TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 809

THE THEORETICAL LATERAL MOTIONS OF AN AUTOMATICALLY CONTROLLED AIRPLANE SUBJECTED TO A YAWING-MOMENT DISTURBANCE

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SUMMARY

The lateral motion resulting from a disturbance of the type produced by asymmetric loss of thrust has been determined for a hypothetical average airplane equipped with an automatic pilot. Plots of the resultant motion and of the various modes that constitute the motion are presented for controls fixed and for various amounts of automatic control. The automatic control is assumed to be of a type that produces aileron deflections proportional to the angle of bank and rudder deflections proportional to the angle of azimuth. The use of an automatic control may introduce either of two modes. The first mode is primarily a rolling oscillation; the second mode is a poorly damped long-period oscillation in azimuth and bank. The motion following any change in trim causes the airplane to reach a state of equilibrium on a different flight heading from that existing before the disturbance and the airplane assumes a new flight attitude.

INTRODUCTION

An investigation of the influence of automatic control on the lateral stability of an airplane has recently been conducted by the NACA (reference 1). The results indicated that, with control, new modes of motion may be substituted for the familiar modes that occur with controls fixed. Thus the nonoscillatory spiral mode, which is always evident as a slow recovery or a divergence when the controls are neutralized during a banked turn, may be replaced by either of two types of oscillation. These oscillations result from the coupling of the spiral mode either with the mode involving damping in rolling or with the azimuth mode, which, for an airplane with fixed controls, is the component of motion associated with the zero root of the stability equation. The nature of these new types of oscillation has been studied in the present paper by calculating the actual motion of the controlled airplane following a disturbance.

METHOD

Several types of disturbance from equilibrium conditions exist for which mathematical solutions of the motion of an airplane can readily be determined. The type of disturbance chosen for this analysis is that resulting from the sudden application of a yawing moment, which tends to turn the airplane from its course. The suddenly applied yawing moment may be considered to represent the effect of an abrupt loss of thrust on one side of a multiengine airplane. The motion following such a type of disturbance is of particular interest when automatic control is employed because usually the primary function of automatic control is to cause the airplane to fly in a fixed direction.

Computations of the motion following the sudden application of a unit yawing-moment disturbance were made for the hypothetical average airplane treated in reference 1. Fixed controls and three arrangements of automatic control were considered. For convenience in the calculations, the moment unit chosen was such that the applied yawing moment was several times as large as that likely to occur because of asymmetric loss of thrust. The size of the applied moment, however, should not affect the usefulness of the results because, for similar disturbances, the magnitudes of the resulting motions are in the same ratios as the magnitudes of the disturbances. The computations were made for conditions representing the average airplane at cruising speed because automatic control is mainly used under such conditions.

The type of automatic control assumed for the airplane produced aileron deflections proportional to the angle of bank and rudder deflections proportional to the angle of azimuth. The amount of aileron deflection applied for a unit angle of bank is expressed by the aileron-control gearing; similarly, the factor of proportionality for the rudder deflection is represented by the rudder-control gearing. For such a type of control, the motion of the airplane will fall in one of three classes, the class being determined by the differences in the components, or modes, entering into the motion (reference 1). The class of motion varies with changes in the magnitude of the control gearings. Values of the control gearings given later in table I are chosen to give the three typical classes of motion. Only negative values of the control gearing were used because the control must oppose the deviation to which it is sensitive if the motion is to be stable. Because previous study indicated that lag does not introduce any new modes of motion, instantaneous control application was assumed. Lag does have a considerable influence on the character of certain of the modes, however, particularly with respect to the damping. These effects are mentioned later in the paper when the various modes are discussed.

The influence of the characteristics of the airplane and the control system on the motion are expressed mathematically by the nondimensional equations

$$\left.\begin{aligned}
&-\beta(y_v - \lambda) - \phi\left(\frac{C_L}{2}\right) - \psi\left(-\lambda + y_{\delta_r}\frac{\partial\delta_r}{\partial\psi}\right) = 0\\
&-\beta(\mu l_v) - \phi\left(l_p\lambda - \lambda^2 + \mu l_{\delta_a}\frac{\partial\delta_a}{\partial\phi}\right) - \psi\,(l_r\lambda) = 0\\
&-\beta(\mu n_v) - \phi\left(n_p\lambda + \mu n_{\delta_a}\frac{\partial\delta_a}{\partial\phi}\right) - \psi\left(n_r\lambda - \lambda^2 + \mu n\delta_r\frac{\partial\delta_r}{\partial\psi}\right) = 1\ (T)
\end{aligned}\right\}\quad(1)$$

Except for the term 1(T), which represents the unit yawing-moment disturbance, the equations of motion are the same as those treated in reference 1. Definitions of the symbols used are given in the appendix. Solutions of the equations to obtain the motion of the airplane were made by the methods of operational calculus (reference 2).

Values of λ, known as stability roots, are obtained as a step in solving equation (1) and are indicative of the modes of motion. Thus, the solution of equation (1) for any of the variables β, ϕ, or ψ is of the form

$$\beta,\ \phi,\ \text{or}\ \psi = c_1 e^{\lambda_1 T} + c_2 e^{\lambda_2 T} + \ldots\ c_5\ e^{\lambda_5 T} + S(T) \qquad (2)$$

where $\lambda_1, \lambda_2, \ldots \lambda_5$ are the stability roots and S(T) is the steady-state motion when the disturbed system has reached a new state of equilibrium. Complex values of λ occur in conjugate pairs and, for every such pair, the corresponding exponential terms in equation (2) can be replaced by an oscillatory component of the motion.

RESULTS

In table I are given values of the stability roots that result when the control gearings are varied, as previously described, to bring about changes in the modes of motion. The roots obtained are based on the stability and the control derivatives and on the value of μ for the average airplane at $C_L = 0.35$. (See the appendix.) Data for controls fixed are given in case 1 for purposes of comparison.

TABLE I. - ROOT COUPLINGS OBTAINED WITH VARIOUS CONTROL GEARINGS

Case	$-\frac{\partial\delta_a}{\partial\phi}$	$-\frac{\partial\delta_r}{\partial\psi}$	$\lambda_{1,2}$	λ_3	$\lambda_{3,4}$	λ_4	$\lambda_{4,5}$	λ_5
1	0	0	-0.409±1.99i	-4.49		-0.00677		0
2	.25	1.00	-.433±2.40i	-4.01			-0.220±0.187i	
3	.50	1.00	-.462±2.41i	-3.35		-.912		-.123
4	.75	1.00	-.499±2.41i		-2.12±0.699i			-.0846

A comparison of case 1 and case 2 in table I shows that the roots λ_4 and λ_5 couple to form a new mode, the $\lambda_{4,5}$ oscillation, due to the introduction of the rudder control. The increased magnitude of the aileron gearing in case 3 causes the coupling to revert to the arrangement for controls fixed. Further increase in aileron control results in the coupling of the λ_3 and λ_4 roots as the $\lambda_{3,4}$ oscillation for case 4.

The motions obtained as solutions of equation (1) for the various cases in table I are as follows:

Case 1

$$\beta = 1.262 + 0.247e^{-0.409T} \cos 1.99(T-0.091) - 0.0003e^{-4.49T} - 1.505e^{-0.00677T}$$

$$\phi = 44.245 + 0.147e^{-0.409T} \cos 1.99(T+0.527) + 0.004e^{-4.49T} - 44.321e^{-0.00677T}$$

$$\psi = -1116.39 + 7.566T - 0.233e^{-0.409T} \cos 1.99(T-0.112) + 0.0001e^{-4.49T} + 1116.62e^{-0.00677T}$$

Case 2

$$\beta = 0.035 + 0.172e^{-0.433T} \cos 2.40(T-0.072) - 0.0006e^{-4.01T} - 0.314e^{-0.220T} \cos 0.187(T-4.829)$$

$$\phi = -0.095 + 0.094e^{-0.433T} \cos 2.40(T+0.503) + 0.007e^{-4.01T} + 1.433e^{-0.220T} \cos 0.187(T-8.190)$$

$$\psi = 0.618 - 0.164e^{-0.433T} \cos 2.40(T-0.100) + 0.0002e^{-4.01T} - 0.623e^{-0.220T} \cos 0.187(T-3.972)$$

Case 3

$$\beta = 0.061 + 0.172e^{-0.462T} \cos 2.41(T-0.089) - 0.002e^{-3.35T}$$
$$+0.058e^{-0.912T} - 0.285e^{-0.123T}$$

$$\phi = -0.083 + 0.104e^{-0.462T} \cos 2.41(T+0.553) + 0.023e^{-3.35T}$$
$$-0.426e^{-0.912T} + 0.462e^{-0.123T}$$

$$\psi = 0.663 - 0.162e^{-0.462T} \cos 2.41(T-0.115) + 0.0009e^{-3.35T}$$
$$+0.031e^{-0.912T} - 0.544e^{-0.123T}$$

Case 4

$$\beta = 0.082 + 0.176e^{-0.499T} \cos 2.41(T-0.110)$$
$$+0.041e^{-2.12T} \cos 0.699(T-0.964) - 0.284e^{-0.0846T}$$

$$\phi = -0.074 + 0.116e^{-0.499T} \cos 2.41(T+0.616)$$
$$-0.305e^{-2.12T} \cos 0.699(T-1.122) + 0.280e^{-0.0846T}$$

$$\psi = 0.707 - 0.165e^{-0.499T} \cos 2.41(T-0.140)$$
$$-0.016e^{-2.12T} \cos 0.699(T-1.488) - 0.543e^{-0.0846T}$$

DISCUSSION

The given solutions of the equations of motion are represented graphically in figures 1 to 4. Time, expressed in nondimensional units in the figures, can be converted to seconds if multiplied by τ (= 0.815).

In figure 1 the motions of the average airplane with controls fixed are shown. When a yawing moment is applied to an airplane under these conditions, the airplane ultimately acquires a steady turning motion of such a value that the moment due to the damping in yawing counteracts the applied moment. The turning is accompanied by inward sideslip so that the rolling moment due to yawing is balanced by the rolling moment due to sideslipping. The yawing moment resulting from the sideslip is overcome by the damping in yawing. The steady-state angle of bank and angle of sideslip are very large because of the nature of the disturbance assumed and because of the absence of corrective control movements. The angle of azimuth increases at a constant rate in the steady state as a result of the steady turning motion. For these reasons steady-state values are not shown in figure 1 although the motion is stable.

The characteristics assigned to the airplane in case 1 were such that the airplane was spirally stable. Many airplanes, however, are spirally unstable. The degree of spiral stability or instability is usually slight for cruising speeds; hence, the initial motion of a spirally unstable airplane would be much the same as that shown in figure 1. The unstable airplane, however, would not approach a new steady-state equilibrium, as already described, but theoretically would continue indefinitely to accelerate in yawing, rolling, and sideslipping motions at a rate depending on the amount of instability.

Figures 2, 3, and 4 show the results of controlling the motion, following an asymmetric loss of thrust, by using various amounts of aileron control and an average value of rudder control. (See table I.) The applied yawing moment is counteracted by rudder deflection instead of by damping in yawing and the motion caused by the disturbance is much reduced. The airplane with automatic control approaches a new equilibrium condition of motion along a straight flight path, with its heading displaced from the desired heading by an angle ψ_0. The magnitude of ψ_0

depends mainly on the amount of control yawing moment that is applied per unit displacement in azimuth, being such that $\psi_0 \mu n_{\delta_r} \frac{\partial \delta_r}{\partial \psi}$ is approximately equal to the disturbing yawing moment.

The airplane flies along the new course at a small angle of sideslip β_0 and at a small angle of bank ϕ_0. Values of β_0, ϕ_0, and ψ_0 for the various control conditions are given in figures 2, 3, and 4. As the magnitude of the aileron gearing is increased, the steady-state bank ϕ_0 becomes smaller. As a result, the airplane acquires more sideslip in order that the side force caused by the rudder control will still be balanced at the smaller angle of bank. The increased sideslip, in turn, introduces a yawing moment that aids the disturbing moment; thus, the heading error ψ_0 increases somewhat as the aileron control is increased. Because the steady-state bank does not decrease at the same rate as the magnitude of the aileron gearing increases, the steady-state deflections of both the aileron and the rudder controls are larger for increased aileron control. Large amounts of aileron control evidently would be undesirable because of the increased drag that would be produced in controlling the airplane after a given asymmetric loss of thrust. Any attempt to improve the performance with one engine dead by decreasing the aileron gearing, however, may be a compromise because of the likelihood of causing a poorly damped $\lambda_{4.5}$ oscillation, which will be discussed in another section of the paper.

Because any change in trim involves the application of a constant moment to the airplane, it is apparent from the preceding discussion that the airplane will maintain its original course and attitude under automatic control only as long as the trim remains unaltered. Likewise, the airplane must be trimmed before being connected to the automatic control if its course and attitude are to correspond to those selected by means of the adjustments on the control. Corrections for initial departure from trim are usually made by slight adjustments of the automatic pilot a few minutes after it is engaged.

In spite of the widely differing modes of which the motions are composed (cf. figs. 6, 7, and 8 or the roots in table I), the motions for the different cases are quite similar, probably because of the fixed value of rudder-control gearing used. The progressive reduction in banking deviation as the aileron control is increased is the only outstanding distinction of the various motions. If a rolling-moment disturbance, which the ailerons play a large part in counteracting, were assumed, the resulting motions would undoubtedly show much larger changes in character for the different control conditions considered, although no new modes would be involved for a change in the type of disturbance assumed.

The various modes that combine to give the motions plotted in figures 1 to 4 are shown in figures 5 to 8. The discussion of the figures, for convenience, will be by modes instead of on the basis of changes in the control conditions.

For all values assumed for the control gearings, the $\lambda_{1,2}$ mode occurs as a short-period lateral oscillation. With no lag in control operation, the character of the mode is little affected by the amount of automatic control and is much the same as the corresponding lateral oscillation with controls fixed (reference 3). If control lag exists, the mode will become unstable as the magnitude of the rudder gearing is increased. (See reference 1.)

The λ_3 mode, which is present except in case 4, is due to the pronounced tendency of the airplane to resist any motion relative to the air in a direction normal to the wing surface. The mode represents a very small component of the motions in all of the cases in the present study in which it occurs. The mode never becomes unstable at normal flight speeds.

One of the modes that does not occur with the controls fixed is the $\lambda_{3,4}$ mode. (See fig. 8.) It is essentially a rolling oscillation having a moderately long period (approximately 7.3 sec) when the aileron gearing has the value assumed $(-\partial\delta_a/\partial\phi = 0.75)$. The rolling motion is accompanied by a little sideslip toward the high wing but involves practically no change in heading. The lack of azimuth deviation accounts for the negligible effect of the $\partial\delta_r/\partial\psi$ gearing on the $\lambda_{3,4}$ mode, which is mentioned in reference 1.

The origin and the behavior of the $\lambda_{3,4}$ mode as a result of changes in the aileron gearing can be explained as follows. Motions in rolling can usually be considered to be nearly independent of motions in the other degrees of freedom because of the heavy damping in rolling. Thus, for rolling motions, the airplane with controls fixed behaves like a simple torsional oscillatory system that possesses a large amount of damping and little restoring moment; the λ_3 mode occurs as a rapid aperiodic subsidence in rolling. The aileron gearing increases the restoring moment of the system relative to the damping and for sufficiently large values of the aileron gearing the rolling motion becomes oscillatory, which results in the $\lambda_{3.4}$ mode.

Previous study of the influence of automatic control (reference 1) indicated that the period of the rolling oscillation rapidly becomes shorter as $-\partial\delta_a/\partial\phi$ is increased beyond the value assumed in this paper. Lag usually exists in the aileron control so the damping also decreases with increased $-\partial\delta_a/\partial\phi$ and an unstable oscillation resembling wing flutter finally results. With no lag, as assumed in the present calculations, the oscillation is heavily damped for all values of $-\partial\delta_a/\partial\phi$. Thus, for $-\partial\delta_a/\partial\phi = 0.75$, the motion is negligible after about one-fourth of an oscillation.

The λ_4 mode is the familiar aperiodic spiral motion, which has been discussed at length in many previous papers on lateral stability. (See, for example, reference 3.) With controls fixed, the mode is usually either poorly damped (fig. 5) or slightly unstable and involves a slow turning motion of the airplane. The damping of the mode rapidly increases as $-\partial\delta_a/\partial\phi$ is increased and the mode becomes predominantly a convergence in bank. (See fig. 7.) The large magnitude of the λ_4 mode in figure 5 is due to the type of disturbance assumed in the calculations and to the lack of any attempt to control the resulting motion. Obviously, immediate corrective measures must be taken to keep the motion within reasonable limits if such a type of disturbance occurs in flight.

The $\lambda_{4.5}$ mode, previously mentioned by Garner in reference 4, occurs for a very limited range of values of the control gearings. The mode does not occur with controls fixed but will be present for the average airplane if a small value of aileron gearing $(-\partial\delta_a/\partial\phi$ not exceeding approximately 0.3) is used in conjunction with rudder gearing. The amount of rudder gearing used appears to be unimportant for the existence of the mode, but some rudder control must be present. The motion represented by the $\lambda_{4.5}$ mode is an oscillation in azimuth and bank about the desired course, accompanied by considerable outward sideslip. (See fig. 6.) As a result of the oscillation, when the airplane is checked in a turn away from the course in one direction it immediately commences to turn off course in the opposite direction. A necessary condition for a turn without sideslip is that the bank lag the azimuth deviation by three-fourths of a period. In the case considered, however, the deviation in bank lags the azimuth motion by about five-eighths of a period. An effective lead of one-eighth period therefore exists in the banking motion and the turn is accompanied by outward sideslip.

The oscillation resembles the phugoid oscillation in longitudinal motion in that the period is long (approximately 27.4 sec for the aileron gearing assumed). The damping is somewhat greater than that normally existing for the phugoid (reference 5) so that the motion is negligible after about one-half an oscillation. Because of the long period, however, the motion actually persists for a considerable interval of time. Therefore, it appears desirable to improve the damping of the mode, because the initial magnitude of the motion is rather large. Reference 1 shows that stability of the azimuth oscillation depends on the value of the aileron gearing and that the stability improves as $-\partial\delta_a/\partial\phi$ is increased. Before any appreciable increase in damping is obtained, however, the oscillation separates into the λ_4 and the λ_5 modes. Lag in control operation appears to have negligible effect on the oscillation.

The predicted behavior of the $\lambda_{4.5}$ mode has been observed under certain flight conditions in a low-wing 8-place modern transport with an automatic pilot. The mode occurred as an unstable oscillation at speeds somewhat below the cruising speed when the aileron gearing was reduced to zero by closing the aileron speed valve. The influence of flight speed on the stability of the mode may be explained as follows: The point at which reduction of the magnitude of $-\partial\delta_a/\partial\phi$ results in decreasing the damping of the $\lambda_{4.5}$ mode to zero depends on the inherent spiral stability present. (See reference 1.) It is well known that the spiral stability of an airplane decreases with a decrease in the flight speed; hence, as the speed is reduced, less reduction of $-\partial\delta_a/\partial\phi$ is required to cause instability of the $\lambda_{4.5}$ mode.

The λ_5 mode is usually described as a convergence in azimuth. The neutral stability of the mode for an airplane with fixed controls is evidenced by the fact that the airplane, after being displaced in azimuth, has no tendency to return to the original course. With automatic control, the mode occurs as a slow aperiodic motion, predominantly in azimuth and bank but also involving considerable sideslip. (See fig. 7.) The poor damping of the motion is very slightly improved as the rudder control is increased and is decreased at about the same rate when the aileron control is increased. The amount of bank included in the mode is decreased so that the azimuth component becomes a relatively more prominent part of the mode as the magnitude of the aileron gearing is increased. (Cf. figs. 7 and 8.)

CONCLUSIONS

1. Of the modes that may occur either with controls fixed or with automatic control, the lateral oscillation and the rolling convergence appear to undergo little change in character with a variation in the amount of control. The spiral mode becomes a well-damped convergence in bank as the amount of aileron control is increased. The azimuth convergence, neutrally stable with fixed controls, becomes a poorly damped motion chiefly involving azimuth and bank with automatic control.

2. Either of two new modes may be introduced by the use of automatic control. The first mode is an oscillation consisting of almost pure rolling and is the result of the coupling of the damping-in-roll root and the spiral-convergence root. The second mode is a long-period oscillation about the desired heading and results from the coupling of the spiral-convergence root and the azimuth-convergence root

3. After any disturbance that causes a change in trim, such as an asymmetric loss of thrust, an airplane controlled by an automatic pilot of the type considered in this study will not return to the course and the flight

attitude desired. The airplane will assume a new course and attitude, displaced from the original flight condition by an amount depending on the magnitude of the disturbance, so that an error in heading will result even for small changes in trim.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., February 7, 1941.

APPENDIX

(Numerical values of symbols for the average airplane are given when the symbols are constants for $C_L = 0.35$.)

X, Y, Z — reference axes or forces along the respective axes. (See fig. 9.) The axes are fixed relative to the airplane and are so orientated that, in steady flight, the X, or longitudinal, axis is directed along the flight path and the Y axis is directed horizontally along the span to the right. The Z axis is perpendicular to the X and the Y axes and is directed downward in normal level flight.

L — moment about X axis

N — moment about Z axis

k_X — radius of gyration of airplane about X (4.95 ft)

k_Z — radius of gyration of airplane about Z (5.85 ft)

V — resultant linear velocity of airplane center of gravity (150 fps)

p — component of angular velocity of airplane about X, radians per second

r component of angular velocity of airplane about Z, radians per second

β, ϕ, ψ angles of sideslip, bank, and azimuth, respectively, measured from undisturbed flight attitude, radians

β_0, ϕ_0, ψ_0 steady-state values of β, ϕ, and ψ following a disturbance

μ relative-density factor $(m/\rho Sb = 3.82)$

m mass of airplane (49.7 slugs)

S wing area (171 sq ft)

ρ density of air (0.00238 slug/cu ft)

τ time-conversion factor $(m/\rho SV = 0.815)$

T time in nondimensional units $(\sec/\tau)$

$1(T)$ function of time; equal to zero when $T < 0$, acquiring the value unity instantaneously at $T = 0$, and equal to unity when $T > 0$

λ differential operator (d/dT)

λ_1 to λ_5 roots of stability equation (reference 1)

C_L lift coefficient $\left(\dfrac{\text{lift}}{\frac{1}{2}\rho V^2 S}\right) = 0.35$

C_Y lateral-force coefficient $\left(\dfrac{Y}{\frac{1}{2}\rho V^2 S}\right)$

C_l rolling-moment coefficient $\left(\dfrac{L}{\frac{1}{2}\rho V^2 Sb}\right)$

C_n yawing-moment coefficient $\left(\dfrac{N}{\frac{1}{2}\rho V^2 Sb}\right)$

b wing span (32.0 ft)

δ_a sum of up and down aileron deflection, radians; positive when right aileron is up

δ_r rudder deflection, radians; positive when trailing edge is deflected to right

$\frac{\partial \delta_a}{\partial \phi}$ aileron-control gearing

$\frac{\partial \delta_r}{\partial \psi}$ rudder-control gearing

$$y_v = \frac{1}{2} \frac{\partial C_Y}{\partial \beta} = -0.140$$

$$l_v = \frac{1}{2} \frac{b^2}{k_X^2} \frac{\partial C_l}{\partial \beta} = -1.42$$

$$l_p = \frac{1}{4} \frac{b^2}{k_X^2} \frac{\partial C_l}{\partial \frac{pb}{2V}} = -4.43$$

$$l_r = \frac{1}{4} \frac{b^2}{k_X^2} \frac{\partial C_l}{\partial \frac{rb}{2V}} = 0.905$$

$$n_v = \frac{1}{2} \frac{b^2}{k_Z^2} \frac{\partial C_n}{\partial \beta} = 0.960$$

$$n_p = \frac{1}{4} \frac{b^2}{k_Z^2} \frac{\partial C_n}{\partial \frac{pb}{2V}} = -0.169$$

$$n_r = \frac{1}{4} \frac{b^2}{k_Z^2} \frac{\partial C_n}{\partial \frac{rb}{2V}} = -0.744$$

$$y_{\delta_r} = \frac{1}{2} \frac{\partial C_Y}{\partial \delta_r} = -0.0347$$

$$l_{\delta_a} = \frac{1}{2} \frac{b^2}{k_X^2} \frac{\partial C_l}{\partial \delta_a} = 2.10$$

$$n_{\delta_a} = \frac{1}{2} \frac{b^2}{k_Z^2} \frac{\partial C_n}{\partial \delta_a} = -0.106$$

$$n_{\delta_r} = \frac{1}{2} \frac{b^2}{k_Z^2} \frac{\partial C_n}{\partial \delta_r} = -0.474$$

$S(T)$ steady-state solution of equations of motion

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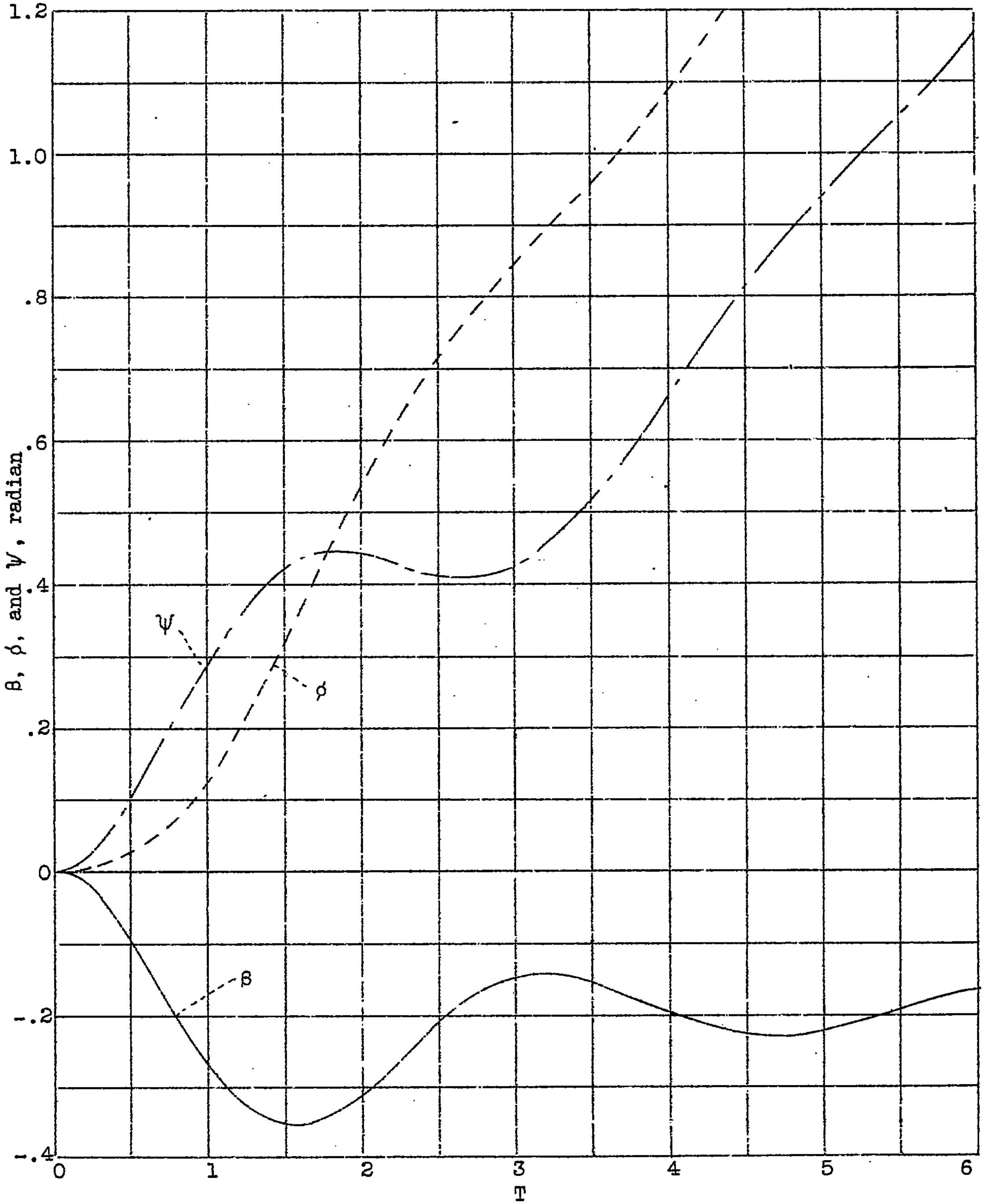

Figure 1.- Resultant motions for case 1.

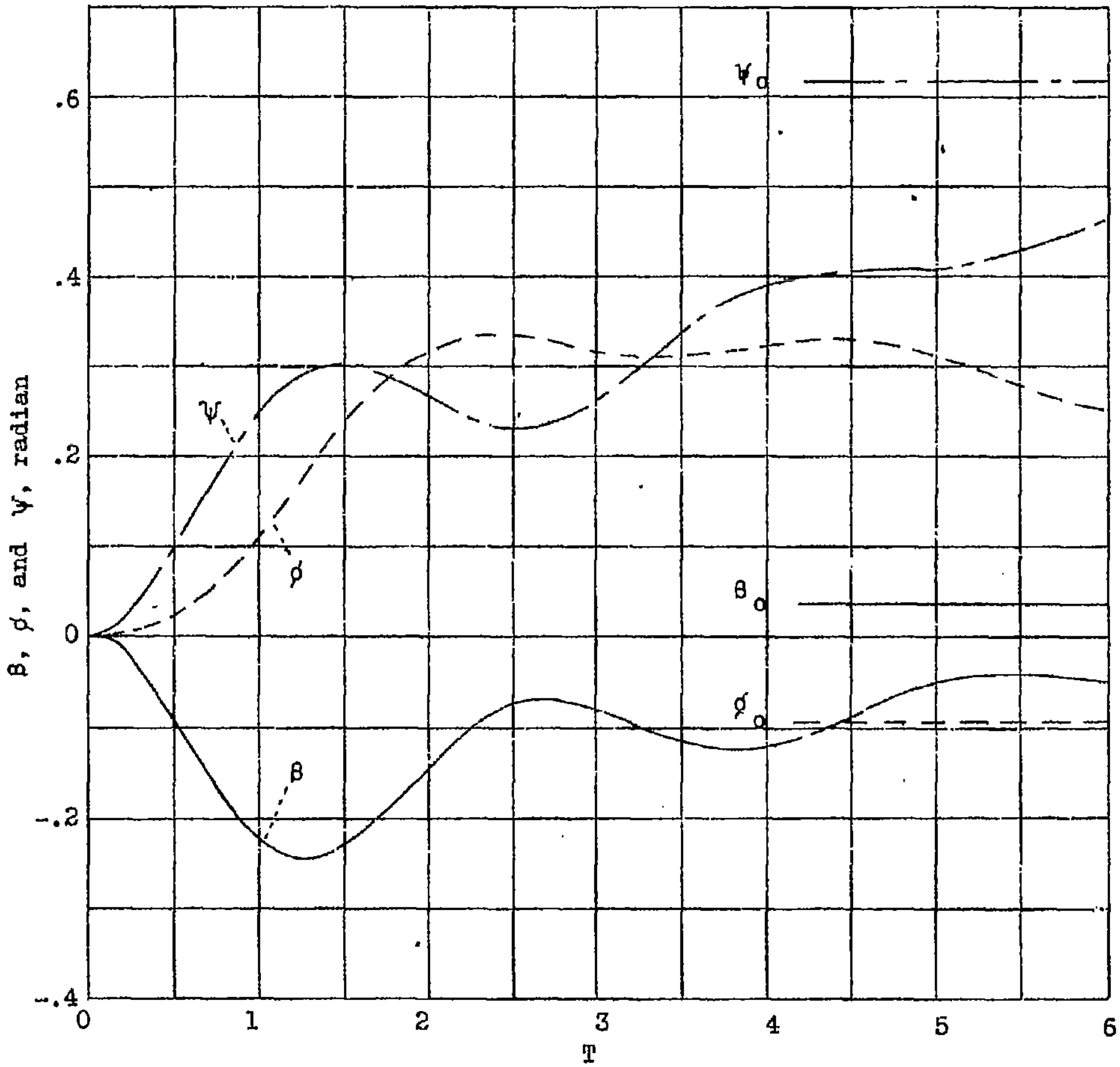

Figure 2.- Resultant motions for case 2.

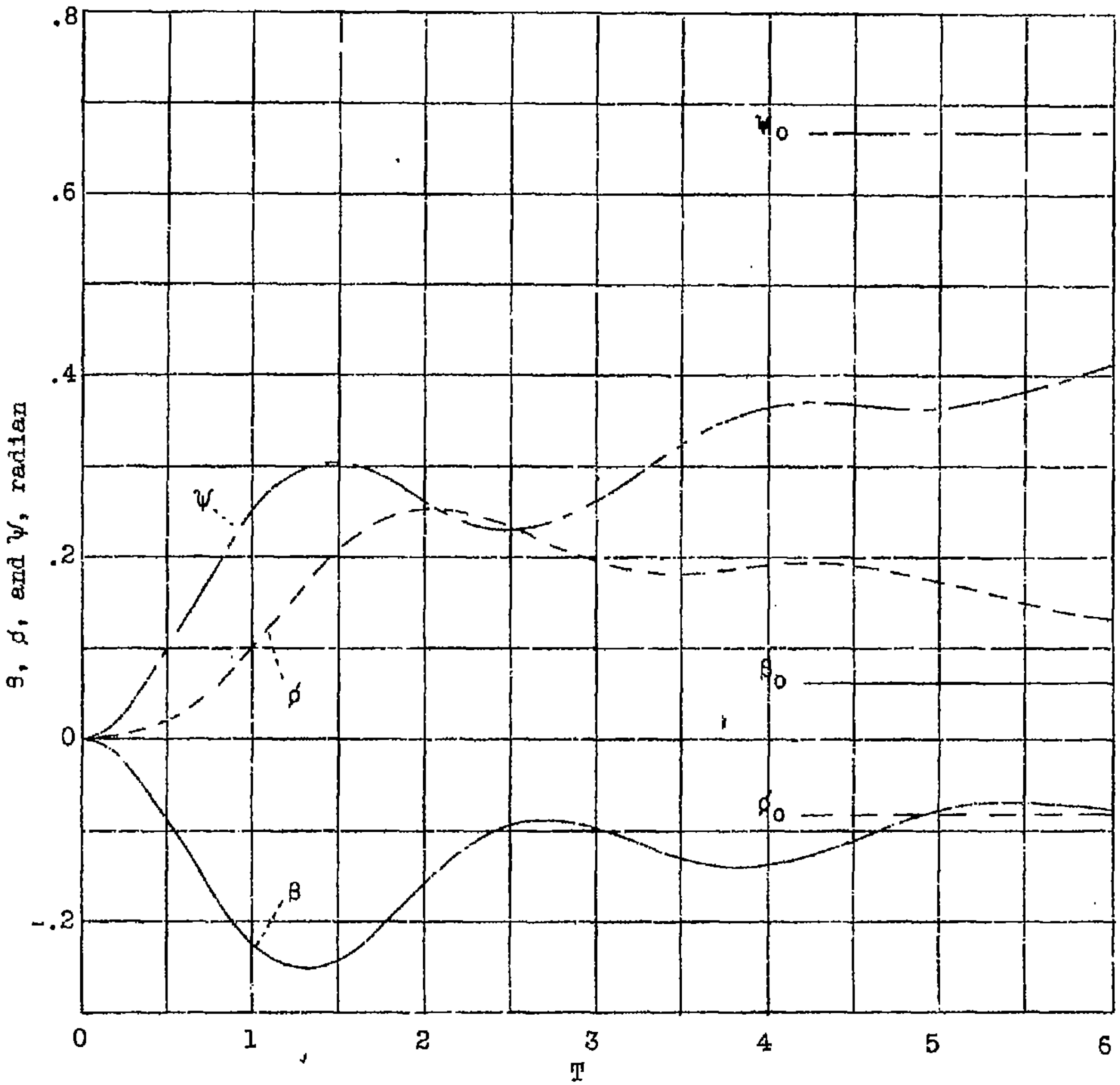

Figure 3.- Resultant motions for case 3.

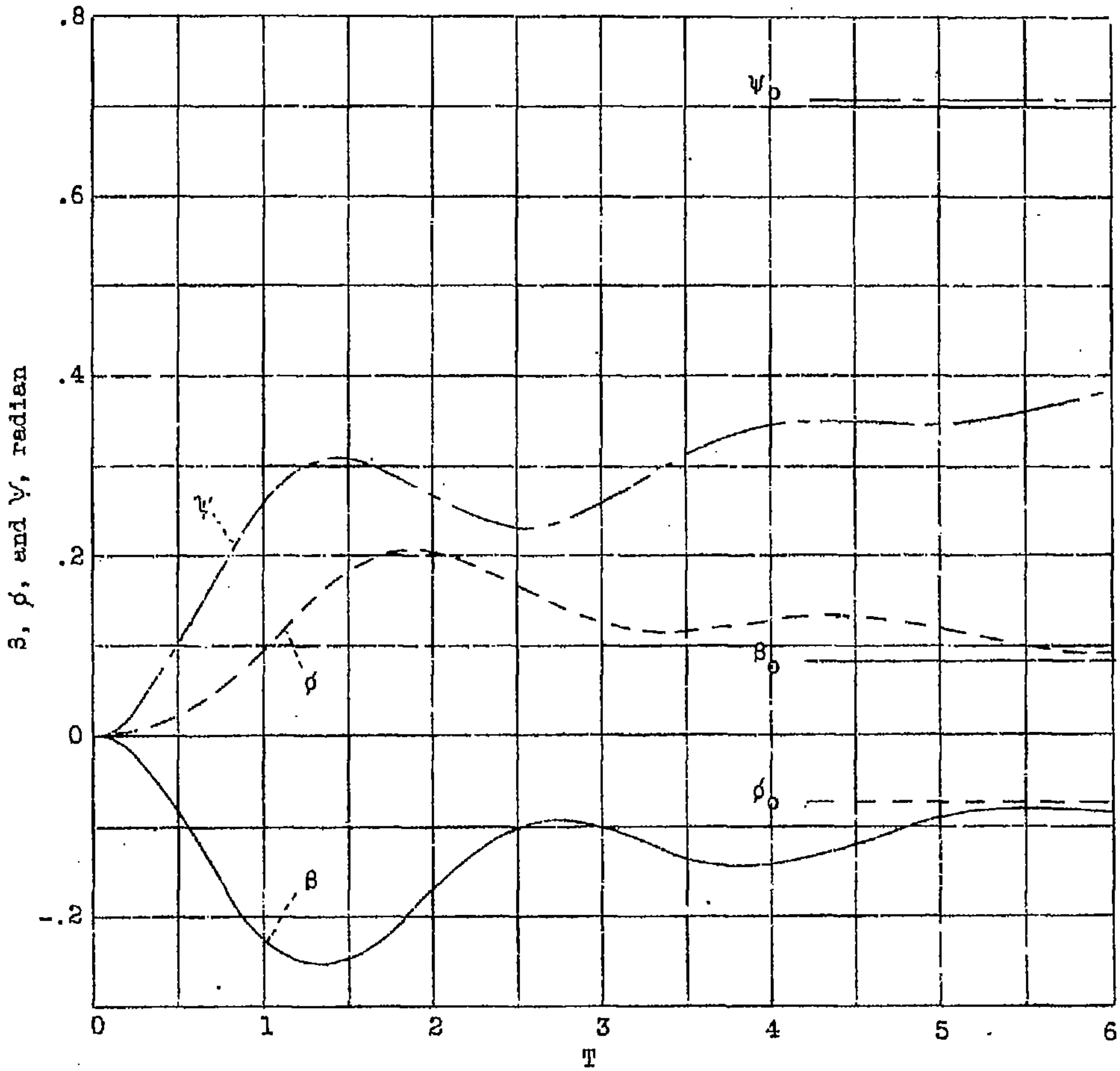

Figure 4.- Resultant motions for case 4.

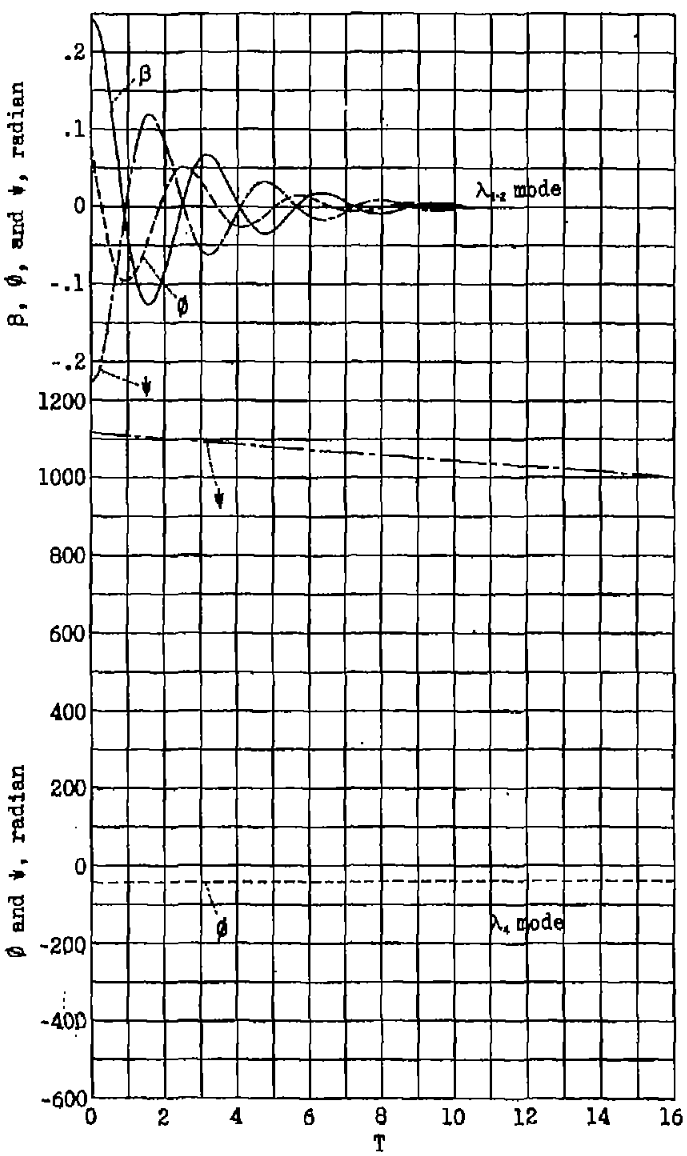

Figure 5.- Modes of motion for case 1. β, ϕ, and ψ are negligible for λ_3 mode. Starting value of β is -1.505 for λ_4 mode.

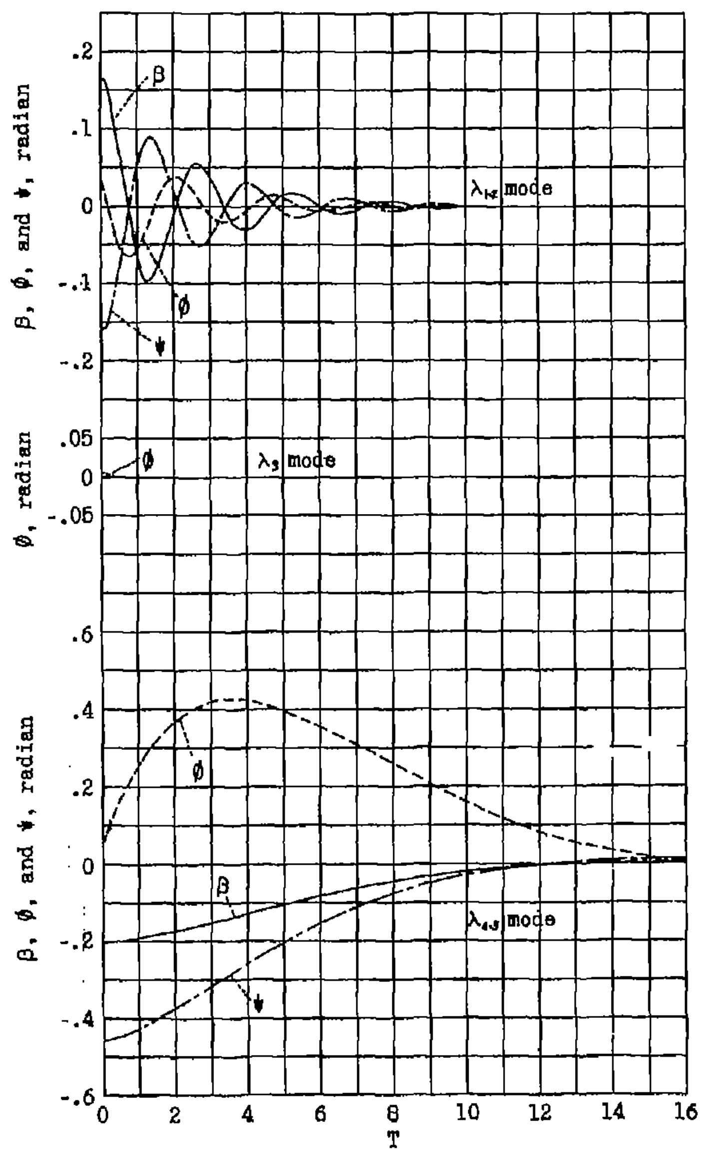

Figure 6.- Modes of motion for case 2. β and ψ are negligible for λ_3 mode.

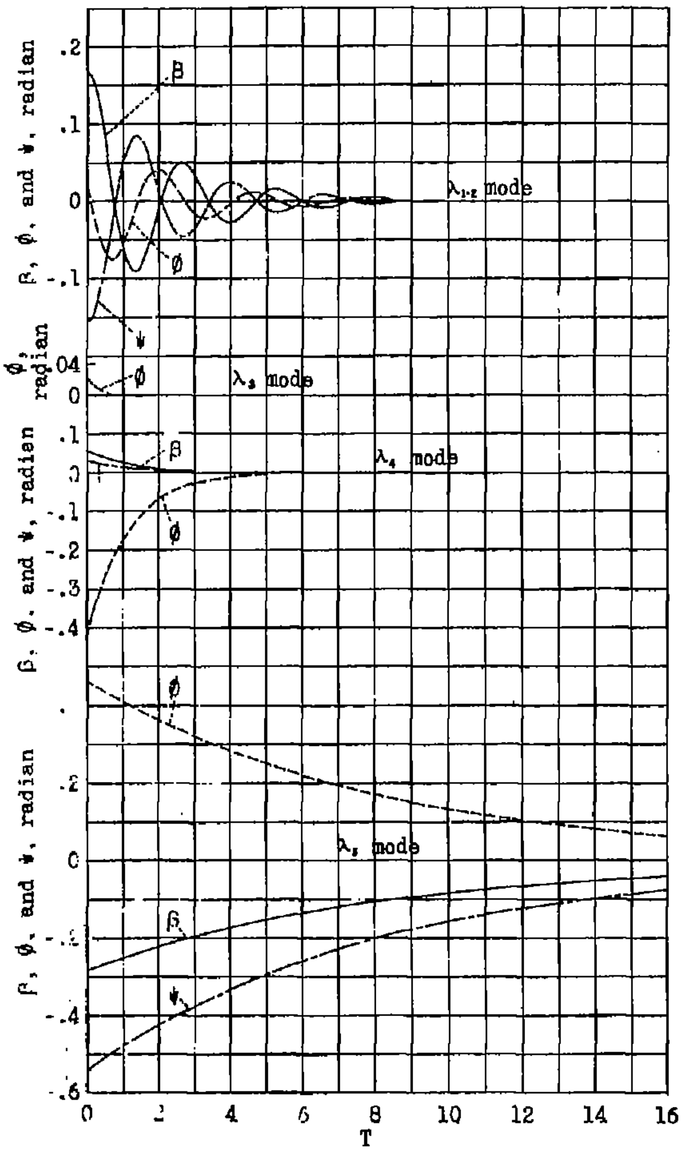

Figure 7.- Modes of motion for case 3. β and ψ are negligible for λ_3 mode.

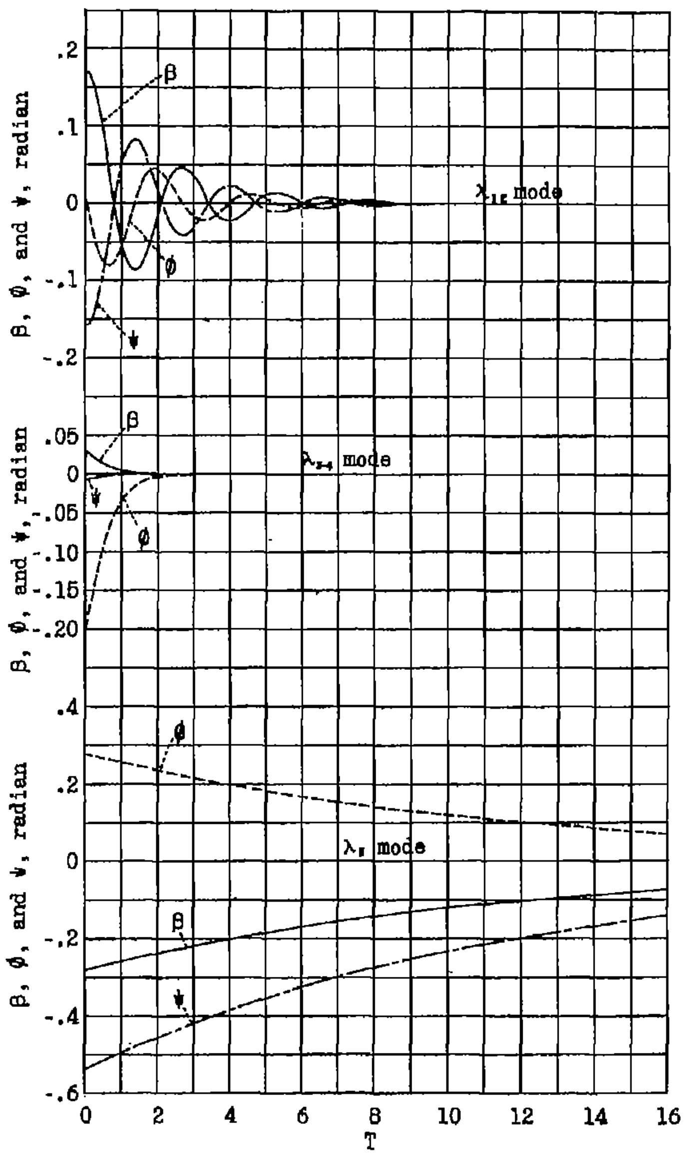

Figure 8.- Modes of motion for case 4.

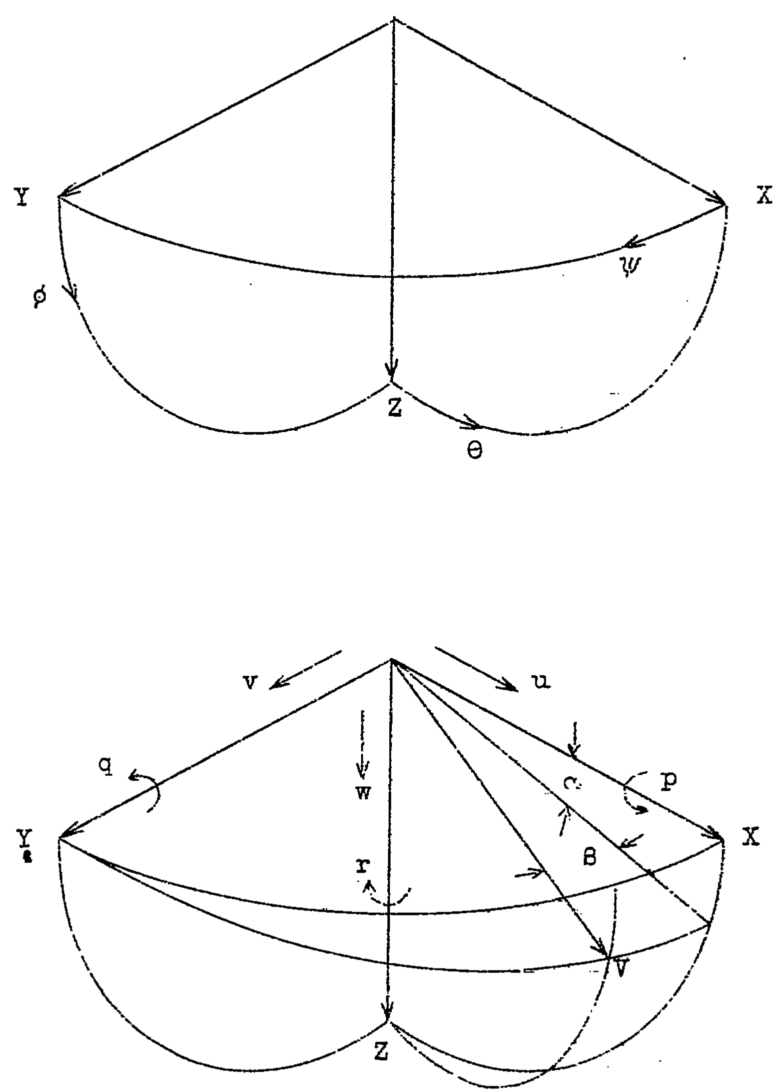

Figure 9.- Positive senses of axes and motions.