures for the effective stiffness of beams are to be obtained from cantilever tests. Though reasonable figures for stiffness in bending were obtained from cantilever tests, attempts to correlate the apparent torsional stiffnesses with theory were unsuccessful. How much this lack of success was due to defects of test procedure and how much to incompleteness of present theory could not be determined.

Stanford University,
Stanford University, Calif., March 15, 1943.

APPENDIX

This appendix is limited to a few sample log sheets used in analyzing the test data which are referred to in the section on Apparatus and Test Procedure. Curves used in analyzing the test data are given as figures A1, A2, and A3.

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3. Niles, Alfred S., and Newell, Joseph S.: Airplane Structures. Vol. I. John Wiley & Sons, Inc., 3d ed., 1943.

4. Timoshenko, S.: Strength of Materials. Pt. II. D. Van Nostrand Co., Inc., 2d ed., 1941, pp. 282-293.

Table 1

Specimen Dimensions

Specimen	B	S	t	D	P	D^2/Pb
1 (a)	2	3	4	5	6	7
-0	2-1/2	7/8	0.0802	1-7/16	2-1/4	0.380
-1	2-1/2	7/8	0.0622	1-7/16	2-1/4	0.377
-2	2-1/2	7/8	0.0485	1-7/16	2-1/4	0.375
-3	2-1/2	7/8	0.0421	1-7/16	2-1/4	0.374
-4	2-1/2	7/8	0.0319	1-7/16	2-1/4	0.372
-5	2-1/2	7/8	0.0256	1-7/16	2-1/4	0.371
-6	2-1/2	1-1/4	0.0513	1-1/2	3-1/4	0.283
-7	2-1/2	1	0.0485	1-1/2	3-1/4	0.282
-8	2-1/2	3/4	0.0514	1-1/2	3-1/4	0.283
-9	2-1/2	5/8	0.0496	1-1/2	3-1/4	0.282
-10	2-1/2	1-1/4	0.0512	1-1/4	4-1/4	0.150
-11	2-1/2	1	0.0507	1-1/4	4-1/4	0.150
-12	2-1/2	3/4	0.0509	1-1/4	4-1/4	0.150
-13	2-1/2	5/8	0.0479	1-1/4	4-1/4	0.150
-14	2-1/2	1-1/4	0.0500	1-3/4	3-1/4	0.384
-15	2-1/2	1	0.0495	1-3/4	3-1/4	0.384
-16	2-1/2	3/4	0.0513	1-3/4	3-1/4	0.385
-17	2-1/2	5/8	0.0487	1-3/4	3-1/4	0.384
-18	2-1/2	1-1/4	0.0507	1-1/4	3-1/4	0.196
-19	2-1/2	1	0.0506	1-1/4	3-1/4	0.196
-20	2-1/2	3/4	0.0512	1-1/4	3-1/4	0.196
-21	2-1/2	5/8	0.0504	1-1/4	3-1/4	0.196
-22	2-1/2	1-1/4	0.0515	1-1/4	2-1/4	0.284
-23	2-1/2	1	0.0488	1-1/4	2-1/4	0.283
-24	2-1/2	3/4	0.0509	1-1/4	2-1/4	0.284
-25	2-1/2	5/8	0.0500	1-1/4	2-1/4	0.283
-26	2-1/2	1-1/4	0.0517	no holes		0
-27	2-1/2	1	0.0500	------ do --------		0
-28	2-1/2	3/4	0.0516	------ do --------		0
-29	2-1/2	5/8	0.0505	------ do --------		0
+1	2-1/2	7/8	0.0522	1-7/16	2-5/8	0.322
+2	2-1/2	3/4	0.0541	1-5/16	2-5/8	0.268
+3	2-1/4	1-1/8	0.0514	1-3/16	2-5/8	0.244
+4	1-3/4	7/8	0.0595	15/16	2-5/8	0.198
+5	1-1/4	5/8	0.0590	11/16	2-5/8	0.151

Table 1 - Continued

1(a)	2	3	4	5	6	7
+6	2-1/2	7/8	0.0524	1-7/16	4-1/2	0.188
+7	2-1/2	3/4	0.0521	1-5/16	4-1/2	0.157
+8	2-1/4	1-1/8	0.0512	1-3/16	4-1/2	0.143
+9	1-3/4	7/8	0.0586	15/16	4-1/2	0.116
+10	1-1/4	5/8	0.0595	11/16	4-1/2	0.088
+11	2-1/2	7/8	0.0524	1-5/16	2-1/4	0.313
+12	2-1/2	1-1/4	0.0518	1-5/16	2-1/4	0.313
+13	2-1/2	1	0.0511	1-5/16	2-1/4	0.313
+14	2-1/2	3/4	0.0521	1-5/16	2-1/4	0.313
+15	2-1/2	1/2	0.0518	1-3/16	2-1/4	0.256
+16	2-1/4	1-1/8	0.0515	1-3/16	2-1/4	0.285
+17	1-3/4	7/8	0.0594	15/16	2-1/4	0.231
+18	1-1/4	5/8	0.0590	11/16	2-1/4	0.176
+19	2-1/2	7/8	0.0807	1-7/16	2-1/4	0.380
+20	2-1/2	7/8	0.0730	1-7/16	2-1/4	0.379
+21	2-1/2	7/8	0.0648	1-7/16	2-1/4	0.380
+22	2-1/2	1-1/4	0.0515	1-3/4	3-1/4	0.385
+23	2-1/2	1-1/8	0.0521	1-3/4	3-1/4	0.385
+24	2-1/2	1	0.0511	1-3/4	3-1/4	0.385
+25	2-1/2	7/8	0.0523	1-3/4	3-1/4	0.385
+26	2-1/2	3/4	0.0518	1-3/4	3-1/4	0.385
+27	2-1/2	5/8	0.0519	1-3/4	3-1/4	0.385
+28	2-1/2	1-1/4	0.0518	1-5/16	3-1/4	0.217
+29	2-1/2	1-1/8	0.0520	1-5/16	3-1/4	0.217
+30	2-1/2	1	0.0520	1-5/16	3-1/4	0.217
+31	2-1/2	7/8	0.0523	1-5/16	3-1/4	0.217
+32	2-1/2	3/4	0.0516	1-5/16	3-1/4	0.217
+33	2-1/2	5/8	0.0519	1-5/16	3-1/4	0.217
+34	2-1/2	1-1/4	0.0518	no holes		0
+35	2-1/2	1-1/8	0.0523	------- do -------		0
+36	2-1/2	1	0.0511	------- do -------		0
+37	2-1/2	3/4	0.0523	------- do -------		0
+39	2-1/2	1-1/4	0.0519	1-5/16	2-1/4	0.313
+40	2-1/2	1-1/8	0.0521	1-5/16	2-1/4	0.313
+41	2-1/2	1	0.0522	1-5/16	2-1/4	0.313
+42	2-1/2	3/4	0.0523	1-5/16	2-1/4	0.313
+43	2-1/2	5/8	0.0519	1-5/16	2-1/4	0.313

[a] Notation:

− Furnished by the former Northrop Aircraft, Inc., now the El Segundo division of the Douglas Aircraft Co., Inc.

\+ Furnished by the Boeing Aircraft Co.

Table 2

Observed Values of EI_{xx} and Y_o

Channel	Observed values of $EI_{xx} \div 1000$					Obs. Y_o in inches	
	from beam tests of			from column tests of		from column tests of	
	A and S	W(2LB)	W(PB)	A and S	W	A and S	W
(a) 1	2	3	4	5	6	7	8
-0	176			178		0.287	
-1	143			145		0.302	
-2	114	115.5	120.0	110	114.1	0.315	0.326
-3	104			106		0.310	
-4	75.9	82.1	81.8				
-5	63.6						
-6	313			309		0.454	
-7	167	170.4	163.5	165	169.3	0.338	0.361
-8	80.8			80.2		0.247	
-9	47.9						
-10	362						
-11	192			191		0.299	
-12	88.5			85.7		0.215	
-13	50.6	52.3	51.6				
-14	279						
-15	149	155.5	156.5	153	157.5	0.395	0.419
-16	66.9			66.8	70.9	0.312	0.317
-17	42.2	44.8	44.3				
-18	344			338		0.428	
-19	189	184.3	173.5	189		0.311	
-20	84.4			81.5	99.8	0.223	0.234
-21	51.0						
-22	336	338.5	360.0	311		0.419	
-23	168	178.9	180.0				
-24	81.9			81.5	99.8	0.231	0.244
-25	50.6						
-26	422			391		0.332	
-27	222	223.7	230.3	222		0.246	
-28	101			101		0.186	
-29	59.4						

Table 2 - Continued

1(a)	2	3	4	5	6	7	8
+1		126.5	125.2		138.0		0.323
+2		83.8	91.1		105.0		
+3	239	250.6	252.5	243		0.407	
+4	130			137	152.2	0.330	0.355
+5	43.0	44.4	44.7	44.3		0.244	
+6	130						
+7	88.3						
+8	249				259.0		0.394
+9	136				155.0		0.348
+10	48.5	50.7	49.8		67.0		0.251
+11		141.4	139.8		143.1		0.294
+12		343.2	346.9				
+13		182.1	184.5		195.6		0.348
+16	239						
+17	124				149.8		0.345
+18	41.7	42.8	42.8		61.7		0.249
+19	175			181		0.313	
+20	157			158		0.250	
+22	287			276		0.513	
+23	210						
+24		158.0	160.0		157.2		0.407
+25	106				118.4		0.363
+26		73.0	73.0				
+27	43.0	43.8	43.8				
+28	342						
+29		272.3	266.6		270.5		0.392
+30		194.0	187.0		197.0		0.334
+31	130	132.5	144.0				
+32					97.1		0.255
+33		52.8	53.1		53.8		0.183
+34					353.0		0.363
+37	103						
+39					261.0		0.447
+40		262.3	257.0		261.0		0.398
+41		217.8	201.2				
+43		51.5	53.3		65.6		0.197

A and S - Allen and Silliman   W - Wellman   (2LB) - two-load bending   PB - pure bending

[a] Notation:
- \- Furnished by the former Northrop Aircraft, Inc., now the El Segundo division of the Douglas Aircraft Co., Inc.
- \+ Furnished by the Boeing Aircraft Co.

Table 3

Investigation of Local Deformation

Channel No. (a)	Stiffness EI in lbs-in²	
	From deflections at load points	From deflections 2 inches inside of load points
-22	330,000	325,000
-23	167,000	169,000
-24	81,200	81,600
-25	51,200	50,200

[a] Notation:
- \- Furnished by the former Northrop Aircraft, Inc., now the El Segundo division of the Douglas Aircraft Co., Inc.
- \+ Furnished by the Boeing Aircraft Co.

Table 4

EI_{xx} from Beam Tests with Varying Specimen Positions

Channel (a)	EI_{xx} in Pure Bending				EI_{xx} in "Two Load Bending"				Max. percent diff.
	Back down		Flanges down		Back down		Flanges down		
	P.C.	H.C.	P.C.	H.C.	P.C.	H.C.	P.C.	H.C.	
-7	163.5	161.7	169.4	180.0	170.4	165.8	161.5	169.2	5.5
-15	156.5	160.0	163.5	169.4	155.5	162.5	156.4	158.4	4.5
-17	44.3	44.2	47.0	46.0	44.8	44.5	46.2	46.0	3.9
-19	173.5	201.3	190.6	174.5	184.3	194.0	186.0	175.7	10.5
+10	49.8	48.3	50.0	52.0	50.7	50.3	49.6	51.6	4.0
+31	144.0	134.0	140.5	132.7	132.5	133.0	132.5	130.1	2.5
								Av.	5.15

P.C. - Pitch centered H.C. - Hole centered

[a] Notation:
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- \+ Furnished by the Boeing Aircraft Co.

Table 5
Test Results
Bending with Load Parallel to Back

Channel	Observed Values of EI_{yy}				Shear-center distance		Observed minus computed shear center distance	Observed M_t/θ
	Pure bending	Simple bending	Two-load bending	Cantilever bending	Observed	Computed		
1(a)	2	3	4	5	6	7	8	9
-0 C	3,120	2,037		1,732	0.348	0.282	0.066	
-2 S	1,705	1,410		1,653	0.322	0.288	0.034	48.4
-3 W	2,395	1,159	1,211	1,324	0.381	0.289	0.092	61.7
-5 C	1,004	754.4		678.0	0.330	0.292	0.038	
-6 W	1,800	1,140	1,292	1,972	0.501	0.459	0.042	134.2
-7 S	2,105	1,640		1,761	0.388	0.344	0.044	52.6
W	4,740	1,364	1,770	1,805	0.474	0.344	0.130	67.7
-9 C	1,494	1,304		1,046	0.198	0.179	0.019	
-10 W	3,580	2,625	2,295	2,239	0.576	0.459	0.117	113.2
-12 C	1,814	1,475		1,234	0.228	0.232	-0.004	
-13 C	1,535			1,069	0.196	0.179	0.017	
S	1,441	1,279		1,343	0.214	0.179	0.035	28.1
-15 W	2,340	1,118	1,422	1,776	0.485	0.344	0.141	82.9
-16 W	1,900	1,206	1,203	1,459	0.291	0.232	0.059	52.4
-17 C	1,219	1,003		965.0	0.154	0.179	-0.025	
-18 C	2,520	1,957		1,618	0.474	0.459	0.015	
-19 S	2,070	2,052		1,958	0.387	0.344	0.043	57.8
-20 W	3,700	1,364	2,045	1,622	0.311	0.232	0.079	55.0
-22 S	2,434	2,063		2,205	0.501	0.459	0.042	92.9
W	3,800	3,280	835	2,192	0.378	0.459	-0.081	114.9

Table 5 - Continued

1 (a)	2	3	4	5	6	7	8	9
-24 W	2,565	1,600	1,630	1,603	0.347	0.232	0.115	75.7
-25 S	1,482	1,279		1,375	0.154	0.179	-0.025	37.6
-26 C				1,761	0.483	0.459	0.024	
-27 C				1,504	0.369	0.344	0.025	
-29 C	1,554			1,112	0.201	0.179	0.022	
+1 W	2,900	1,615	1,565	1,735	0.351	0.287	0.064	82.9
+2 W	2,100	1,447	1,405	1,578	0.262	0.231	0.031	51.0
+3 S	1,809	1,568		1,727	0.436	0.412	0.024	77.4
+4 W	1,158	1,039		833.0	0.410	0.317	0.093	52.0
+5 C	340.8	304.5		258.8	0.205	0.223	-0.018	
+6 S	1,881	1,691		1,824	0.308	0.287	0.021	65.0
+7 C	1,769	1,553		1,247	0.253	0.232	0.021	
+8 W	2,240	2,235	2,045	1,459	0.531	0.412	0.119	77.0
+9 C	922.0	875.0		699.0	0.286	0.317	-0.031	
W	1,090	986.0	924.0	852.0	0.367	0.317	0.050	55.6
+10 C	352.0	311.4		272.4	0.192	0.223	-0.031	
W	340.0	274.0	306.0	303.0	0.250	0.223	0.027	24.8
+11 W	2,485	1,645	1,642	1,670	0.436	0.287	0.149	69.2
+12 S	2,752	2,304		2,247	0.523	0.459	0.064	90.9
+13 W	3,600	1,725	1,845	1.913	0.436	0.343	0.093	84.1
+14 C				1,261	0.257	0.232	0.025	
+15 C	1,308	1,200		911.0	0.104	0.127	-0.023	
+16 C	1,841	1,420		1,267	0.384	0.412	-0.028	
S	1,736	1,557		1,629	0.444	0.412	0.032	79.4
+17 W	1,223	1,149	1,067	861.0	0.425	0.317	0.108	51.0
+18 W	497.0	274.0	318.0	296.0	0.309	0.223	0.086	22.4
+19 C				1,705	0.282	0.282	0	
+20 C				1,664	0.346	0.283	0.063	
+21 C	2,788			1,499	0.390	0.285	0.105	
S	2,763	2,052		2,046	0.390	0.285	0.105	86.4

C - Carah and Park S - Scarbrough W - Wellman

[a] Notation: - Furnished by the former Northrop Aircraft, Inc., now the El Segundo division of the Douglas Aircraft Co., Inc.
+ Furnished by the Boeing Aircraft Co.

Table 5 - Continued

1(a)	2	3	4	5	6	7	8	9
+22 S	2,360	1,630		2,023	0.487	0.459	0.028	74.5
+23 S	2,008	1,611		1,922	0.456	0.401	0.055	67.4
+24 W	4,420	1,450	1,593	1,748	0.466	0.343	0.123	78.4
+25 W	2,190	1,196	1,355	1,592	0.366	0.287	0.079	47.0
+26 W	2,075	994.0	1,193	1,325	0.296	0.232	0.064	47.4
+27 S	1,279	1,126		1,278	0.173	0.178	-0.005	31.8
+29 W	4,160	2,110	2,055	2,087	0.491	0.401	0.090	78.4
+30 W	2,795	1,682	1,820	1,895	0.456	0.343	0.113	73.3
+31 S	1,932	1,593		1,835	0.303	0.287	0.016	50.4
+32 W	2,480	1,469	1,543	1,656	0.276	0.232	0.044	49.2
+33 W	1,800	1,173	1,305	1,366	0.198	0.178	0.020	20.3
+34 C	2,560	2,292		1,762	0.467	0.459	0.008	
S	2,646			2,418	0.496	0.459	0.037	100.5
W	3,270	1,970	2,835	2,323	0.556	0.459	0.097	95.1
+35 C	2,356			1,629	0.393	0.401	-0.008	
+36 C	2,195			1,533	0.337	0.343	-0.006	
+37 C				1,273	0.218	0.232	-0.014	
+39 W	1,770	1,230	1,254	2,151	0.571	0.459	0.112	128.3
+40 W	6,000	1,742	2,445	2,072	0.531	0.401	0.130	98.7
+41 W	3,990	1,635	2,150	2,015	0.451	0.343	0.108	98.0
+42 C	2,011			1,229	0.228	0.232	-0.004	
+43 W	1,450	1,135	1,211	1,434	0.221	0.178	0.043	49.8

C - Carah and Park S - Scarbrough W - Wellman

[a] Notation:
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- \+ Furnished by the boeing Aircraft Co.

Table 6

Critical Loads

(a) Channel Number	Length (in.)	P_{cr}^{*} (lb.)(b)	P_{cr}^{**} (lb.)(b)	P_{cr}^{***} (lb.)(b)	$\frac{P_{cr}^{*} - P_{cr}^{**}}{P_{cr}^{*}}$ (b)
- 5	32.06	34	20	24	0.412
- 9	31.75	58	33	76	.431
-12	31.12	75	60	90	.200
+ 5	31.75	32	23	37	.281
+ 9	32.09	100	77	67	.130
+10	32.06	43	25	32	.419

(b)
- P_{cr} is the experimental buckling load when applied at the shear center.
- P_{cr}^{**} is the experimental buckling load when applied at the centroid of the section.
- P_{cr}^{***} is the computed value of the critical load, to be applied at the shear center.

[a] Notation:
- \- Furnished by the former Northrop Aircraft Co., Inc., now the El Segundo division of the Douglas Aircraft Co., Inc.
- \+ Furnished by the Boeing Aircraft Co.

Table 7

Observed and Computed Values of I_{xx}/t and Y_o

	Obs. I_{xx}/t from.		Computed I_{xx}/t from			Obs.	Computed Y_o from		
Channel	Beam tests	Column tests	D_e=D	D_e=D'	D_e=D"	Y_o	D_e=D	D_e=D'	D_e=D"
1 (a)	2	3	4	5	6	7	8	9	10
-0 A	0.2132	0.2155	0.1930	0.2132	0.1930	0.287	0.303	0.274	0.303
-1 A	0.2248	0.2263	0.2004	0.2215	0.2004	0.302	0.296	0.266	0.296
-2 A	0.2281	0.2202	0.2062	0.2281	0.2062	0.315	0.291	0.261	0.291
W	0.2311								
-3 A	0.2399	0.2445	0.2092	0.2311	0.2092	0.310	0.288	0.258	0.288
W		0.2631				0.326			
-4 A	0.2310		0.2137	0.2361	0.2137				
W	0.2498								
-5 A	0.2410		0.2160	0.2388	0.2160				
-6 A	0.5924	0.5848	0.5400	0.6310	0.5430	0.454	0.468	0.405	0.464
-7 A	0.3342	0.3303	0.2925	0.3439	0.2949	0.338	0.352	0.299	0.346
W	0.3409	0.3389				0.361			
-8 A	0.1510	0.1515	0.1294	0.1526	0.1312	0.247	0.245	0.203	0.242
-9 A	0.0937		0.0781	0.0913	0.0789				
-10 A	0.6860		0.5830	0.6860	0.6170				
-11 A	0.3678	0.3658	0.3153	0.3695	0.3320	0.299	0.327	0.271	0.310
-12 A	0.1688	0.1635	0.1414	0.1649	0.1489	0.215	0.223	0.181	0.210
-13 A	0.1025		0.0851	0.0987	0.0890				
W	0.1060								
-14 A	0.5420		0.4875	0.5650	0.4875				
-15 A	0.2902	0.3001	0.2619	0.3053	0.2619	0.395	0.384	0.338	0.384
W	0.3050	0.3089				0.419			

Table 7 - Continued

1 (a)	2	3	4	5	6	7	8	9	10
-16 A	0.1265	0.1264	0.1166	0.1364	0.1166	0.312	0.270	0.233	0.270
W		0.1342				0.317			
-17	0.0842		0.0705	0.0825	0.0705				
W	0.0893								
-18 A	0.6585	0.6473	0.5850	0.6760	0.6030	0.428	0.437	0.376	0.421
-19 A	0.3625	0.3626	0.3185	0.3645	0.3266	0.311	0.327	0.276	0.315
W	0.3537								
-20 A	0.1601	0.1545	0.1411	0.1617	0.1461	0.223	0.224	0.186	0.215
W		0.1892				0.234			
-21 A	0.0982		0.0846	0.0966	0.0876				
-22 A	0.6635	0.5862	0.5830	0.6535	0.5870	0.419	0.437	0.390	0.435
W	0.6380								
-23 A	0.3341		0.3172	0.4050	0.3188				
W	0.3560								
-24 A	0.1562	0.1554	0.1416	0.1579	0.1426	0.231	0.223	0.194	0.222
W		0.1903				0.244			
-25 A	0.0982		0.0896	0.0940	0.0852				
-26 A	0.7924	0.7343	0.7420	0.7420	0.7420	0.332	0.332	0.332	0.332
-27 A	0.4310	0.4311	0.3991	0.3991	0.3991	0.246	0.241	0.241	0.241
W	0.4341								
-28 A	0.1923	0.1906	0.1756	0.1756	0.1756	0.186	0.161	0.161	0.161
-29 A	0.1143		0.1041	0.1040	0.1041				
+1 W	0.2352	0.2566	0.2050	0.2327	0.2050	0.323	0.292	0.254	0.292
+2 W	0.1502	0.1884	0.1378	0.1561	0.1396	0.257	0.230	0.196	0.226
+3 A	0.4516	0.4590	0.4110	0.4748	0.4205	0.407	0.403	0.351	0.395
W	0.4732								
+4 A	0.2121	0.2235	0.1818	0.2159	0.1902	0.330	0.322	0.275	0.311
W		0.2483				0.355			

A - Allen and Silliman W - Wellman

[a] Notation:

- Furnished by the former Northrop Aircraft, Inc., now the El Segundo division of the Douglas Aircraft Co., Inc.

+ Furnished by the Boeing Aircraft Co.

Table 7 - Continued

1 (a)	2	3	4	5	6	7	8	9	10
+5 A	0.0708	0.0729	0.0606	0.0746	0.0652	0.244	0.239	0.200	0.226
W	0.0730								
+6 A	0.2409		0.2050	0.2471	0.2155				
+7 A	0.1645		0.1250	0.1629	0.1459				
+8 A	0.4721		0.4110	0.4930	0.4358		0.403	0.336	0.382
W		0.4911				0.394			
+9 A	0.2252		0.1824	0.2236	0.1965		0.322	0.265	0.302
W		0.2568				0.348			
+10 A	0.0792		0.0606	0.0770	0.0664		0.239	0.195	0.223
W	0.0826	0.1093				0.251			
+11 W	0.2618	0.2651	0.2131	0.2381	0.2131	0.294	0.280	0.247	0.280
+12 W	0.6432		0.5730	0.6410	0.5730				
+13 W	0.3460	0.3717	0.3092	0.3446	0.3092	0.348	0.334	0.296	0.334
+16 A	0.4507		0.4110	0.4668	0.4310				
+17 A	0.2027		0.1817	0.2129	0.1879		0.322	0.279	0.314
W		0.2449				0.345			
+18 A	0.0686		0.0606	0.0738	0.0640		0.239	0.202	0.229
W	0.0704	0.1015				0.249			
+19 A	0.2106	0.2178	0.1927	0.2135	0.1927	0.313	0.303	0.274	0.303
+20 A	0.2089	0.2101	0.1963	0.2170	0.1963	0.250	0.300	0.271	0.300
+22 A	0.5410	0.5203	0.4860	0.5650	0.4860	0.513	0.503	0.450	0.503
+23 A	0.3913		0.3630	0.4223	0.3630				
+24 W	0.3000	0.2987	0.2611	0.3045	0.2611	0.407	0.385	0.339	0.385
+25 A	0.1968		0.1796	0.2102	0.1796		0.327	0.285	0.327
W		0.2198				0.363			
+26 W	0.1368		0.1168	0.1366	0.1168				
+27 A	0.0804		0.0698	0.0816	0.0698				
W	0.0819								
+28 A	0.6412		0.5730	0.6665	0.5910				
+29 W	0.5082	0.5050	0.4280	0.4975	0.4420	0.392	0.388	0.331	0.377
+30 W	0.3552	0.3678	0.3098	0.3585	0.3190	0.334	0.334	0.282	0.323

Table 7 - Continued

1 (a)	2	3	4	5	6	7	8	9	10
+31 A	0.2413		0.2134	0.2472	0.2203				
W	0.2460								
+32 W		0.1827	0.1384	0.1596	0.1439	0.255	0.229	0.189	0.221
+33 W	0.0988	0.1006	0.0828	0.0951	0.0847	0.183	0.180	0.147	0.173
+34 W		0.6617	0.7425	0.7425	0.7425	0.363	0.332	0.332	0.332
+37 A	0.1912		0.1756	0.1756	0.1756				
+39 W		0.4882	0.5730	0.6400	0.5730	0.447	0.444	0.399	0.444
+40 W	0.4885	0.4864	0.4282	0.4791	0.4282	0.398	0.389	0.347	0.389
+41 W	0.4049		0.3098	0.3454	0.3098				
+43 W	0.0964	0.1227	0.0826	0.0919	0.0826	0.197	0.180	0.156	0.180

A - Allen and Silliman W - Wellman

[a] Notation:

- Furnished by the former Northrop Aircraft, Inc., now the El Segundo division of the Douglas Aircraft Co., Inc.

+ Furnished by the Boeing Aircraft Co.

Table 8

Comparison of Observed and Computed Results

Notation

I_e = moment of inertia indicated in column test
I_b = " " " " " bending "
I_c = " " " computed from formula
D_e = effective diameter of lightening hole
D = actual " " " "
D' = $(0.2 + 1.5\ D^2/Pb)D$
D" = $(0.7 + D^2/Pb)D$ but not greater than D

A, Allen and Silliman tests
W, Wellman tests
C, Groups A and W combined
S, Group C omitting W tests where A are available
M, I_e from W and I_b from A tests

Item	Assumed D_e	Test group	Number of tests	Extremes min.	Extremes max.	Arithmetic mean	Algebraic mean	Algebraic median
1	2	3	4	5	6	7	8	9
$(I_e/I_b-1)\times 100$		A	25	-11.6	+5.3	2.42	-0.71	-0.2
		W	14	-0.7	44.1	11.10	10.81	2.7
		M	13	1.4	48.0	16.68	16.68	14.0
		S	41	-11.6	44.1	6.74	4.47	0.7
$(I_e/I_c-1)\times 100$	D	A	25	-1.0	22.9	11.37	11.29	11.6
		W	26	-14.8	80.4	28.60	26.61	23.4
		C	51	-14.8	80.4	20.15	19.10	15.9
		S	44	-14.8	80.4	19.28	18.06	14.5
	D'	A	25	-10.3	8.5	3.82	-1.34	-1.6
		W	26	-23.7	42.0	12.75	9.64	9.15
		C	51	-23.7	42.0	8.37	4.26	1.1
		S	44	-23.7	42.0	8.10	3.47	0.25

Table 8 - Continued

1	2	3	4	5	6	7	8	9
	D"	A	25	-1.0	17.5	9.69	9.59	9.1
		W	26	-14.8	64.6	25.71	23.73	23.4
		C	51	-14.8	64.6	17.86	16.80	14.2
		S	44	-14.8	64.6	16.90	15.67	12.75
$(I_b/I_c-1)\times 100$	D	A	50	5.4	31.5	13.50	13.50	12.3
		W	29	8.8	36.3	16.96	16.96	16.2
		C	79	5.4	36.3	14.77	14.77	14.0
		S	63	5.4	31.5	14.16	14.16	13.4
	D'	A	50	-17.4	9.7	3.80	-0.90	-1.1
		W	29	-12.1	17.3	3.92	1.69	0.4
		C	79	-17.4	17.3	3.84	0.05	-0.3
		S	63	-17.4	17.3	3.78	-0.15	-0.5
	D"	A	50	5.0	19.5	10.93	10.93	10.55
		W	29	7.5	30.7	15.07	15.07	14.7
		C	79	5.0	30.7	12.45	12.45	11.5
		S	63	5.0	30.7	11.93	11.93	11.2
$(Y_{o\ obs} - Y_{o\ comp})$	D	A	25	-0.050	0.042	0.0142	0.0003	0.004
		W	26	-0.010	0.047	0.0196	0.0181	0.019
		C	51	-0.050	0.047	0.0170	0.0094	0.009
		S	44	-0.050	0.042	0.0153	0.0070	0.008
	D'	A	25	-0.021	0.079	0.0393	0.0376	0.039
		W	26	0.031	0.084	0.0590	0.0590	0.0595
		C	51	-0.021	0.084	0.0493	0.0485	0.051
		S	44	-0.021	0.079	0.0467	0.0457	0.0485
	D"	A	25	-0.050	0.042	0.0141	0.0045	0.007
		W	26	0.003	0.047	0.0344	0.0344	0.022
		C	51	-0.050	0.047	0.0194	0.0147	0.014
		S	44	-0.050	0.046	0.0175	0.0120	0.0115

Table 9

Comparison of Computed and Observed Stiffnesses
Pure Bending

Channel	Computed I/t		Obs I/t	Percent error		D^2/Pb
	Full back	Full hole	P.B.	Full back	Full hole	
1 (a)	2	3	4	5	6	7
-0 C	3.488	3.240	3.777	8.3	16.5	0.380
-2 S	3.657	3.409	3.415	-6.6	0.2	0.375
-3 W	3.690	3.442	5.526	49.8		0.374
-5 C	3.777	3.529	3.810	0.9	8.0	0.371
-6 W	4.750	4.469	3.408	-28.2	-23.7	0.283
-7 S	4.025	3.744	4.213	4.7	12.5	0.282
W	4.025	3.744	9.490	135.9		0.282
-9 C	2.908	2.627	2.923	0.5	11.4	0.282
-10 W	4.750	4.587	6.794	43.1		0.150
-12 C	3.274	3.111	3.458	5.6	11.0	0.150
-13 C	2.917	2.754	3.111	6.2	13.0	0.150
S	2.917	2.754	2.921	0.1	6.0	0.150
-15 W	4.017	3.571	4.590	14.3		0.384
-16 W	3.270	2.824	3.596	10.0		0.385
-17 C	2.915	2.469	2.430	-16.5	-1.5	0.384
-18 C	4.756	4.593	4.826	1.5	5.0	0.196
-19 S	4.013	3.850	3.972	-1.0	3.2	0.196
-20 W	3.270	3.107	7.020	114.7		0.196
-22 S	4.747	4.584	4.590	-3.3	0.1	0.284
W	4.747	4.584	7.164	51.1		0.284
-24 W	3.274	3.111	4.890	49.4		0.284
-25 S	2.908	2.745	2.879	-1.0	4.9	0.283
-29 C	2.905	2.905	2.987	2.8	2.8	0
+1 W	3.637	3.389	5.396	48.4		0.324
+2 W	3.257	3.068	3.770	15.7		0.268
+3 S	3.425	3.286	3.418	-0.2	4.0	0.244
+4 W	1.539	1.470	1.890	22.8		0.198
+5 C	0.528	0.501	0.561	6.3	12.0	0.151
+6 S	3.637	3.389	3.489	-4.1	3.0	0.189
+7 C	3.267	3.078	3.245	0.9	5.5	0.157
+8 W	3.427	3.288	4.250	24.0		0.143
+9 C	1.544	1.475	1.529	-1.0	3.6	0.116
W	1.544	1.475	1.806	17.0		0.116

Table 9 - Continued

1 (a)	2	3	4	5	6	7
+10 C	0.527	0.500	0.574	8.9	14.8	0.088
W	0.527	0.500	0.555	5.3	11.0	0.088
+11 W	3.637	3.448	4.608	26.7		0.313
+12 S	4.747	4.558	5.160	8.7	13.2	0.313
+13 W	4.009	3.820	6.840	70.6		0.313
+15 C	2.532	2.393	2.452	-3.2	2.5	0.256
+16 C	3.425	3.286	3.470	1.3	5.6	0.285
S	3.425	3.286	3.272	-4.5	-0.4	0.285
+17 W	1.539	1.470	2.000	30.0		0.231
+18 W	0.527	0.500	0.818	55.2		0.176
+21 C	3.569	3.321	4.180	17.1		0.380
S	3.569	3.321	4.142	16.0		0.380
+22 S	4.747	4.301	4.450	-6.2	3.4	0.385
+23 S	4.376	3.930	3.741	-14.5	-4.8	0.385
+24 W	4.009	3.563	8.394	109.6		0.385
+25 W	3.637	3.191	4.062	11.7		0.385
+26 W	3.267	2.821	3.890	19.1		0.385
+27 S	2.899	2.453	2.391	-17.5	-2.5	0.385
+29 W	4.376	4.187	7.870	80.0		0.217
+30 W	4.008	3.819	5.221	30.2		0.217
+31 S	3.637	3.448	3.584	-1.5	4.0	0.217
+32 W	3.267	3.078	4.670	42.9		0.217
+33 W	2.899	2.710	3.369	16.2		0.217
+34 C	4.747	4.747	4.800	1.1	1.1	0
S	4.747	4.747	4.960	4.5	4.5	0
W	4.747	4.747	6.134	29.2		0
+35 C	4.376	4.376	4.370	-0.1	-0.1	0
+36 C	4.008	4.008	4.169	4.0	4.0	0
+39 W	4.747	4.558	3.310	-30.2	-27.4	0.313
+40 W	4.376	4.187	11.185	155.8		0.313
+41 W	4.008	3.819	7.424	85.0		0.313
+42 C	3.267	3.078	3.732	14.2		0.313
+43 W	2.899	2.710	2.711	-6.5	0.1	0.313

C - Carah and Park   S - Scarbrough   W - Wellman

[a] Notation: - Furnished by the former Northrop Aircraft, Inc., now the El Segundo division of the Douglas Aircraft Co., Inc.
+ Furnished by the Boeing Aircraft Co.

Table 10. Comparison of Computed and Observed Deflections
Simple, Cantilever, and Two Load Bending

Channel	Simple Bending			Cantilever Bending			Two Load Bending		
	Comp. $Et\delta_t$	Obs. $Et\delta_t$	Percent error	Comp. $Et\delta_t$	Obs. $Et\delta_t$	Percent error	Comp. $Et\delta_t$	Obs. $Et\delta_t$	Percent Error
1 (a)	2	3	4	5	6	7	8	9	10
-0 C	26,950	27,690	2.7	68,530	104,130	53.3			
-2 S	24,880	23,360	-6.1	53,950	53,840	-0.2			
-3 W	25,410	25,520	0.4	51,670	56,340	9.1	8,521	8,972	5.3
-5 C	24,790	23,880	-3.7	63,210	84,970	34.4			
-6 W	18,400	31,630	72.0	39,170	46,140	17.8	6,284	10,257	63.2
-7 S	20,310	20,100	-1.0	47,260	50,520	6.9			
W	20,960	25,000	19.3	45,670	47,640	4.3	7,228	7,077	-2.1
-9 C	27,490	26,730	-2.8	78,330	106,670	36.2			
-10 W	16,870	13,710	-18.7	38,060	40,520	6.5	5,901	5,759	-2.4
-12 C	23,350	24,270	3.9	68,730	92,880	35.1			
-13 C				76,890	100,810	31.1			
S	25,060	25,430	1.5	62,960	65,470	4.0			
-15 W	24,540	31,110	26.8	48,340	49,420	2.2	8,125	8,985	10.6
-16 W	28,480	29,880	4.9	58,170	62,360	7.2	9,560	11,009	15.6
-17 C	30,940	34,140	10.3	80,970	113,490	40.2			
-18 C	17,220	18,220	5.8	48,230	70,520	46.2			
-19 S	19,250	16,750	-13.0	46,560	47,410	1.8			
-20 W	23,760	26,390	11.1	54,730	55,920	2.2	8,378	6,461	-22.9
-22 S	17,860	16,960	-5.0	40,570	42,860	5.7			
W	18,430	11,040	-40.0	39,200	41,660	6.3	6,292	15,916	153.0
-24 W	24,900	22,380	-10.1	55,510	56,320	1.5	8,666	8,063	-7.0
-25 S	26,640	26,580	-0.2	64,290	66,740	3.8			
-26 C				47,420	66,040	39.4			
-27 C				55,780	74,820	34.2			
-29 C				76,600	102,180	33.4			
+1 W	23,750	22,720	-4.3	50,930	53,320	4.7	8,134	8,611	5.9
+2 W	24,740	26,280	6.2	55,590	60,780	9.3	8,638	9,939	15.1
+3 S	22,990	22,280	-3.1	54,870	54,610	-0.5			
+4 W	48,550	40,280	-17.0	114,880	126,560	10.2	17,334	15,397	-11.2
+5 C	134,450	136,300	1.4	417,680	512,820	22.8			
+6 S	20,900	21,050	0.7	51,100	52,710	3.1			
+7 C	23,450	23,900	1.9	68,910	94,000	36.4			
+8 W	22,660	16,100	-29.0	52,200	62,180	19.1	7,997	6,461	-19.2
+9 C	47,500	47,020	-1.0	144,130	188,490	30.7			
W	47,500	41,790	-12.0	113,880	121,900	7.0	17,059	16,373	-4.0
+10 C	122,660	133,100	8.5	418,020	490,920	17.4			
W	122,660	152,700	24.5	329,770	348,420	5.7	48,654	50,234	3.2
+11 W	23,450	22,390	-4.5	50,720	55,580	9.6	8,061	8,235	2.2
+12 S	18,470	15,270	-17.3	41,040	42,310	3.1			
+13 W	21,710	20,830	-4.1	46,330	47,390	2.3	7,422	7,156	-3.6
+14 C				70,610	92,960	31.6			
+15 C	30,510	30,320	-0.6	89,140	127,830	43.5			
+16 C	24,460	25,500	4.3	67,400	91,440	35.6			
S	23,700	22,490	-5.1	55,420	58,050	4.7			
+17 W	49,090	36,320	-26.0	115,260	122,160	6.0	17,464	14,365	-17.7
+18 W	123,850	151,400	22.2	330,640	353,430	6.9	48,952	47,838	-2.3
+19 C				68,670	106,510	55.1			
+20 C				67,830	98,760	45.5			
+21 C				67,070	97,220	45.0			
S	25,660	21,460	-16.4	55,410	58,120	4.9			
+22 S	21,340	21,480	0.7	43,220	46,730	8.1			
+23 S	22,520	21,980	-2.4	46,400	49,780	7.3			
+24 W	24,640	24,780	0.6	48,470	51,840	7.0	8,153	8,259	1.3
+25 W	26,370	30,740	16.6	52,860	58,270	10.2	8,792	9,971	13.4
+26 W	28,510	36,610	28.4	58,220	69,290	19.0	9,573	11,206	17.1
+27 S	30,220	31,320	3.6	67,160	74,560	11.0			
+29 W	18,690	17,320	-7.3	41,580	44,130	6.1	6,500	6,530	0.5
+30 W	20,130	21,710	7.9	45,190	48,620	7.6	7,026	7,373	4.9
+31 S	21,170	22,320	5.4	51,310	52,330	2.0			
+32 W	23,990	24,680	2.9	54,920	55,180	0.5	8,445	8,626	2.1
+33 W	26,630	31,100	16.7	61,620	67,310	9.2	9,420	10,269	9.0
+34 C	16,140	15,880	-1.5	47,420	66,120	39.4			
S				38,850	39,320	1.2			
W	16,140	18,480	14.5	37,530	39,500	5.3	5,718	4,714	-17.6
+35 C				51,320	72,310	40.9			
+36 C				55,910	75,070	34.2			
+37 C				68,270	92,490	35.5			
+39 W	19,060	29,670	55.8	39,660	42,770	7.8	6,450	10,690	65.8
+40 W	20,270	21,030	3.7	42,730	44,580	4.3	6,897	5,501	-20.3
+41 W	21,720	22,380	3.0	46,340	45,910	-0.9	7,423	6,267	-15.6
+42 C				70,610	95,820	35.7			
+43 W	28,210	32,170	14.0	62,770	64,180	2.2	9,814	11,072	12.8

C - Carah and Park   S - Scarbrough   W - Wellman

[a] Notation: - Furnished by the former Northrop Aircraft, Inc. now the El Segundo division of the Douglas Aircraft Co., Inc.
+ Furnished by the Boeing Aircraft Co.

Table 11

Torsional Stiffness of Specimens

Specimen	GJ			L_e		M_t/θ		
	Ob-served	Computed $w_e=w$	Computed $w_e=w-D$	$w_e=w$	$w_e=w-D$	Computed $w_e=w$	Computed $w_e=w-D$	Ob-served
1 (a)	2	3	4	5	6	7	8	9
-2 S	1,507	588	377	15.82	12.91	37.1	29.2	48.4
-3 W	1,720	385	248	11.02	8.51	35.0	29.1	61.7
-6 W	3,740	824	564	8.78	6.81	93.8	82.8	134.2
-7 S	1,637	624	404	13.70	10.89	45.6	37.1	52.6
W	1,888	624	404	11.00	8.98	56.8	45.0	67.7
-10 W	3,155	819	603	9.02	7.19	90.8	83.9	113.2
-13 S	874	495	319	20.24	17.76	24.5	18.0	28.1
-15 W	2,310	662	390	11.23	8.24	59.0	47.3	82.9
-16 W	1,460	650	347	15.52	11.89	41.9	29.2	52.4
-19 S	1,800	707	499	14.29	12.00	49.5	41.6	57.8
-20 W	1,532	646	431	15.49	13.19	41.7	32.7	55.0
-22 S	2,891	833	614	11.26	9.34	74.0	65.8	92.9
W	3,200	833	614	8.83	7.27	94.4	84.5	114.9
-24 W	2,110	636	424	15.41	13.11	41.2	32.4	75.7
-25 S	1,172	562	362	20.72	18.30	27.1	19.8	37.6
+1 W	2,310	731	468	14.64	12.04	49.9	38.9	82.9
+2 W	1,420	761	495	16.16	13.73	47.0	36.0	51.0
+3 S	2,410	741	534	14.02	11.87	52.8	45.0	77.4
+4 W	1,450	874	623	18.54	16.90	47.2	36.9	52.0
+6 S	2,023	739	474	17.74	15.01	41.7	31.5	65.0

Table 11 - Continued

1 (a)	2	3	4	5	6	7	8	9
+8 W	2,145	733	528	11.22	9.33	65.3	56.6	77.0
+9 W	1,550	836	597	18.38	16.71	45.5	35.7	55.6
+10 W	692	604	418	23.18	22.24	26.1	18.8	24.8
+11 W	1,930	739	497	14.67	12.36	50.4	40.2	69.2
+12 S	2,830	848	614	11.33	9.30	74.8	66.0	90.9
+13 W	2,345	728	503	11.63	9.46	62.6	53.2	84.1
+16 S	2,470	746	537	14.06	11.90	53.0	45.1	79.4
+17 W	1,420	870	620	18.51	16.89	47.0	36.7	51.0
+18 W	624	589	407	23.14	22.19	25.4	18.3	22.4
+21 S	2,690	1,387	886	19.38	16.73	71.6	52.9	86.4
+22 S	2,320	833	526	11.25	8.44	74.0	62.4	74.5
+23 S	2,100	817	500	12.93	9.76	63.2	51.2	67.4
+24 W	2,185	728	428	11.62	8.58	62.6	49.8	78.4
+25 W	1,310	735	414	14.66	11.28	50.1	36.7	47.0
+26 W	1,322	670	357	15.63	12.01	42.8	29.8	47.4
+27 S	990	629	314	21.11	17.27	28.8	18.2	31.8
+29 W	2,185	812	576	10.28	8.34	79.0	69.0	78.4
+30 W	2,043	767	530	11.83	9.68	64.8	54.8	73.3
+31 S	1,569	735	494	17.72	15.21	41.5	32.4	50.4
+32 W	1,372	662	430	15.60	13.13	42.5	32.8	49.2
+33 W	640	629	393	17.94	15.49	35.1	25.3	23.0
+34 S	3,130	848	848	11.32	11.32	74.9	74.9	100.5
W	2,650	848	848	8.91	8.91	95.2	95.2	95.1
+39 W	3,575	853	617	8.93	7.21	95.5	85.6	128.3
+40 W	2,750	817	579	10.31	8.37	79.2	69.2	98.7
+41 W	2,730	777	537	11.88	9.72	65.4	55.2	98.0
+43 W	1,388	629	393	17.94	15.49	35.1	25.4	49.8

S - Scarbrough W - Wellman

[a] Notation: - Furnished by the former Northrop Aircraft, Inc., now the El Segundo division of the Douglas Aircraft Co., Inc.
+ Furnished by the Boeing Aircraft Co.

TABLE A-1

Sample Data Sheet

Bending Test

Channel No. +31, Test No. 51

Tare Load = 10 lbs.

No.	Load W (lbs.)	Dial Readings (in thousandths of an inch)					Differences in Dial Reading		Deflection, δ (in. x 1000)
		1	2	3	4	5	2-3	4-3	
1	0	0	0	0	0	0	0	0	0
2	10	99.2	89.1	78.1	89.0	99.6	11.0	10.9	11.0
3	20	99.1	78.8	56.3	78.8	99.5	22.5	22.5	22.5
4	45	98.9	56.0	5.5	55.8	99.5	50.5	50.3	50.4
5	55	99.0	47.0	85.5	46.5	99.5	61.5	61.0	61.3
6	65	99.1	38.5	65.9	37.5	99.6	72.6	71.6	72.1
7	75	99.4	30.0	46.5	28.8	99.9	83.5	82.3	82.9
8	65	99.0	37.9	64.9	37.1	99.6	73.0	72.2	72.6
9	55	98.9	46.5	84.5	46.1	99.5	62.0	61.6	61.8
10	45	98.8	55.0	4.5	55.1	99.4	50.5	50.6	50.6
11	20	99.0	78.2	55.8	78.3	99.2	22.4	22.5	22.5
12	10	99.0	88.4	77.5	88.5	99.3	10.9	11.0	11.0
13	0	99.0	99.2	99.5	99.4	99.4	- 0.3	- 0.1	- 0.2

TABLE A-2

SAMPLE DATA SHEET
SHEAR CENTER TEST

Channel No. -26 Test No. 15-A

No.	Load W (lb.)	Dial Readings - (inches)				Shear-Center	
		Right	Left	δ-up	δ-down	d' * (in.)	d ** (in.)
1	0	0	0	0	0.0030		
2	1	0.0050	0.0050	0.0057	.0095	0.422	0.448
3	2	.0105	.0105	.0120	.0158	.436	.462
4	3	.0160	.0162	.0185	.0223	.453	.478
5	4	.0215	.0217	.0251	.0287	.457	.483
6	5	.0271	.0270	.0314	.0350	.456	.482
7	6	.0328	.0329	.0388	.0416	.457	.483
8	7	.0383	.0386	.0446	.0479	.457	.483
9	8	.0440	.0439	.0510	.0543	.458	.484
10	9	.0497	.0496	.0573	.0605	.458	.484
11	10	.0554	.0556	.0642	.0667	.457	.483
12	11	.0612	.0612	.0708	.0730	.458	.484
13	12	.0678	.0675	.0782	.0792	.458	.484
						Average	d = .483 in.

* d' is the distance from the back of the web to the shear center.

** d is the distance from the center-line of the web to the shear center.

Calculation of Analytical Shear-Center Distance

$$d = \frac{b^2 h^2 t}{4I}$$

h = 2.448 in.
b = 1.224 in.
t = 0.052 in.
I = 0.2545 in.^{4}

$$d = \frac{1.224^2 \times 2.448^2 \times 0.052}{4 \times 0.2545} =$$

0.460 in.

$$d_e/d_t = \frac{0.483}{0.460} = 1.050$$

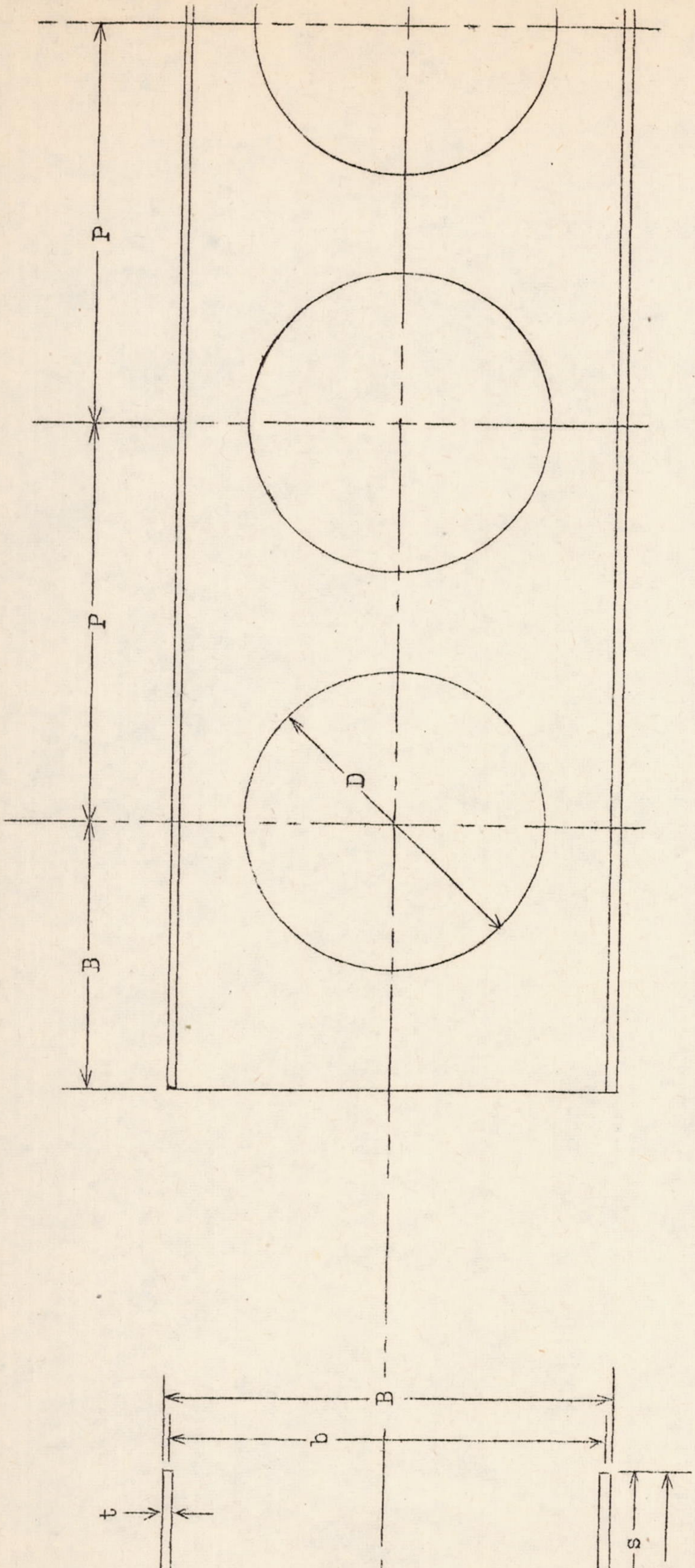

Figure 1.- Specimen dimensions.

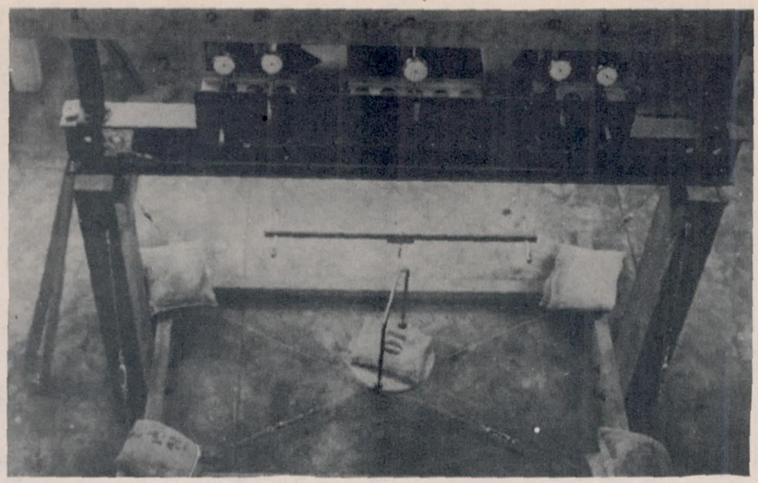

Figure 2.- Setup for Allen and Silliman test in pure bending.

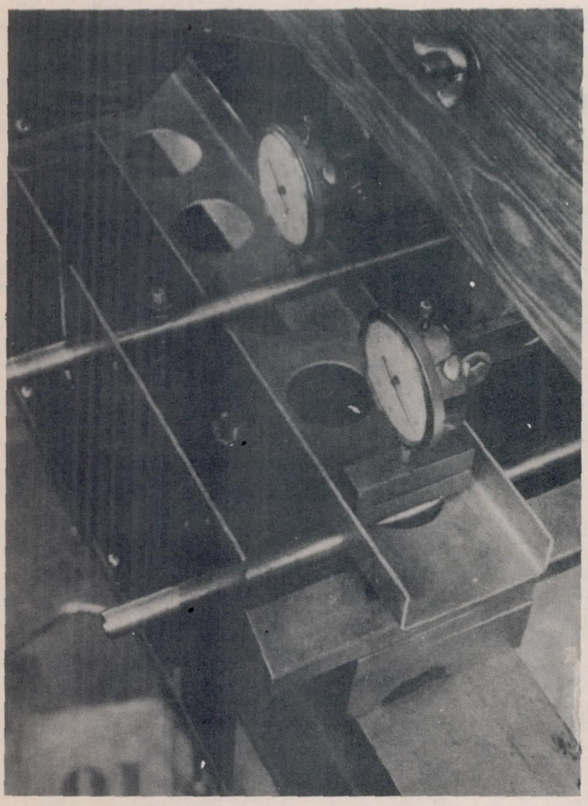

Figure 5.- Beam-support detail used in Allen and Silliman tests.

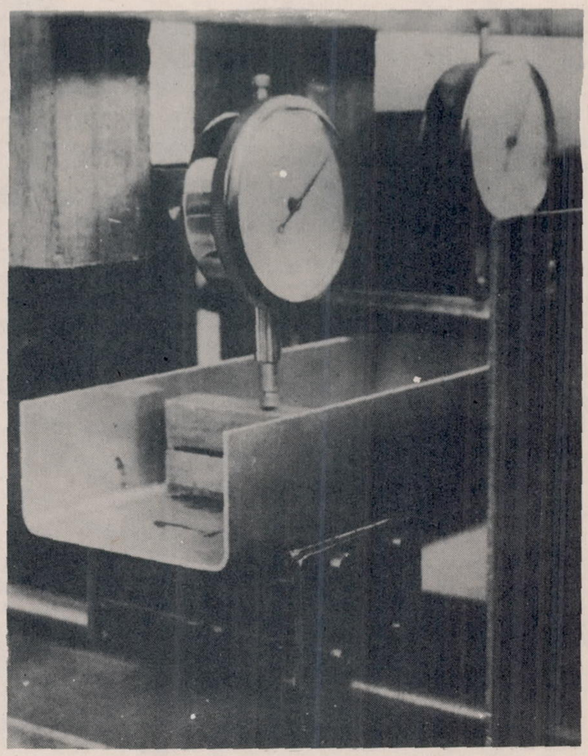

Figure 6.- Roller pad assembly.

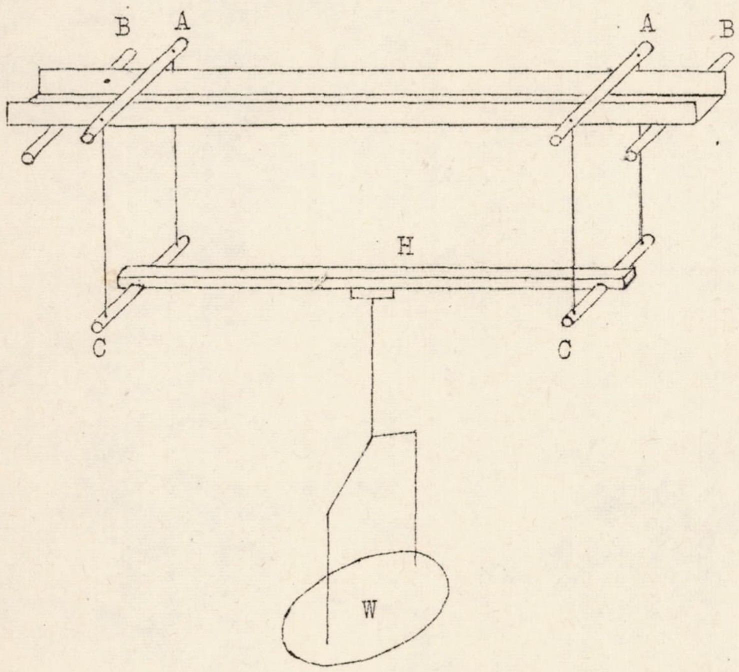

Figure 3.- Loading arrangement for pure bending tests.

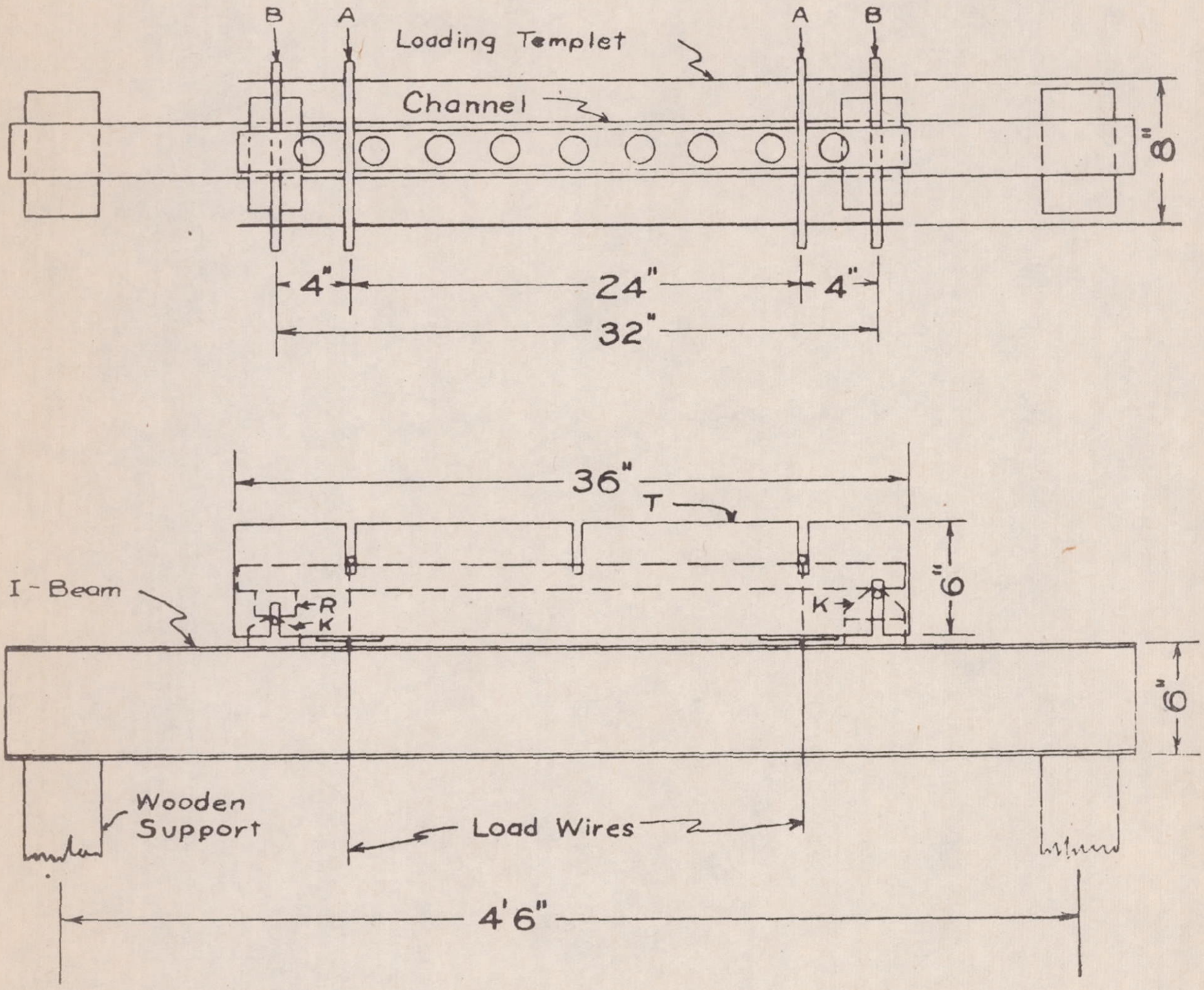

Figure 4.-

Apparatus Used for Pure Bending Tests

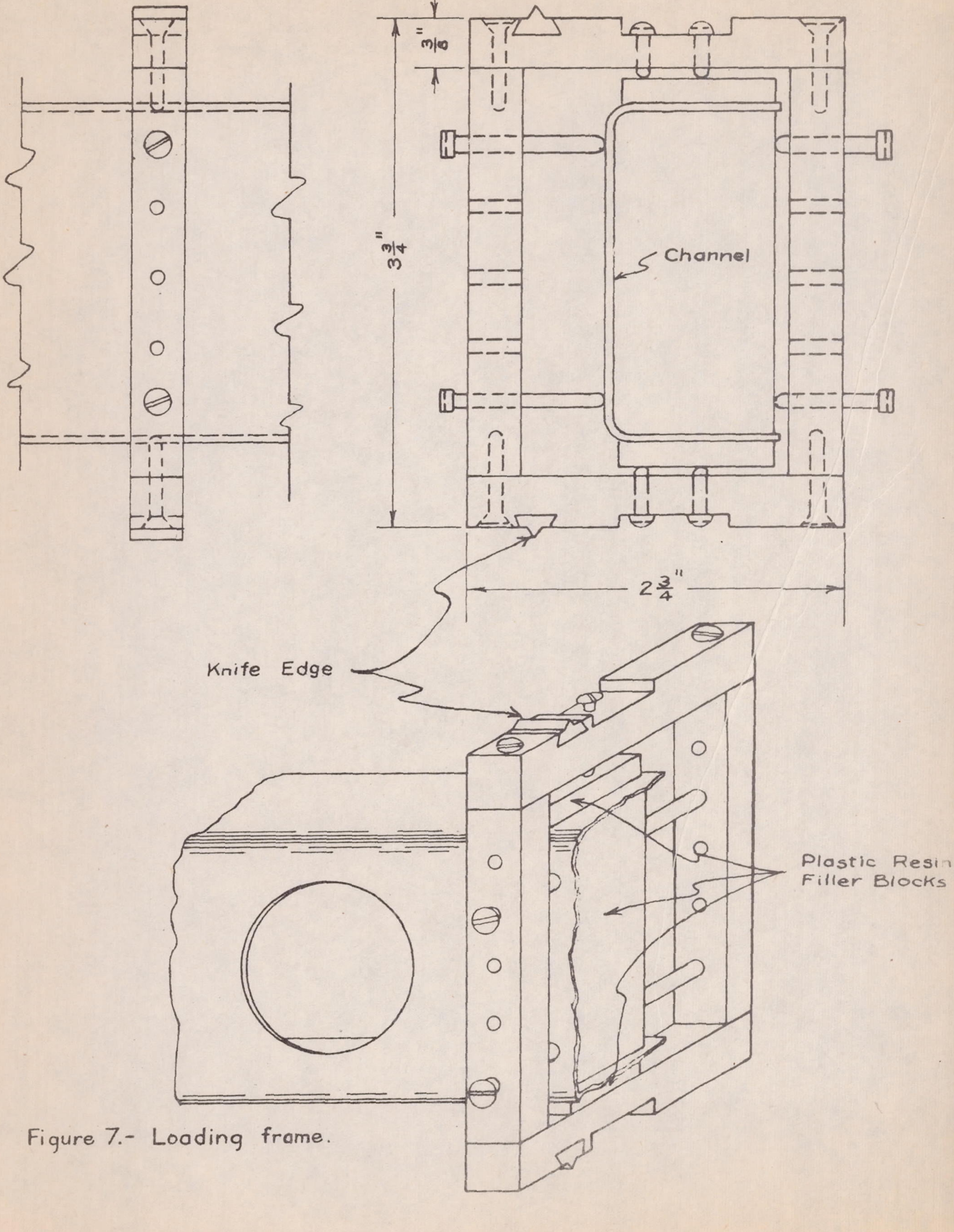

Figure 7.- Loading frame.

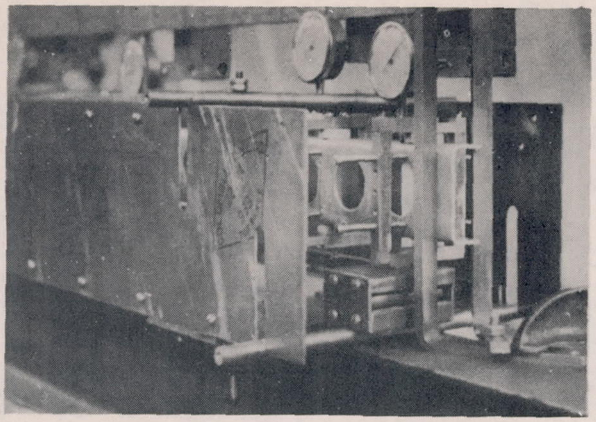

Figure 8.- Beam-support detail used in Carah and Park tests.

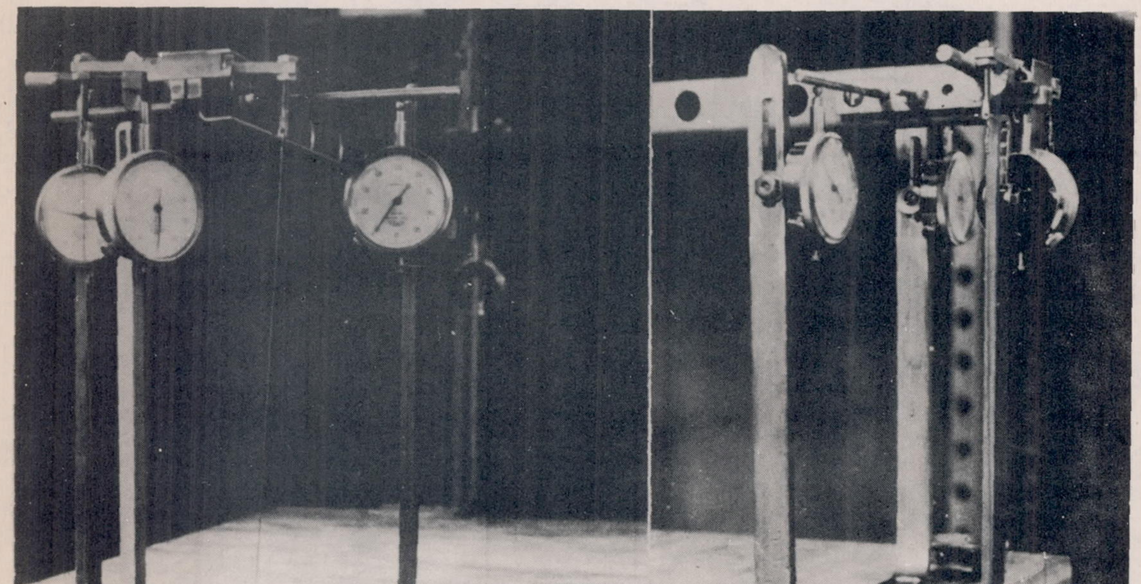

Figure 9.- Two views of free end of beam used in Carah and Park cantilever tests.

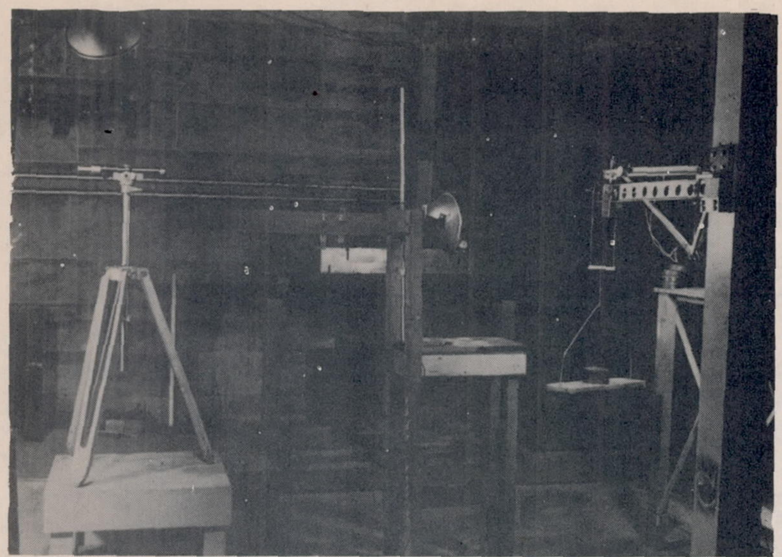

Figure 10.- Setup for Wellman cantilever-beam test.

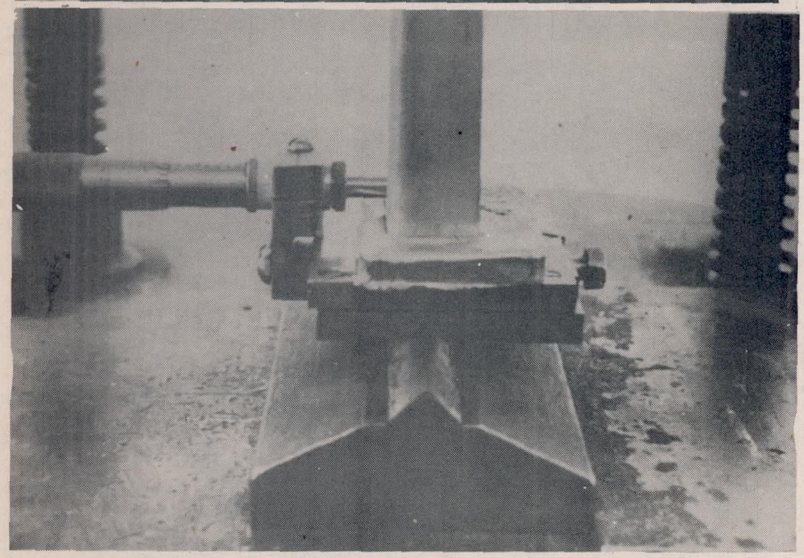

Figure 14.- Column end fitting used in Allen and Silliman tests.

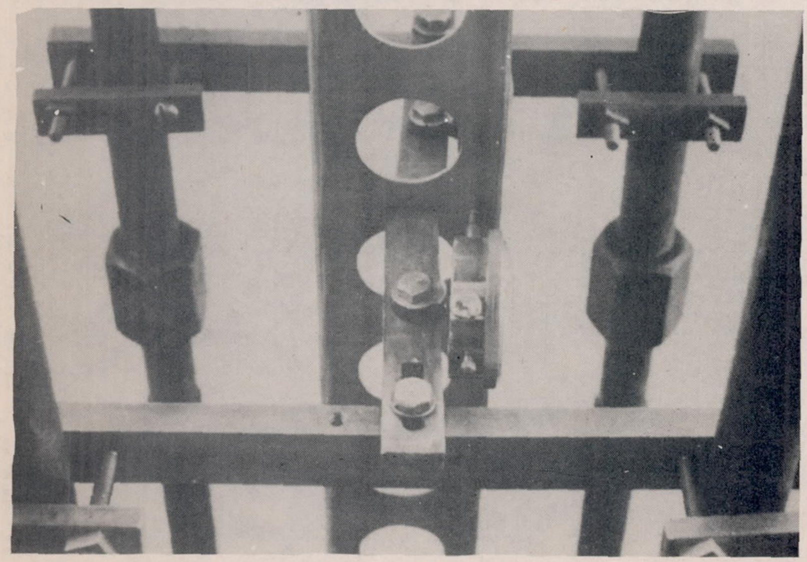

Figure 15.- Apparatus for measuring midspan deflection used in Allen and Silliman column tests.

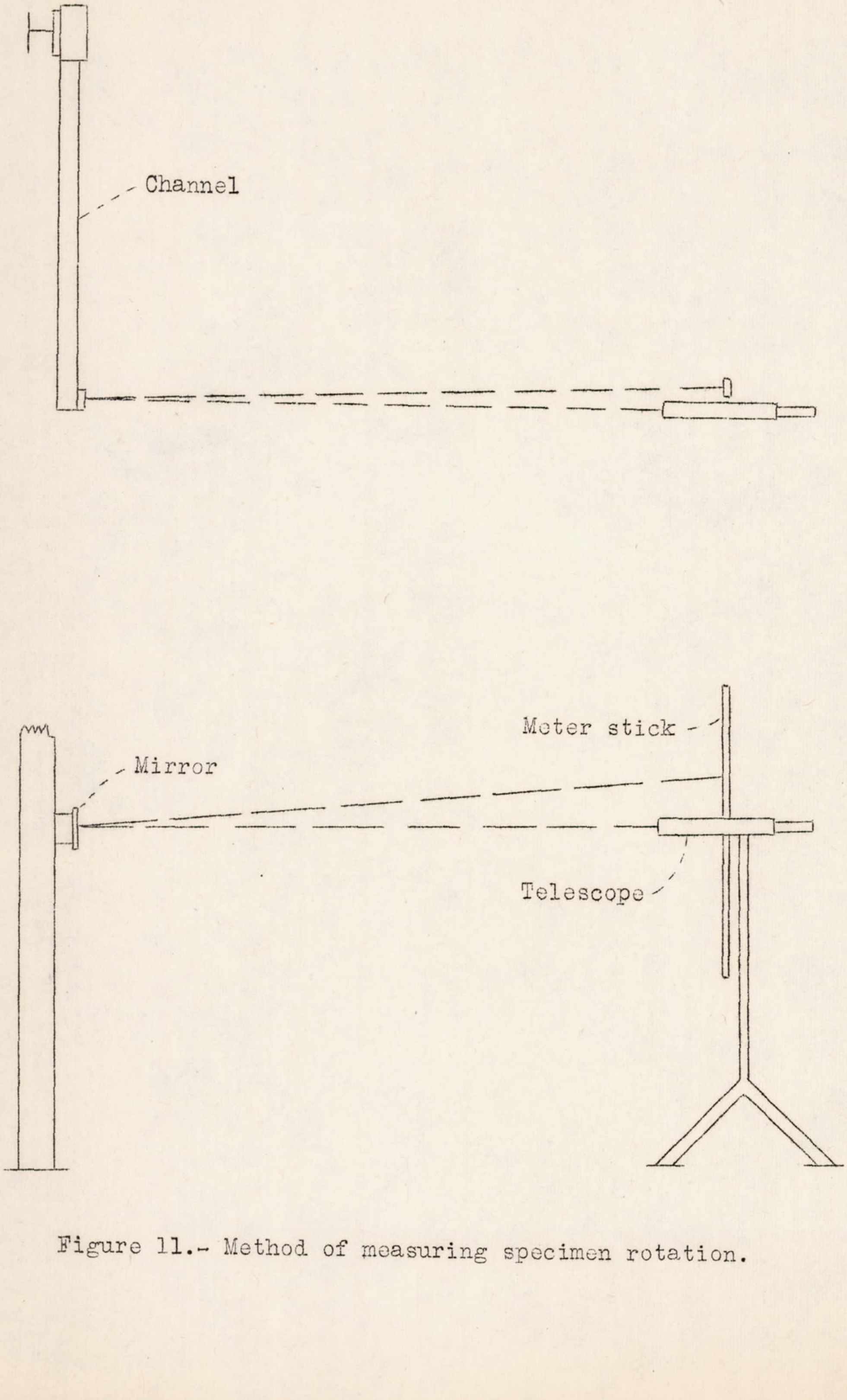

Figure 11.- Method of measuring specimen rotation.

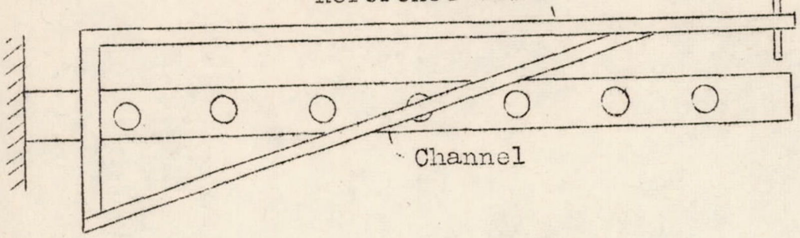

Figure 12.- Arrangement of reference bar.

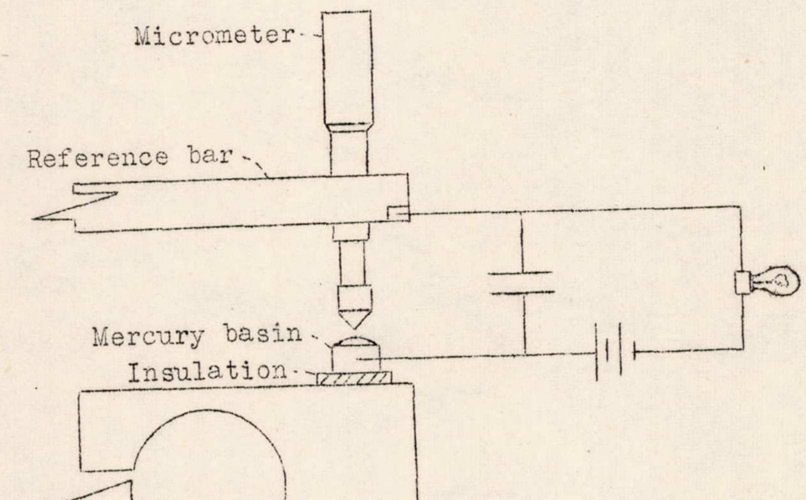

Figure 13.- Electrical system for determining deflection.

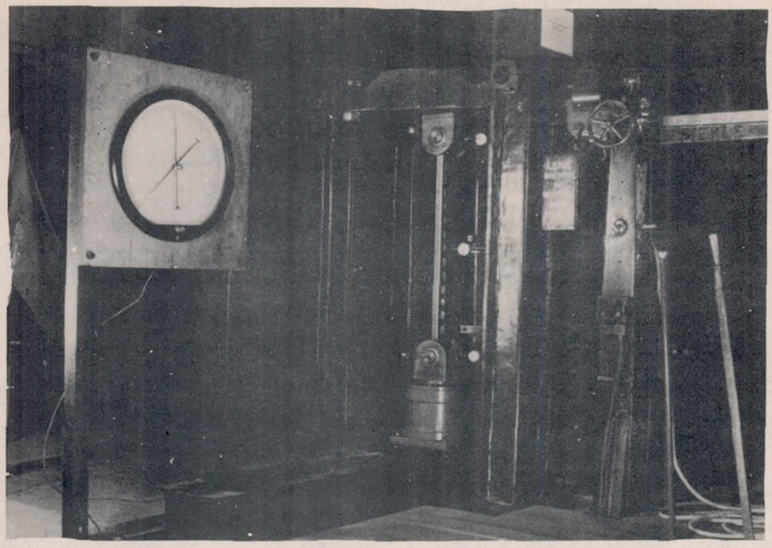

Figure 16.— General view of setup used in Wellman column test.

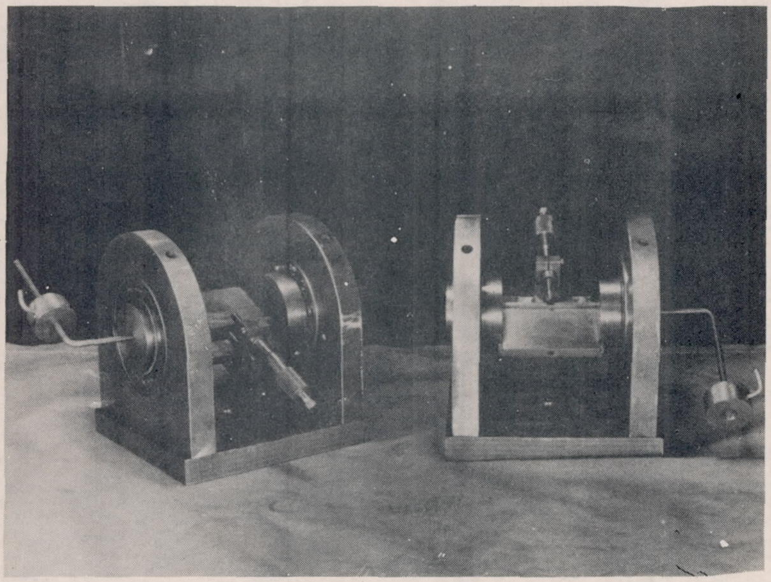

Figure 17.— Column end fittings used by Wellman.

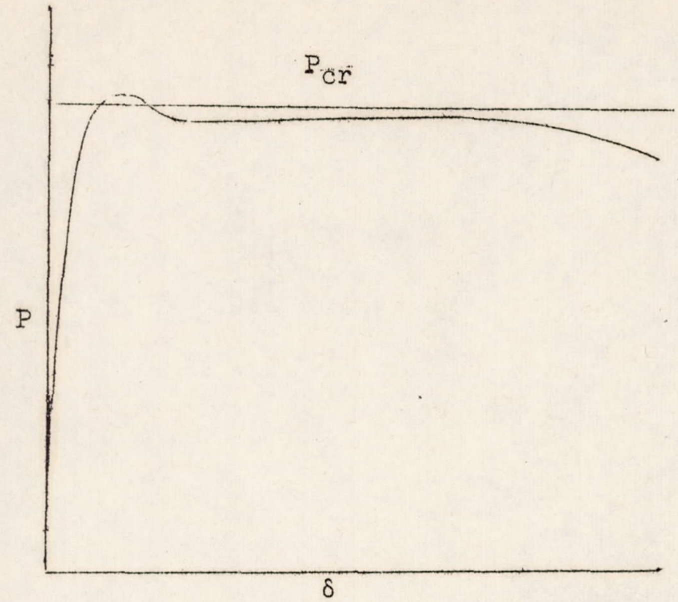

Figure 18.- Typical load against deflection curve of channel under axial compression.

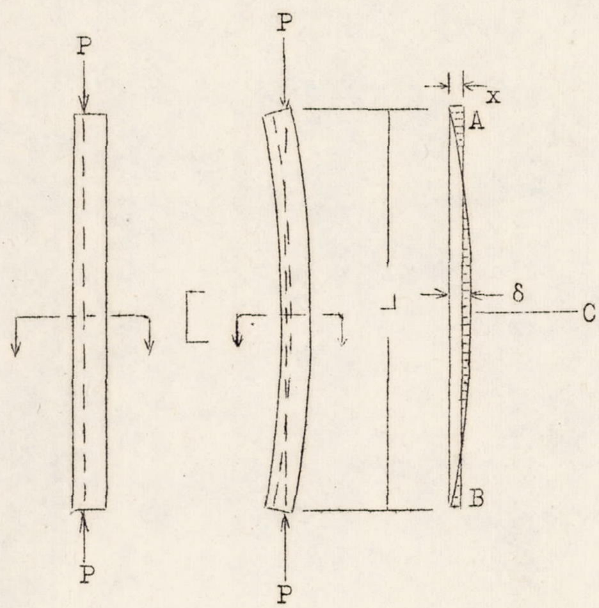

Figure 22.- Effect of initial curvature.

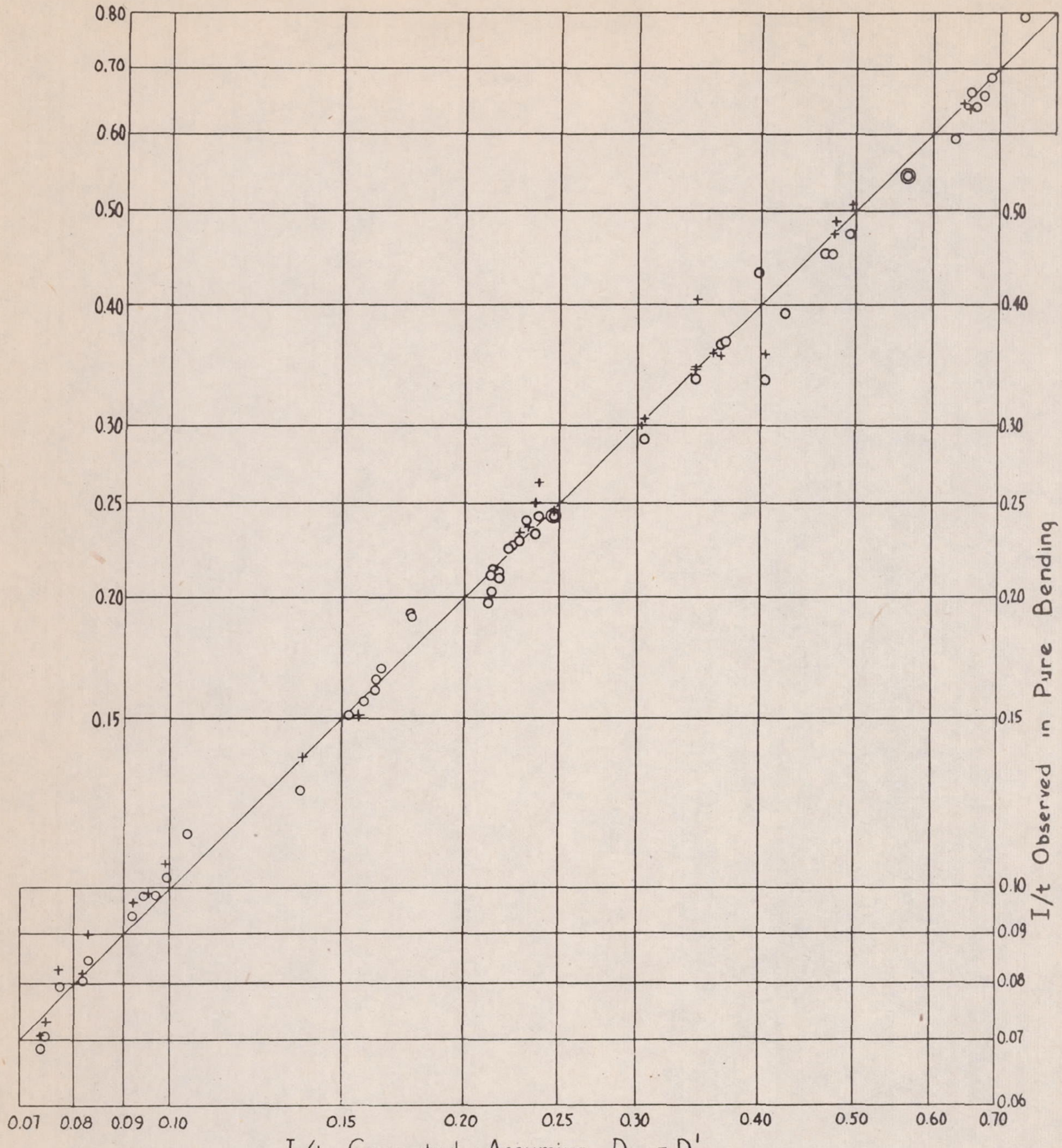

Figure 19.-

Comparison of I/t Observed in Pure-Bending Tests and I/t Computed on Assumption That $D_e = D'$

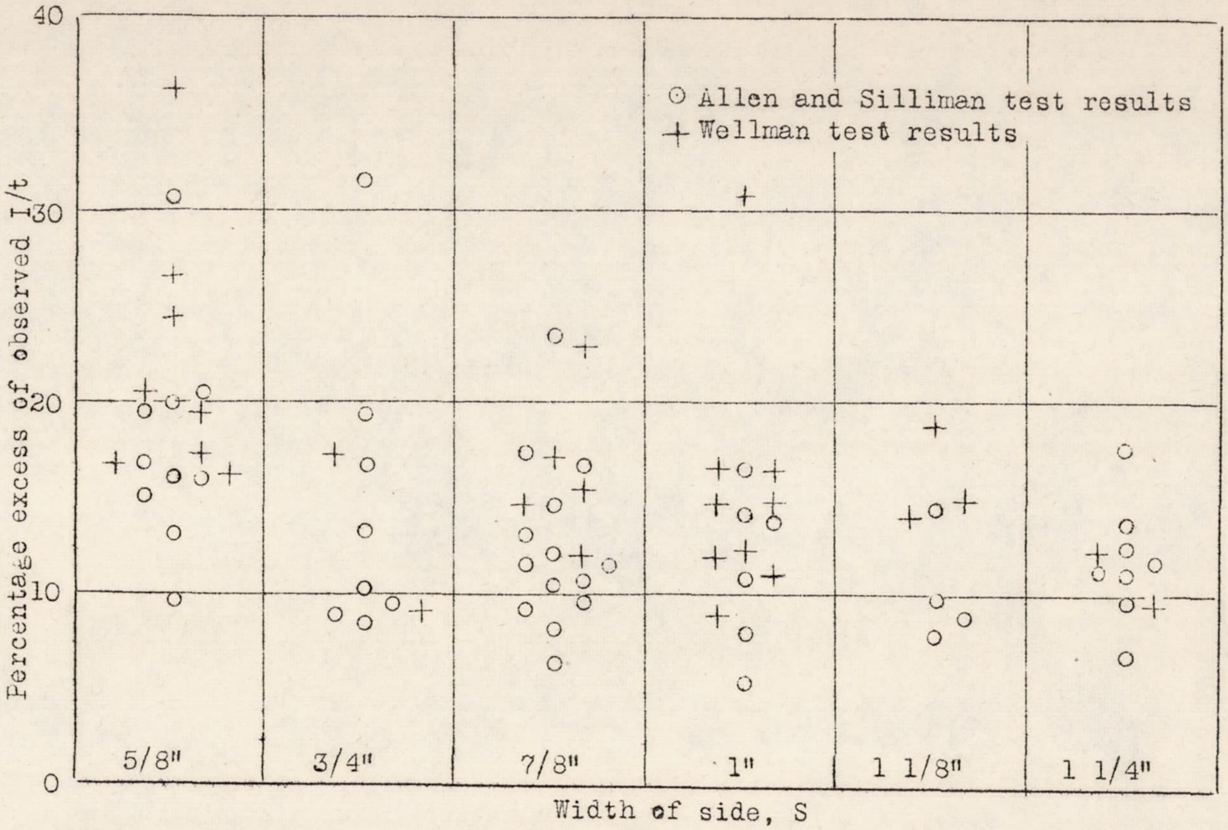

Figure 20.- Percentage excess of observed I/t from pure-bending tests above I/t computed assuming $D_e = D$, grouped according to nominal width of side S.

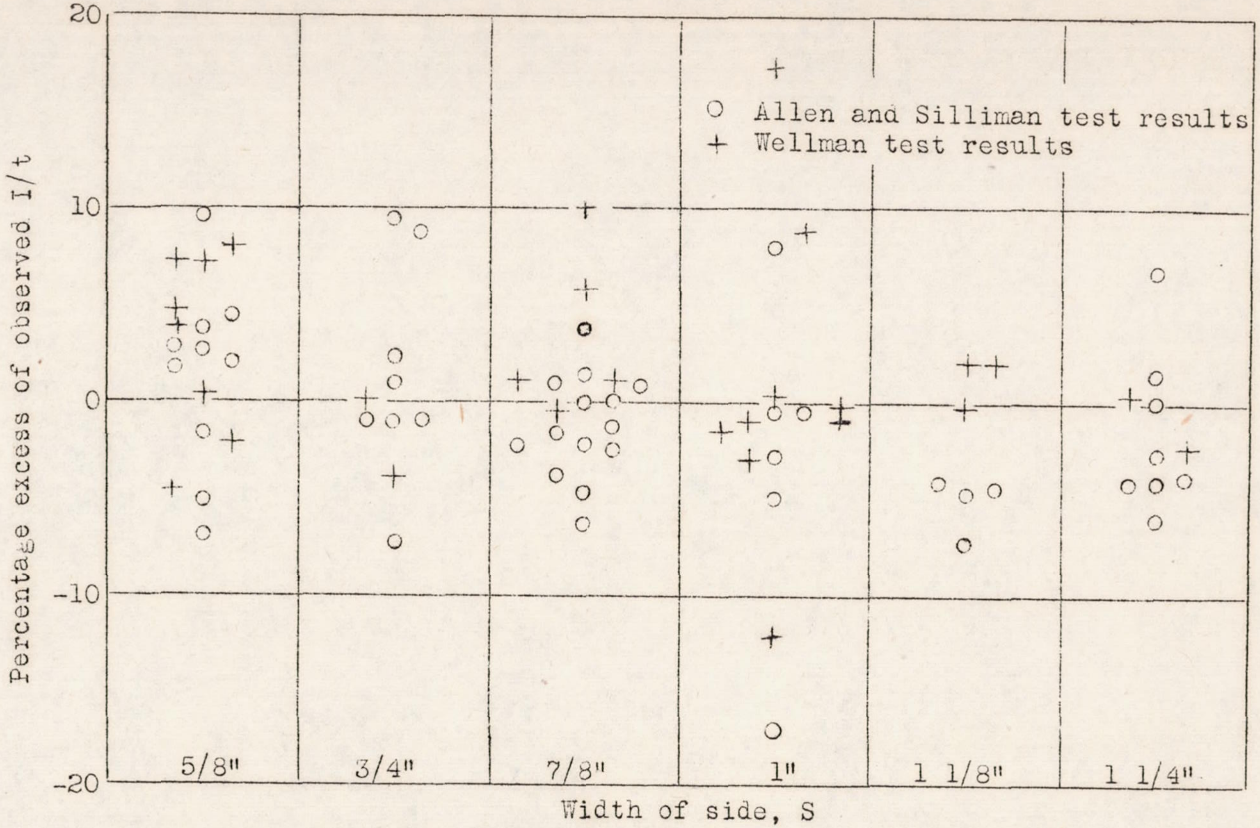

Figure 21.- Percentage excess of observed I/t from pure-bending tests above I/t computed assuming $D_e = D'$, grouped according to nominal width of side S.

(1 block = 10 divisions on 1/25 Engr. scale)

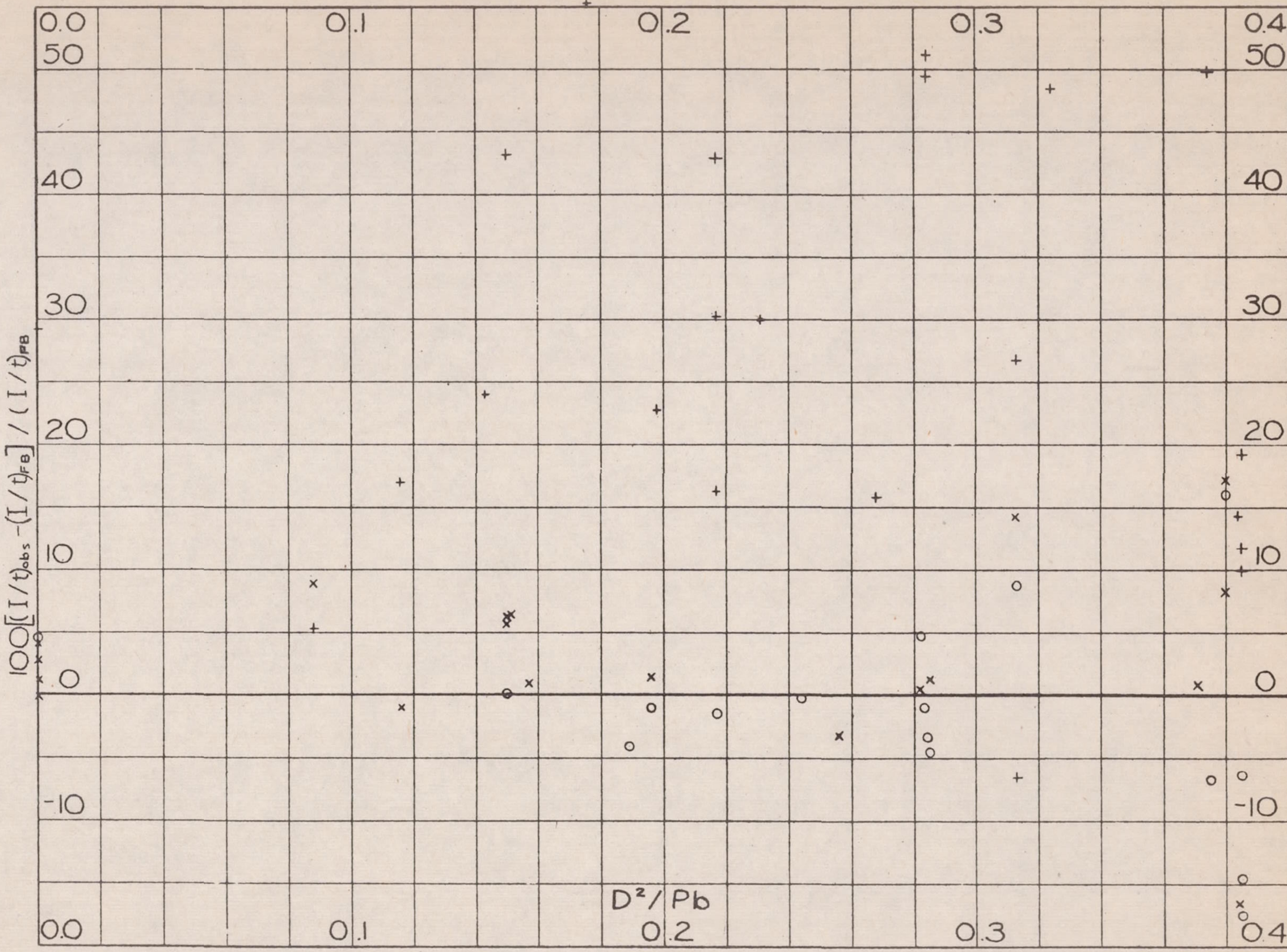

Figure 23.-Comparison of Observed and Computed I/t from Pure Bending Tests

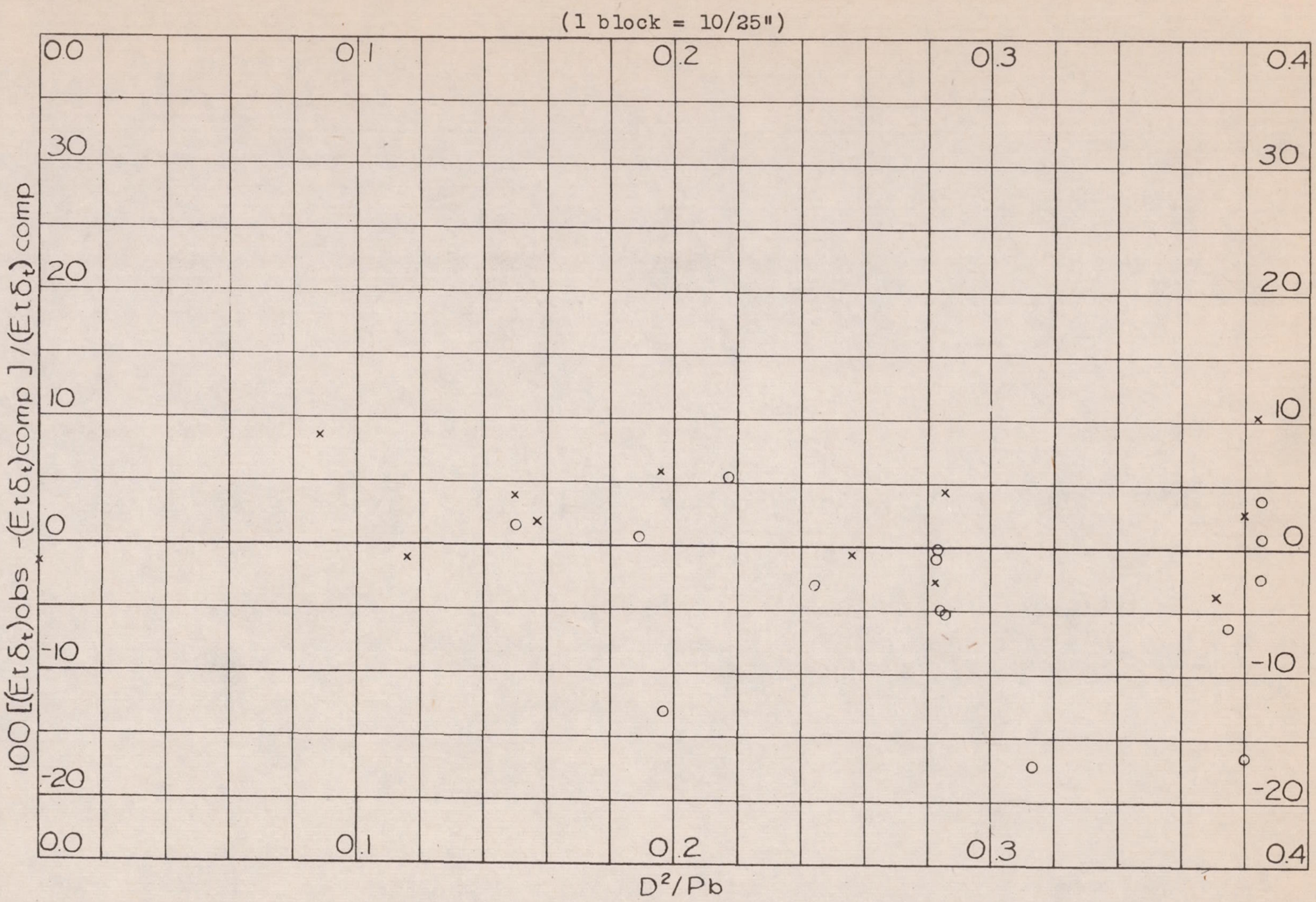

Figure 24.-
Comparison of Observed and Computed $Et\delta_t$ from Simple Bending Tests

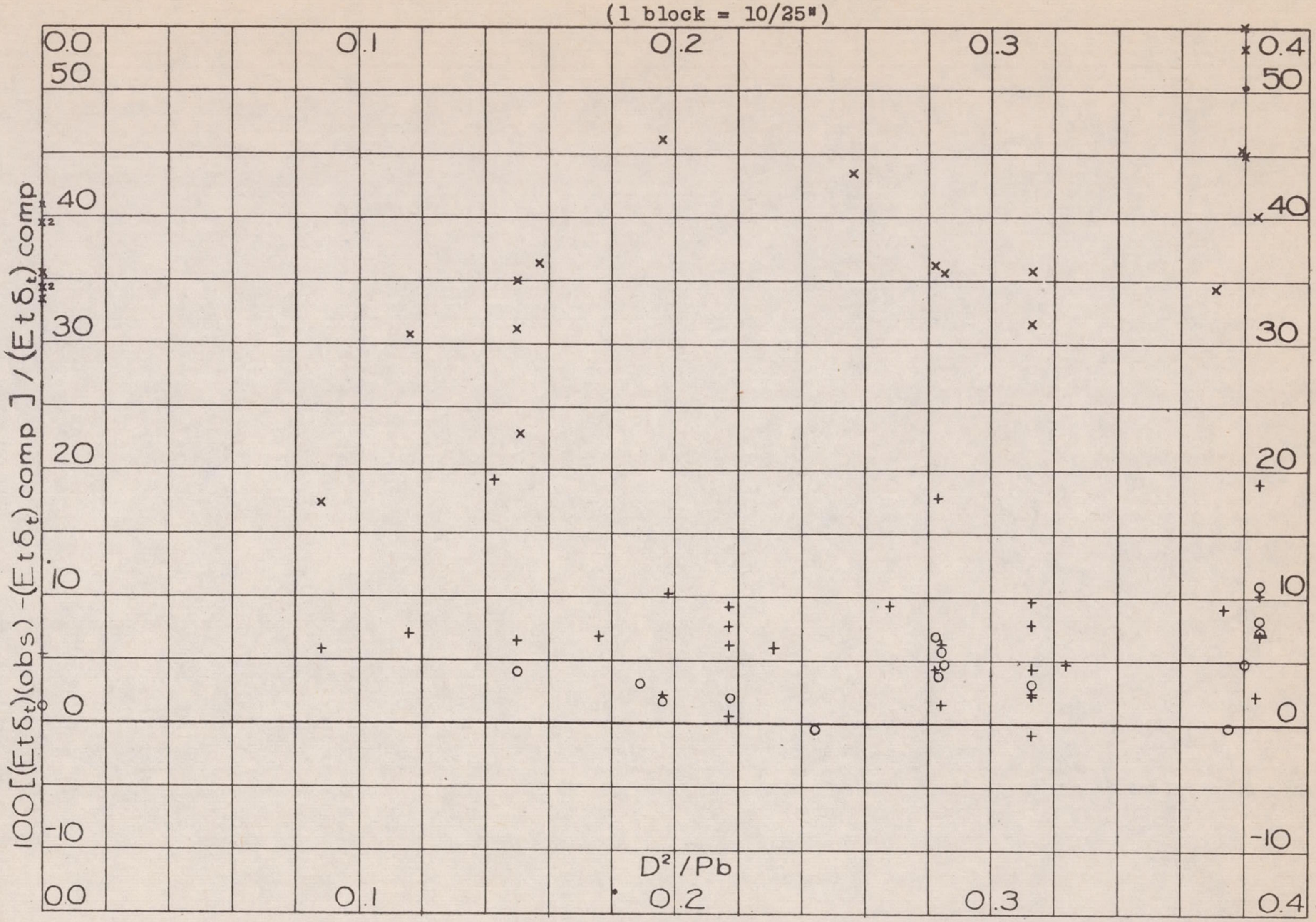

Figure 25.-
Comparison of Observed and Computed $Et\delta_t$ from Cantilever Bending Tests

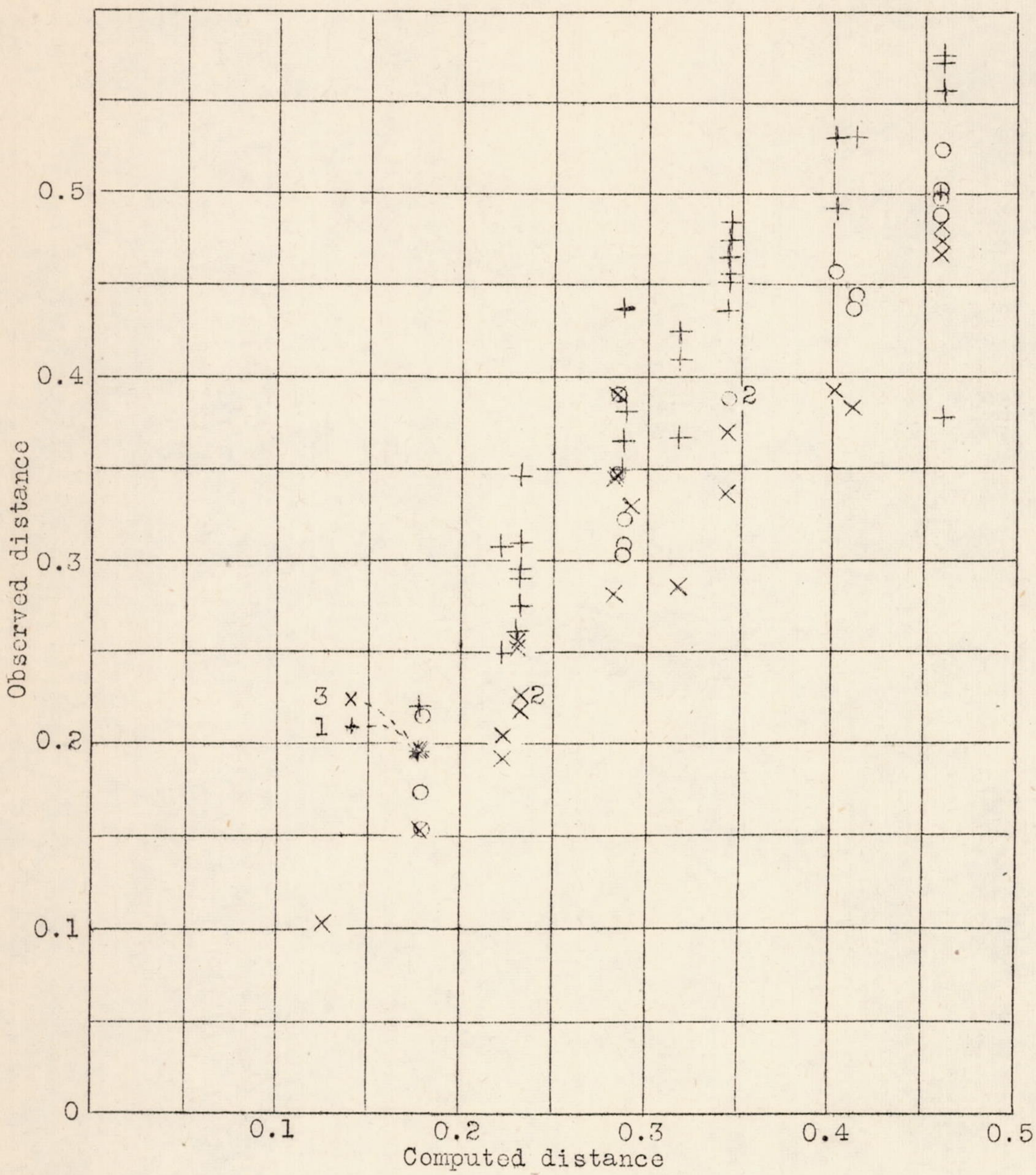

Figure 26.- Comparison of observed and computed distances in inches from midline of web to shear center.

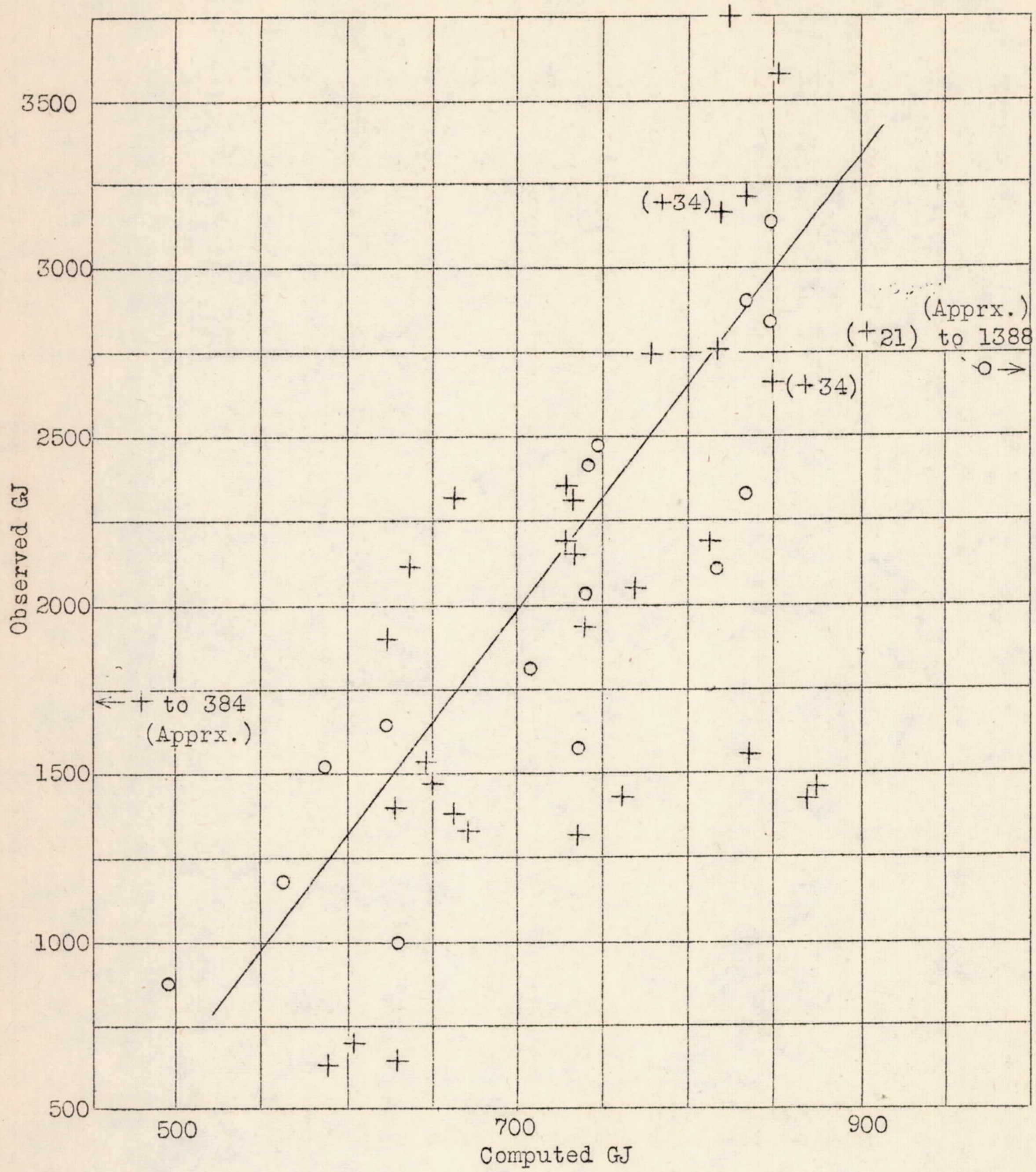

Figure 27.- Comparison of observed GJ with GJ computed assuming w_e=w.

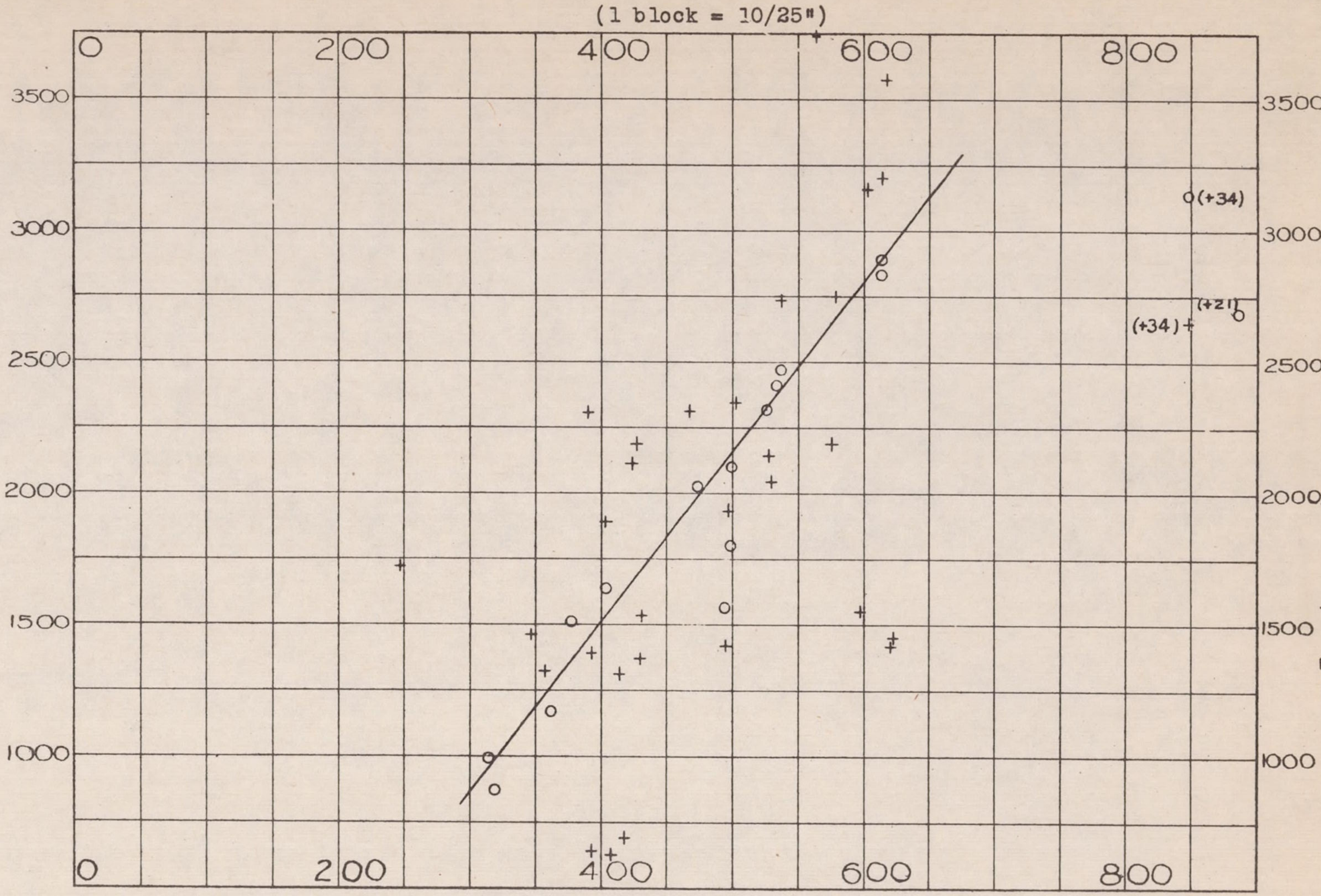

Figure 28.- Comparison of GJ, observed, with GJ, computed, Assuming $w_e = w - D$

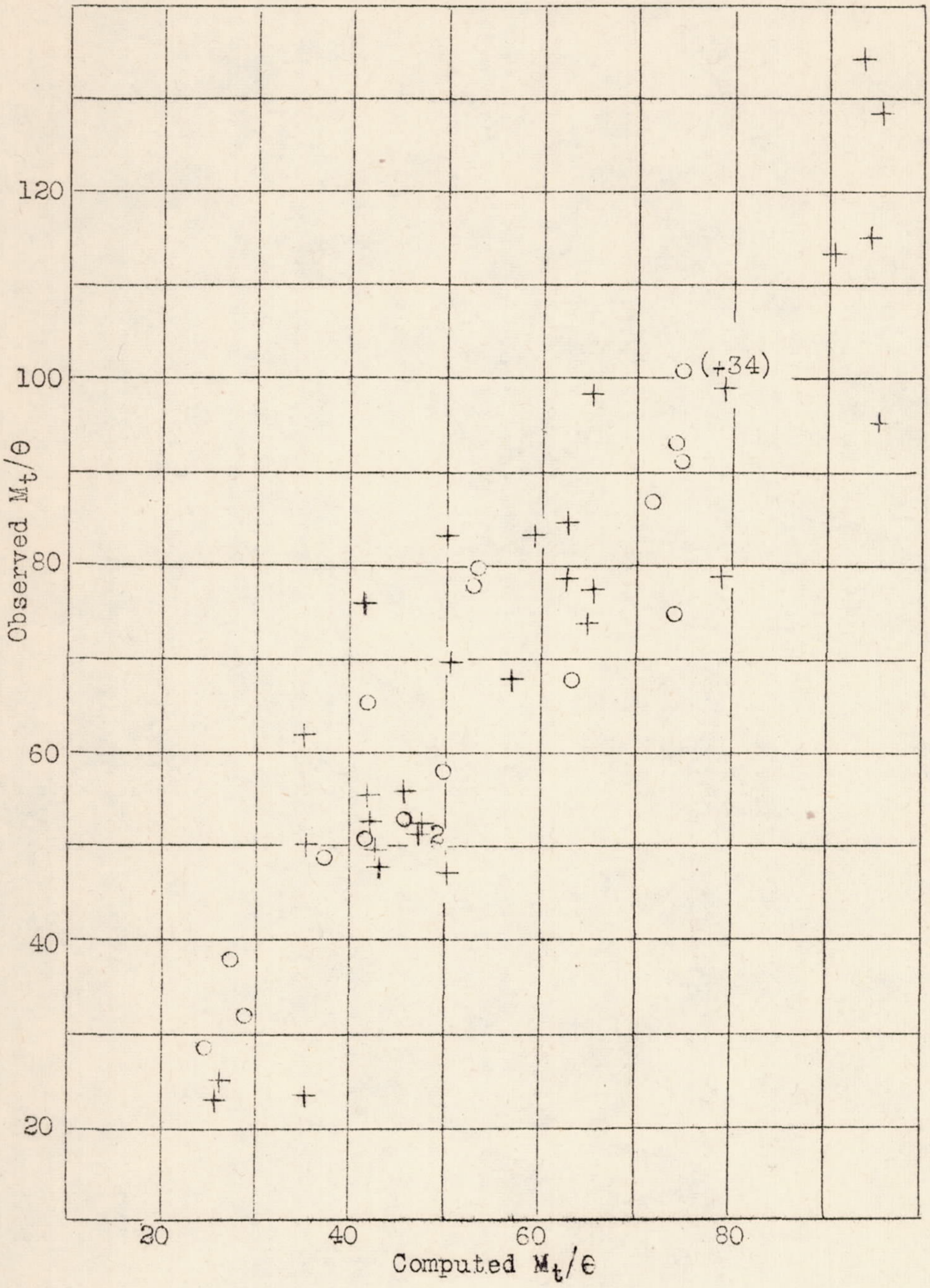

Figure 29.- Comparison of observed M_t/θ and M_t/θ computed assuming $w_e = w$.

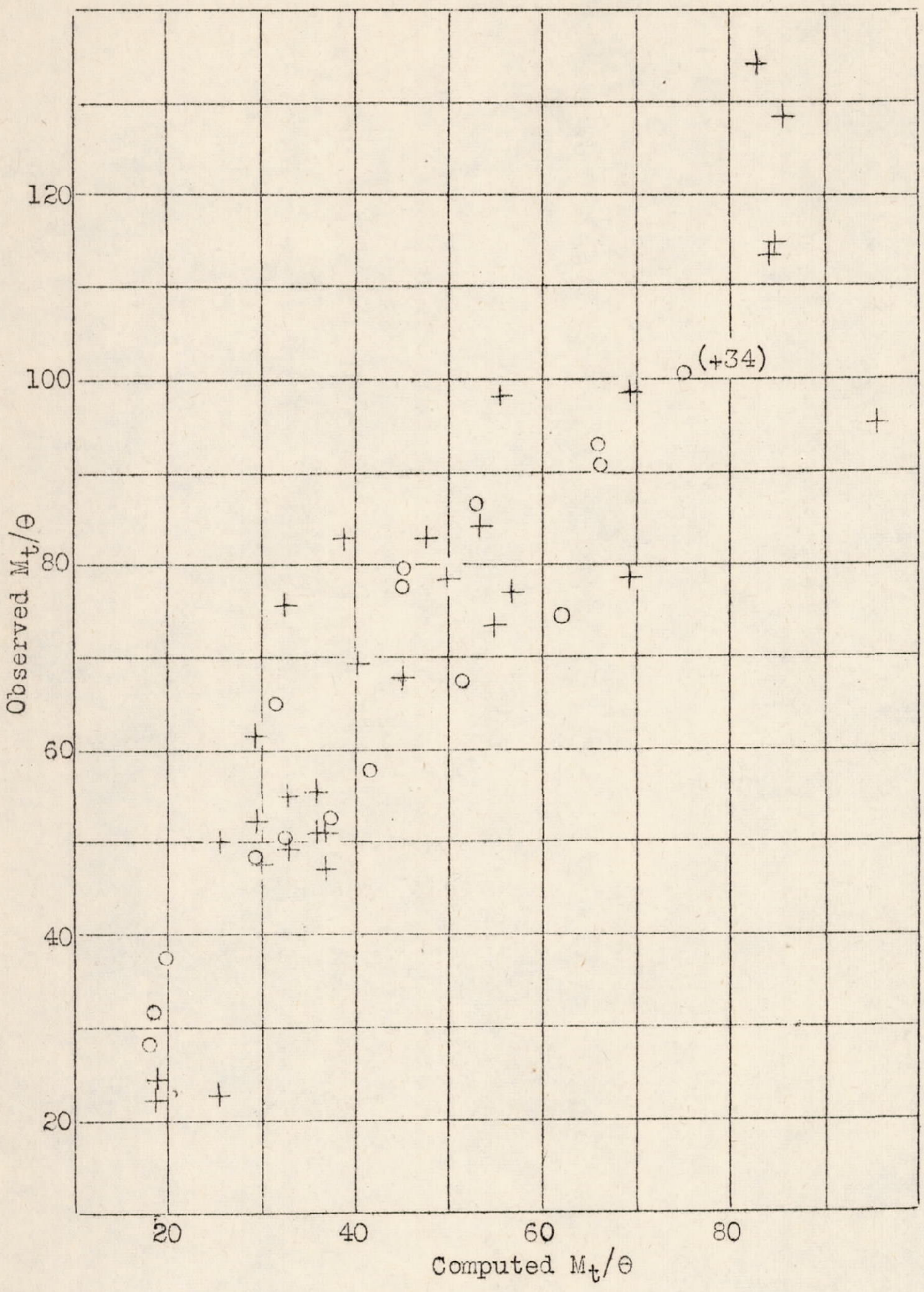

Figure 30.- Comparison of observed M_t/θ and M_t/θ computed assuming $w_e = w - D$.

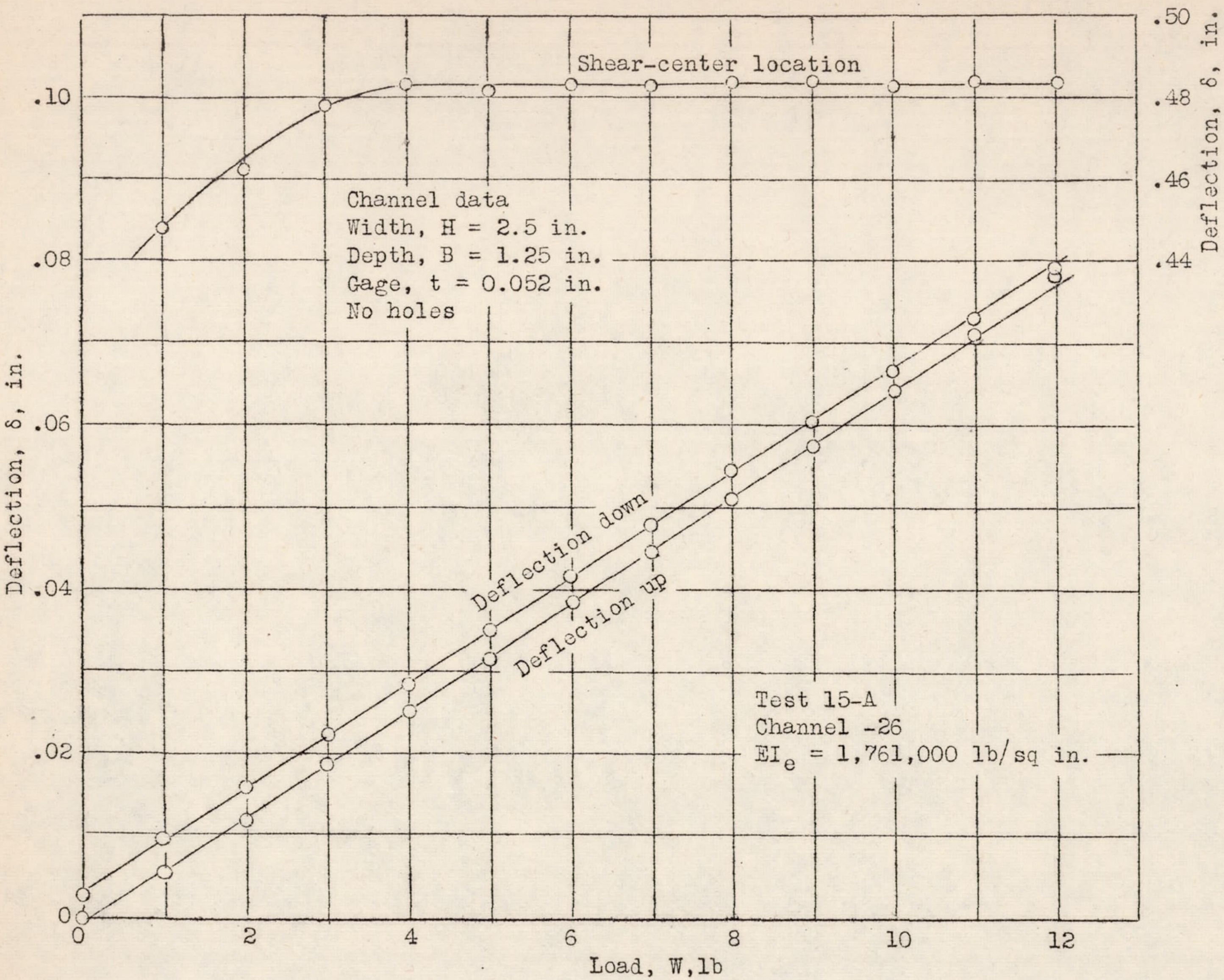

Figure A-1.- Load-deflection and shear-center location curves from shear-center tests.

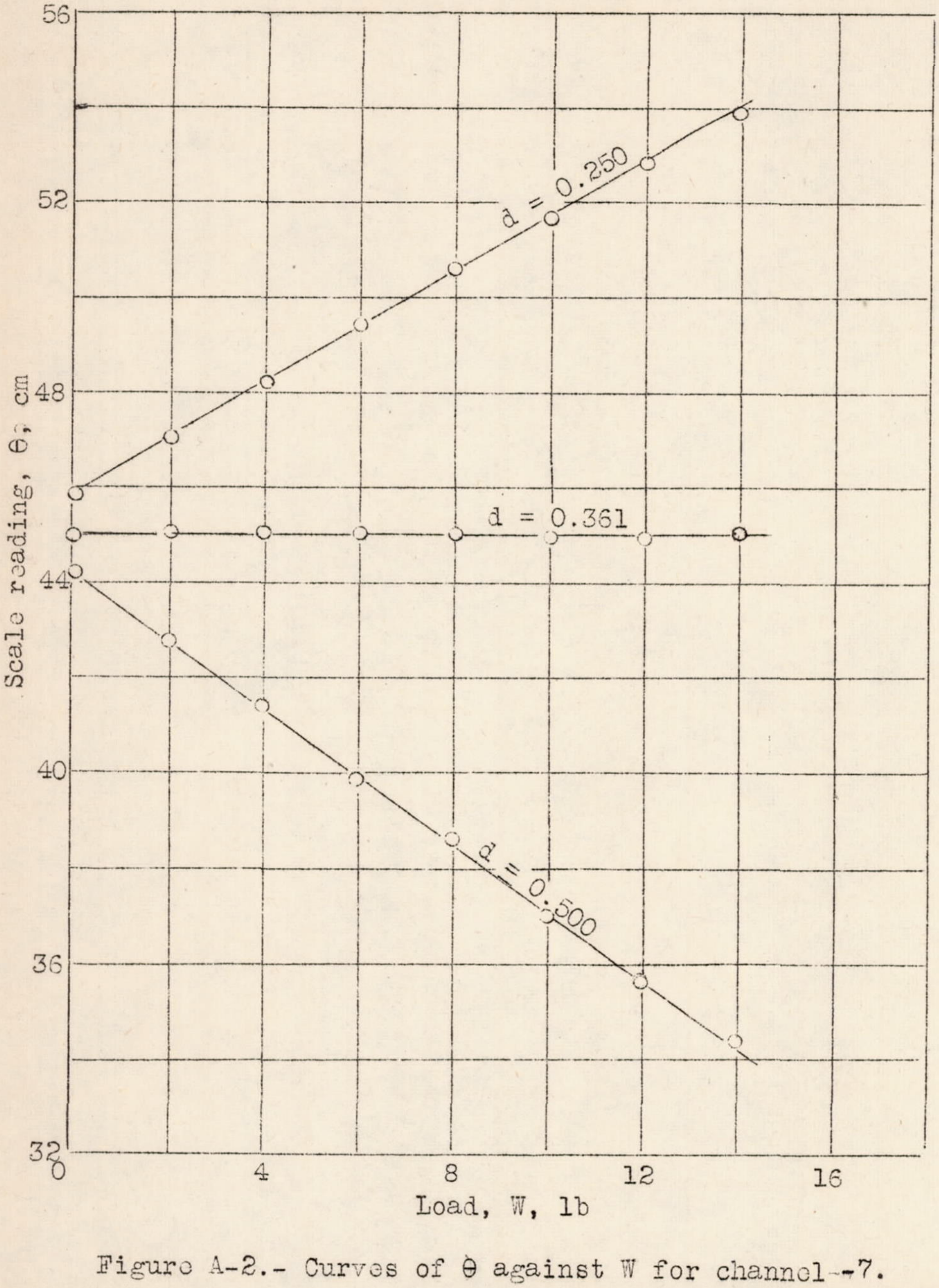

Figure A-2.- Curves of θ against W for channel--7.

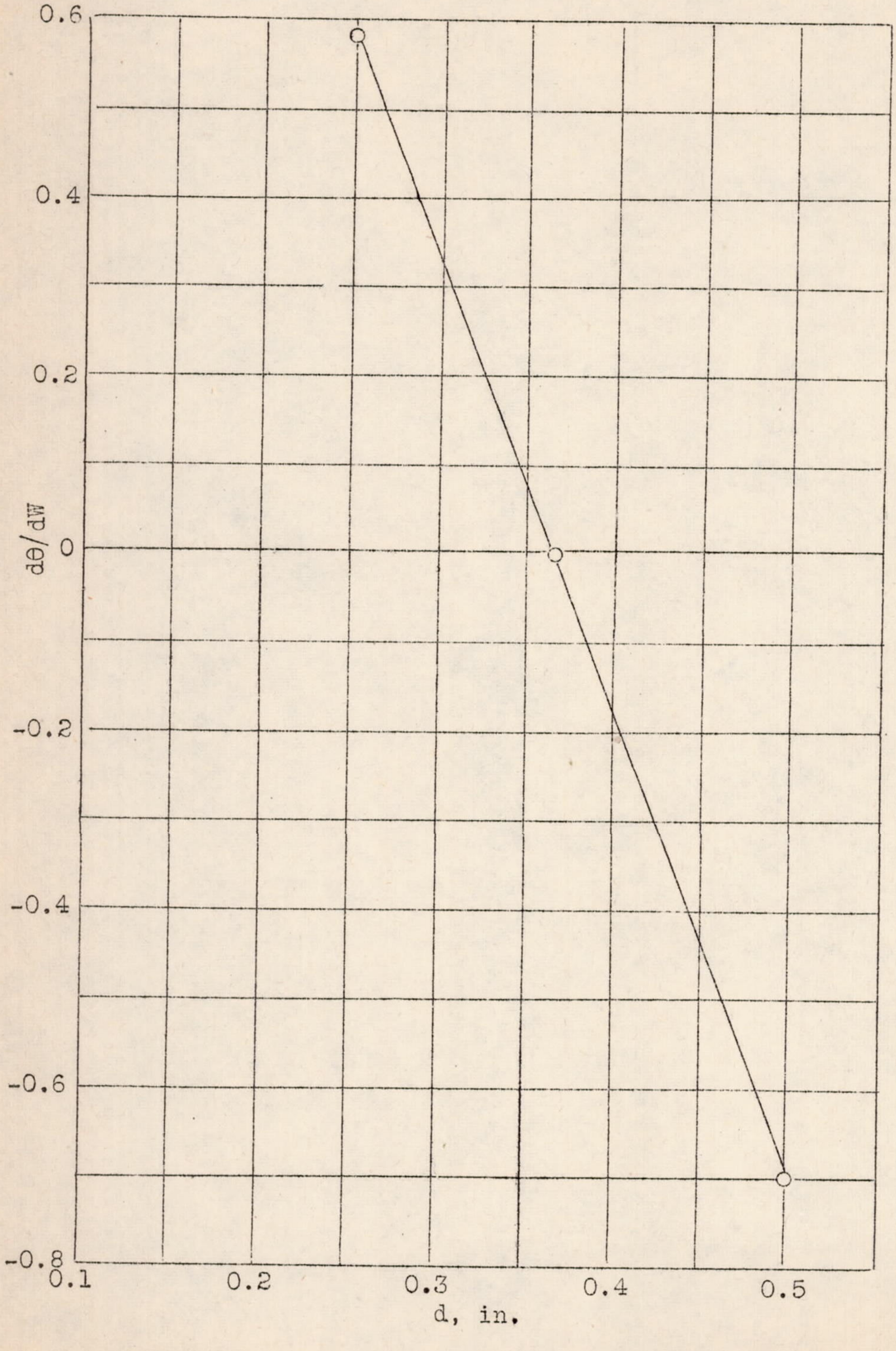

Figure A-3.- Curve of dθ/dW against d for channel -7.