NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

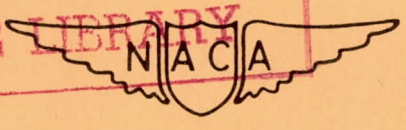
TECHNICAL NOTE

No. 1176

STRESSES AROUND RECTANGULAR CUT-OUTS WITH REINFORCED COAMING STRINGERS

By Paul Kuhn, Norman Rafel, and George E. Griffith

Langley Memorial Aeronautical Laboratory
Langley Field, Va.



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SUMMARY

Strain measurements and strength tests were made on six skin-stringer panels under axial load. Three of these panels had short rectangular cut-outs, and three a long one; the width of cut-out was about one-half of the width of the panel. Three types of coaming stringers were used: without reinforcement, with riveted-up reinforcement, or with integral reinforcement. The strain measurements were found to be in good agreement with a previously published theory adapted where necessary by making overlapping assumptions.

INTRODUCTION

In reference 1 a method was given for calculating the stresses around rectangular cut-outs in skin-stringer panels under axial load, and strain measurements made in the elastic range were presented to substantiate the theory. In the present paper, this work is extended to panels in which the coaming stringers (stringers bordering the cut-out) are reinforced in the region of the cut-out and are tapered to the basic stringer section. The tests as well as the analysis cover the elastic range and the ultimate load.

SYMBOLS

A_1 effective cross-sectional area of all continuous stringers, exclusive of main stringer bordering cut-out, square inches

A_2 effective cross-sectional area of main continuous stringer bordering cut-out, square inches

A_3 effective cross-sectional area of all discontinuous stringers, square inches

b_1 distance from A_2 to centroid of A_1, inches

b_2 distance from A_2 to centroid of A_3, inches

L half-length of cut-out, inches

σ normal stress (in stringers), kips per square inch

τ shear stress (in sheet), kips per square inch

P load on (whole) panel, kips

Subscripts

all allowable

max maximum

ult ultimate

TEST SPECIMENS AND PROCEDURE

Specimens.- The test specimens were six doubly symmetrical panels of 24S-T aluminum alloy with sixteen stringers. They were divided into two groups of three panels; the panels of the first group had a very short cut-out; those of the second group had a long cut-out. The cut-outs were rectangular and interrupted six stringers of each panel. In one of the three panels of each group (panels 1 and 4), the coaming stringers (stringers bordering the cut-out) had the same cross section as the other stringers; in the other two panels of the group, the coaming stringers were reinforced in the region of the cut-out, the cross-sectional area of the reinforcement being about equal to the area of the interrupted stringers. The reinforced stringers were either built up by riveting straps to the basic stringers (panels 2 and 5) or were machined in one piece (panels 3 and 6). The general arrangement and pertinent details of the panels are shown in figure 1. The dimensions used for the calculations are given in table 1.

Procedure in the elastic range.- For the tests in the elastic range, the panels were subjected to a uniformly distributed tensile load at each end by means of a whipple-tree arrangement. One whipple-tree was anchored and the other one was loaded by means of a hydraulic jack. The load was measured by a ring dynamometer accurate to about 1/2 percent. Strain readings were taken at an initial load

of 1 kip and at increments of 5 kips up to a load of 21 kips. Tuckerman optical strain gages of 2-inch gage length were used for all tests in the elastic range. Load-strain plots were made for all gage stations; it was almost always possible to draw straight lines through all tests points except the initial-load point, to an accuracy equivalent to 40 psi (twice the smallest reading of the gages). If the straight line missed the initial-load point by more than 100 psi, a check run was made, and the trouble was usually eliminated; if the trouble persisted, the readings at this station were discarded.

Procedure for strength tests.- For the strength tests, the panels were subjected again to uniformly distributed tensile loads by means of whipple-trees. The loads were applied by a 1200-kip-capacity testing machine accurate to about 1/2 percent. No reduction in load was made at any time during the test. Strains were measured with electric resistance-type gages (Baldwin-Southwark gages) of $\frac{1}{2}$-inch or 1-inch gage length at load increments varying from 5 to 20 kips, the largest increment being used in the middle of the elastic range. The accuracy of the strain readings is believed to be within about 20×10^{-6} at low strains, about 2 percent at intermediate strains and somewhat less at high strains. The loss in accuracy at high strains results from the fact that electric gages give inaccurate results on first application of strain (reference 2); no corrections were applied for this effect.

METHODS OF ANALYSIS

Elastic range.- The analysis for stresses in the elastic range was made by the simplified three-stringer method given in reference 1. This method is based on the assumption that all stringers are of constant cross section, whereas panels 2, 3, 5, and 6 have coaming stringers that are reinforced along the edges of the cut-out. For these panels, two sets of calculations were made; one was based on the assumption that the reinforced section of the coaming stringer was continuous throughout the length of the panel; the other one on the assumption that the basic (nonreinforced) section was continuous. The results from the first set of calculations were assumed to apply in the region lying between two transverse lines drawn through the middle of the tapered section of the coaming stringers; the results from the second set were assumed to apply in the regions between the transverse lines just mentioned and the ends of the panel. For convenience, the regions defined in this manner will be referred to as "region of the

cut-out" and "region away from the cut-out," respectively. The constants defining the three-stringer structure are given in table 2.

Ultimate strength.- Any perforated tension specimen will fail across a critical section that is determined by the stress distribution resulting from the geometry of the specimen and by the stress-strain characteristics of the material. There are two limiting "theories of failure." On the one hand, it may be assumed that all stress concentrations persist until the moment of failure, and that failure takes place when the peak stress in the specimen exceeds the allowable value for the material. This theory will be referred to as "the brittle-failure theory." On the other hand, it may be assumed that all stress concentrations are eliminated by yielding before failure takes place; this theory will be referred to as "the plastic-failure theory."

When the brittle-failure theory is used, the stress distribution is calculated by the elastic theory, and the peak stresses are located. In stiffened panels, there is a peak normal stress in a stringer and a peak shear stress in the sheet. The stringer stress is evidently a simple stress. The shear stress in the sheet is affected to some extent by superposed longitudinal and transverse normal stresses, but in most cases these superposed stresses may be neglected. Failure would therefore be expected when either the peak stringer stress or the peak shear stress in the sheet exceeds the allowable value.

The procedure for applying the plastic-failure theory is well known from the design of riveted joints in mild steel. Possible paths of fracture are considered, and the strength of the total assembly is computed by adding the strengths of the elements of each path, obtained by multiplying the cross-sectional area of each element by the tensile, shear, or bearing strength of the material, as the case may be. In this investigation a variation of this theory was employed. It was assumed that yielding eliminated the general nonuniformity of stress distribution resulting from the existence of the cut-out but did not eliminate the highly localized stress concentrations arising from the presence of rivet holes.

In practice, it may sometimes be necessary to use an intermediate theory based on the elastic theory of stress distribution but modified for partial yielding. The tests made in this investigation agreed fairly well with one of the two limiting theories previously described and consequently furnished no basis for establishing such an intermediate theory. This result was brought about by the fact that the coaming stringers were of two extreme types: they had either no

reinforcement at all or were reinforced enough to make up (nearly) for the loss of panel area caused by the cut-out.

RESULTS AND DISCUSSION

Elastic Behavior

General remarks.- The results of the tests and the calculations for the elastic range are presented in figures 2 and 3 in the form of spanwise-stress plots for a load of 20 kips in each case; the calculated maximum stresses are also shown in table 2. The experimental stresses shown were obtained from measured strains with the value of Young's modulus assumed to be 10,600 ksi. As explained in the section "Methods of Analysis," the calculated curves for the panels with reinforced coaming stringers have two branches; the branch in the region of the cut-out is calculated on the assumption that the reinforced section is continued to the ends of the panel; the branch away from the cut-out is calculated on the assumption that the basic stringer section continues through the middle of the panel without reinforcement. For each panel the calculated curves for the continuous stringers (except the coaming stringers) are identical, because these stringers are represented by a single stringer in the three-stringer method of analyzing cut-out panels; the curves for the interrupted stringers of a given panel are also identical for the same reason.

Stringer stresses in panels with constant-section coaming stringers.- In panels 1 and 4, the (constant-section) coaming stringers exhibit high stress peaks at the transverse ribs that bound the cut-outs; the agreement between experimental and theoretical peak stresses is good. The rate at which the stresses decrease from their peak values with increasing distance from the cut-outs is larger for the experimental than for the theoretical values. The chordwise distribution of the stresses in the other continuous stringers is fairly uniform in panel 1 and consequently in fairly good agreement with the calculated curve; in panel 4, the chordwise distribution is not so uniform at stations 0 and 7, and the agreement is therefore close only on one stringer near the middle of the group (stringer 3), whereas stringers 1 and 4 show deviations of opposite sign. Similar deviations are obvious near the ends of the interrupted stringers, but here the stresses are low and consequently of no practical concern.

Stringer stresses in panels with riveted-up reinforcements on coaming stringers.- In panels 2 and 5 with the riveted-up rein-

forcements, the agreement between experimental and calculated stresses is satisfactory, on the whole, in the regions of practical interest. (See fig. 2.) The coaming stringers show local stress peaks not predictable by the simple theory used herein at stations just beyond the end of the reinforcements (station 8 on panel 2 and station 32 on panel 5), presumably as a result of the sudden change in cross section of the stringer. On panel 2 with the short cut-out and the sharply tapered reinforcement, this local peak stress is higher than the average stress in the stringer at the edge of the cut-out. The average stress referred to is the average over the thickness of the pack, and the method of obtaining it will be explained presently.

The measured stresses in the tapered region of the coaming stringer of panel 2 are considerably lower than the calculated values. These stresses are of necessity measured on the outside faces of the outermost straps. There are two factors apparent that may contribute to this discrepancy. In panels 1 and 4 with coaming stringers of constant section, the experimental stresses are also low in the corresponding regions; this fact indicates that inaccuracy of the theory may be one reason. The other apparent reason is that a riveted connection is not so stiff as a solid metal-to-metal connection would be, and consequently, the outer straps carry less stress than the inner straps.

Within the region of the cut-out, it was possible to measure the stresses in each individual strap of the coaming stringer by putting gages on the exposed edges of the straps. Figure 2 shows for station 0 the stresses in the innermost strap, the outermost strap, and the average stress for the entire pack. Two sets of measured values are shown; one was obtained when the straps were fastened with $\frac{1}{8}$-inch rivets, the other one after these rivets had been replaced by $\frac{3}{16}$-inch rivets. With the larger rivets, the spread between the stresses in the outermost and the innermost straps was almost cut in two. With either size of rivet, however, the average stress in the pack was only slightly lower than the theoretical value for panel 2 at the relatively low loads used; at the higher loads, larger differences would, of course, be expected. In panel 5, the distance between the center line of the cut-out and the end of the shortest strap was sufficient to eliminate the effects of finite rivet stiffness at station 0.

Stringer stresses in panels with integrally reinforced coaming stringers.- For panels 3 and 6, in which the coaming stringers had integral reinforcements, the agreement between experimental and calculated stresses is again quite satisfactory in the regions of

practical interest. Panel 3 with the short cut-out and sharp taper again exhibits the high local stress peak just beyond the end of the reinforcements and the very low stresses in the region of the taper. Somewhat perplexing is the fact that the stress in the coaming stringer at station 0 is appreciably less in panel 3 than in panel 2. This result indicates that at low stresses, at least, the wide integral reinforcement was less effective than the riveted-up reinforcement.

Shear stresses.- The shear stresses in all panels are shown in figure 3. The agreement between experimental and calculated stresses is very satisfactory.

Ultimate Strengths

Panels with coaming stringers of constant section.- In panels 1 and 4, the (constant section) coaming stringers showed a very high stress peak at the transverse rib. The electrical gages at this station indicated no appreciable deviation from a straight-line relationship between strain and load up to the highest loads at which readings were taken (about 0.9 P_{ult}). The panels might therefore be expected to fail according to the brittle-failure theory, that is, when the theoretical maximum stress reached the allowable value. The allowable stress was determined by tests on sample stringers with rivet holes, in order to include the effect of stress concentration due to such holes; the ultimate stress based on the net area was found to be 64.0 ksi (average of 3 tests) against 70.3 ksi as determined by standard tensile specimens. With an allowable stress of 64.0 ksi, failure for panel 1 was predicted at 71.5 kips against an actual failing load of 77.0 kips; failure for panel 4 was predicted at 84.8 kips against an actual failing load of 87.0 kips. (See table 3.) The predictions were therefore conservative by about 7 percent for panel 1 and $2\frac{1}{2}$ percent for panel 4. The fracture should be a tear across the net section of the panel at (or near) the rib line, and figure 4 shows that this type of failure was actually observed.

As table 2 shows, the calculated ratios of peak shear stress to peak stringer stress were 0.58 for panel 1 and 0.62 for panel 4. If the allowable values are taken as 37 ksi for shear and 62 ksi for tensile stress, the corresponding ratio of the allowable stresses is 0.60. Consequently, there was a theoretical possibility that in panel 4 the failure might be precipitated by shear failure of the sheet. Analysis of the stresses near the cut-out showed, however, that the sheet-bearing stress under the rivet at the corner of

cut-out exceeded the allowable value at about one-half of the ultimate load in panel 1 as well as in panel 4. Progressive bearing failure relieved the peak shear stress sufficiently to preclude shear failure.

The stringer stresses were not appreciably affected by the redistribution of shear stress because they depend on the integrated effect of the shear stresses. When the peak shear stresses are reduced materially, the transfer of load from the cut stringers to the continuous stringers takes place at a greater distance from the cut-out; the chordwise distribution of the stringer stresses in the net section may therefore be expected to become somewhat more uniform as the load increases. This consideration may explain why the strength predictions were conservative.

Panels with reinforced coaming stringers.- In panels 2, 3, 5, and 6 with reinforced coaming stringers, the chordwise distribution of the stringer stresses in the net section was much more uniform within the elastic range than in panels 1 and 4 with constant-section stringers. A convenient measure of the nonuniformity at the rib station (where all the stringer stresses reach their peak values) is the ratio of stress in the coaming stringer to average stress in the other continuous stringers (stress in substitute single stringer). For panel 1 (short cut-out, no reinforcement) this ratio was 1.87, for panels 2 and 3 (short cut-out, reinforced stringers) the ratio was only 1.14; for panel 4 (long cut-out, no reinforcement) the ratio was 1.41, for panels 5 and 6 (long cut-out, reinforced stringers) it was only 1.12.

The uniformity of the chordwise distribution was further increased, at loads above about one-half of the ultimate, by an increasing loss of effectiveness of the reinforced portion, compensated by increased stresses in the other continuous stringers, mainly the nearest one. For the stringers with long taper (panels 5 and 6), this effect was small, but for the stringers with short taper, it was quite pronounced, particularly for the built-up reinforcement of panel 2. (See fig. 5.)

In the spanwise direction, the stringer stresses in the panels with reinforced stringers were also much more uniform than in the panels without reinforcements. In the panels with reinforced stringers, the area of the net section of the panel (alongside the cut-out) was only slightly less than the area of the full section; in the panels without reinforcement, the area of the net section was about 5/8 of the area in the gross section, because 6 out of 16 stringers were cut.

These considerations show that, in the elastic range, distribution of the stringer stresses in the reinforced panels was not too far from the uniform distribution assumed in the plastic-failure theory and that the distiibution tended to become even more uniform at higher loads. There is considerable justification, then, for using the plastic-failure theory in predicting the strength of the reinforced panels.

Examination of possible critical sections, or paths of failure, on the basis of the plastic-failure theory showed that the panels should fail by tearing of the four continuous stringers at or near the transverse rib, by tearing of the coaming stringer in the unreinforced section, and shearing or bearing failure of the sheet or rivets under the tapered portions of the coaming stringers. This type of failure was observed in all reinforced panels (fig. 4). The tearing of the coaming stringers took place just beyond the ends of the reinforcements, presumably because local stress concentrations existed at these points, as shown by the strain measurements in the elastic range. It may seem from figure 4 that panel 5 (long cut-out with built-up reinforcement on stringers) did not fail in the predicted manner, the coaming stringers being torn underneath the top straps at the end of the cut-out instead of being torn just beyond the end of the tapered section. Calculation showed that the strength of the panel for this path of failure should be about 5 percent higher than for the path of failure described for the other panels. Examination of the panel disclosed that the failure had apparently started in the predicted manner by tearing of one end of one coaming stringer at the point where the reinforcement began; however, for some unknown reason, failure then took place along a different path, and as a result, the failure at the end of the reinforcement can be seen only by inspection of the original panel. The strengths predicted by adding the appropriate strengths of elements (table 4) were slightly unconservative as shown by the last column of table 4, the average ratio of observed to predicted failing load being 0.99 and the lowest value 0.95. Predictions based on the brittle-failure theory would have been from 16 to 27 percent conservative.

The ratios of peak shear stress to peak stringer stress were so high in panels 2, 3, 5, and 6 (table 2) that shear failures in the sheet might be expected. No shear failures developed, however; only bearing failures under the rivets were observed for the reason given in the discussion of the strength tests of panels 1 and 4 with constant-section coaming stringers.

CONCLUSIONS

The following conclusions were drawn from the tests on six axially loaded skin-stringer panels with rectangular cut-outs:

1. The observed stresses in the elastic range agreed fairly well with those predicted by a previously published theory. (The theory was adapted to panels with reinforced coaming stringers by making two overlapping assumptions.)

2. The ultimate strengths of the two panels with constant-section coaming stringers differed from those predicted by the "brittle-failure theory" (that is to say, by using the elastic theory) by 2.5 to 7 percent, the predictions being conservative.

3. The ultimate strengths of the four panels with reinforced coaming stringers differed not more than ±5 percent from the strengths predicted by the "plastic-failure theory" commonly used in the design of perforated plates made of ductile material. This conclusion should not be generalized to apply to other panels unless the cross-sectional area of the reinforcements of the coaming stringers is roughly equal to the cross-sectional area of the material removed by the cut-out.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., August 19, 1946

REFERENCES

1. Kuhn, Paul, Duberg, John E., and Diskin, Simon H.: Stresses around Rectangular Cut-Outs in Skin-Stringer Panels under Axial Loads - II. NACA ARR No. 3J02, 1943.

2. Campbell, William R.: Performance Tests of Wire Strain Gages. I - Calibration Factors in Tension. NACA TN No. 954, 1944.

TABIE 1

PANEL DIMENSIONS

Panel	Sheet thickness (in.)	Gross sheet area (sq in.)	Average gross area of one stringer (sq in.) (a)	Average gross area of one coaming stringer (sq in.) (b)
1	0.0315	0.835	0.0975	0.0975
2	.0315	.835	.0973	.3669
3	.0316	.837	.0975	.3810
4	.0318	.843	.0964	.0981
5	.0316	.837	.0951	.3876
6	.0318	.843	.0992	.3702

[a]Not including coaming stringer.
[b]In region of cut-out.

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TABLE 2

CONSTANTS USED IN THEORETICAL CALCULATIONS AND MAXIMUM THEORETICAL STRESSES

Panel	A_1 (sq in.)	A_2 (sq in.)	A_3 (sq in.)	b_1 (in.)	b_2 (in.)	L (in.)	σ_{max} ksi (c)	τ_{max} ksi (c)	$\frac{\tau_{max}}{\sigma_{max}}$ (c)
1	0.746	0.159	0.632	7.301	6.164	1.5	17.9	10.3	0.58
2	.747	[a].158 [b].428	.624	7.449	6.019	1.5	10.7	8.1	.76
3	.746	[a].157 [b].442	.626	7.315	6.165	1.5	11.3 10.5	7.9	.70
4	.747	.160	.625	7.305	6.160	14.5	15.1	9.4	.62
5	.735	[a].157 [b].450	.622	7.438	6.012	14.5	9.6	7.6	.79
6	.754	[a].154 [b].432	.634	7.315	6.165	14.5	9.7	7.6	.78

[a]For region away from cut-out (basic stringer).
[b]For region of cut-out (reinforced stringer).
[c]Calculated for P = 20 kips.

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TABLE 3 .- ULTIMATE STRENGTH CALCULATIONS FOR CONSTANT-SECTION COAMING STRINGERS

Panel	σ_{max} (ksi) (a)	Predicted P_{ult} (kips) (b)	Observed P_{ult} (kips)	Observed P_{ult} / Predicted P_{ult}
1	17.9	71.5	77.0	1.08
4	15.1	84.8	87.0	1.03

[a]Calculated for P = 20 kips

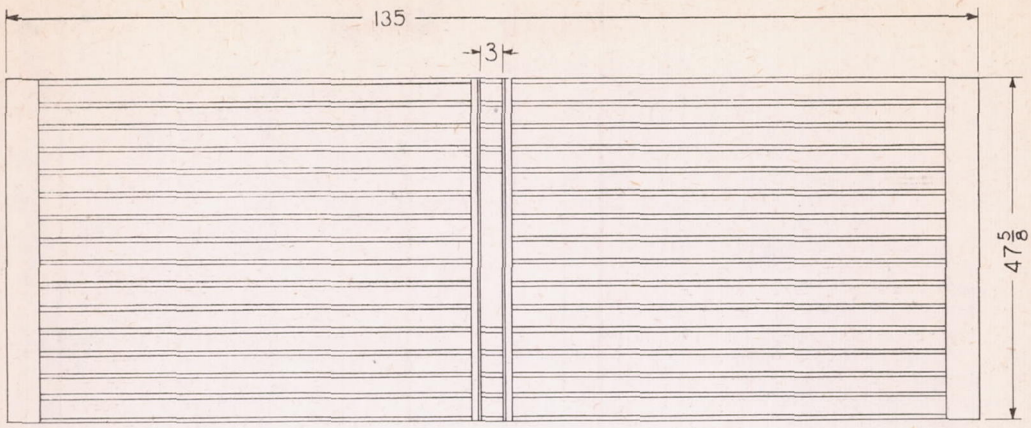
[b]Predicted $P_{ult} = \frac{64}{\sigma_{max}} \times 20$ kips

TABLE 4 .- ULTIMATE STRENGTH CALCULATIONS FOR REINFORCED SECTION COAMING STRINGERS

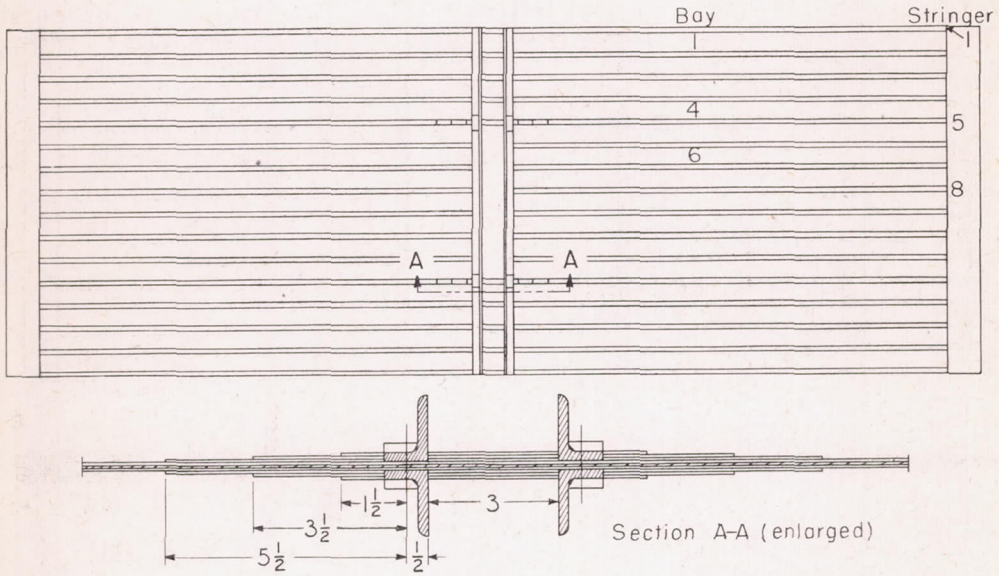
Panel	Total net tensile area (sq in.) (a)	Tensile strength (based on σ_{all} = 64 ksi) (kips)	Bearing strength of sheet against rivets (kips)	Predicted P_{ult} (kips)	Observed P_{ult} (kips)	Observed P_{ult} / Predicted P_{ult}
2	1.498	95.9	[b]5.4	101.3	98.6	0.97
3	1.522	97.4	3.6	101.0	96.0	.95
5	1.505	96.3	10.6	106.9	105.4	.99
6	1.544	98.8	9.2	108.0	113.4	1.05

[a](Sheet area from coaming stringers to sides of panel plus area of 10 stringers minus rivet holes.

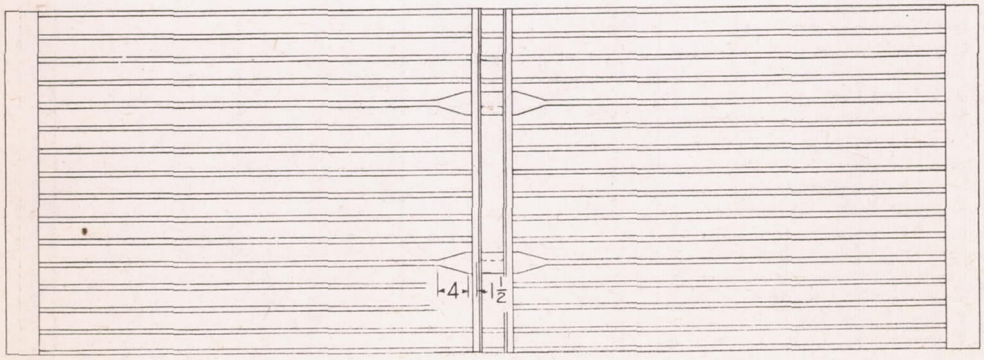
[b]Diameter of rivets in coaming stringer, $\frac{3}{16}$ inch.



(a) Panel 1.



(b) Panel 2.



(c) Panel 3.

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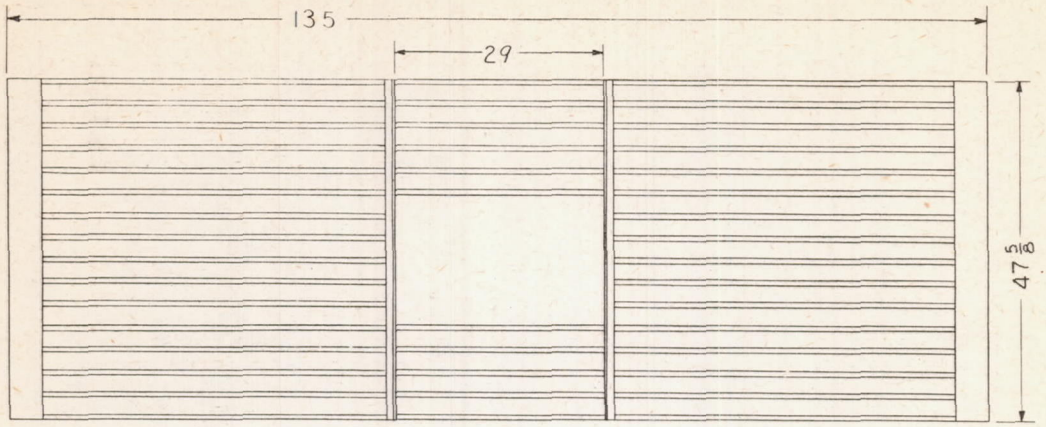
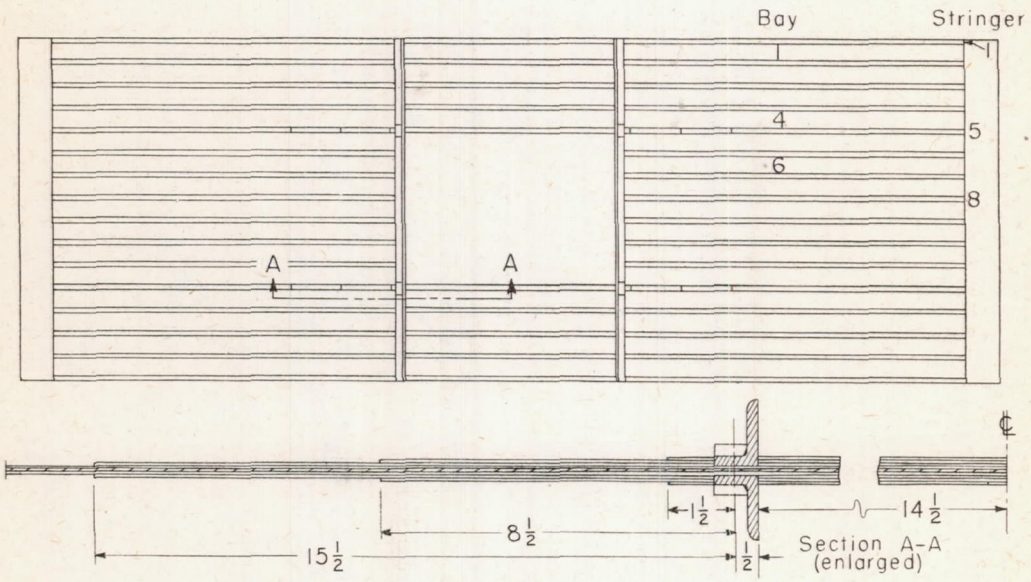
Figure 1.— Test panels.



(d) Panel 4.



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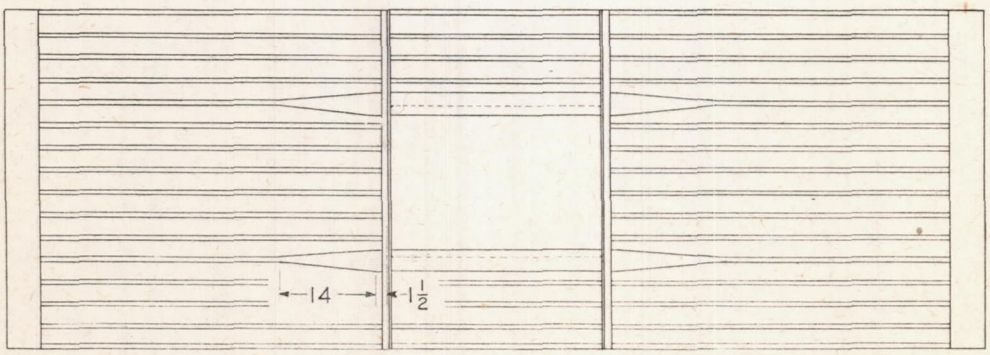

(f) Panel 6.

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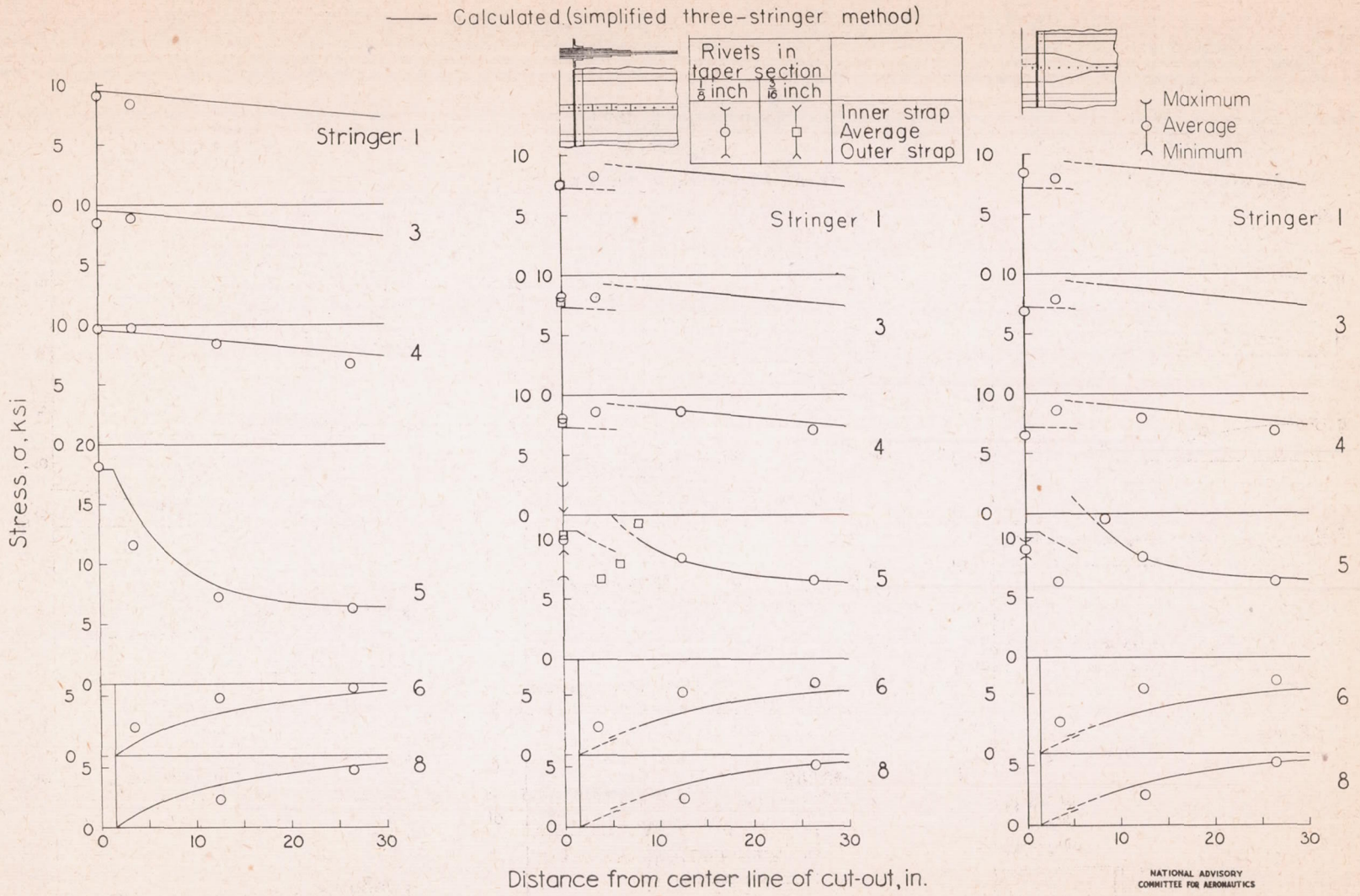
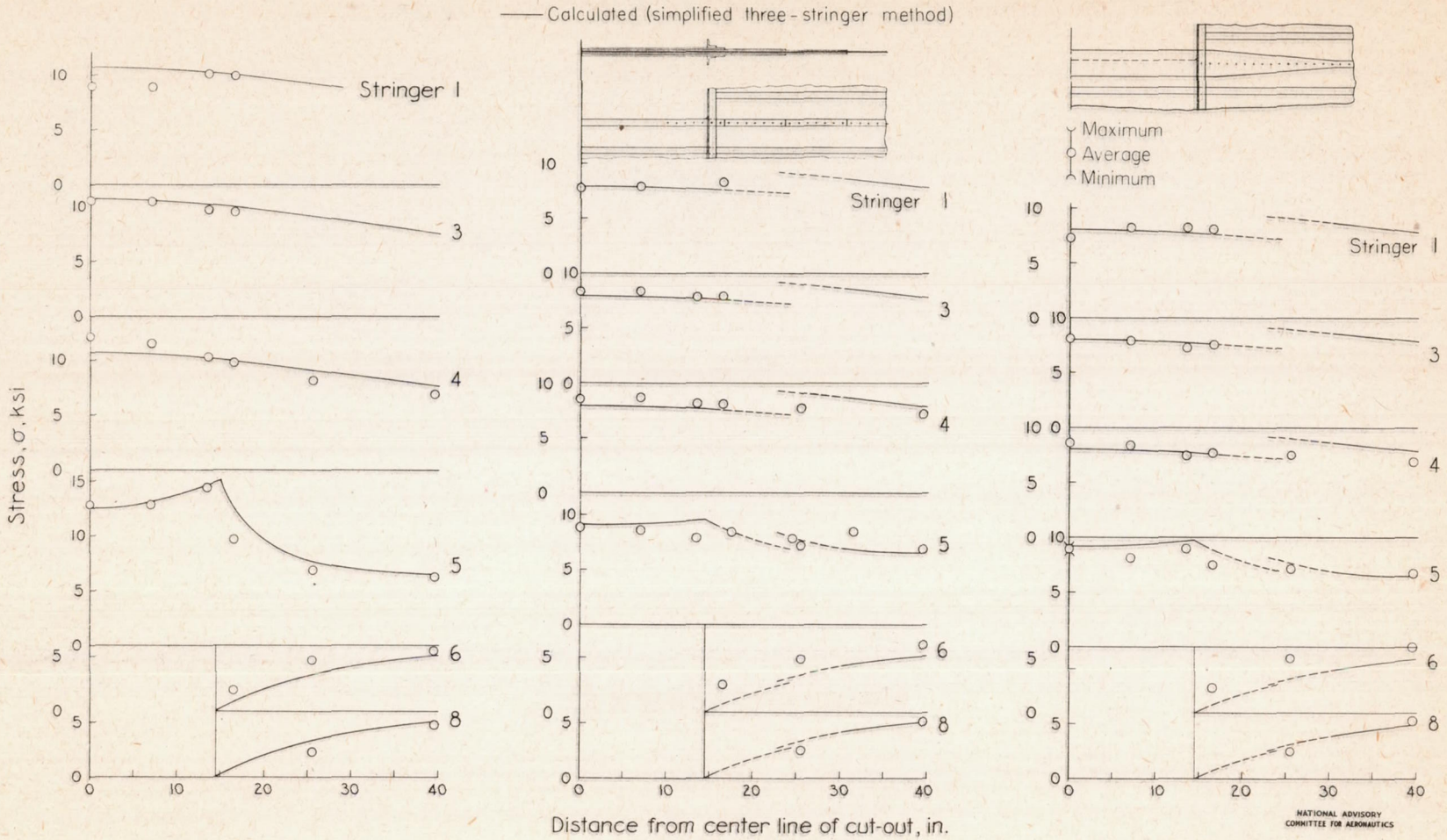
Figure 1.- Concluded.



(a) Panel 1. (b) Panel 2. (c) Panel 3.

Figure 2.- Stringer stresses in panels for P = 20 kips.

Fig. 2a-c

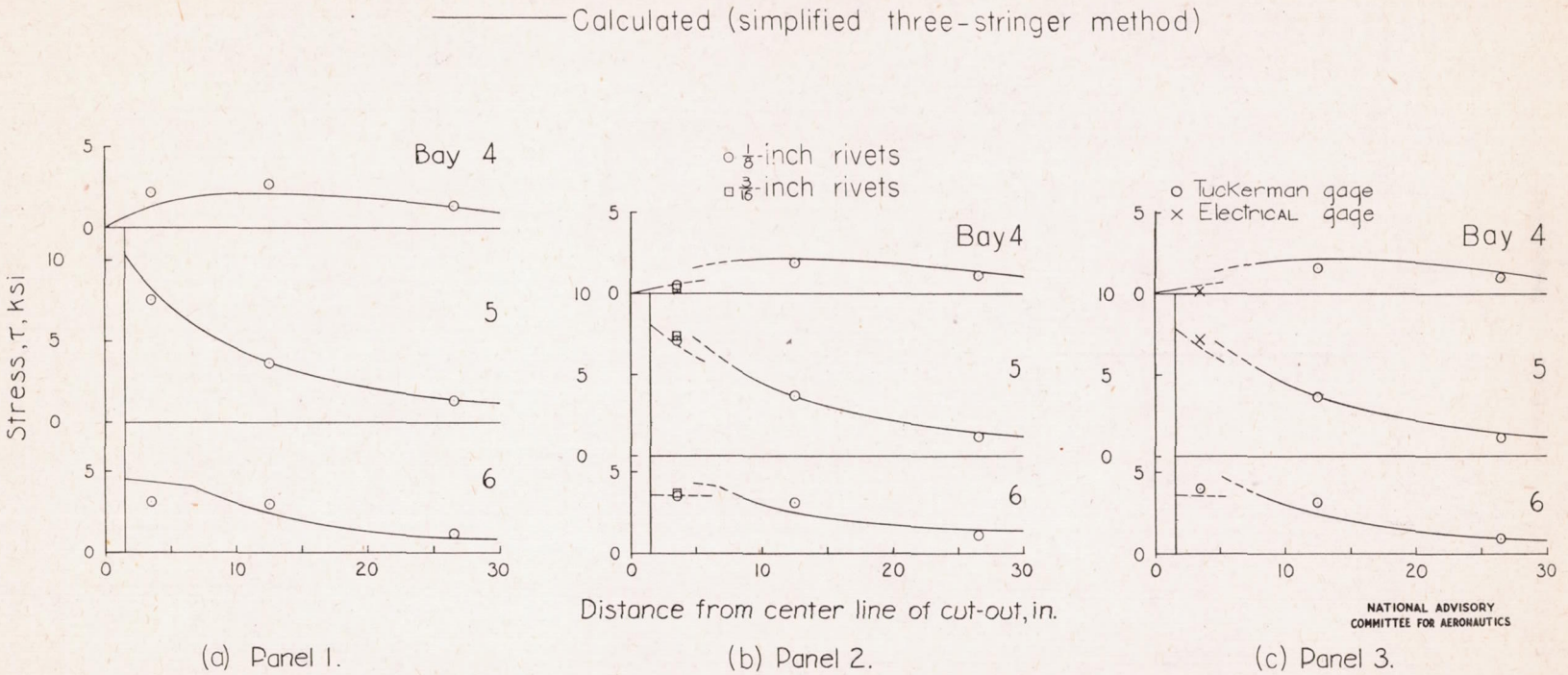


(d) Panel 4.

(e) Panel 5.

(f) Panel 6.

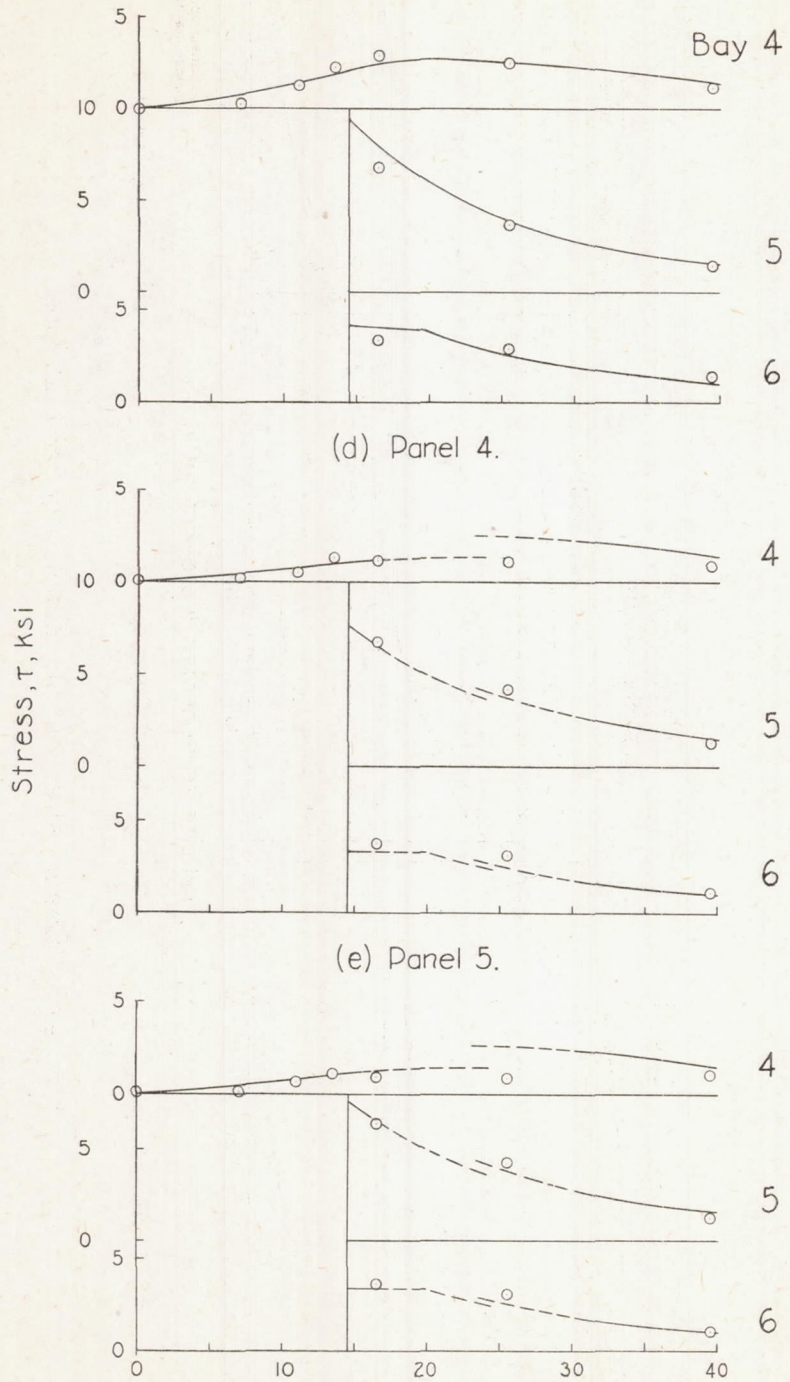
Figure 2.— Concluded.



(a) Panel 1. (b) Panel 2. (c) Panel 3.

Figure 3.– Shear stresses in panels for P = 20 kips.

—— Calculated (simplified three-stringer method)



(d) Panel 4.

(e) Panel 5.

(f) Panel 6.

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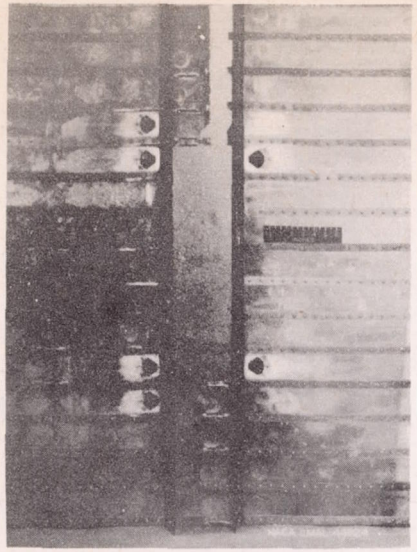
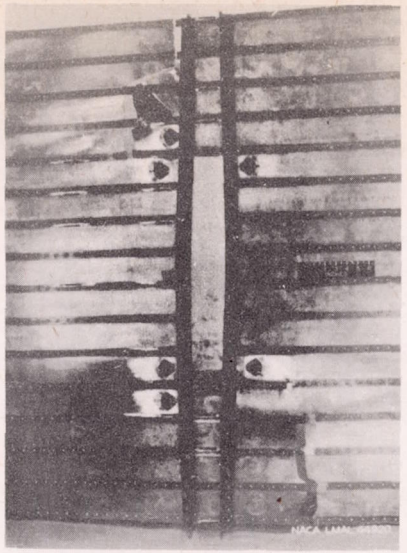
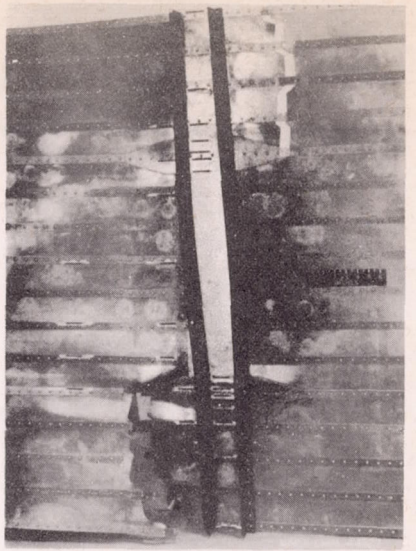
Figure 3.- Concluded.



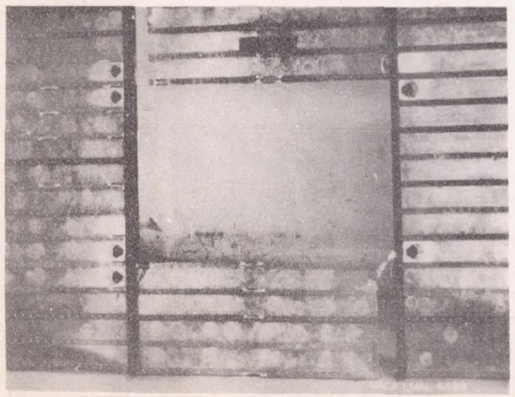
(a) Panel 1.



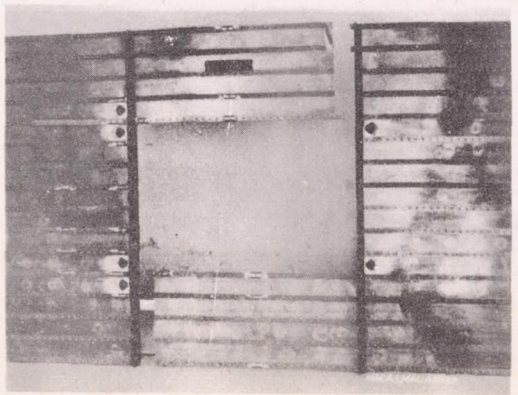
(b) Panel 2.



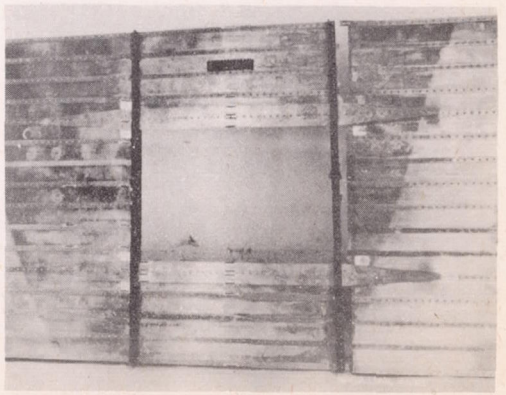
(c) Panel 3.



(d) Panel 4.

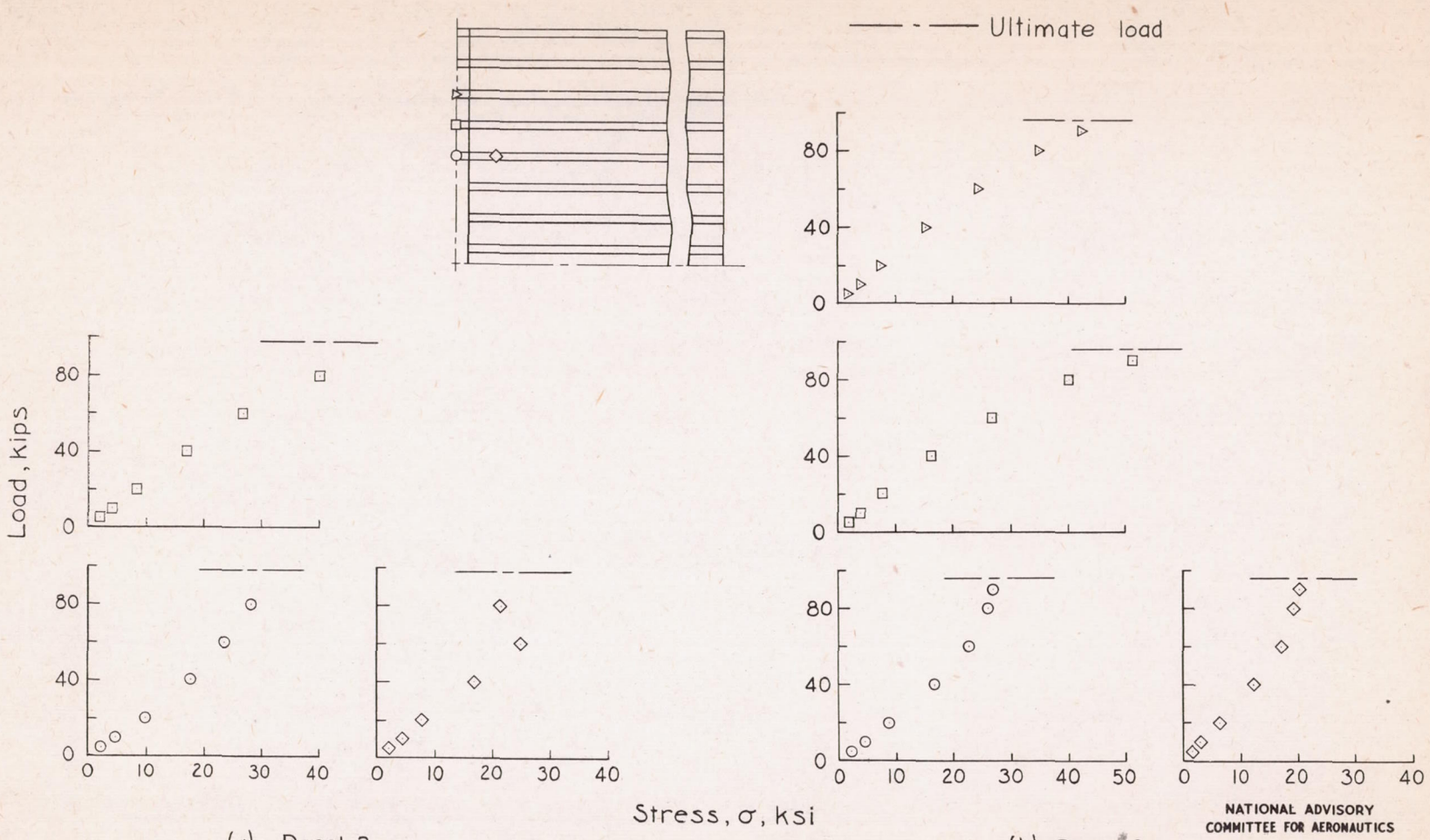


(e) Panel 5.



(f) Panel 6.

Figure 4.- Test panels after failure.



(a) Panel 2. (b) Panel 3.

Figure 5.- Load-stress plots for panels 2 and 3 showing reduction of stress in reinforced coaming stringers at high loads.

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