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No. 1106

PERFORMANCE PARAMETERS FOR JET-PROPULSION ENGINES

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SUMMARY

Performance parameters for jet-propulsion engines have been
developed from the concepts of flow similarity, inertia forces, elas-
tic forces, viscous forces, and thermal expansions of the working
fluid. The analysis relates performance to the geometry of the
boundaries, Mach number, Reynolds number, and total-temperature ratio.
The results of performance tests with turbojet engines are expressed
in terms of two types of generalizing parameter: dimensionless param-
eters and "corrected" parameters that are not dimensionless.

INTRODUCTION

It is common practice to express the performance of jet-
propulsion engines in terms of generalized parameters derived by
dimensional analysis. The grouping of variables into these parameters
is useful in that the number of variables involved in experiment or
analysis is effectively reduced. Generalization of the data by the
use of these parameters permits results of specific tests with a
specific engine to be used for estimating performance at other
conditions.

A derivation of performance parameters has been given by Warner
and Auyer in reference 1. These parameters were in wide use at an
earlier date and, as mentioned in reference 1, were originally used
by the British. Silverstein of the NACA found from experiments with
turbojet engines that the parameters could be used to correlate per-
formance characteristics of the engine except for the characteristics
that involve fuel flow; namely, fuel flow, fuel-air ratio, and spe-
cific fuel consumption.

Some of the factors that may influence jet-engine performance
but are not considered in the derivation of the listed parameters
are viscous friction, variations of the mass flow of gas resulting

from addition of fuel, combustion efficiency, and variations of compressibility. (Compressibility is herein defined as $\dfrac{d\rho/\rho}{dp/p}$ where ρ is density and p is pressure.) Before the approximations involved in the omission of these factors can be evaluated, a more complete analysis based upon the important internal fluid forces is necessary.

This report presents an analysis of the performance of jet-propulsion engines based upon the physical concepts of forces and expansions: inertia, elastic, viscous forces, and thermal expansion. The considerations of flow similarity are first applied, dimensionless parameters consisting of ratios of the forces and expansions are derived, and the more commonly used parameters are derived from the dimensionless parameters.

SYMBOLS

Except where special units are given, any set of consistent units may be used.

A   cross-sectional area of flow passage

a   speed of sound

c_p specific heat at constant pressure

C_T thrust coefficient

D   diameter

F_n thrust

$f_1, f_2, f_3 \ldots$ indicate functional relations

g   ratio of mass units to gravitational units

h   heating value of fuel

k   constant of proportionality

L   linear dimension

M   Mach number

N   engine speed

2

P   power

p   pressure, total or static

Q   heat units added per unit mass of fluid

R   gas constant

T   temperature, total or static

V   linear velocity

v   specific volume

W_a mass rate of air flow

W_f mass rate of fuel flow

γ ratio of specific heats of fluid

δ ratio of pressure to standard NACA sea-level pressure

η_c combustion efficiency

θ ratio of temperature to standard NACA sea-level temperature

μ coefficient of viscosity of gas

ρ density

τ temperature ratio

    Numerical subscripts indicate stations in the flow system except where the subscript is on the symbol f.

    Subscript x applies to the cross section used for computing the thrust coefficient.

    Subscript std indicates values corresponding to the NACA standard sea-level atmosphere.

SIMILARITY IN FLUID FLOW

Conditions of Similarity

    The frictionless, adiabatic flows of incompressible fluids in two systems are said to be similar if corresponding streamlines in

the two systems are geometrically similar. If the streamlines are similar, the physical boundaries of the flow are geometrically similar for the two systems. As a result of the similarity of flow, the ratio of velocities and pressure gradients at any two points in one system equals the ratio of the respective quantities at corresponding points in the other system.

In the more general case where friction loss and heat exchange accompany the flow of compressible fluids, the requirement that corresponding streamlines be similar is not a sufficient restriction to insure that the velocities, pressure gradients, and other conditions of the fluid be proportional in two systems. As an example, in the case where heat is added to a fluid flowing in parallel streamlines, a momentum pressure drop will accompany the heat addition although no pressure drop would accompany the adiabatic flow of the fluid in parallel streamlines. It therefore appears that the concept of similarity of flow requires further refinement.

In reference 2, the condition of kinematic similarity is given as a refinement to the condition of geometrical similarity. Kinematic similarity is obtained when the flow patterns are geometrically similar and the ratio of the velocities at corresponding points in two systems are equal. Dynamic similarity is also given in reference 2 as a further refinement of the concept of similarity of fluid flow. Dynamic similarity is obtained when the following conditions are met for two systems being compared: the flow in the systems must be geometrically similar; the ratio of velocities at two points in one system must equal the ratio of velocities at corresponding points in the other system; the ratio of homologous masses are equal; and the ratio of homologous forces are equal. If three of these conditions are satisfied, the fourth condition will be automatically satisfied for reasons given in reference 2.

In the case of fluid flow with the addition or extraction of heat, the conditions for dynamic similarity fix the ratios of forces and performance characteristics within two systems. The requirements for dynamic similarity are, however, inadequate for fixing the thermal effects because these requirements can be fulfilled even though the distributions and magnitudes of the heat additions to the two systems are quite different. An example of this difference is the case of two geometrically similar systems in which incompressible fluids are flowing and heat is added to one of the systems but not to the other. The heat addition may change the temperatures but not the geometry, the velocities, the masses, or the forces.

For the purposes of this analysis, the flow of fluids in two systems will be considered to be dynamically and thermally similar when the corresponding streamlines in the two systems are geometrically

4

similar; the ratio of the velocities at any two points in one system equals the ratio of velocities at corresponding points in the other; and the ratios of homologous masses (densities), forces, and temperatures are equal.

It is not convenient to deal with the velocities, the densities, the pressures, or the temperatures at all points in the fluid; it is possible, however, to relate the flow pattern to the conditions around the boundary and thereby to simplify the analysis. Proof of this statement is not given here, but the truth of the statement is fairly evident as a consequence of the principle that if identical causes act in two cases, the results will be identical.

Internal Fluid Forces and Flow

Dimensional analysis shows that the conditions for similarity listed in the preceding section of this report are satisfied in two geometrically similar systems when the ratios of inertia, viscous, and elastic forces at a selected pair of corresponding points have equal values; the total temperatures of the fluids at corresponding points are proportional; and the compressibilities of the fluids at corresponding points are equal. The succeeding paragraphs discuss some of the ratios relating the internal fluid forces, the temperatures, or the compressibilities.

One of the most widely used ratios of forces that has been used in similarity studies is Reynolds number $\dfrac{L\,V\,\rho}{\mu}$, which is dimensionless and is proportional to the ratio of inertia to viscous forces within the fluid. From dimensional analysis it has been deduced that, for geometrically similar systems in which there is incompressible isothermal fluid flow, the flow patterns will be similar if the Reynolds numbers for all systems are equal.

A second dimensionless quantity, the compressibility of the fluid, is defined by the following equation:

$$\text{Compressibility} = -\frac{p}{v}\left(\frac{\partial v}{\partial p}\right)$$

If the fluid is a gas and compression is isentropic

$$\frac{p}{v}\left(\frac{\partial v}{\partial p}\right) = -\frac{1}{\gamma}$$

Therefore

$$\text{Compressibility} = \frac{1}{\gamma}$$

The ratio of specific heats γ is therefore a measure of the compressibility of any fluid that acts in accord with the perfect gas law. Dimensional analysis shows that flow similarity can exist in two fluid-flow systems only when the values of γ at corresponding points are equal.

The specific-heat ratio γ is not a convenient parameter for expressing the performance of jet-propulsion engines because none of the principal variables (speed, pressure, density, temperature, or heat input) enter into the value of γ except for a second-order effect of temperature. Consequently, other parameters have been developed that are closely related to the principal variables and, at the same time, are dependent upon the compressibility.

One such dimensionless parameter is Mach number V/a, which is proportional to the square root of the ratio of inertia forces to elastic forces. Dimensional analysis shows that for compressible, nonviscous fluids, the flow patterns in two geometrically similar systems will be similar if the Mach numbers for both systems are equal.

A fourth dimensionless number, which is referred to herein as the "total-temperature ratio," is important in the case where heat exchange accompanies the flow of compressible fluids. The total-temperature ratio τ is the ratio of the total temperature at a point downstream from where the heat is added to the fluid to the total temperature at some point upstream from where heat is added. This parameter is very simple to use in cases where heat is added to the gas at a point or where the velocities of the fluid are low. In the more general case where the addition of heat is distributed over a region where the velocities vary appreciably, not only is the over-all total-temperature ratio important but the distribution of the heat addition to the flow pattern is important. The total-temperature ratio may be thought of as a measure of the expansion of a gas that results from the addition of heat. In the case of compressible, nonviscous fluids flowing at such low velocities that inertia forces are negligible, the flow patterns in two geometrically similar systems will be similar if the total-temperature ratio for a selected pair of points in both systems are equal and the distribution of the heat addition in the two systems is similar.

In the more general case of the flow of compressible, viscous fluids accompanied by heat transfer, the flow pattern in two systems will be similar if the following five conditions are met:

1. The boundaries are geometrically similar.

2. The fluids are perfect gases.

3. The Mach numbers at a selected pair of corresponding points are equal.

4. The Reynolds numbers at a selected pair of corresponding points are equal.

5. The total-temperature ratios at a selected pair of corresponding points are equal and the distributions of heat addition for the two systems are similar.

As a result of the preceding statements the following equation relating velocities at various points in a fluid flow system to Mach number, Reynolds number, and total temperature can be written

$$\frac{V_1}{V_0} = f_1 [M_0, R_0, \tau]$$

$$= f_1 \left[\frac{V_0}{a_0}, \frac{L V_0 \rho_0}{\mu_0}, \frac{T_2}{T_0} \right] \tag{1}$$

Similarly, p_1/p_0, T_1/T_0, and ρ_1/ρ_0 can be related to the same three parameters that appear on the right-hand side of equation (1).

DIMENSIONLESS PARAMETERS

Ram Jets

A ram jet is one of the simplest devices in which inertia, elastic, viscous, and thermal effects must be considered. Consider the flow pattern around and through a ram jet as shown in the following sketch.

Station 0   Station 1   Station 2

7
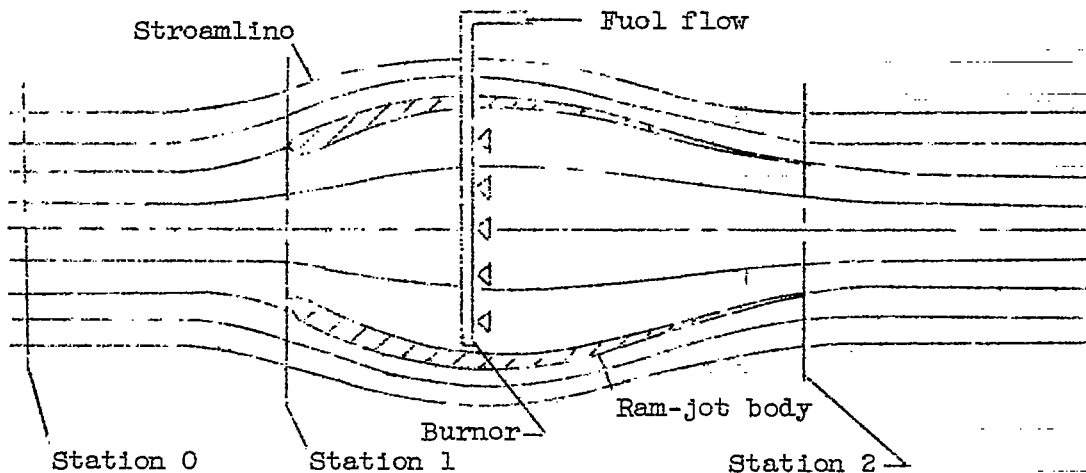
When there is no fuel flowing, the flow pattern is a function only of Mach number and Reynolds number. As a specific example

$$\frac{V_1}{V_0} = f_2 \left[\frac{V_0}{a_0}, \ \frac{L\ V_0\ \rho_0}{\mu_0} \right]$$

If heating of air is accomplished by burning of fuel but the mass of the fuel is small compared with the mass of air flowing through the ram jet, equation (1) applies. Equation (1) will be used as the basis for deriving parameters that involve important performance variables, such as air flow, fuel flow, and thrust. In order to simplify the equations, the symbol Z will be used as a shorthand notation for

$$\left[\frac{V_0}{a_0}, \ \frac{L\ V_0\ \rho_0}{\mu_0}, \ \frac{T_2}{T_0} \right]$$

such that

$$\frac{V_1}{V_0} = f_1 [Z]$$

Air flow. - Air flow is related to the Mach number at some station within the engine, such as the inlet, as follows:

$$\frac{V_1}{a_1} = \frac{W_a}{\rho_1\ A_1\ a_1} = f_3 [Z] \tag{2}$$

Usually ρ_0 and a_0 are known and ρ_1 and a_1 are dependent variables. The relations of ρ_1 to ρ_0 and a_1 to a_0 are

$$\frac{\rho_1}{\rho_0} = f_4 [Z]$$

$$\frac{a_1}{a_0} = f_5 [Z]$$

These equations can be used to eliminate ρ_1 and a_1 from equation (2) and the following result is obtained:

$$\frac{W_a}{\rho_0\ A_1\ a_0} = f_6 [Z]$$

Temperature is involved in ρ_0 and a_0 but, by rearrangement, the following more useful modification of the air-flow parameter can be obtained:

$$\frac{W_a \, a_0}{\gamma \, L^2 \, p_0} = f_7 \, [Z] \tag{3}$$

The air-flow parameter is, therefore, $\dfrac{W_a \, a_0}{\gamma \, L^2 \, p_0}$

Fuel flow. - Fuel is the means by which heat is introduced into the engine. The fuel flow is therefore related to T_2/T_0 because this ratio is a measure of the heating effect of the fuel. The fuel-flow parameter is derived from T_2/T_0 as follows:

$$\frac{T_2}{T_0} = 1 + \frac{Q}{c_p \, T_0} = 1 + \frac{h \, \dfrac{W_f}{W_a} \, \eta_c}{c_p \, T_0} \tag{4}$$

Air flow is a dependent variable whose value can be obtained from equation (3). If equation (3) is used to eliminate W_a from equation (4) and the resulting expression for T_2/T_0 is substituted into equation (1) the following result is obtained:

$$\frac{V_1}{V_0} = f_8 \left[\frac{V_0}{a_0}, \ \frac{L \, V_0 \, \rho_0}{\mu_0}, \ \frac{\gamma - 1}{\gamma} \left(\frac{h \, W_f}{L^2 \, p_0 \, a_0} \right) \eta_c \right] \tag{5}$$

The dimensionless parameter involving fuel flow is, therefore,

$$\frac{\gamma - 1}{\gamma} \left(\frac{h \, W_f}{L^2 \, p_0 \, a_0} \right) \eta_c$$

Net thrust. - Net thrust, if the mass of the fuel is neglected, is given by the equation

$$F_n = W_a \, (V_3 - V_0) = W_a \, V_0 \left(\frac{V_3}{V_0} - 1 \right)$$

but

$$\frac{V_3}{V_0} = f_9 \, [Z]$$

$$\frac{W_a \, a_0}{\gamma \, L^2 \, p_0} = f_7 \, [Z]$$

and V_0/a_0 is part of Z. Therefore

$$\frac{F_n}{\gamma L^2 p_0} = \left(\frac{W_a\, a_0}{\gamma L^2 p_0}\right)\left(\frac{V_0}{a_0}\right)\left(\frac{V_3}{V_0} - 1\right)$$

$$= f_{10}\,[Z] \tag{6}$$

A dimensionless parameter $\dfrac{F_n}{\gamma L^2 p_0}$ has been developed. The dimensionless parameter $\dfrac{F_n}{\gamma A_x p_0}$ is sometimes substituted in which A_x is the maximum cross-sectional area of the engine.

Another parameter frequently used is the thrust coefficient C_T, which is defined by the equation

$$C_T \equiv \frac{F_n}{\frac{1}{2}p_0 V_0^2 A_x} = \frac{2F_n}{\gamma p_0 A_x M_0^2}$$

Power. - The propulsive power P is given by the equation

$$P = k\, F_n V_0$$

Substitute the value of F_n from equation (6);

$$\frac{P}{\gamma L^2 p_0 V_0} = f_{11}\,[Z]$$

Multiply both sides of the equation by V_0/a_0 and rearrange the equation to get

$$\frac{P}{\gamma L^2 p_0 a_0} = f_{12}\,[Z] \tag{7}$$

Thrust specific fuel consumption. - The thrust specific fuel consumption equals the quotient of fuel flow and thrust. From equation (5), the parameter involving fuel flow is $\dfrac{\gamma - 1}{\gamma}\left(\dfrac{h W_f}{L^2 p_0 a_0}\right)\eta_c$. Divide this parameter by equation (6) to get

$$\frac{(\gamma - 1)\, h\, \eta_c}{a_0}\left(\frac{W_f}{F_n}\right) = f_{13}\,[Z]$$

where W_f/F_n is the thrust specific fuel consumption.

Power specific fuel consumption. - The power specific fuel consumption is the quotient of fuel flow and power. Divide the parameter $\frac{\gamma - 1}{\gamma} \left(\frac{h \, W_f}{L^2 \, p_0 \, a_0} \right) \eta_c$ by equation (7) to obtain

$$(\gamma - 1) \, h \, \eta_c \left(\frac{W_f}{P} \right) = f_{14} \, [Z]$$

where W_f/P is the power specific fuel consumption.

Turbojets

Turbojets and ram jets differ essentially in that the turbojet has moving parts that absorb or deliver energy to the working fluid; whereas the ram jet has no moving parts. Notwithstanding this difference, the analysis just given for ram jets applies also to the turbojet engine because the motion of the rotating parts is completely determined by the geometry of the engine, the inertia forces, the elastic forces, and the viscous forces; these forces are, in turn, a function of Reynolds number and Mach number at some point in the fluid and of total-temperature ratio across the engine. In other words, the parameters that control the forces acting on a rigid body in a moving fluid are the same parameters that control the type and magnitude of motion of the moving parts of a turbojet. The analysis just developed for ram jets applies equally well to turbojets.

In the case of a turbojet, an additional dependent variable is introduced, engine speed. A dimensionless parameter involving engine speed is $\frac{NL}{a_0}$. Derivation of this parameter will not be given but it is closely related to the Mach number of the compressor blades. This parameter can be equated to a function of Mach number, Reynolds number, and total-temperature ratio or fuel-flow parameter as follows:

$$\frac{NL}{a_0} = f_{15} \left[\frac{V_0}{a_0}, \; \frac{L \, V_0 \, \rho_0}{\mu_0}, \; \frac{T_2}{T_0} \right]$$

$$= f_{16} \left[\frac{V_0}{a_0}, \; \frac{L \, V_0 \, \rho_0}{\mu_0}, \; \frac{\gamma - 1}{\gamma} \left(\frac{h \, W_f}{L^2 \, p_0 \, a_0} \right) \eta_c \right] \qquad (8)$$

Because engine speed is a readily measured quantity and the determination of combustion efficiency is uncertain, it is frequently desirable to describe performance as a function of speed instead of fuel flow as is done in equation (5). Rearrange equation (8) in the · form

$$\frac{\gamma - 1}{\gamma}\left(\frac{h\,W_f}{L^2\,p_0\,a_0}\right)\eta_c = f_{17}\left[\frac{V_0}{a_0},\ \frac{L\,V_0\,\rho_0}{\mu_0},\ \frac{NL}{a_0}\right]$$

Substitute this value of $\dfrac{\gamma - 1}{\gamma}\left(\dfrac{h\,W_f}{L^2\,p_0\,a_0}\right)\eta_c$ into equation (5)

$$\frac{V_2}{V_0} = f_{18}\left[\frac{V_0}{a_0},\ \frac{L\,V_0\,\rho_0}{\mu_0},\ \frac{NL}{a_0}\right] \tag{9}$$

The combination of parameters appearing on the right-hand side of equation (9) will be used extensively in succeeding paragraphs.

It can be readily shown that the parameters containing thrust, power, thrust economy, and power economy are related, respectively, to the parameters in equation (9) by the equations:

$$\frac{F_n}{\gamma\,L^2\,p_0} = f_{19}\left[\frac{V_0}{a_0},\ \frac{L\,V_0\,\rho_0}{\mu_0},\ \frac{NL}{a_0}\right] \tag{10}$$

$$\frac{P}{\gamma\,L^2\,p_0\,a_0} = f_{20}\left[\frac{V_0}{a_0},\ \frac{L\,V_0\,\rho_0}{\mu_0},\ \frac{NL}{a_0}\right] \tag{11}$$

$$\frac{(\gamma - 1)\,h\,\eta_c}{a_0}\frac{W_f}{F_n} = f_{21}\left[\frac{V_0}{a_0},\ \frac{L\,V_0\,\rho_0}{\mu_0},\ \frac{NL}{a_0}\right] \tag{12}$$

$$(\gamma - 1)\,h\,\eta_c\,\frac{W_f}{P} = f_{22}\left[\frac{V_0}{a_0},\ \frac{L\,V_0\,\rho_0}{\mu_0},\ \frac{NL}{a_0}\right] \tag{13}$$

Test Results with Turbojets

Tests have been run at the Cleveland laboratory of the NACA to check the applicability of equations (10) to (13). A turbojet engine was tested at pressures and temperatures simulating altitudes between 10,000 and 30,000 feet and at a constant simulated flight Mach number. The data are plotted in figure 1 with the thrust parameter $\dfrac{F_n}{\gamma\,A_x\,p_0}$ for the ordinate and the speed parameter $\dfrac{NL}{a_0}$ for the abscissa. The data fall close to a single curve except for slight deviations at high values of the speed parameter. It is concluded that, for the range of

values covered in this investigation, the effect of variations in Reynolds numbers upon performance is negligible because Reynolds number varies with changes of altitude and there was no noticeable effect of altitude upon the relation of thrust parameter to speed parameter.

Figure 2 shows the effect of speed parameter on the thrust parameter for three values of Mach number. This figure is a complete representation of equation (10) for the range of values of the parameters covered in the experiments. The performance of other similar engines of different sizes and at any operating condition that is represented within the range of parameters given can be estimated from the figure.

In figure 3, which is similar to figure 2, the relations of the air-flow parameter and compressor pressure ratio to the engine-speed parameter and flight Mach number are shown. Other characteristics, such as power and specific air consumption, can be treated in a similar manner.

Performance characteristics that involve fuel flow, such as specific fuel consumption, fuel-air ratio, and fuel consumption are of special interest because combustion efficiency η_c appears in all parameters in which fuel flow appears. As a specific example, figure 4 shows the relation of the fuel-flow parameter $\eta_c \dfrac{(\gamma - 1)\ h\ W_f}{a_0\ F_n}$ and a modified fuel-flow parameter $\dfrac{(\gamma - 1)\ h\ W_f}{a_0\ F_n}$ to the speed parameter $\dfrac{NL}{a_0}$. The modified fuel-flow parameter does not contain combustion efficiency. Results of tests at three altitudes plotted in terms of these parameters show that separate curves for each altitude were determined by the data when the modified parameter was used but only one curve represented the data when the parameter involving η_c was used.

APPROXIMATIONS AND SPECIAL CONSIDERATIONS

Fuel mass. - It has been assumed here that the sole effect of fuel addition to the flowing air is a heating effect. Another effect of the fuel is to increase the mass of the gas. When this additional effect is significant, equation (1) must be modified as follows:

$$\frac{V_1}{V_0} = f_{23}\left[M_0,\ R_0,\ \frac{T_2}{T_0},\ \frac{W_f}{W_a}\right]$$

Usually W_f/W_a is small enough to neglect the mass effect.

Another effect of fuel is to change the composition of the gas and thereby change the value of γ. The effect of variations of γ will be discussed in the following paragraph.

Specific-heat variations. - Two similar engines can be compared only when the values of γ are the same at corresponding points in both engines because, it will be recalled, equal values of γ are required for similarity of flow. Although γ actually changes appreciably as a result of the addition of fuel, it has been found that, as far as the results of tests with turbojets are concerned, the variation can be neglected.

If two similar engines are tested at different air temperatures, the values of γ will differ in the two engines, particularly in the high-temperature portions of the gases. Again, it has been found that variations of γ with temperature are not significant.

Combustion efficiency. - Some of the factors involved in the relation of combustion efficiency to the condition of the gas are the kinetics of chemical reactions and the mechanism of fuel vaporization. Each of these factors is related to temperature and pressure, but this relation cannot be expressed in terms of Mach number, Reynolds number, and total-temperature ratio. As a consequence, combustion efficiency is not related to these parameters. It is concluded, therefore, that the combustion efficiency must be experimentally determined at each condition where it is desired to estimate performance, or some independent method of estimating combustion efficiency must be developed.

Point of heat addition. - The performance of the jet engine can be modified by burning fuel at places other than in the usual combustion chamber. In such cases, two similar engines can be compared only when the distribution of heat addition to the engines is similar for the two cases.

Propeller turbine. - The analysis given in this report applies also to the propeller turbine. The speed of such engines is often controlled by varying the propeller pitch. This change of pitch is a change of the geometry of the engine combination and a new parameter, propeller blade angle, should be added to equation (1).

DIMENSIONAL GENERALIZING FACTORS

The incorporation of several variables into a single parameter constitutes a generalization of the relation among variables. The generalized parameter is often dimensionless, but not necessarily so, and it is frequently desirable to use the parameters that are not dimensionless. One reason for this preference is that dimensionless

parameters are not usually easily identified with particular variables. Another reason is that, even in the case where the parameter is easily identified with one variable, the numerical values of the parameter differ greatly from the actually observed numbers representing the magnitude of the variable.

One method of obviating the difficulties connected with dimensionless parameters is the use of so-called correction or reduction factors, as will be illustrated by example.

First, consider parameters identified with engine speed. The dimensionless parameter $\dfrac{NL}{a_0}$ (equation (8)) has already been used. Another parameter frequently used is $N/\sqrt{\theta_0}$. It will be proved that these two parameters are directly proportional to each other.

The parameter $\dfrac{NL}{a_0}$ can be rewritten as $\left(\dfrac{N}{\sqrt{T_0}}\right)\left(\dfrac{L}{\sqrt{\gamma R}}\right)$. Multiply and divide the parameter by the square root of the standard sea-level temperature T_{std}

$$\frac{NL}{a_0} = \frac{N}{\sqrt{\dfrac{T_0}{T_{std}}}} \cdot \frac{L}{\sqrt{\gamma R T_{std}}} = \frac{N}{\sqrt{\theta_0}}\left(\frac{L}{a_{std}}\right)$$

In performance tests of a given engine, L is, of course, a constant and a_{std} is a constant by definition. The proportionality between $\dfrac{NL}{a_0}$ and $N/\sqrt{\theta_0}$ is therefore proved. The parameter $N/\sqrt{\theta_0}$ is referred to as "corrected" engine speed.

Other similar parameters are derived in table I along with the commonly accepted names for the parameters. The corrected parameters have the same significance as have the corresponding dimensionless parameters, although the corrected parameters are not dimensionless. For example, the corrected speed $N/\sqrt{\theta_0}$ was derived from the Mach number of the compressor blade tips and, therefore, the corrected speed is a measure of the ratio of inertia and elastic forces in the fluid. An example of the use of corrected parameters is shown in

figure 5. The corrected thrust and corrected thrust specific fuel consumption of a turbojet engine was plotted against corrected engine speed. The data are the same as are plotted in figures 1 and 3.

Aircraft Engine Research Laboratory,
    National Advisory Committee for Aeronautics,
        Cleveland, Ohio, May 7, 1946.

REFERENCES

1. Warner, D. F., and Auyer, E. L.: Contemporary Jet-Propulsion Gas Turbines for Aircraft. Mech. Eng., vol. 67, no. 11, Nov. 1945, pp. 707-714.

2. Anon.: Hydraulic Models. Manual of Eng. Practice No. 25, Comm. Hydraulics Div. on Hydraulic Res., Am. Soc. Civil Eng., July 23, 1942, pp. 3-4.

TABLE I - DERIVATION OF CORRECTED PARAMETERS

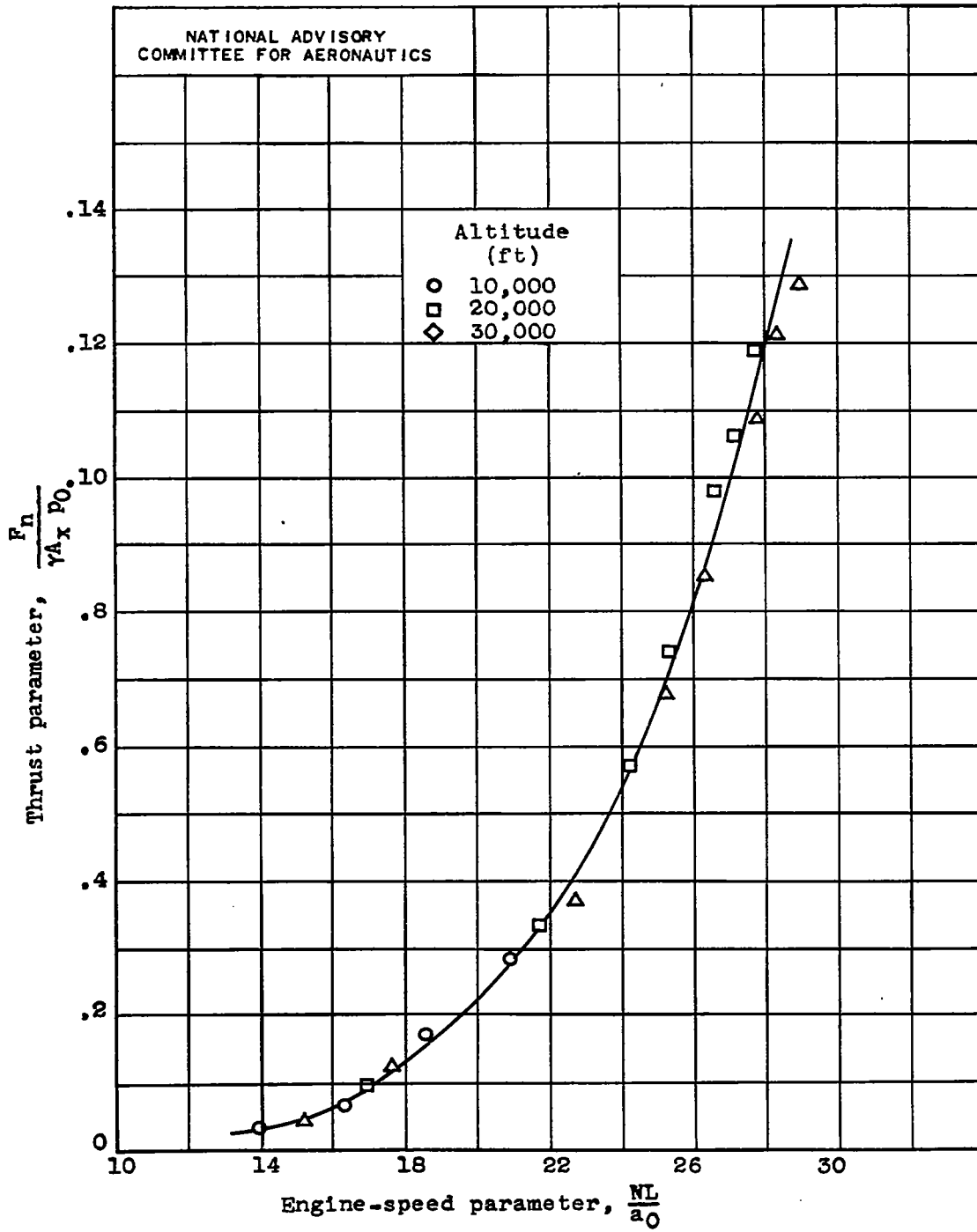| Dimensionless parameter	Constant of proportionality	Corrected parameter	Designation
$\dfrac{V}{a}$	a_{std}	$\dfrac{V}{\sqrt{\theta}}$	Corrected airspeed
$\dfrac{NL}{a}$	$\dfrac{a_{std}}{L}$	$\dfrac{N}{\sqrt{\theta}}$	Corrected engine speed
$\dfrac{W_a a}{\gamma L^2 p}$	$\gamma L^2 \dfrac{p_{std}}{a_{std}}$	$\dfrac{W_a \sqrt{\theta}}{\delta}$	Corrected air flow
$\dfrac{\gamma-1}{\gamma}\dfrac{hW_f}{L^2 pa}$	$\dfrac{\gamma}{\gamma-1}\dfrac{L^2}{h}\,p_{std}\,a_{std}$	$\dfrac{W_f}{\delta\sqrt{\theta}}$	Corrected fuel flow
$(\gamma-1)\dfrac{h}{a^2}\dfrac{W_f}{W_a}$	$\dfrac{a_{std}^2}{(\gamma-1)h}$	$\dfrac{W_f/W_a}{\theta}$	Corrected fuel-air ratio
$\dfrac{F_n}{\gamma L^2 p}$ or $\dfrac{F_n}{\gamma A_x p}$	$\gamma L^2 p_{std}$ or $\gamma A_x p_{std}$	$\dfrac{F_n}{\delta}$	Corrected net thrust
$\dfrac{P}{\gamma L^2 pa}$	$\gamma L^2 p_{std}\,a_{std}$	$\dfrac{P}{\delta\sqrt{\theta}}$	Corrected net thrust power
$\dfrac{(\gamma-1)h\eta_c}{a}\dfrac{W_f}{F_n}$	$\dfrac{a_{std}}{(\gamma-1)h\eta_c}$	$\dfrac{W_f}{F_n\sqrt{\theta}}$	Corrected net thrust specific fuel consumption
$(\gamma-1)h\eta_c\dfrac{W_f}{P}$	$\dfrac{1}{(\gamma-1)h\eta_c}$	$\dfrac{W_f}{P}$	Net power specific fuel consumption
$\dfrac{T}{T_0}$	T_{std}	$\dfrac{T}{\theta}$	Corrected temperature
$\dfrac{p}{p_0}$	p_{std}	$\dfrac{p}{\delta}$	Corrected pressure

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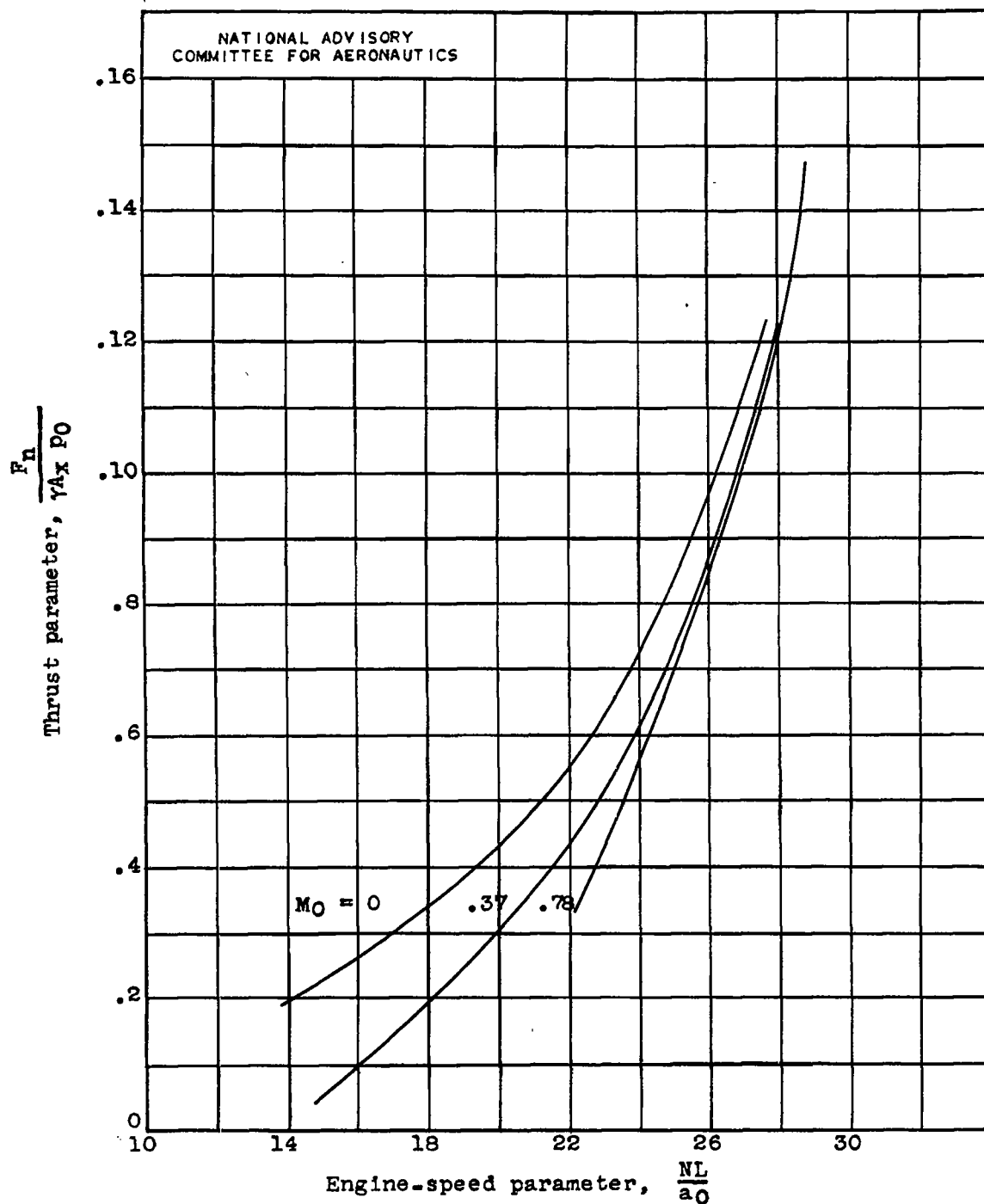Figure 1.- Generalization of static thrust
characteristics of a turbojet engine.

Fig. 2   NACA TN No. 1106

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$M_O = 0$ .37   .78

Thrust parameter, $\dfrac{F_n}{\gamma A_x \, p_O}$

Engine-speed parameter, $\dfrac{NL}{a_O}$

Figure 2.– Effect of flight Mach number on generalized
thrust characteristics of a turbojet engine.

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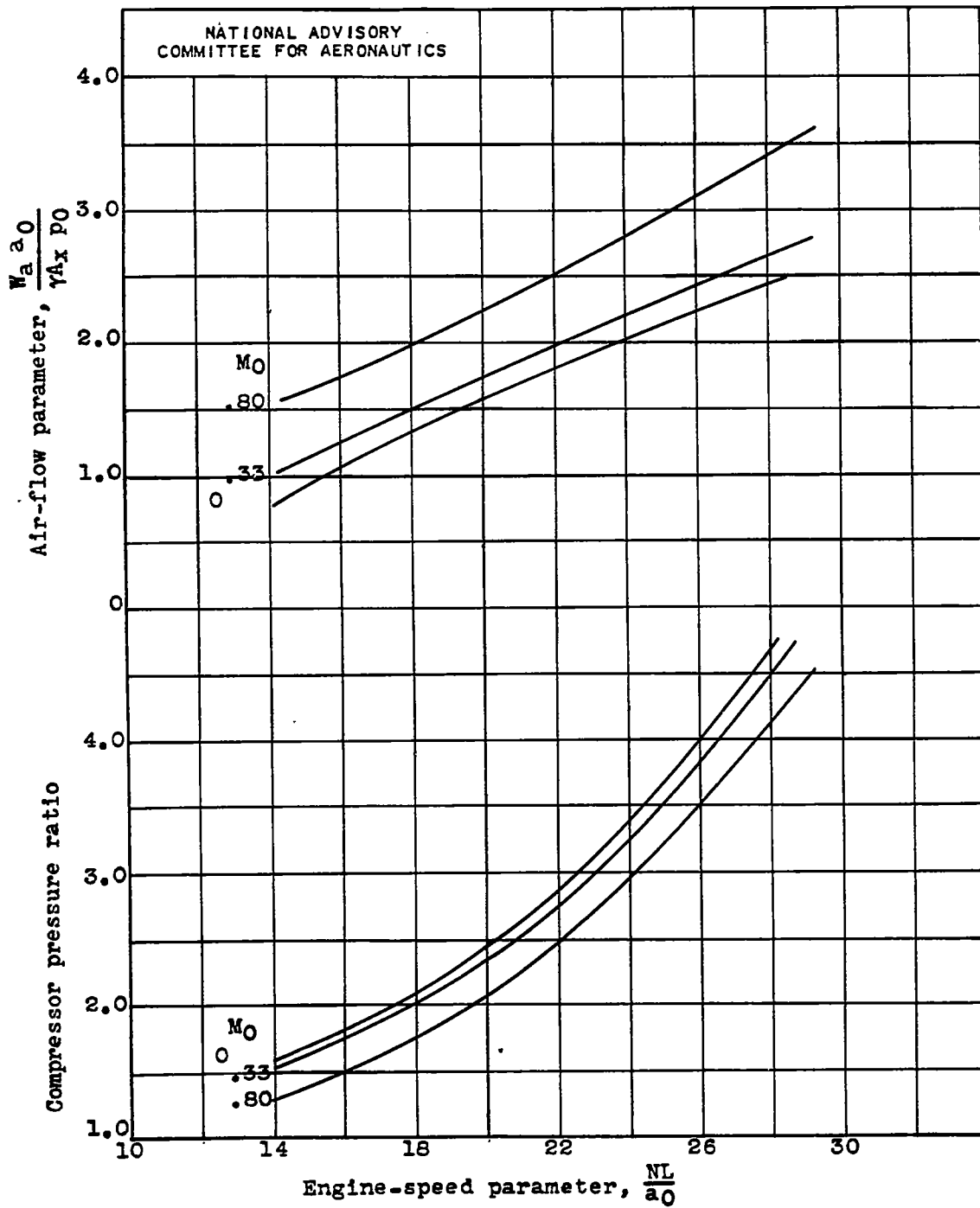
Figure 3.— Generalization of air-flow and compression-
pressure characteristics of a turbojet engine.
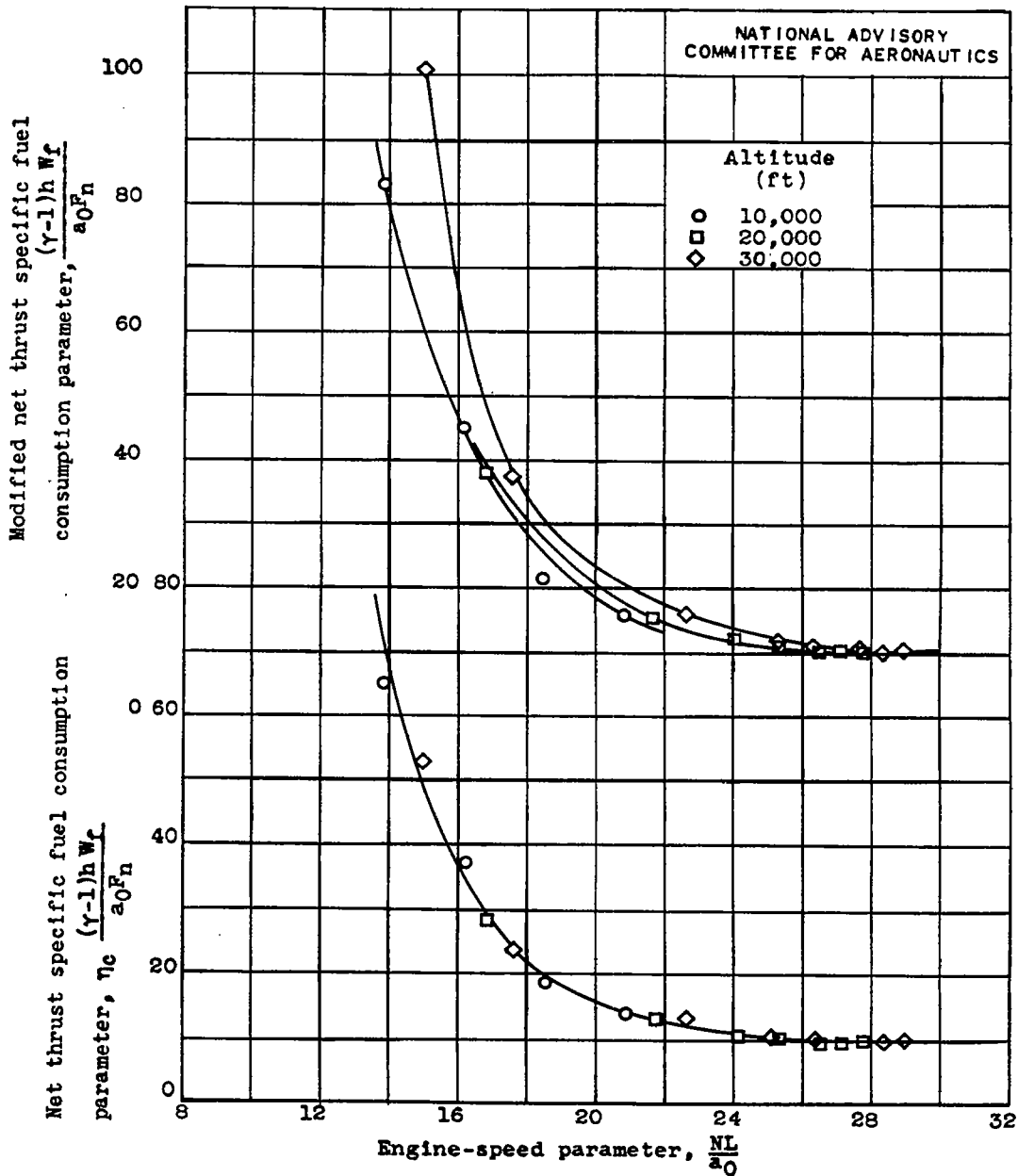
Fig. 4

NACA TN No. 1106

Figure 4.- Effect of altitude on generalization of specific
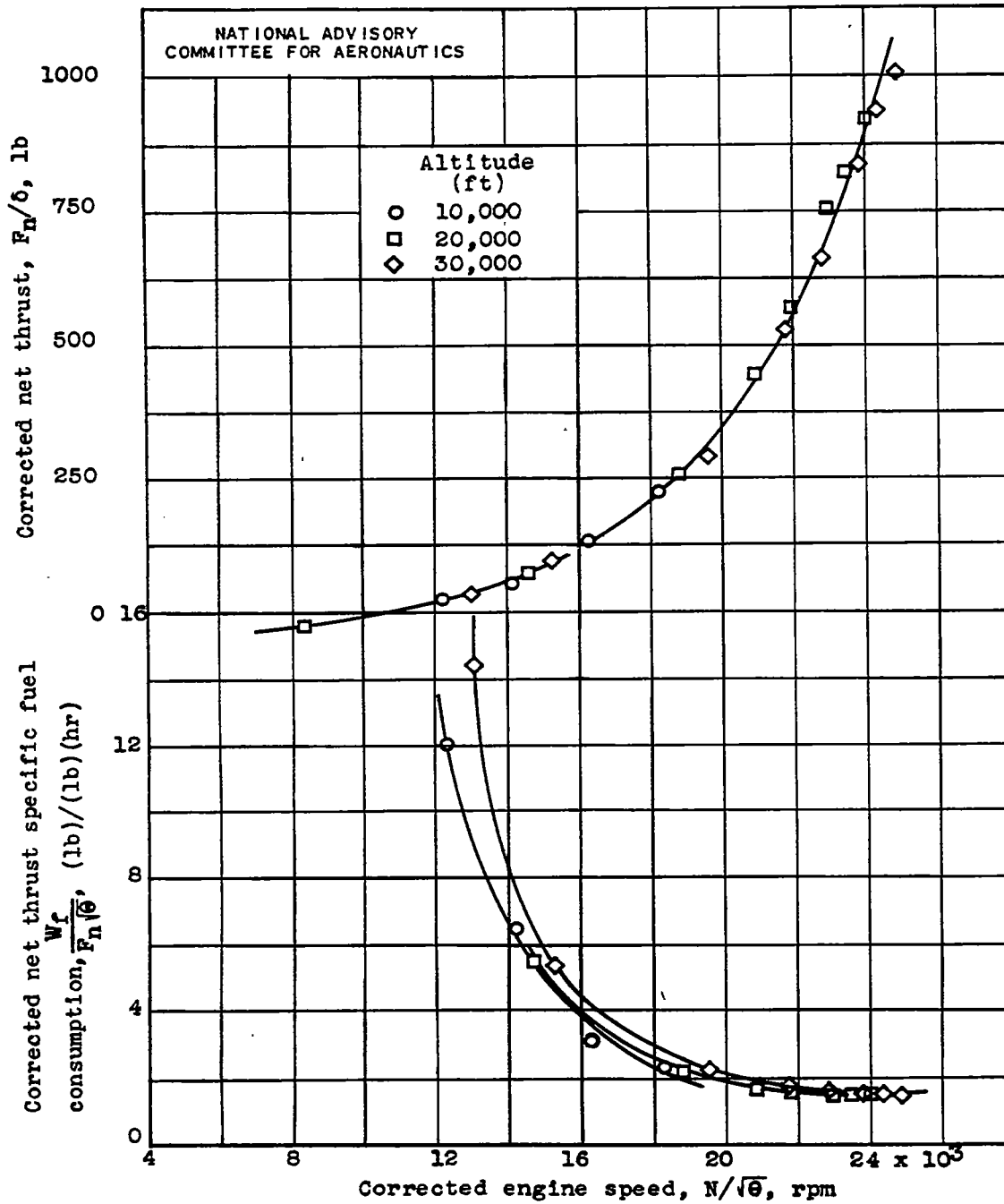fuel consumption of a turbojet engine.

Figure 5.— Variation of corrected thrust and corrected
thrust specific fuel consumption with corrected engine
speed of a turbojet engine. Static test conditions.