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THERMODYNAMIC CHARTS FOR THE COMPUTATION OF COMBUSTION AND MIXTURE TEMPERATURES AT CONSTANT PRESSURE

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SUMMARY

Charts are presented for calculating the combustion temperatures and the temperature changes involved in constant-pressure thermodynamic processes of air and the products of combustion of air and hydrocarbon fuels. Examples are given in which the charts are applied to the calculation of the quantity of fuel required for combustion to a given temperature, the increase in the temperature of products of combustion by the introduction and combustion of additional fuel, the change in enthalpy of the products of combustion with change in temperature, the equilibrium temperature resulting from the mixing of two gases representing products of combustion, and the temperature resulting from the afterburning of richer-than-stoichiometric exhaust gases with additional air. The charts are applicable only to processes in which the final fuel-air ratio is leaner than stoichiometric and at temperatures where dissociation is unimportant. They may be applied to steady-state flow processes.

INTRODUCTION

The accurate calculation of the change in energy of a fluid as its temperature or composition is changed is difficult especially when the range of temperature variation is large. Calculations of this type arise in connection with the phenomena of combustion, afterburning of rich-mixture exhaust gas, and the mixing of air and combustion gases. Because of the difficulties imposed by the use of exact specific heats, many approximations have been used and in some cases these approximations have led to large errors.

Graphical methods of solving problems that involve a change in the heat content of air or the products of combustion of air and hydrocarbon fuels at constant pressure have been devised at the NACA Cleveland laboratory. Because the total temperature of a gas is a direct measure of its total energy content, the charts may be used for any steady-state flow process by using the total temperatures of the fluid. The charts are applied in this report, by means of examples, to the calculation of the quantity of fuel required for combustion to a given temperature, the increase in temperature of products of combustion by the introduction and burning of additional fuel, the change in the enthalpy of a burned mixture with a change in temperature, the equilibrium temperature resulting from the mixing of two gases each of which is the product of combustion, and the temperature resulting from the afterburning of richer-than-stoichiometric exhaust gases with air.

SYMBOLS

C_p	molal specific heat at constant pressure, (Btu)/(lb mole)($^{\circ}$R)
c_{pf}	specific heat at constant pressure of fuel, (Btu)/(lb)($^{\circ}$R)
e_c	lower internal energy of combustion of fuel, (Btu)/(lb)
f	fuel-air ratio
$\bar{f}$	mean fuel-air ratio of mixture
f'	chart fuel-air ratio; function of T_a and T_b
H	molal enthalpy of gas, (Btu)/(lb mole)
h_a	enthalpy of air, (Btu)/(lb)
$\bar{h}_a$	mean value of enthalpy of air mixture, (Btu)/(lb of mixture)
h_b	enthalpy of leaner-than-stoichiometric burned mixture of fuel and air, (Btu)/(lb)
h_{br}	sensible enthalpy of richer-than-stoichiometric exhaust gas excluding energy of chemical organization, (Btu)/(lb)
h_c	lower enthalpy of combustion of fuel, (Btu)/(lb)
$(h_c)_{eff}$	effective lower enthalpy of combustion of fuel in richer-than-stoichiometric exhaust gas, (Btu)/(lb)

$-h_{chem}$	enthalpy available through combustion with excess air of 1 + f pounds of richer-than-stoichiometric exhaust gas, (Btu)/(lb)
J	mechanical equivalent of heat, (778 ft-lb)/(Btu)
K	correction factor to change in total enthalpy of richer-than-stoichiometric exhaust gas with changes in temperature for changes in fuel-air ratio and hydrogen-carbon ratio of fuel
K_h	correction factor to fuel-air ratio for change in lower enthalpy of combustion of fuel
K_m	correction factor to fuel-air ratio for change in hydrogen-carbon ratio of fuel
M_a	molecular weight of air, 28.972 (lb)/(lb mole)
m	hydrogen-carbon ratio of fuel
P	pressure (lb)/(sq ft)
R	universal gas constant, 1545.7 (ft-lb)/(lb mole)($^{\circ}$R)
R_a	gas constant for air, 53.352 (ft-lb)/(lb)($^{\circ}$R)
R_b	gas constant of burned mixture of fuel and air, (ft-lb)/(lb)($^{\circ}$R)
r	ratio of weights of original air in two burned mixtures
T	total temperature, ($^{\circ}$R)
T_a	total air temperature, ($^{\circ}$R)
$\overline{T}_a$	total mean temperature of air mixture, ($^{\circ}$R)
T_b	total temperature of burned mixture, ($^{\circ}$R)
T_e	total exhaust-gas temperature, ($^{\circ}$R)
T_i	total initial temperature, ($^{\circ}$R)
T_f	total final temperature, ($^{\circ}$R)

T_r	reference temperature, ($^{\circ}$R)
$\underline{V}$	specific molal volume, (cu ft)/(lb mole)
W_a	weight of original air in burned mixture, (lb)
η_c	combustion efficiency ✓
Ω	total enthalpy of richer-than-stoichiometric exhaust gas referred to enthalpy of air at 540° R as zero of enthalpy
ω	total enthalpy of richer-than-stoichiometric exhaust gas at 1980° R
$(O_2)_a$	concentration of oxygen in air, (mole oxygen)/(mole air)
(H_2)	concentration of hydrogen in exhaust gas, (mole hydrogen)/(mole of original air)

The subscripts 1, 2, etc. are used alone or in addition to any regular subscript to indicate successive values taken on by a single quantity. For example, T_{b1} and T_{b2} designate, respectively, the temperature of a burned mixture, T_b at points 1 and 2.

The subscripts x, y, etc. are used to identify two or more fluids that are to be mixed.

The total temperature of a gas is defined as the temperature attained by the gas in a steady-state flow process when it is brought to rest with the conversion of all its kinetic energy to thermal energy.

SOURCE OF THERMODYNAMIC DATA

The specific-heat data used in the preparation of the constant-pressure combustion charts for hydrocarbon fuels and air were taken from reference 1 and extended to higher temperatures by the use of data contained in references 2 to 8. The charts are exact only at temperatures below those at which dissociation becomes important. In most calculations, dissociation may be neglected for all temperatures below about 3200° R. The charts have been extended beyond 3200° R, without, however, considering dissociation, in order that approximate calculations may be made with the chart for a higher final temperature.

The charts are based on a reference temperature of 540° R (80.3° F). The heat of combustion of the fuel, therefore, is to be corrected to the basis of the 540° R and to the normal state of the fuel at that temperature before combustion calculations are made. These corrections are discussed in appendix A.

PRINCIPLE OF THE CONSTANT-PRESSURE COMBUSTION CHARTS

Thermodynamics of constant-pressure combustion. - In order to avoid the use of complicated subscripts the notation

$$x\Big]_{y}^{z}$$

is used to mean "the value of x at z minus the value of x at y."

The lower enthalpy of combustion of a fuel at constant pressure h_c is defined as the heat $(-h_c)$ removed during the combustion at constant pressure of a mixture of fuel and oxygen when the initial and final temperatures are equal and the products of combustion are all in the gaseous phase. The first law of thermodynamics applied to an ideal constant-pressure combustion for leaner-than-stoichiometric mixtures leads to

$$h_a\Big]_{T_r}^{T_a} - fh_c = (1 + f)\ h_b\Big]_{T_r}^{T_b} \tag{1}$$

The form of equation (1) and of all subsequent equations implies that the value of $-h_c$ has been corrected by adding to $-h_c$ the quantity of heat that must be added to the fuel to bring the fuel to its initial condition from its standard phase at the reference temperature T_r of 540° R. (See appendix A.) When this correction has been applied, it is unnecessary to include an explicit term for the condition of the fuel in the equations for the conservation of energy.

For leaner-than-stoichiometric mixtures, the term $(1 + f)\ h_b$ is given by

$$(1 + f)\ h_b = h_a + f\,\frac{Am + B}{m + 1} \tag{2}$$

where

$$A = \left(H_{H_2O} - \frac{1}{2} H_{O_2}\right) \Big/ 2.016$$

$$B = \left(H_{CO_2} - H_{O_2}\right) \Big/ 12$$

(see reference 1, equation (20).) The term $\frac{Am + B}{m + 1}$ in equation (2) accounts for the difference between the enthalpy of the carbon-dioxide and water vapor in the burned mixture and the enthalpy of the oxygen removed from the air by their formation.

The substitution of equation (2) in equation (1) yields the following relation:

$$f = \frac{h_a\Big]_{T_a}^{T_b}}{-h_c - \frac{Am + B}{m + 1}\Big]_{T_r}^{T_b}} \qquad (3)$$

Two types of chart have been prepared by the use of equation (3). The first is an alinement chart on which the fuel-air ratio is represented by the slope of a line. This chart permits the solution of all problems involving enthalpy changes when dissociation may be neglected but its general use may be inconvenient because a geometric construction or the use of a parallel rule is required to determine the slope of the fuel-air-ratio line.

The second set of charts, which have a somewhat limited range of application, show: (a) temperature rise during combustion as a function of the fuel-air ratio and initial air temperature; (b) temperature rise during combustion as a function of the fuel-air ratio and the final mixture temperature. These curves have been prepared for a single fuel. Correction factors permit the calculation of the fuel-air ratio for other fuels with different hydrogen-carbon ratios and lower heating values.

Construction of the constant-pressure alinement chart. - The alinement-type constant-pressure chart is shown in figure 1. The linear scale to the left is the enthalpy of air, which was arbitrarily taken as zero at 540° R. The horizontal lines marked with a temperature scale are drawn at the ordinate equal to the enthalpy of air at that temperature minus its enthalpy at 540° R. The abscissa is the lower enthalpy of combustion of the fuel h_c.

Along each temperature line the quantity

$$\left.\frac{Am + B}{m + 1}\right]_{T_r}^{T}$$

is laid off to the left of the zero of the abscissa for three values of the hydrogen-carbon ratio m (0.10, 0.15, and 0.20) of the fuel. Thus, vertical lengths on the chart represent quantities in the numerator of equation (3); horizontal lengths represent quantities in the denominator of equation (3).

The values of h_a, A, and B are given in table I as functions of the temperature.

USES OF THE CONSTANT-PRESSURE ALINEMENT CHART

Constant-pressure combustion. - The use of the constant-pressure alinement chart to determine the temperature attained by an ideal constant-pressure combustion process is illustrated in figure 2(a) by the following example:

Initial air temperature T_a, 900° R
Final combustion temperature T_b, 1860° R
Lower enthalpy of combustion h_c, -18,900 Btu per pound
Hydrogen-carbon ratio of fuel m, 0.175

A line is drawn from the point a (located at $T_a = 900^\circ$ R, $h_c = -18{,}900$ Btu/lb) to the point b (at the intersection of the 1860° R temperature line with the $m = 0.175$ line (interpolated)). For the purpose of illustration, the right triangle abc is completed. The side bc of the triangle is equal to the term

$$\left.h_a\right]_{T_a}^{T_b}$$

which is the numerator of equation (3), and the side ac of the triangle is equal to the term

$$-h_c - \left.\frac{Am + B}{m + 1}\right]_{T_r}^{T_b}$$

$C_{12}H_{26}$ .182

which is the denominator of equation (3). The slope of the line ab is therefore proportional to the fuel-air ratio required for the combustion. The scales of the ordinate and the abscissa in figures 1 and 2 have been so chosen that the fuel-air ratio is one-fortieth of the actual geometric slope. Two alternative methods are provided for measuring the slope of line ab; hence the fuel-air ratio:

(a) Draw a line through the reference point A (located at $T = 540^\circ$ R, $h_c = -8000$ Btu/lb) parallel to the line ab. The fuel-air ratio is found at the intersection of this line with the extreme right-hand fuel-air-ratio scale. The required fuel-air ratio for the example is 0.0138.

(b) Extend the line ab to the right until it intersects the abscissa of h_c to the right of the zero line at d ($h_c = -18{,}900$ Btu/lb). Read the fuel-air ratio f_2 opposite this intersection (0.01828) and the fuel-air ratio f_1 opposite the ordinate T_a of 900° R (0.00448). The required fuel-air ratio f is the difference between the fuel-air ratios f_2 and f_1.

$$f = 0.01828 - 0.00448 = 0.0138$$

The inverse problem of finding the initial or final temperature, if one temperature and the fuel-air ratio are known, is solved in a similar manner except that the slope of the fuel-air ratio line is used to determine the unknown temperature.

Reheating by burning additional fuel. - The method of calculation of the temperature resulting from burning additional fuel in the products of combustion is shown in figure 2(b). Assume that the combustion gas of the example described in figure 2(a) is cooled to a temperature of 1620° R. Determine the amount of fuel required to raise the temperature of the gas to a temperature of 1900° R by burning additional fuel.

Draw a line from the point a (located at $T_b = 1620^\circ$ R, $m = 0.175$) with a slope of $f = 0.0138$ (parallel to a'A) until it intersects the vertical line for a value of h_c of -18,900 Btu per pound at b. This point establishes an initial temperature, approximately 630° R, compatible with a combustion temperature T_b of 1620° R and the original fuel-air ratio f of 0.0138. Draw a line from b to c (located at $T_b = 1900^\circ$ R, $m = 0.175$) and the parallel line Ac'. The slope of Ac' is the total fuel-air ratio required (0.0183). The required increment in fuel per pound of original air f is the difference between these two fuel-air ratios: $\Delta f = 0.0183 - 0.0138 = 0.0045$.

The increment of fuel required may also be determined by extending the lines ab and bc to the second abscissa of h_c, using the scale to the right of the zero line, at d and e, respectively, (at $h_c = -18,900$ Btu/lb). The difference between the ordinates d and e is Δf, as indicated on figure 2(b).

Change of enthalpy of a burned mixture with change in temperature. - The use of the constant-pressure alinement chart to determine the heat abstracted from a burned mixture of combustion gases during a change in temperature is illustrated in figure 2(c) by the following example:

Initial temperature T_i, 2600° R
Final temperature T_f, 1000° R
Fuel-air ratio f, 0.030
Hydrogen-carbon ratio m, 0.175

Locate the points a and b at $T_i = 2600°$ R and $T_f = 1000°$ R on the m = 0.175 line. Draw a line with slope equivalent to f = 0.030 through each point a and b intersecting the $h_c = 0$ axis at a' and b', respectively. The required change in enthalpy is the difference between the ordinate of a' and b', -469 Btu per pound of original air or -455.5 Btu per pound of burned mixture. This result follows immediately from a consideration of equation (2). The distance ac in figure 2(c) is equal to

$$h_a\Big]_{T_c}^{T_f}$$

and the distance cd is equal to

$$f\,\frac{Am + B}{m + 1}\Big]_{T_i}^{T_f}$$

by the construction. The sum of ac and cd is then given by

$$(ac) + (cd) = h_a\Big]_{T_i}^{T_f} + f\,\frac{Am + B}{m + 1}\Big]_{T_i}^{T_f}$$

$$= (1 + f)\,h_b\Big]_{T_i}^{T_f} \qquad (4)$$

By the construction shown in figure 2(c) the line segment $a'b' = ac + cd$ because the lines aa' and db' are parallel.

Temperature of a mixture of combustion gases. - The temperature of a mixture of two gases each produced by the combustion of a lean mixture of air and a hydrocarbon fuel may be calculated by the construction shown in figure 2(d). It is assumed that the two fuels have the same hydrogen-carbon ratio. The mixture of the following two gases may be considered as an example.

(a) Fluid x, the products of combustion of 1 pound of air W_{ax} with 0.0138 pound of fuel, $f_x = 0.0138$, at a temperature of 1620° R.

(b) Fluid y, the products of combustion for 3 pounds of air W_{ay} with 0.021 pound of fuel at a fuel-air ratio f_y of 0.007 at a temperature of 1200° R.

The hydrogen-carbon ratio of the two fuels is 0.175.

Draw the line ca' with the slope $f_x = 0.0138$ through the point a located at $T_b = 1620°$ R and $m = 0.175$. The point a' is the intersection of this line with the line $h_c = 0$. Draw the similar line cb' through the point b located at $T_b = 1200°$ R and $m = 0.175$ with the slope $f_y = 0.007$. Draw a horizontal line from b' to d on the line $r = -1$ and a horizontal line from a' to e on the line $r = W_{ay}/W_{ax} = 3$, where W_a is the weight of original air in each fluid. In order to obtain the greatest possible accuracy the weight ratio r should be expressed as a number greater than 1. The line $a'e$, corresponding to $r \geqq 1$ must always be drawn to the left from the point a' on the $h_c = 0$ axis corresponding to the fluid having the smaller weight.

Draw the line de, which intersects the line $h_c = 0$ at g; draw the line gc. The temperature of the gas mixture (1310° R) is found at i where the line gc intersects the hydrogen-carbon-ratio line at $m = 0.175$. The slope of the line cg is equal to the mean fuel-air ratio of the mixture. The mean fuel-air ratio can be determined by drawing the line Ag' parallel to the line cg. The construction just described solves the problem of determining the temperature of the gas mixture because the line de divides the vertical line segment $a'b'$ into two segments with lengths proportional to r.

The distance $a'c'$ is proportional to f_x; the distance $b'c'$ is similarly proportional to f_y. The distance $a'b'$ is then proportional to $f_x - f_y$ and the distance b'g is therefore proportional to $(f_x - f_y)/(1 + r)$. The distance gc' is then proportional to $f_y + \frac{f_x - f_y}{1 + r} = \frac{f_x + r\,f_y}{1 + r}$ and the slope of the line cg corresponds to the mean fuel-air ratio $\overline{f}$ given by the equation

$$\overline{f} = \frac{f_x + r\,f_y}{1 + r}$$

The line cg divides any vertical line drawn from a'c to b'c into segments whose lengths are proportional to r. From the construction illustrated in figure 2(c), the vertical distance from i (in fig. 2(d)) to the line a'c is proportional to the change in enthalpy of fluid x in Btu per pound of original air; similarly the vertical distance from i to the line b'c is proportional to the change in enthalpy of fluid y. The construction, therefore, satisfies the condition that the change in enthalpy of the fluids shall be equal but opposite in sign when equilibrium is attained; namely,

$$\Big[(1 + f_x)\ \Delta h_b\Big]_{\text{fluid } x} = -r\Big[(1 + f_y)\ \Delta h_b\Big]_{\text{fluid } y}$$

because the length of the two segments have the ratio r.

Afterburning of richer-than-stoichiometric exhaust gases. - Consideration of the law of the conservation of energy yields the following equation for the temperature produced by the combustion of 1 + f pounds of a richer-than-stoichiometric exhaust gas with r pounds of air, when the final fuel-air ratio $f/(1 + r)$ is leaner than stoichiometric:

$$(1 + f)\ h_{br}\Big]_0^{T_e} - h_{chem} + r\ h_a\Big]_0^{T_a} = (1 + f + r)\ h_b\Big]_0^{T_b} \qquad (5)$$

The value of fuel-air ratio f used in equation (5) is the original fuel-air ratio of the richer-than-stoichiometric exhaust gas. From equation (2) the enthalpy of the final mixture at temperature T_b is given by the equation

$$(1 + f + r)\ h_b\Big]_0^{T_b} = (1 + r)\ h_a\Big]_0^{T_b} + f\ \frac{Am + B}{m + 1}\Big]_0^{T_b} \qquad (6)$$

When the term $(1 + f + r)\, h_b\Big]_0^{T_b}$ is eliminated from equations (5) and (6), the following more convenient form is obtained:

$$\Omega = h_a\Big]_{T_r}^{T_b} + r\, h_a\Big]_{T_r}^{T_b} + f\, \frac{Am + B}{m + 1}\Bigg]_{T_r}^{T_b} \tag{7}$$

where

$$\Omega = (1 + f)\, h_{br}\Big]_0^{T_e} - h_{chem} - h_a\Big]_0^{T_r} - f\, \frac{Am + B}{m + 1}\Bigg]_0^{T_r} \tag{8}$$

is the total enthalpy of the richer-than-stoichiometric exhaust-gas referred to the enthalpy of air at the reference temperature T_r as the zero of enthalpy.

The afterburning calculation may also be treated as the combustion of a fuel with the effective lower enthalpy of combustion $(h_c)_{eff}$ given by

$$(h_c)_{eff} = \frac{\Omega - h_a\Big]_{T_r}^{T_a}}{f} \tag{9}$$

When the value of Ω from equation (9) is substituted in equation (7), the relation reduces to

$$\frac{f}{1 + r} = \frac{h_a\Big]_{T_a}^{T_b}}{-\, (h_c)_{eff} - \frac{Am + B}{m + 1}\Bigg]_{T_r}^{T_b}} \tag{10}$$

The mean fuel-air ratio $\bar{f}$ for this combustion is given by the equation

$$\bar{f} = \frac{f}{1 + r} \tag{11}$$

where f is the original fuel-air ratio of the richer-than-stoichiometric exhaust gas.

A derivation of equations (7) and (8) and the equation for Ω in terms of the thermodynamic properties of the component gases is given in appendix B.

The value of Ω may be obtained from figure 3 in which Ω is given as the sum of two numbers ω and $K\Delta\omega$. The change in Ω as the temperature is changed is equal to $\Delta\omega$ for a fuel with a hydrogen-carbon ratio of 0.175 at a fuel-air ratio of 0.08. The correction factor K accounts for changes in the hydrogen-carbon ratio and the fuel-air ratio.

The use of the constant-pressure alinement chart to determine the temperature of a mixture of exhaust gas and air with afterburning is illustrated in figure 4 by the following example: Determine the temperature attained by the afterburning and subsequent mixing of 1.08 pounds of richer-than-stoichiometric exhaust gas, fuel-air ratio f of 0.08, hydrogen-carbon ratio m of 0.175, and an exhaust-gas temperature T_e of 1980° R, with 2 pounds of air at 411° R. (Note that $r = 2$.)

From figure 3, the value of $\omega = 767.5$ and $K\Delta\omega = 0$. The value of Ω is therefore 767.5 Btu per pound of air. The necessary construction is shown in figure 4. The point a' is located on the line $h_c = 0$ with the ordinate $h_a = \Omega = 767.5$. (It will often be necessary to extend the scale of ordinates beyond 900 Btu/lb of air.) From this point, the final temperature is determined exactly as in the case of a mixture of two leaner-than-stoichiometric combustion gases.

Draw a line through a' with the slope of f equal to the fuel-air ratio of the richer-than-stoichiometric exhaust gas ($f = 0.08$) intersecting at c the line bc drawn at the air temperature $T_a = 411°$ R. Draw a horizontal line from a' to e under $r = 2$. Draw a horizontal line from b to the point d at $r = -1$. Draw the line ed, which intersects $h_c = 0$ at g. Draw the line cg. The final combustion temperature is found at i where the line cg intersects the hydrogen-carbon-ratio line $m = 0.175$. The final temperature obtained by afterburning and subsequent mixing is 1425° R.

The point c is located at the value of the abscissa equal to the value of $(h_c)_{eff}$ given by equation (9), as can be seen from the construction. The slope of the line cg corresponds to the mean fuel-air ratio $\overline{f}$ (equation (11)).

The highest temperature of afterburning that can be calculated with this chart is that for a stoichiometric final mixture. The temperature for burning just to stoichiometric mixture for the example cited is 2560° R (2100° F) and is obtained with approximately 10 percent air at 411° R.

Heat balance of an internal-combustion engine. - The quantity of heat in the exhaust-gas of an internal-combustion engine to be used in constructing a heat balance is given by

$$(1 + f)\, h_b \Big]_{T_r}^{T_e}$$

for leaner-than-stoichiometric mixtures and by the quantity Ω for richer-than-stoichiometric mixtures. In each case this quantity is the enthalpy per pound of original air in the burned mixture.

SPECIAL COMBUSTION CHARTS

Two special charts for making combustion calculations have been prepared. (See figs. 5 and 6.) These charts show the temperature rise ΔT during ideal constant-pressure combustion for a fuel with a lower enthalpy of combustion h_c of -18,700 Btu per pound and a hydrogen-carbon ratio m of 0.175. The plots of the two correction factors K_m and K_h that permit the calculation of the fuel-air ratio for other fuels with different hydrogen-carbon ratios and lower heating values are included as inserts. The fuel-air ratio f is the product of the chart fuel-air ratio f' and the two factors K_m and K_h

$$f = f' K_m K_h$$

The two correction factors have been so adjusted that the correction is exact for the average variation of the lower heating value with the hydrogen-carbon ratio of the gasolines, kerosenes, and light fuel oils currently available. The assumed average relation is

$$h_c = -(15{,}935 + 15{,}800\, m) \qquad (12)$$

The corrections are also exact for a hydrogen-carbon ratio of 0.175 for any lower heating value. Small errors exist for other combinations of heating value and hydrogen-carbon ratio. For example, the fuel quantity calculated for a combustion temperature of 3000° R for a fuel with a hydrogen-carbon ratio of 0.084 will be in error about 1 percent for every 1500 Btu per pound that the lower heating value of the fuel varies from the value given by equation (12).

USES OF THE SPECIAL COMBUSTION CHARTS

The use of these combustion charts is illustrated by the following examples using a fuel with a hydrogen-carbon ratio m of 0.100 and a lower enthalpy of combustion h_c of -18,300 Btu per pound.

Constant-pressure combustion. - Find the combustion temperature T_b resulting when 0.02 pound of fuel is burned with 1 pound of air at an initial temperature of 600° R. First estimate the combustion temperature from figure 5 using an approximate chart fuel-air ratio f' equal to $-fh_c/18,700 = 0.02 \times 18,300/18,700 = 0.0196$. From figure 5, for the approximate chart fuel-air ratio f' of 0.0196 and an initial temperature T_a of 600° R, ΔT is 1352° R; therefore T_b is approximately $600^\circ + 1352^\circ$ R = 1952° R. Determine the correction factors K_m and K_h from the correction inserts and calculate the chart fuel-air ratio.

$$K_h = 1.023$$

$$K_m = 0.9890$$

$$f' = f/K_h\,K_m = 0.02/(1.023)(0.989) = 0.1977$$

Repeat the calculation of ΔT using the new chart fuel-air ratio of 0.1977. At this chart fuel-air ratio $\Delta T = 1366^\circ$ R; therefore $T_b = T_a + \Delta T = 600 + 1366 = 1966^\circ$ R. The corresponding combustion temperature calculated from figure 1 by the method shown in figure 2(a) is 1966° R.

Reheating by burning additional fuel. - If the gas of the foregoing example is cooled to 1800° R and reheated to a temperature of 2300° R by burning additional fuel, determine how much additional fuel is required. This calculation is made by finding an initial air temperature consistent with the hot-gas temperature, fuel-air ratio, hydrogen-carbon ratio and lower enthalpy of combustion of the fuel. This calculated initial temperature is used to calculate the fuel-air ratio required to produce the desired reheat temperature. The additional fuel required per pound of original air is the difference between the new fuel-air ratio and the original fuel-air ratio. Determine the correction factors K_m and K_h from the known values of h_c and m from the correction insert of figure 6 at 1800° R. These values are 1.0230 and 0.9904, respectively. Calculate the initial chart fuel-air ratio f_1'

$$f_1' = f_1/K_m\,K_h = 0.02/(1.023)(0.9904) = 0.01974$$

and find the corresponding ΔT for $T_b = 1800^\circ$ R from figure 6; the value of ΔT is seen to be 1397° R. The corresponding initial temperature is

$$T_a = T_b - \Delta T = 1800 - 1397 = 403^\circ \text{ R}$$

Determine the chart fuel-air ratio f' required to produce a combustion temperature $T_b = 2300^\circ$ R by burning fuel with an initial air temperature T_a of 403° R (fig. 5):

$$\Delta T = 1897^\circ \text{ R}$$

$$f_2' = 0.0282$$

Calculate the correction factors K_m and K_h corresponding to a temperature of 2300° R

$$K_m = 0.9855$$

$$K_h = 1.0236$$

The combustion fuel-air ratio f_2 is therefore

$$f_2 = f'_2 K_m K_h = (0.0282)(1.0236)(0.9855) = 0.02844$$

The additional fuel required is then

$$\Delta f = f_2 - f_1 = 0.02844 - 0.02 = 0.00844 \text{ pound of fuel per pound of original air.}$$

The corresponding increment in fuel-air ratio calculated from figure 1 is 0.00840.

Temperature of a mixture of combustion gases. - The temperature of a mixture of combustion gases may be calculated by finding a consistent initial air temperature for each gas of the mixture, as in the first part of the preceding example. The temperature of the mixture of the two masses of air used in the original combustion is then calculated and also the mean fuel-air ratio of the entire gas mixture. The final mixture temperature is then found by burning the averaged mixture. The enthalpy of dry air is shown in figure 7 as an aid to calculating the air-mixture temperature using the equation

$$(W_{ax} + W_{ay})\,\overline{h}_a = W_{ax}\,h_{ax} + W_{ay}\,h_{ay}$$

As an example compute the mixture temperature of the following combustion gases:

Fluid x, the product of combustion of 1 pound of air W_{ax} with 0.0138 pound of fuel, hydrogen-carbon ratio m of 0.1, fuel-air ratio f_x of 0.0138, at a temperature T_{bx} of 1620° R.

Fluid y, the product of combustion of 3 pounds of air W_{ay} with 0.021 pound of fuel, hydrogen-carbon ratio m of 0.1, fuel-air ratio f_y of 0.0070, at a temperature T_{by} of 1100° R.

Because the mixture temperature is independent of the heating value of the fuel, it is not necessary to apply the correction for the heat of combustion but only that for the hydrogen-carbon ratio.

The chart fuel-air ratios of the fluids, the temperature changes and the corresponding initial temperatures are:

$$f_x' = 0.0138/K_m = (0.0138/0.9918) = 0.01391$$

$$\Delta T_x = 987$$

$$T_{ax} = 1620 - 987 = 633^\circ \text{ R}$$

$$f_y' = 0.0070/K_m = 0.0070/0.9960 = 0.00703$$

$$\Delta T_y = 525$$

$$T_{ay} = 1100 - 525 = 575^\circ \text{ R}$$

In this case the two initial temperatures are so close that their weighted average may be taken as the mean air mixture temperature $\overline{T}_a$

$$\overline{T}_a = (3\ T_{ay} + T_{ax})/4 = 590^\circ \text{ R}$$

In general, $\overline{T}_a$ must be found by calculating the enthalpy of the air for each T_a from figure 7 and determining the weighted average enthalpy, which is then used to find the mean temperature from figure 7.

Assume that the mixture is now burned from an initial air temperature of 590° R. The estimated final temperature is obtained by using an estimated value of K_m equal to the weighted average value for the two fluids at their initial temperatures

$$K_m = (3 \times 0.9960 + 0.9918)/4 = 0.9950$$

The mean fuel-air ratio of the mixture $\bar{f}$ is 0.0087

$$f' = 0.0087/0.9950 = 0.008743$$

From figure 5, ΔT is 647° R; the mixture temperature T_b is therefore $590^{\circ} + 647^{\circ} = 1237^{\circ}$ R. As a check, the value of K_m for this value of T_b is 0.9951, which is sufficiently close to the assumed value. The corresponding mixture temperature calculated from figure 1 is 1240° R.

This method can be applied to the calculation of the amount of fuel necessary for reheating by burning additional fuel and to the calculation of mixture temperatures only if the apparent initial temperatures corresponding to the given state of the burned mixtures are real physical temperatures. The calculations can be made from figure 1 without regard to this limitation.

Aircraft Engine Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, October 10, 1945.

APPENDIX A

CORRECTION OF HEATS OF COMBUSTION TO THE REFERENCE TEMPERATURE OF 540° R

The heating value usually determined from measurements in a combustion bomb is the lower internal energy of combustion at constant volume e_c rather than the lower enthalpy of combustion. For most uses, the differences between these two heats of combustion are negligible.

The difference $(-e_c) - (-h_c)$ is due to the work done displacing the atmosphere $P\Delta V$ which is equal to $\Delta(PV)$ for the processes at constant pressure. The gas constant of the burned mixture is given by equation (21) of reference 1,

$$(1 + f)\,R_b = R_a + f\,\frac{R}{4.032}\,\frac{m}{m + 1}$$

and the external work done in a constant-temperature process (in Btu/lb of fuel) therefore is

$$\frac{\Delta(PV)}{fJ} = \frac{1}{4.032}\,\frac{RT}{J}\,\frac{m}{m + 1} = 0.493\,\frac{m}{m + 1}\,T$$

For example, the difference $(-e_c) - (-h_c)$ is 40 Btu per pound for octane at 540° R. The minimum probable error in very precise measurements is about 25 Btu per pound. In most ordinary determinations, the probable error is between 50 and 100 Btu per pound and therefore is larger than the difference between $-e_c$ and $-h_c$.

The heat of combustion is stated for a reference temperature T_r, usually 15° C to 25° C (59° F to 77° F). The variation of the heat of combustion in this range is, however, very much smaller than the uncertainty in the measurements. For hydrocarbon fuels, the lower heating value increases with temperature according to the relation,

$$\frac{d(-h_c)}{dT} = c_{pf} - \frac{am + b}{m + 1} = c_{pf} - \frac{2.242\,m + 0.1573}{m + 1}$$

at temperatures in the neighborhood of 540° R

where

$$a = \left(c_{p_{H_2O}} - 1/2\ c_{p_{O_2}}\right) \Big/ 2.016$$

$$b = \left(c_{p_{CO_2}} - c_{p_{O_2}}\right) \Big/ 12$$

c_{pf} specific heat at constant pressure of fuel

Assuming that $c_{pf} = 0.5$ Btu per pound per °R, the increase in the lower heating value with an increase in reference temperature is less than 0.2 Btu per pound per °R.

If the fuel is introduced, however, to the system at a temperature other than 540° R, the heat of combustion must be corrected by adding to the lower heating value the quantity of heat that must be added to the fuel to bring the fuel to its initial temperature and condition from 540° R. This correction is approximately 0.5 Btu per °F for fuel in the liquid phase. If the fuel is vaporized, the latent heat of vaporization must be added if the lower heating value was given for fuel in the liquid phase.

APPENDIX B

THE AFTERBURNING OF RICHER-THAN-STOICHIOMETRIC EXHAUST GASES

Consideration of the law of the conservation of energy yields the following equation for the temperature T_b produced by the combustion of $1 + f$ pounds of richer-than-stoichiometric exhaust gas with r pounds of air:

$$(1 + f)\, h_{br}\Big]_0^{T_e} - h_{chem} + r\, h_a\Big]_0^{T_a} = (1 + f + r)\, h_b\Big]_0^{T_b} \qquad (5)$$

From equation (2)

$$(1 + f + r)\, h_b\Big]_0^{T_b} = (1 + r)\, h_a\Big]_0^{T_b} + f\, \frac{Am + B}{m + 1}\Big]_0^{T_b} \qquad (6)$$

provided that the final mixture ratio is leaner than stoichiometric.

When equation (6) is substituted into equation (5) and the term $(1 + r)\, h_a\Big]_{T_a}^{0}$ added to each side of the equation the following equation is obtained

$$(1 + f)\, h_{br}\Big]_0^{T_e} - h_{chem} - h_a\Big]_0^{T_a} = (1 + r)\, h_a\Big]_{T_a}^{T_b} + f\, \frac{Am + B}{m + 1}\Big]_0^{T_b} \qquad (13)$$

If the term $f\, \frac{Am + B}{m + 1}\Big]_0^{T_r}$ is subtracted from each side of equation (13) and it is noted that

$$h_a\Big]_0^{T_a} = h_a\Big]_0^{T_r} + h_a\Big]_{T_r}^{T_a}$$

the following equation is obtained:

$$(1+f)h_{br}\Big]_{0}^{T_e} - h_{chem} - h_a\Big]_{0}^{T_r} - f\,\frac{Am+B}{m+1}\Big]_{0}^{T_r} - h_a\Big]_{T_r}^{T_a} = (1+r)h_a\Big]_{T_1}^{T_b} + f\,\frac{Am+B}{m+1}\Big]_{T_r}^{T_b} \qquad (14)$$

The value of the first four terms of the left-hand side of equation (14) depends on the temperature and the composition of the hot exhaust gas. The calculation of the sum of these terms can be simplified by a suitable collection of terms. For convenience let

$$\Omega = (1+f)\,h_{br}\Big]_{0}^{T_e} - h_{chem} - h_a\Big]_{0}^{T_r} - f\,\frac{Am+B}{m+1}\Big]_{0}^{T_r} \qquad (8)$$

The value of the first two terms $(1+f)\,h_{br}\Big]_{0}^{T_e} - h_{chem}$ may be calculated from equations (25) and (26) and table II of reference 1:

$$(1+f)\,h_{br} - h_{chem} = h_a - \frac{(O_2)_a}{M_a}\,C + f\,\frac{D+mE}{m+1} + \frac{(H_2)}{M_a}\,F \qquad (15)$$

where

$$C = H_{O_2} + 2H_{CO} - 2H_{CO_2}$$

$$D = \left(2H_{CO} - H_{CO_2}\right)\Big/12$$

$$E = \left(H_{H_2O} + H_{CO} - H_{CO_2}\right)\Big/2.016$$

$$F = H_{CO_2} + H_{H_2} - H_{CO} - H_{H_2O}$$

The enthalpy as defined by equation (26) in reference 1 includes the chemical enthalpy. When equations (8) and (15) are combined,

$$\Omega = h_a\Big]_{T_r}^{T_e} - \frac{(O_2)_a}{M_a}\,C\Big]_{0}^{T_e} + f\,\frac{D\Big]_{0}^{T_e} - B\Big]_{0}^{T_r} + m\left(E\Big]_{0}^{T_e} - A\Big]_{0}^{T_r}\right)}{m+1} + (H_2)\,\frac{F}{M_a}\Big]_{0}^{T_e} \qquad (16)$$

Equation (16) may be simplified by writing

$$\Omega = U + f\,\frac{mX + Y}{m + 1} + (H_2)\,W \qquad (17)$$

where

$$U = h_a\Big]_{T_r}^{T_e} - \frac{(O_2)\,a}{M_a}\,C\Big]_0^{T_e}$$

$$X = E\Big]_0^{T_e} - A\Big]_0^{T_r}$$

$$Y = D\Big]_0^{T_e} - B\Big]_0^{T_e}$$

$$W = F/M_a\Big]_0^{T_e}$$

The values of U, X, Y, and W, which are functions only of T_e, are given in table II for dry air. The values of (H_2) computed by means of equation (25) of reference 1 are given in table III.

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TABLE I - FACTORS FOR COMPUTING THE ENTHALPY OF LEAN-MIXTURE COMBUSTION GASES

T (°K)	T (°R)	h_a (Btu/lb)	A	B
200	360	85.99	---------	--------
300	540	129.13	1,195.8	26.042
400	720	172.48	1,602.6	60.767
500	900	216.40	2,016.9	105.33
600	1080	261.13	2,442.8	157.18
700	1260	306.85	2,884.9	214.38
800	1440	353.62	3,343.3	276.07
900	1620	401.38	3,821.0	341.50
1000	1800	450.04	4,321.6	410.00
1100	1980	499.54	4,842.9	481.42
1200	2160	549.73	5,388.6	554.75
1300	2340	600.58	5,956.6	630.25
1400	2520	651.90	6,548.9	707.92
1500	2700	703.86	7,152.6	785.50
1600	2880	756.19	7,780.8	865.00
1700	3060	808.94	8,423.1	945.33
1800	3240	862.04	9,081.3	1026.58
1900	3420	915.47	9,754.2	1108.25
2000	3600	969.17	10,440.2	1190.25

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TABLE II - FACTORS FOR COMPUTING THE ENTHALPY OF RICH-MIXTURE EXHAUST GAS

T (°K)	T (°R)	U	X	Y	W
300	540	-1756.90	60,115.4	20,198.8	-610.76
400	720	-1716.47	60,622.0	20,267.1	-602.85
500	900	-1673.67	61,074.9	20,324.6	-590.97
600	1080	-1628.85	61,497.5	20,375.4	-576.84
700	1260	-1582.24	61,909.3	20,422.4	-561.91
800	1440	-1534.01	62,318.2	20,467	-546.44
900	1620	-1484.37	62,731.2	20,511	-531.03
1000	1800	-1433.51	63,156.2	20,554	-516.02
1100	1980	-1381.52	63,592	20,597	-501.21
1200	2160	-1328.68	64,048	20,640	-487.51
1300	2340	-1275.49	64,517	20,683	-473.56
1400	2520	-1220.61	65,006	20,725	-460.51
1500	2700	-1165.61	65,505	20,768	-447.61

TABLE III - HYDROGEN CONTENT OF EXHAUST GAS

Fuel-air ratio	Hydrogen-carbon ratio of fuel						
	0.084	0.100	0.125	0.150	0.175	0.189	0.200
	Concentration of hydrogen, mole per mole or original air						
0.07	-------	-------	-------	0.00100	0.00398	0.00597	0.00764
.08	0.00323	0.00543	0.00984	.01530	.02190	.02560	.02881
.09	.01218	.01662	.02478	.03408	.04431	.05035	.05518
.10	.02380	.03091	.04332	.05677	.07094	.07909	.08551
.11	.03803	.04809	.06499	.08260	.10061	.11077	.11870
.12	.05471	.06779	.08918	.11083	.13250	.14456	.15391

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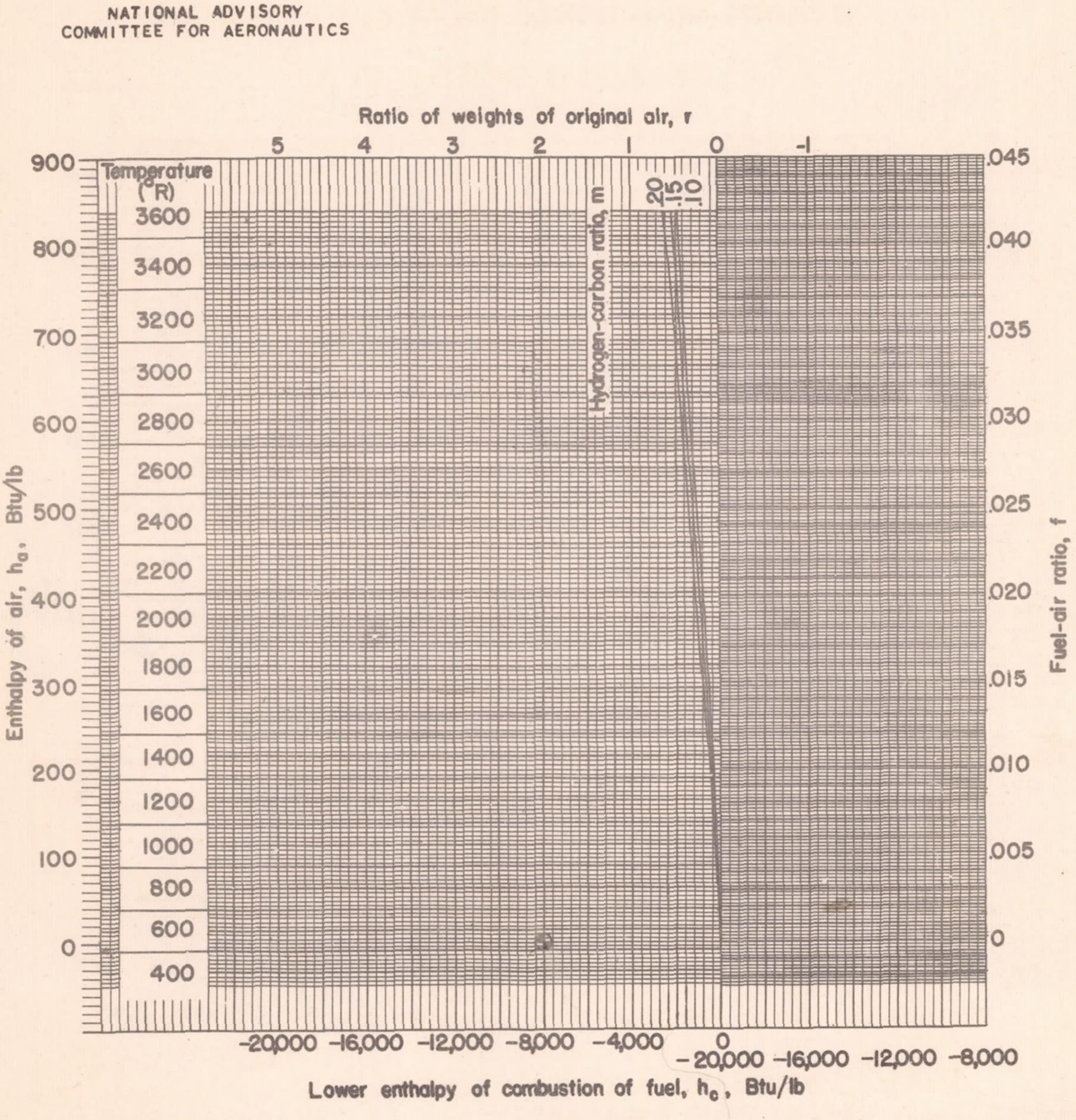
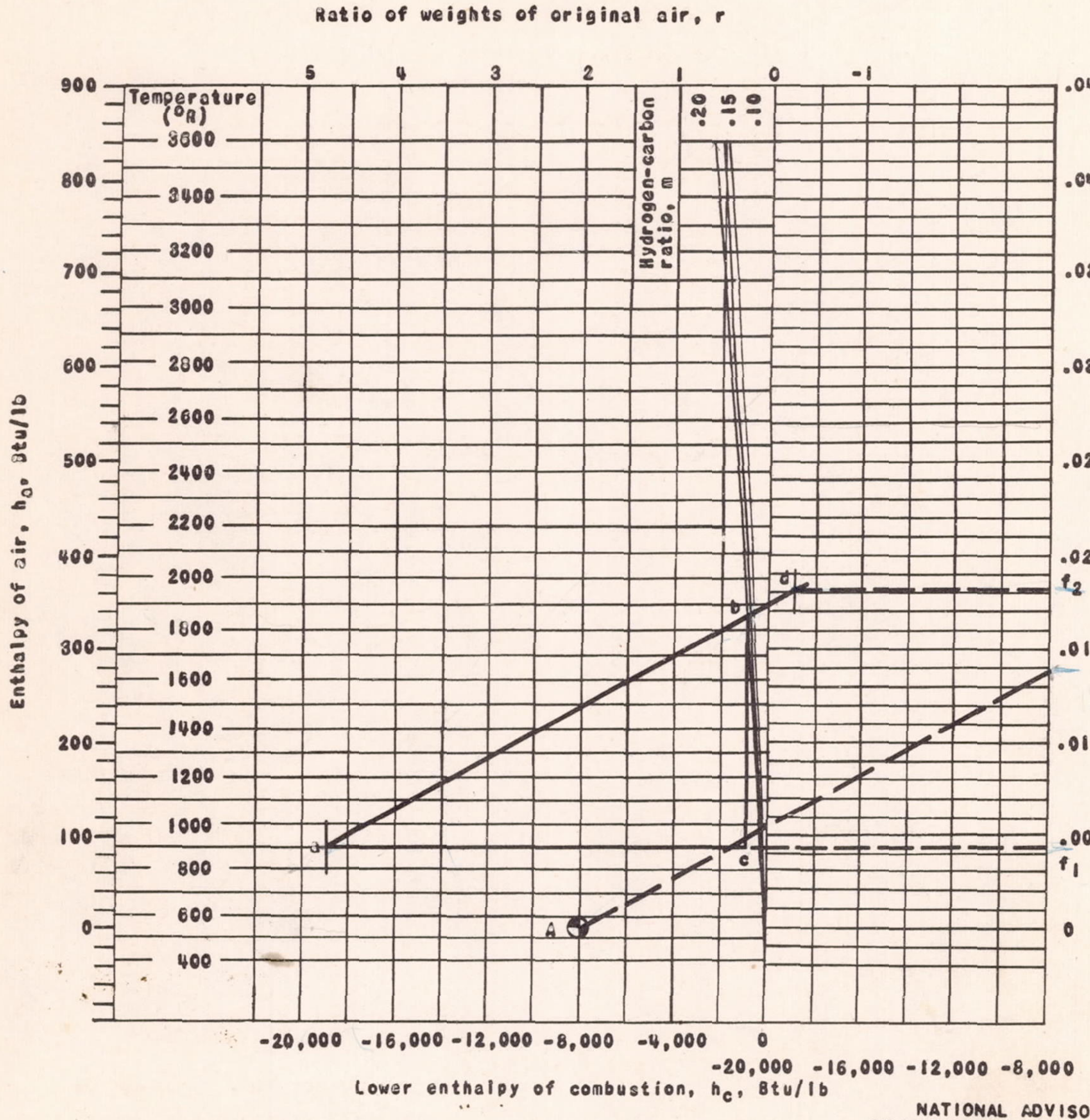


Figure 1. - Constant-pressure alinement chart. (A 25- by 25-in. print of this figure is attached.)



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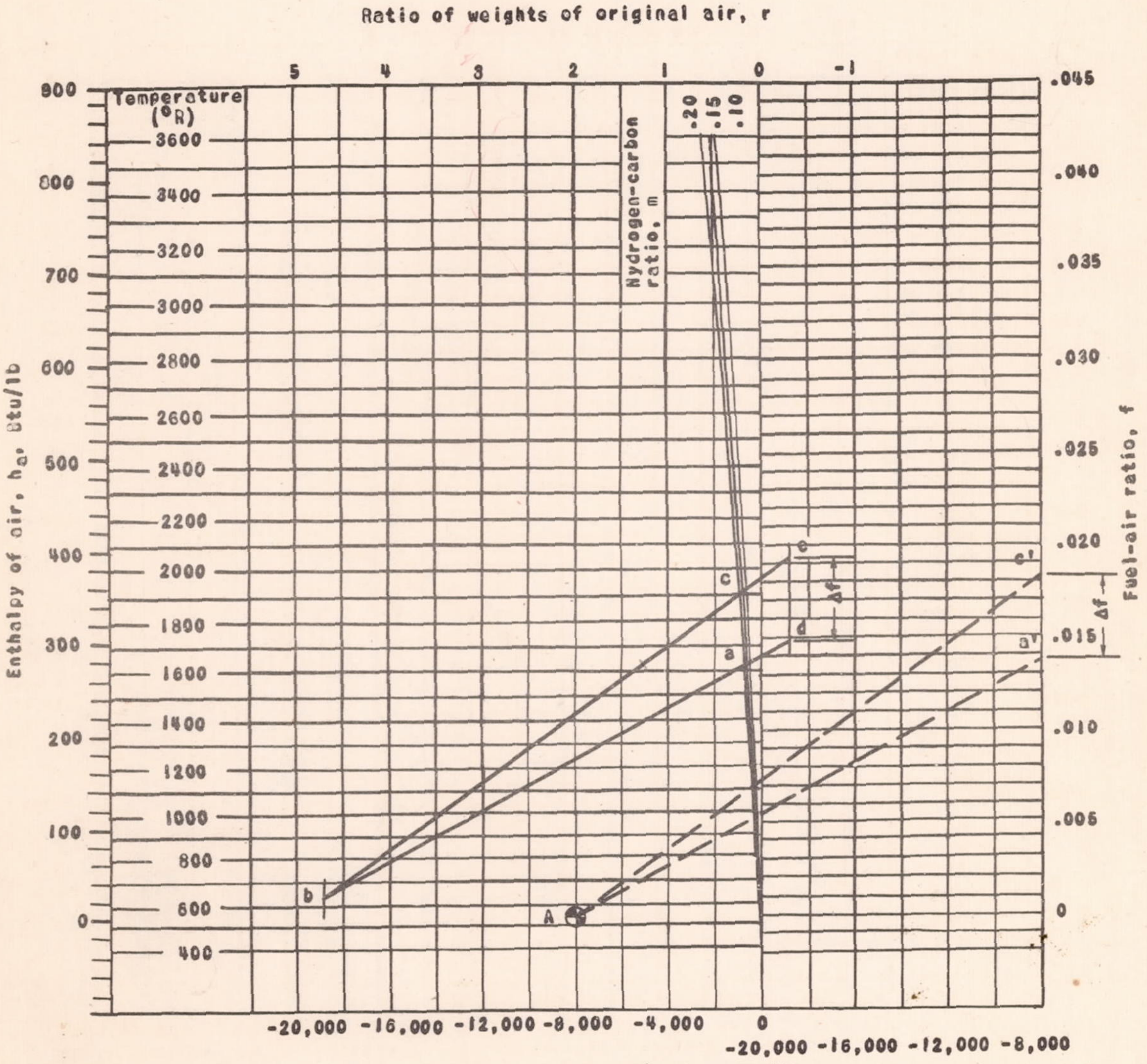
(a) Constant-pressure combustion.

Figure 2. - Use of the constant-pressure alinement chart.

$T_a = 900$
$T_b = 1860$
1840
.173



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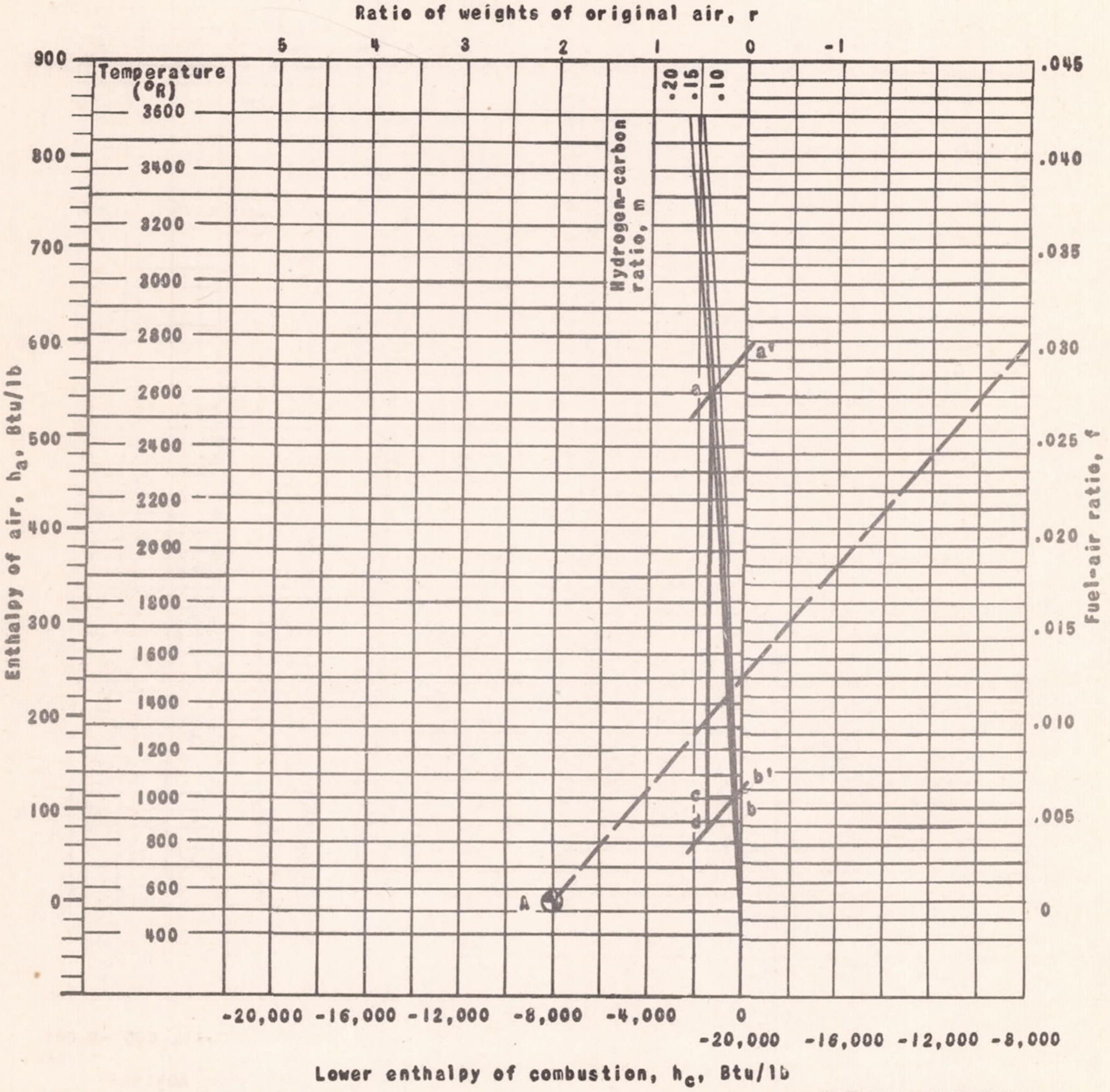
(b) Reheating by burning additional fuel.

Figure 2. - Continued.



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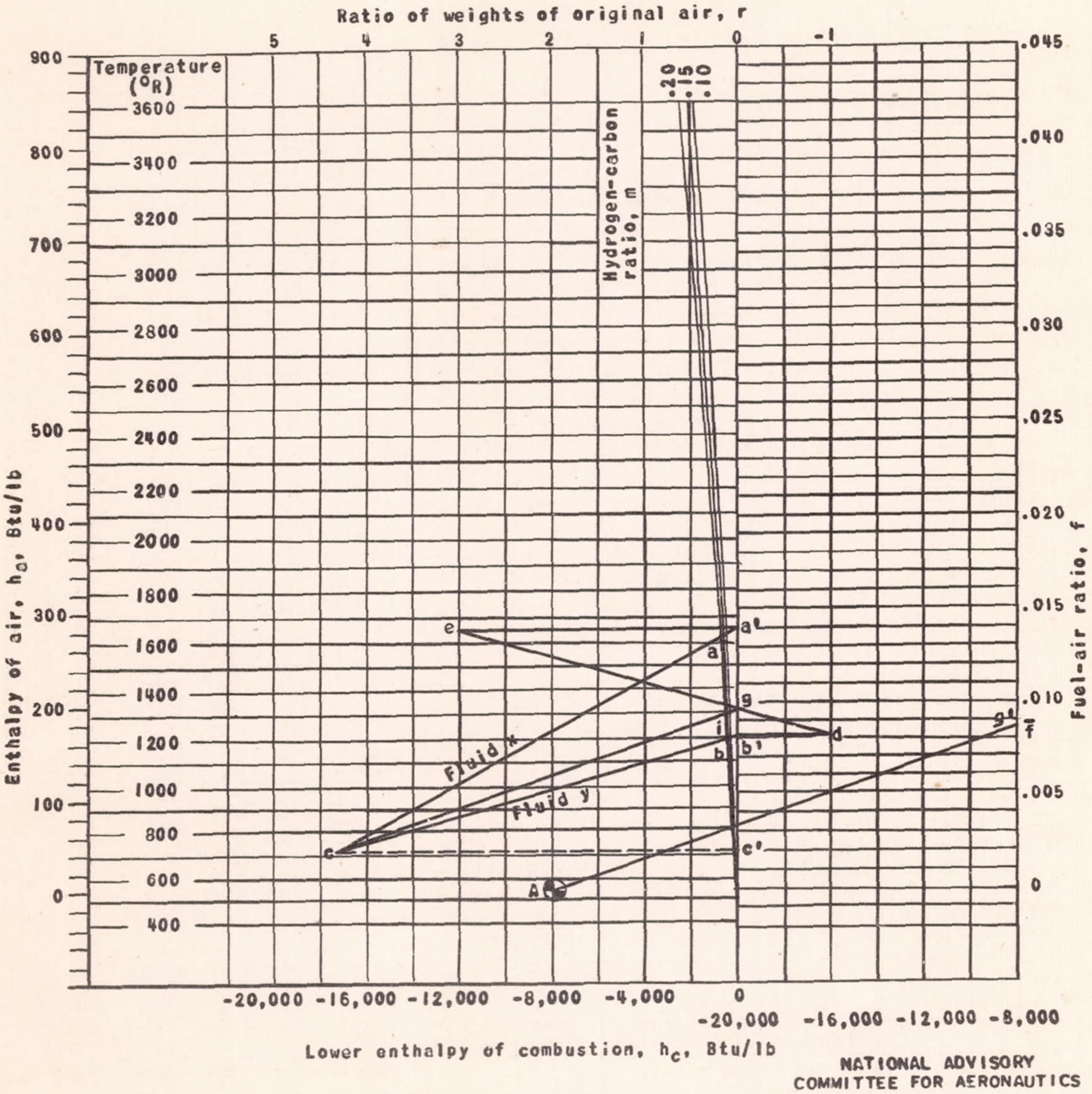
(c) Change in enthalpy of a burned mixture with a change in temperature.

Figure 2. - Continued.



(d) Temperature of a mixture of combustion gases.
Figure 2. - Concluded.

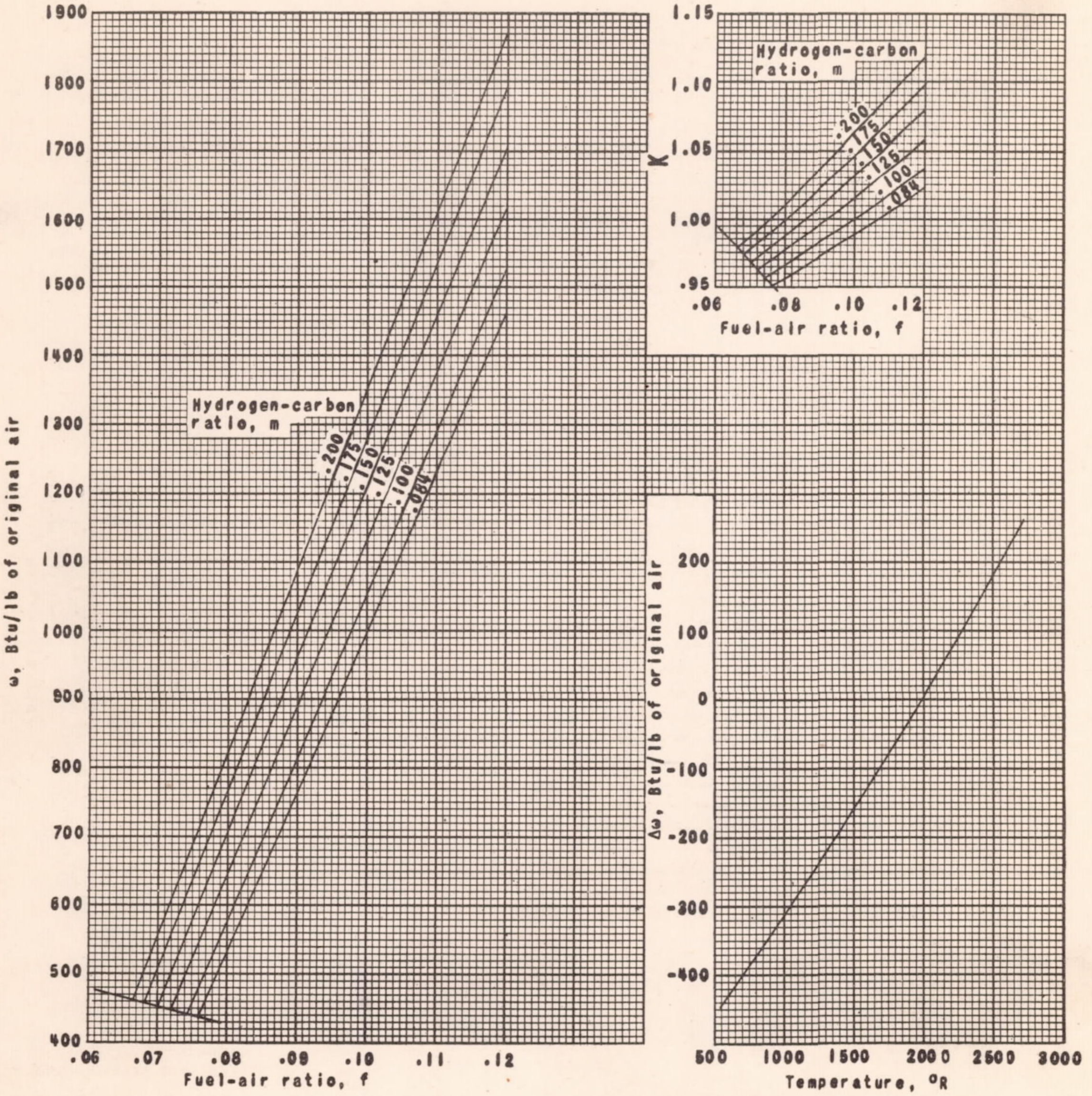


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Figure 3. - Total enthalpy Ω of richer-than-stoichiometric exhaust gas. $\Omega = \omega + K\Delta\omega$. (A $17\frac{1}{2}$- by 19-in. print of this figure is attached.)

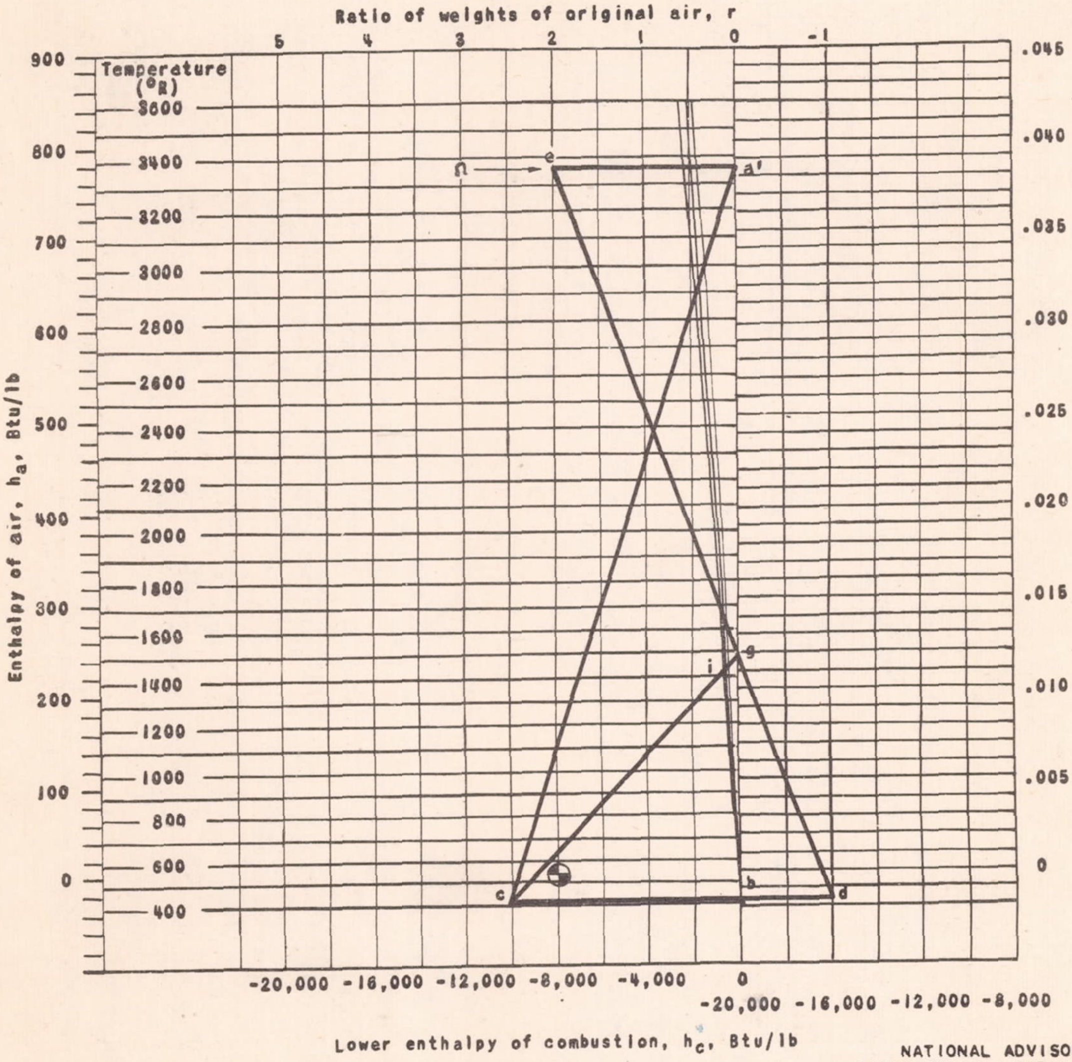

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Figure 4. - Use of the constant pressure alinement chart to determine the temperature of after burning of richer-than-stoichiometric exhaust gas.

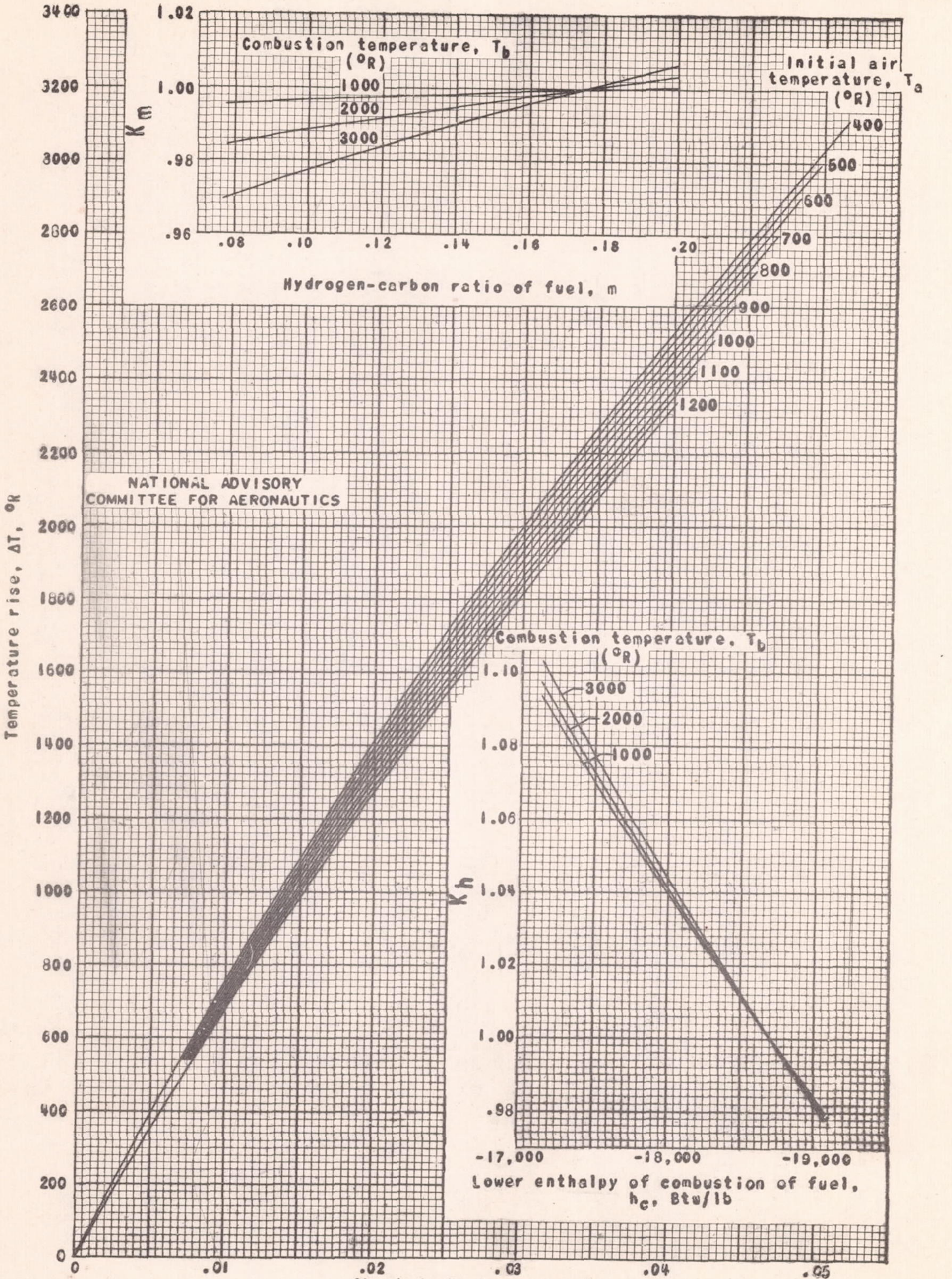

Figure 5. - Temperature rise during ideal constant-pressure combustion as a function of initial temperature; $f = f'K_mK_h$. (A 15- by 21-in. print of this figure is attached.)

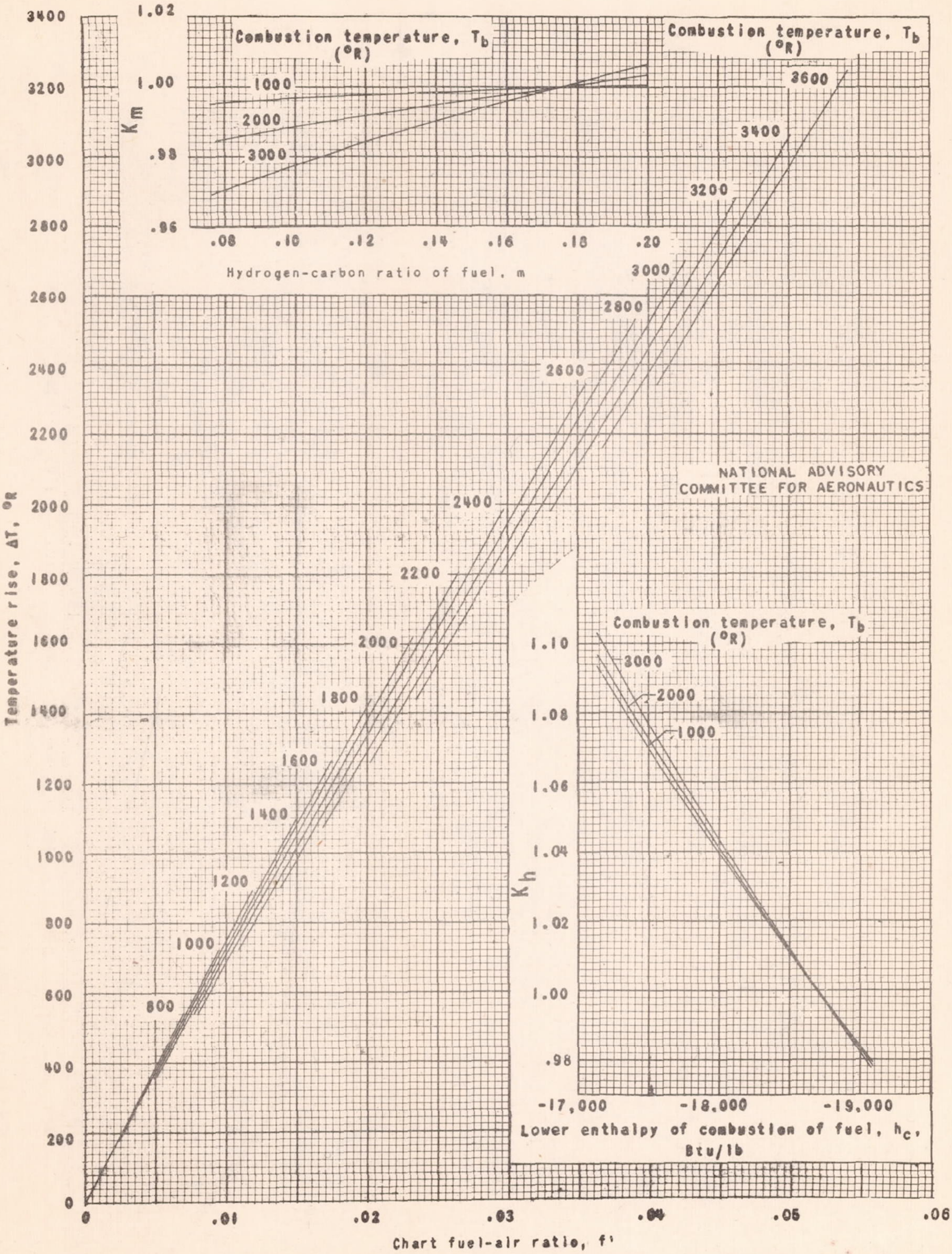

Figure 6. - Temperature rise during ideal constant pressure combustion as a function of the combustion temperature; $f = f'K_mK_h$. (A 16- by 21-in. print of this figure is attached.)

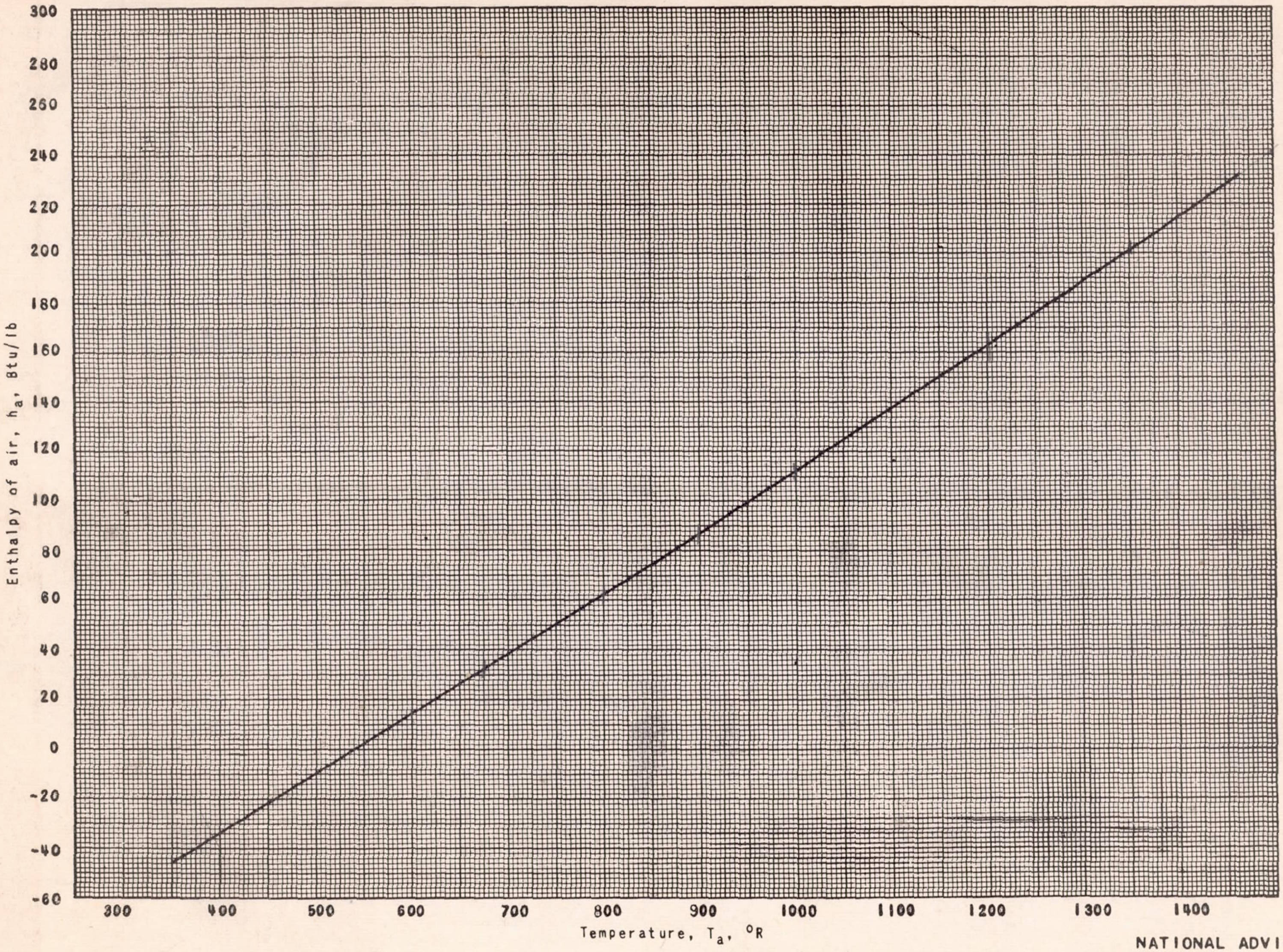

Figure 7. - Enthalpy of air.

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