## NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

## TECHNICAL NOTE

No. 1037<br>COLUMN STRENGTH OF ALUMIIUM ALIOY<br>14S-T' $\# X T R U D R D ~ S H A P E S ~ A N D ~ R O D ~$<br>By J. R. Leary and Marshall Holt<br>Aluminum Company of America



Washington
May 1946

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NATIONAL ADVISORY COMMITTRE FOR ARRONAUTICS
TEGENIGAI NOTE NO, 1027
OOLUMN STRENGTH OF ALUMINUM ALIOY
146-T EXTRUDED SHAPES AND ROD
By J. E. Leary and Marahall Halt
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## INTRODUCTION

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Considerable interest is being shown in the use of aluminum alloy l4S-T in heavy-duty gtructural applications as well as in aircraft. This alloy, once considered primarily a forging alloy, is now being produced in a variety of forma, such as extruded shapes, rolled shapes, and alclad sheet and plate. With the expanding uses of this material it has seemed desirable to determine somo of its structural characteristios, and one of the important items is column strength. The column test data presented herein have been obtained on extruded shapes and on rolled and drawn rod of this alloy.
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OBJECT

It war the object of this investigation to determine the column strength of aluminum alloy l4S-T on the basis of tests of extruded shapes and rolled and drawn rod.

## SPECIMENS AND METHOD OF TEST

Extruded ghapes of $14 S-T$ were selected to represent the following three thickness ranges covered by the specification:

| Thickness range | Section |
| :--- | :--- |
| 0.725 to 0.499 in. | $2 \frac{1}{2}-$ by $2 \frac{1}{2}-$ by $1 / 4-i n$. angle |
| 0.500 to 0.749 in. | $4-$ by $9 / 16-i n$. zee |
| 0.750 in. and over | $5 / 8-$ by 2latin. bar |

In addition, tests were made on 1 -inch diameter relled and drawn rod.

The nominal elements of these sections are:

| Section | Dimensions (in.) | $\begin{gathered} \text { Die } \\ \text { number } \end{gathered}$ | $\begin{gathered} \text { Nem- } \\ \text { inal } \\ \text { thick- } \\ \text { ness } \\ (i n .) \end{gathered}$ | $\begin{gathered} \text { Area } \\ (\mathrm{sqin.}) \end{gathered}$ | $\begin{gathered} \text { Least } \\ \text { radius } \\ \text { of } \\ \text { gyra- } \\ \text { tien } \\ (i n,) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Angle | 2] by $2 \frac{7}{3}$ by $1 / 4$ | 78-H | 1/4: | 1,194 | 0.489 |
| Zee | 4 by $9 / 16$ | 771-F | 9/16 | 5.289 | . 675 |
| Bar | $5 / 8$ by $2 \frac{1}{4}$ | 22513-EG | 5/8 | 1.406 | . 181 |
| Bar | 1 by 2 | 22513-7Y | 1 | 2.000 | . 289 |
| Rod | I-in. diam. | Rolled-drawn | -- | . 785 | . 250 |

The column specimens tested are described in table I. The actual average area was determined for each specimen from the weight, length, and nominal specific gravity (0.IOl lb per cu in.). The crookedness was obtained by inserting thickness gages between the specimen and a plane surface upon which it rested. The ratio of length to crookedness is greater than 1000 except for the four specimens out from the $5 / 8-b y 2 \frac{3}{4}-i n c h$ bar marked No, 16 and specimen 18-20 from the 1 - by 2-inch bar. Experience has indicated that the strengths of the specimens with this ratio less than 1000 are significantly reduced by the crookedness. The original angle of twist was determ mined from measurements obtained by inserting thicknese gages under one corner of an outatanding leg of the angle or one corner of the bar when the other three corners touched the surface plate. The ends of the specimens were finished flat and parallel by turning on an arbor in a lathe,

The tests, except those on the three shortest zee specimens, were made in an Amsler testing machine of 300,000-pound' maximum capacity with intermediato load ranges of 30,000 , 100,000, and 200,000 pounds (type 150 SZBDA, serial No. 5254), This machine is of the four-column type, and the guides on the movable head are adjustable to allow a minimum of lateral
motion of the movable platen for the satisfactory operation of the machine. When testing the shortor specimens in the $300,000-\mathrm{pound}$ capacity machine, the platens were protected by hardened steel disks 9 inches in diameter, the faces of which had boen finished flat and parallel by precision grinding. The three shortest specimens of zee sections were tested in the 3, 000, 000-pound capacity Templin Precigion Metal Working Machine (Balawin-Southwark Shop Order No. 63430).

All specimens were tested as columns with flat ends. During each test on either machine the platens were fixed in position to prevent tipping, but before tho test they were alined parallel within 0.0003 inch in 12 inches by means of special leveling rings. The platen in the lower head is supported by a pair of tapered rings which vary unfformly in thickness so that, by rotating one ring relative to the other and both rings relative to the lower head, this platen can be tipped and alined parallel to the upper platen.

The mechanical properties of the material are shown in table II. The tensile values given in all cases surpass the specified minimum properties for $14 S-T$ extruded shapes for the particular thickness range. The compressive stress-strain relations, as determined by the movement of the platens during the tests of specimens of the full cross section, are shown in figure l. It is recognized that the relative movement of the heads of the machine includes strains other than those in the specimen; so these curves have been corrected to give an initial slope equal to the nominal modulus of elesticity of the material, $10,600,000$ psi. (See efference l.)

## RESULTS AND DISCUSSION

The results of the column tests are given in table ind figures 2, 3, 4, and 5. All specimens oxcopt tho $2 \frac{1}{3}-$ by 2 竞 by $1 / 4$-inch angle failed by sidewise bending, ant the fest results, except for the angle, follow the Euler and tangentmodulus column curves fairly well. The equations of these curves are of the same form, the differance baing in the interpretation of the term $E$ which is the effective modulus of elasticity. Tho oquation is;

$$
\begin{equation*}
\frac{F}{A}=\frac{\pi^{2}}{\left(\frac{K I}{r}\right)^{2}} \tag{1}
\end{equation*}
$$

where
$P$ total Ioad, pounds
A cross-sectional area, square inches
(\#) effective modulus of elasticity, pounds per square inch.
Euler's interpretation for stresses in the elastic range uses $\ddagger$ equal to the initial value, $10,600,000$ psi. Bngesseris interpretation for stresses above the olastic range uses an effective modulus which is lase than the initial modulus and whioh varies with the stress. In this case the tangent modulus was taken as the effective modulus, and the compressive-stress-tangent-modulus relations are shown in figure 6.

K coefficient describing the end concitions, taken hare as 0.5 (flat ends assumed equivalent to fixed ends)
I. length of specimen, inches
$r$ least rađius of gyration, inches
The straight-line column curves obtained by the procedure outlined in reference 2 are shown in figures 2,3 , and 4 for the sections that failed by sidewise bending. The equation is of the form,

$$
\begin{equation*}
\frac{P}{A}=B-D\left(\frac{K I}{I}\right) \tag{2}
\end{equation*}
$$

where
B Intercept of the straight line on the axis of wero slenderness ratio
$D$ slope of the straieht line, such that the straight line ia tangent to the Euler curve
and the other terim are as defined above. The relation between $B$ and the compressive yield strength of the material is given in the above referenoe as:

$$
\begin{equation*}
B=C Y S\left(1+\frac{C Y S}{.200000}\right) . \tag{3}
\end{equation*}
$$

in which OYS is compressive yield strength, pounds per square inch. This equation is to be used only in the range. of effective slenderness ratios up to that at the point of tangency of the straight line and Buler curves. Beyona the point of tangency the Euler curve is applicable.

The agreement between the test results and the combination of the straight line and the Filer curves indicates that the combination is probably satisfactory for the design of 14S-T structures for stressea less than the compressivo jiela strength. It will be noticed that the trends of the tangent modulus curves and of the data points in some cases suggest the possible use of an empirical curve of the parabolic type also but not to the same extent as in the case of 75STT, which has a higher yield atrength.

As noted above, some of the equal-leg angle specimens did not fail by sidewise bending, Instead, the shortor ones failed by a combination of aidewise bending and twiating about a longitudinal axis. On the basis of elastic action, the strengths of this latter group of specimens could be computed by the following equation:

where
$p=\frac{P}{A}$ average stress at failure, pounds per square inch
$p$ polar radius of gyration about the shoar center, inches
Po polar fadius of gyration about the centroid, inches
$x_{0}$ distance between shear center and centroid, inches
Q Euler column strength for bending about the principal axis of maximum stiffness, pound per square inch, computed by equation (I)

T column strength for pure twisting failure, pounds per square inch

Further explanation of some of the terms in equation (4) is given in appendix A.

The curve of equation (4) is shown in figure 5. The Euler curve for bending failure and the curve for combined elastic bending and twisting failure for $2 \frac{1}{2}$ by $2 \frac{1}{2}$ by $1 / 4-i n c h$ angles intersect at an effective sienderness ratio equal to about 50. On the basis of elastic action, it would, therefore, be expected that the specimens longer than this would fail by bending and shorter ones would fail by combined bonding and twisting.

It is seen in figire 5 that the test results in the region where combined bending and twisting failures occur are above the elastic limit stress anc that the áata points ife somewhat below the computed curve based on elastic action. In the case of bending failures, inelsstic action can be taken care of by using the tangent modulus as tho effective modulus in the Euler equation. The case of twisting failures is not so simple because of the biexial stress conditions in the twisting problem. The use of the tangent modulus in oquation (4) leads to a computed curve that lies below the test results. Better agreement with the test resuits would, therefore, be obtained by using an effective modulus between the tangent modulus and the initial motulus. An effective motulus that results in feasonably good agreement with the test data can be obtained from either of the following relations:

$$
\begin{gather*}
\bar{B}=\mathbb{E} \sqrt{E^{\prime}}=\sqrt{E E^{\prime}}  \tag{5}\\
\bar{E}_{1}=E^{3} \sqrt{\frac{E^{1}}{E^{\prime}}}=\sqrt[3]{E^{2} E^{1}}
\end{gather*}
$$

where
$\overline{\#}, \bar{E}_{1}$ effective modulus, pounds per square inch

E initial modulus, pounas per square inch
\#'
tangent modulus, pounds per square inch
The use of equation (6) results in a slightly higher computed curve.

The compressive stress-tancent modulus curve for theso angle specimens and the effective modulus defined by equation
(5) are shown in figure 7. The compressive atress-strain curve determined on a specimen cut from the angle is shown in figure 8.

An approximate method, which is much simpler, for computing the strength of equal-leg ancles which fail hy twisting considers each of the outstanding legs as a flat plate with one longitudinal edge simply supported and the other free. The ultimate strength of the angle is assumed equal to the buckling strength of the plate. Actualiy, there may be a a slight restraint along the supported edge of the plate because of the bulk of material at the junction of the two legs, but in comparison with complete fixation any restroint from this source is undoubtedly slight. On the basis of elastic action, the critical bucking stress is given by the equation,

$$
\begin{equation*}
\sigma=k \frac{\mathbb{B}}{\left(1-\mu^{2}\right)}\left(\frac{t}{b}\right)^{2} \quad(\text { reference } 4) \tag{7}
\end{equation*}
$$

## where

$\sigma$ average stress at failure, pounds per square inch
$k$ factor depending on length-width ratio of the plate and the conditions along the odges and ends
$E$ modulus of elasticity, pounds per square inch
$\mu$ Poisson's ratio
t. thickness of plato, inches
$b$ with of plate, inches
In these tests the condition of the loaded edges of the individual legs was practically equivalent to fixed ends since the individual legs were machined flat and bore on the platens as columns with flat ends. Thus the value of k for use in equation (7) for computing the twisting strength of equal-leg angles cen be obtained from the equation,

$$
\begin{equation*}
k=\frac{\mu^{2}}{12}\left[\frac{4}{\left(\frac{I}{\delta}\right)^{2}}+0.406\right] \quad(\text { feference } 5) \tag{8}
\end{equation*}
$$

Two sets of curves of buckling strength computed by means of equations (7) and (8) are shown in figure 9. In one set the ratio of $b / t$ was taken equal to 10 , which is the ratio of the full width of leg to the thickness, and in the other set the ratio was taken equal to 9 , which is the ratio of the outstanding width to the thickness. As is the case of equation (4), the combination with the suler curve indicates that specimens shorter than about $\mathrm{KI} / \mathrm{r}$ equal to 50 would fail by combined bending and twisting. In this region the data points lie between the two computed curves based on tho offective modulus defined by equation (5).

Kollbrunner (reference 6) employed this method of analysis with his data from column tests on equal-legangles and used the following relation for the effective modulus,

$$
\begin{gather*}
\bar{E}_{2}=\oiint\left(\frac{T+\sqrt{T}}{2}\right) \\
T=\frac{\mathbb{R}^{\prime \prime}}{E}=\frac{A \frac{W^{\prime}}{E}}{\left(1+\sqrt{\frac{E^{\prime}}{T}}\right)^{2}} \tag{9}
\end{gather*}
$$

where
$\mathbb{F}_{2} \quad$ offective modulus, pounds per square inch
Finditial modulus, pounds per square inch
Et double modulus, pounds per square inch
$\mathbb{B}^{1}$ tangent modulus, pounds per square inch
$T$ ratio of double modulus to initirl modulus
This relation for effective modulus was trief out with the data in figures 5 and 9 , but it gave no better agreement with the data than the simple expression of equation (5).

An even simpler approximate method for computing the buckling strength of an outstanding plate is described in the Structural Aluminum Handbook published by Aluminum Company of America (1945). An equivalent slenderness ratio is obtained for the particular wiath-thickness ratio and the buckiing
strength then determined from a column curve for the material. The dotted horizontal line in figure 9 was thus determined, and it is apparent that the Handbook method is on the conservative side.

In order to avoid the twisting type of failure or buckling of the legs at stresses below the tangentmodulus column ourve for bending failures, the width-thickness ratio of the legs would need to be about 7 or less.

## CONCLUSIONS

The following conclusions concerning the column strength of extruded 14S-T shapes and rolled and drawn rod have been drawn from the date and discussion presented in this report.

1. There is good agreoment between the test data and the combination of Euler and tangent-modulus column curves (equation (I)) for specimens that fail by sidewise bending, the coefficient of end restraint, $K$, of the specimens tested as columns with flat ends being taken equal to 0.50 .
2. For the purpose of design of straight, axially loaded columns that fail by sidewise bending and not by twisting or local bucking, the combination of the Euler curve and a straight line tangent to it (equations (l) and (2)) should be satisfactory for ultimate column strengths less than the compressive yield strength.
3. Single-member columns consisting of equal-leg angles of $14 S-T$ and having a width-thickness ratio of the legs equal to 10 are subject to failure by combined bending and twisting about a longitudinal axis at an average stress less than that computod for failure by bending about the axis of least stiffness when the effective slendernese ratios ( $\mathrm{KI} / \mathrm{r}$ ) are less than about 50 .
4. In order to avoid the twisting type of failure or buckling of the legs at stresses below the tangent-modulus column curves for bending failures, the widthothickness ratio based on the outstanding width of the legs would need to be about 7 or less.
5. Thereis good agreement between the test results from the equal-leg angle specimens and the curve of equation (4) for the combination bending and twisting type of failure when
the effective aodulus is as defined by equation (5) or (6). The use of the tangent modulus as the effective modulus gives a computed curve somewhat below the data points.
6. For a simple approximate method of computing the column strength of equal-leg angles, equations (7) and (8) from the theary of flat plates can be used. Tho effective motuli defined by equations (5) and (6) give satisfactory agreement with the data for column strengths above the elastic stress range. The computed strengths are conservative when the full width is used in determining the width-thickness ratio.
7. The approximate method for computing the buckiing strengths of outstanding plates as given in the Structural Aluminum Handbook results in conservative computef strengths for equal-leg angles.

Aluminum Research Laboratories,
Aluminum Company of America,
New Kensirgton, Penna., July 5, 1945.

## AEYENDIXA

Frurther explanation of some of the terms in equation (4):

$$
\begin{equation*}
T=\frac{G C}{I_{p}}+\frac{n^{z_{\pi}} \overline{E^{E}}}{I^{\bar{E}} I_{p}} \Gamma \tag{10}
\end{equation*}
$$

where
$G$ modulus of elasticity in shear, psi
0 torsion factor, in. ${ }^{\prime}$ (sometimes designated as J)
Ip polar moment of inertia of the cross section with respect to the shear center, in. 4
$n$ number of half-wares in the configuration of the deformed member
$\Gamma$ torsion-bending factor, in. (variously designated $G B T$ 02• $\sigma_{B D}$ )

I length of the member, in.

$$
\begin{equation*}
G=\frac{\leftrightarrows}{2(I+\mu)} \tag{11}
\end{equation*}
$$

where
$\mu \quad$ Poisson's ratio; for aluminum alloys the value is usually taken as one-third.

$$
\begin{equation*}
0=\frac{2}{3} d t^{3}-0.210 t^{4}+0.164 t^{4} \tag{12}
\end{equation*}
$$

where
d length of leg, $b, m i n u s$ one-half the thickness of leg, in:
$t$ thickness of leg, in.
8 diameter of largest circle that can be drawn within the cross section at the heel of the angle, in.

$$
\begin{equation*}
\Gamma=\frac{1}{18} a^{3} t^{3} \tag{13}
\end{equation*}
$$

The shear center of an equal-ieg angle is in the heel of the angle at the intersection of the center lines of the two legs. If the effects of the fillet and roundings are neg-... lected, it follows that:

$$
\begin{equation*}
x_{0}=\frac{d}{2 \sqrt{2}} \tag{14}
\end{equation*}
$$

By definition it follows that

$$
\begin{align*}
& I_{p}=I_{x}+I_{y}+A x_{0}^{z}  \tag{15}\\
& \rho^{z}=r_{x}^{2}+r_{y}^{z}+x_{0}^{z} \tag{16}
\end{align*}
$$

TDeveloped from equation (21) of reference 7 .
where
$I_{x}$, $I_{y}$ momenta of inertia about a pair of perpendicular axeg, in.

A cross-sectional area, sq. in.
$r_{x}, r_{y} \quad$ radif of gyration about a pair of perpendicular

It should be pointed out that equations (4) and (10) are valid for any cross section having one axis of symmetry. the Values of the terms as defined by equations (12), (13), and (14) are limited to equal-leg angles.

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[Specimens tested as columne with flat ends]

lomputed from the length and weight of the specimen and the nominal specific gravity of the material
${ }^{2}$ specimens teated as columns with flat ende, $X$ taken as 0.5 .
$3_{\text {ror the }}$ zee, el arookednesa in plane parallel to the flanges; for other aeotione, $e_{1}=$ orookednese in plane of least stiffness; for the zee, ez $=$ crookedness in plane parallel to the wob.

TABLI II. - MEGHANICAL PROPFRTIES OF NATERIAL; IAVESTIGATION OF COLUIN STRENGTH OF 14 S-T

| Section | Dimensions (in.) | Tencile strength <br> (psi) | $\begin{gathered} \text { Tensile } \\ \text { yleld } \\ \text { strength } \\ (\text { get }= \\ 0.2 \%) \\ (\mathrm{psi}) \end{gathered}$ | Filongation in 2 in. (percent) | Type of - tensile specimen ${ }^{1}$ | Compresm sive yield strength (set $=$ $0.2 \%$ ) (D61) | Type of compressive specimen |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Extruded bar | $5 / 8 \times 2 \frac{1}{4}$ | 75,900 | 68,000 | 13.0 | 1/2 in. round | 66,900 | Fuil section |
| Extruded zee | $4 \times 9 / 16$ | 65,860 | 60,500 | 12.0 | 1/2 in. round | 259,800 | Full section |
| Extruded bar | $1 \times 2$ | 75,900 | 67.500 | 11.00 | 1/2 in. round | ${ }^{2} 65,600$ | Pall section |
| Rolled and dram rod | 1 diameter | 69,900 | 62,250 | 13.0 | 1/2 in. round | ${ }^{2} 64,500$ | Full section |
| Txtruded angle | $2 \frac{1}{2} \times 2 \frac{1}{3} \times 1 / 4$ | 62,400 | 56,300 | 10.0 | 1/2 in. wide, full thickness | ${ }^{3} 57,100$ | 5/8 in. wide, full thickness |

${ }^{1}$ Specimens in accordance with ASM Standard Methods of Tension Testing of Metallic Materials ( $\mathrm{E} 3-42$ ).
${ }^{2}$ Determined from stress-strain curves shown in fig. 1.
${ }^{3}$ Determined from stress-strain curve ghown in fig. 8.


Figure 1.- Compregsive gtrasa-gtrain curvea, 246-T. Gtraing obtained fram aeagured relative movenent of the testing machine platens; corrocted to give an inltial slope of $10,600,000 \mathrm{pal}$.








Figure 7．－Comprossive atrees－modulus curves．Rolation darived from comprebsive strebe－strain curve for $8-1 / 2 \times 2-1 / 2 \times 1 / 4-1 n o h$ extruded $149-T$ angle．Compresaive apecimen＝ $5 / 8-1$ noh wide $X$ full thicknerg．istrains measured with Huggenberger tensoneter．


Figure 8.- Compressive stress-strain curve, $145-T$. 2-1/2 $\times 2-\frac{1}{2} / 2$ $x$ 1/4-inch extruded angle. Compressive specimen 5/8. inch wide $x$ full thickness. Strains measured with Euggenberger tensometers, gage length $=0.50-1 \mathrm{nch}$.


Fgure 9.- Column atrength of $2-1 / 2 \times 2-1 / 2 \times 1 / 4$-inoh angle, oxtruded $148-T$. Tested an a oolumn with flat ends, x taken as 0.50 .

