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EXPERIMENTAL AND THEORETICAL STUDIES OF SURGING IN CONTINUOUS-FLOW COMPRESSORS

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SUMMARY

Experiments have been conducted to determine the conditions that cause surging in compressors and to determine the effect of various installations and operating conditions on the character of the velocity and pressure variations occurring during surging. These investigations were made on three compressor units and the variation of static, total, or velocity pressure with time was recorded. In addition to the experimental studies, a simplified analysis was made to determine how instability of flow may occur in a compressor. An examination, based on this analysis, was made of several possible methods of inhibiting the occurrence of surging.

Surging was found to be a periodic variation of pressures and velocities that may occur when the slope of the compressor characteristic curve is positive. A transition region of finite extent and characterized by erratic pulsations of small magnitude separated surging from stable operation. The frequency of the pulsations was always the same throughout the unit; the wave form was frequently nonsinusoidal.

The actual magnitude of the slope at which surging begins is determined by the time required for the static pressure in the external pipes to adjust itself to a change in flow conditions in the compressor passages. The magnitude and the frequency of the pressure pulsations, as well as the point on the characteristic curve at which surging begins, depend on a complex relation of the capacity and the resistance of each component of the total system. Because the occurrence and manifestation of unstable operation is dependent on the dynamic properties of the complete air-flow system, attempts to utilize one of the several methods for inhibiting surge should be made only on the actual installation with which the compressor is to be used.

INTRODUCTION

In modern high-speed superchargers and compressors a periodic pulsating flow known as surging terminates the operating range at low flow rates. In order to obtain the desired flexibility of operation of an aircraft power plant, the range of the compressor, indicated by the ratio of maximum volume flow to minimum volume flow, must be as large as possible. The range of centrifugal compressors rapidly decreases as the impeller tip speed is increased. The demand for increased power-plant performance at high altitudes, however, has placed increasing emphasis on the necessity for high tip speeds. As a result, when a compressor is designed for maximum power, the cruising power may be limited by the occurrence of surging. An extension of the surge-free range of operation of centrifugal compressors is therefore an important part of compressor research.

References 1, 2, and 3 have stated that surging is essentially the result of a flow instability, which occurs only when the slope of the characteristic curve (pressure ratio against flow function) is positive. The slope of the characteristic curve where surging begins, however, is not the same for all compressors; in fact, when a given compressor unit is tested in two installations, the value of the flow function at which surging begins (the surge point) is frequently different. Although the slope of the characteristic curve influences the occurrence of surging, several unknown properties of the complete installation apparently also govern its occurrence.

In the tests reported in reference 3, an attempt was made to influence the position of the surge point by introducing a fluctuating flow at the compressor inlet. This forced periodicity of flow was found to have a negligible effect.

Observations of the characteristics of the pressure pulsations during surging (references 2 and 3) indicated that the frequency and magnitude of the pulsations are somewhat affected by the volume of the compressor and the pipes of the system. In general, an increase in this volume apparently resulted in an increase in the amplitude of the pulsations and a reduction in the frequency.

In the present investigation, a special recording instrument was used to obtain an estimate of the magnitude and frequency of the pressure pulsations during surging. Experimental studies were made of the transition from steady flow to surging and of the phase relation of static, total, and velocity pressures during surging.

Other experiments were made to determine how the frequency and the magnitude of the pulsations were affected by changes in the volume of the compressor system and the tip speed of the impeller.

Several of the records of the pressure pulsations are presented and discussed. A simplified analysis is presented to show how instability of flow may be produced in a compressor and an examination based on this analysis is made of several possible methods of inhibiting the occurrence of surging.

Part of the experimental work was done on two test units at the NACA Langley Field laboratory. More complete tests at the Cleveland laboratory were made on a third test unit, which was especially designed for surge studies.

APPARATUS

Compressor Test Rigs

Three separate test units were used for the experimental investigations. Each unit was set up to conform to the standards and specifications recommended in reference 4. In each instance, the compressor unit was mounted directly on a speed increaser having a step-up ratio of 15:1 and was driven by an aircraft engine or a dynamometer. The air flow was determined by measuring the pressure drop across a calibrated orifice with an NACA micromanometer. Pitot tubes of 0.060-inch steel tubing were installed at the inlet and the outlet measuring stations; static orifices of 0.050-inch diameter were placed a short distance upstream of the pitot tubes. Throttles were placed in the inlet and outlet pipes to regulate the flow.

Tests were first made on unit A, which consisted of a fully shrouded impeller and a vaned diffuser mounted in a standard NACA variable-component test rig having a torus-shaped collector of 15-cubic-foot capacity. (See reference 4.)

Unit B consisted of a mixed-flow impeller and a 20-inch-diameter vaneless diffuser, which were also mounted in a variable-component test rig.

The greatest part of the work was done on test unit C (fig. 1), which was especially designed for investigations of the cause and character of the surge pulsations. Because the volume enclosed in the collector may have had an effect on the results of previous investigations, this unit was designed to make the total volume of the compressor itself as small as possible. A conventional impeller, which was modified by reducing both the diameter at the inlet and the height of the blades at the tips, was used in this rig. The vaneless diffuser, which formed an integral unit with the scroll collector, was designed to avoid the occurrence of separation in the diffuser. The disturbances in the flow system at low volume flows were reduced by designing the lip at the discharge of the collector to have a 0° angle of attack at a low volume flow. A schematic diagram of the rig in which this unit was tested is shown in figure 2.

Instantaneous Pressure Recorder

The pressure and velocity pulsations during surging were measured on a standard NACA differential-pressure recorder. This instrument has been described in references 5 and 6 but, for the convenience of the reader, some of its principles are described again. The instrument with two cells installed is shown in figure 3.

Two simultaneous pressure recordings can be taken by using two cells with a single film unit. Because the pressure cell actually measures differential pressures, the variations in static pressure, total pressure, or velocity pressure may be recorded. Static and total pressures are opposed by some known pressure, usually atmospheric, to obtain a direct trace of pressure variation with time. The variation in velocity pressure is recorded by introducing static pressure on one side of the diaphragm and total pressure on the other; the instrument records the differential pressure and a trace of the velocity-pressure variation is thereby obtained. Also, by recording a differential pressure rather than an absolute pressure, a sensitive cell can be used for a large range of compressor operating conditions.

When used with test units A and B, the instrument comprised two cells: one with a differential-pressure range from 0 to 50 inches of water, the other with a range from 0 to 30 inches of water. Cells with pressure ranges from 0 to 5 inches of water and 0 to 50 inches of water were used for tests on unit C. When the sensitive cell was used, the vibrations of the motor and the gear train that moved the film past the light orifice were recorded along with the pressure variations. Although these vibrations combined with the pressure vibrations to superimpose a low-frequency oscillation on the film record, the accuracy of the frequency measurements was not affected.

A study of the characteristics of the pressure-recording system showed that the indicated magnitudes of the pressure variations were not altogether reliable, being affected by the size, the type, and the length of the pressure tubes. In general, however, the magnitudes of the pressure variations indicated by the traces were roughly proportional to those actually occurring in the system when the installation remained unchanged for the entire series of tests. The recording system was probably accurate enough to register traces similar to the actual pressure waves.

TEST PROCEDURE

The first series of tests was made on test unit A to develop a technique for recording the pressure pulsations and to determine the character of surging. An investigation was also made of the relative frequency, magnitude, and phase relations of total-pressure, static-pressure, and velocity-pressure variations in the inlet and the outlet pipes of the system.

Investigations were made on test unit B to study the characteristics of the pulsations in the several unusual surge ranges encountered during performance tests of this unit. Traces of the velocity pressure in both the inlet and the outlet pipes were taken simultaneously at the standard measuring stations to determine whether the amplitude, the wave form, and the frequency of the pulsations were the same for all surge ranges.

One series of tests on unit C was designed to determine how the volume of the pipes used with the compressor influenced the frequency and magnitude of surging. Inasmuch as theoretically an infinite volume of air is in motion, regardless of the shape and size of the pipes, the volume affecting the surge characteristics is difficult to establish definitely. In a conventional test unit, a drop in pressure from approximately that of the local atmosphere occurs at the inlet throttle; an increase in pressure occurs through the compressor; and a decrease in pressure to approximately that of the local atmosphere occurs at the outlet throttle. The largest part of the pressure cycle is thus obtained between the inlet and outlet throttles and the volume enclosed by the system between these two stations may be considered to be the characteristic volume of the system. This concept, however, may be expected to be most exact when the pressure drop through the throttles is a maximum. As the pressure drop decreases, the influence of the exterior regions becomes more pronounced. The volume was therefore varied by using each of three inlet conditions in conjunction with each of three outlet throttle locations (fig. 2); thus nine different combinations of inlet and outlet conditions were tested.

The three inlet conditions were:

1. Inlet throttle wide open (located 24 diameters upstream from the compressor inlet, station K)

2. Inlet throttle partly closed (station K)

3. Inlet pipes replaced by a nozzle at the compressor inlet (station L)

The three outlet conditions were:

1. Outlet throttle 51 diameters downstream of compressor outlet (station A)

2. Outlet throttle 30 diameters downstream of compressor outlet (station B)

3. Outlet throttle replaced by a 3-inch gate valve at compressor outlet (station D)

All the tests to determine the effect of volume of the system on surging were made at an impeller tip speed of 960 feet per second.

Tests were also made on test unit C to determine the effect of tip speed on the characteristics of surging. For these tests complete compressor data and a number of velocity-pressure traces were taken at several tip speeds.

In order to apply the results of the analysis of surging, the variation of the pressure drop across the throttles with mass flow had to be determined. The throttle setting was maintained constant and the weight flow of air was varied over a wide range by varying the impeller speed. The weight flow of air and the pressure drops across the throttles were recorded. All tests were run at sea-level conditions with inlet-air temperatures from 68° to 100° F.

EXPERIMENTAL RESULTS

Character of transition from stable operation to surging. - The variations of the outlet total pressure with time for test unit A as the outlet throttle was gradually closed until audible surge occurred is shown in figure 4. At point A on the trace, only very small fluctuations in total pressure exist. Point B shows the beginning of small periodic fluctuations that persisted until the throttle was closed to point C, where a larger fluctuation of pressure occurred. As throttling was continued, this fluctuation appeared intermittently until finally, at point D, it became periodic. When the throttle was again opened, this sequence of events was reversed. Before the unit would stop surging, the outlet throttle had to be opened beyond the point at which surging began. This "hysteresis" effect was more pronounced for other operating conditions and often made stable operation difficult to regain without opening the throttle considerably.

The corresponding variation of the inlet total pressure when the inlet throttle was closed until audible surge occurred and then was opened is shown in figure 5. The order of events followed the same general trend as for the outlet trace.

For each of these traces an arbitrary pressure was used to oppose the fluctuating pressure in order to keep the surge trace within the scale of the recorder. For this reason the magnitude scale shows only the relative size of the fluctuation.

Phase relation of various pressures in flow system. - Simultaneous recordings are shown in figure 6(a) of the variation of the outlet velocity and static pressures with time as the flow is throttled to the surge point for test unit A. Traces obtained for the variation of outlet velocity and total pressure with time are shown in figure 6(b). The traces showing the corresponding variation of inlet and outlet velocity pressures are given in figure 6(c). The outlet velocity-pressure trace was kept within the limits of the recorder (the zero line was not placed to utilize the full film width) by reversing the pressure tubes, which inverted the trace on the film. In the outlet pipe, the total, static, and velocity pressures were substantially in phase. The velocity pressure in the inlet pipe was sufficiently in phase with that in the outlet pipe to show that the system was surging as a unit. The traces in which both the total and the static pressures apparently lead the velocity pressure by a small amount indicate an impossible situation and this error is attributed to the inherent deficiency of the instrument in detecting such small phase variations.

Characteristics of pressure pulsations during several surge ranges of test unit B. - In addition to the surge that terminated the range of operation of unit B at low values of volume flow, an audible surging also occurred at two points having relatively high values of volume flow. The characteristic curve determined by the standard compressor tests for this unit at an impeller tip speed of 900 feet per second is shown in figure 7. The solid portions of the curve represent stable operation and the dashed portions, unstable operation. Between points A and B the pulsations were plainly evident; at point C the surge could be picked up only with an instrument; and at D the pulsations were very violent. The pressure fluctuations (fig. 8) show the important parts of velocity-pressure traces made when the inlet throttle was slowly closed from an open position. Figure 8(a) shows the inlet velocity pressure as the unit enters the first surge range. As throttling is continued, the surging changes character from a high-frequency, low-amplitude wave to a wave of lower frequency and greater amplitude at point A shown in figure 8(a) and figure 7. At point B in figure 8(b), the flow has been throttled to such an extent that the trace for the outlet velocity pressure, which was off scale in figure 8(a), now appears in addition to the trace of the inlet velocity pressure. When the volume flow has been sufficiently reduced, the large, low-frequency fluctuations disappear leaving only the high-frequency waves of smaller amplitude. At a still lower value of volume flow indicated at point C on figures 7 and 8(c), a very gentle pulsation develops. This pulsation also disappears as throttling is continued and the operation is stable until a final, violent surge condition develops at the lower end of the operating range. The trace of pressure fluctuations at point D on figure 8(d) clearly shows that the form of this trace is different from the previous traces and does not even remotely resemble a sine wave. The magnitude and the frequency of the pulsations are different in each of the surge ranges. In all cases, the slope of the characteristic curve is positive each time that surging occurs (fig. 7).

Effect of external volume on frequency and amplitude of pressure pulsations during surging. - The compressor of test unit C was constructed to enclose a minimum volume. For this test unit, nine different combinations of inlet and outlet conditions were used. All tests to determine the effects of the external-system volume on surging were run at an impeller tip speed of 960 feet per second and the pressure cell was connected to record the variations in velocity pressure in the outlet pipe.

The lowest frequency and the greatest magnitude of the pressure pulsations were obtained when the external pipes enclosed the largest volume. For this condition the outlet throttle was located 51 diameters downstream of the compressor and the inlet throttle was wide open. The trace obtained from this test is shown in figure 9(a) and the frequency indicated by this trace was 4.8 cycles per second. Complete reversal of flow occurred at this condition and a considerable mass of air was discharged from the orifice tank into the atmosphere during part of the surge cycle.

With the outlet throttle in the same location and with the inlet and the outlet throttles partly closed, thus effectively reducing the volume of the inlet pipe, the frequency was increased to 11 cycles per second and the magnitude of the pressure variation was smaller, as shown in figure 9(b). The wavy envelope shown was due to vibration of the gear train in the pressure recorder. The disturbances at the orifice were considerably reduced from those observed with the inlet throttle wide open.

When the inlet pipe was replaced by the nozzle, the frequency decreased to 8 cycles per second and the magnitude also decreased. This result, shown in figure 9(c), was rather surprising because both the frequency and the magnitude decreased, a trend that did not correspond to any other observations made during similar tests. Some question exists as to what effect the nozzle actually had on the inlet volume because the pressure drop through the nozzle was very small and the volume of the test chamber may have influenced the results to some extent.

Traces obtained for each of the inlet conditions with the outlet throttle in its normal position, which is 30 pipe diameters downstream of the compressor outlet, are shown in figure 10. Again the lowest frequency, 6.5 cycles per second, was obtained with the inlet throttle wide open. Operating the unit with both throttles partly closed increased the frequency to 13.5 cycles per second and decreased the magnitude of pulsation. The effect of the nozzle, as shown by figure 10(c), was quite different from that previously noted in that the frequency rather than decreasing as before now increased to 60 cycles per second. In addition, a large decrease in magnitude was observed. The wavy envelope of the traces is again due to vibrations of the gear train in the pressure recorder.

Throttling at the scroll outlet with a standard 3-inch gate valve caused the effect of changes in inlet volume to become almost imperceptible. The traces corresponding to these conditions are shown in figure 11. When the nozzle was used (fig. 11(c)), the amplitude of the pulsation appeared to increase considerably. Audible observation, however, did not confirm this increase. The reliability of this trace is also doubtful because some question exists as to whether the inlet volume was the volume of the nozzle or the volume of the outside atmosphere. The lowest frequency, 56 cycles per second, was again noted for the open inlet pipe and for the other two conditions a frequency of 60 cycles per second was observed.

The results of this phase of the investigation show that the volume of the system has a very definite effect on the magnitude and the frequency of the pressure pulsations. In general, this trend agrees with the observations of previous investigations; that is, a large external-system volume results in a low-frequency, high-amplitude vibration; whereas a small volume results in a high-frequency, low-magnitude vibration. No simple relation, however, could be found to express the frequency as a function of the inlet and the outlet volumes, which indicates that factors other than the effective volume also control the surging characteristics. The pressure drop across the throttle and the inertia of the air in motion must be considered.

The following table shows the effects of volume on frequency of surge pulsations for test unit C:

Inlet volume (cu ft) ↓ / Outlet volume (cu ft) →	0.460	2.071	3.652
	Frequency (cps)		
0	60.0	60.0	8.0
3.034	60.0	13.5	11.0
5.114	56.0	6.5	4.8

Effect of impeller tip speed on magnitude and frequency of pressure pulsations during surging. - In the course of these investigations, additional tests were conducted on test unit C to determine the effect of tip speed on the magnitude and the frequency of the pressure pulsation. The records taken at 960 and 1200 feet per second with the outlet throttle 30 diameters downstream of the compressor outlet and the inlet throttle 24 diameters upstream of the compressor inlet are presented in figure 12. Although the amplitude of the pulsations is approximately doubled, the frequency is changed very little by an increase in tip speed. This result is representative of tests on other compressors.

ANALYSIS OF SURGE

Instability of Flow in Compressors

The results of the present and previous investigations (references 1, 2, and 3) have shown that surging occurs only when the slope of the characteristic curve is positive and that the volume of air enclosed by the system has a definite effect on the frequency and the magnitude of the pressure variations. In addition, the variations in pressure and velocity during surging need not have a sinusoidal variation with time. These observations indicate that surging is a self-induced vibration resulting from an inherent instability of the flow in some part of the operating range. The slope of the characteristic curve is a criterion of the stability of the flow. Inasmuch as the frequency and the magnitude of the pulsations are affected by the volume, this quantity might also be expected to affect the stability. Vibrations of this type seldom have purely sinusoidal qualities.

The general method used to study the stability of a given motion is to assume that a small deviation from the steady form of motion is produced and then to investigate whether the ensuing reactions tend to oppose the deviation or accentuate it (references 7, pp. 32-35, and 8). The reactions are brought about by the inertia, elastic, and frictional forces induced by the deviation. The inertia forces are a function of the rate of change of the velocity of the motion; the elastic forces are a function of the magnitude of the deviation itself; the frictional forces are a function of the velocity of the motion. Because stability of the motion is the issue rather than the motion itself, only the linear terms are considered.

In a compressor operating at constant speed, the inertia forces are due to changing the velocity of the total mass of air in motion. Elastic forces result from the elastic property of air. Frictional forces are (1) the true frictional force caused by skin friction and throttling, and (2) the force that causes the pressure rise in the compressor, which is, in a sense, a negative friction force. These forces are interrelated in a complex manner and efforts to develop an analytical expression to describe the influence of these forces for a general compressor system indicated that the equations become greatly involved. By the use of simplifying assumptions, however, an approximation was developed (see the appendix) to determine the relative influence of the various forces affecting the stability of the operation of a compressor. The derivation is somewhat limited because it is based on a simple system of an inlet pipe, an inlet throttle, and a compressor unit. Further limitations are imposed on the development because of the following simplifying assumptions:

(1) The difference between the static pressure and the total pressure is small.

(2) The pressure and the mass distribution in the inlet pipe vary linearly along the pipe length.

(3) The volume of the compressor is negligible.

Two simultaneous differential equations can be set up from these assumptions to obtain the expressions

$$\delta M_i = K_1\, e^{st} \sin\,(\omega + \phi_1)$$

and

$$\delta M_c = K_2\, e^{st} \sin\,(\omega + \phi_2)$$

where

δM_i	small change of mass flow rate through inlet throttle
δM_c	small change of mass flow rate through compressor
K_1, K_2	constants determining relative amplitude of vibratory motion
ϕ_1, ϕ_2	constants governing phase relation of vibratory motion
s	stability term
t	time
ω	frequency term

Only the exponent s need be examined to investigate the stability of the system. When $s < 0$, the system is in stable equilibrium; when $s \geqq 0$, instability results. The frequency term ω is of no interest for this discussion because the method of small deviations does not apply after instability occurs and the amplitude of the motion becomes large.

When s and ω are expressed in terms applicable to the compressor system being considered,

$$s = -2a^2 \left[\frac{1 - \dfrac{\gamma A v x y}{a^2 L}}{\gamma v (x-y)}\right] \tag{1}$$

and

$$\omega = 2\sqrt{\frac{a^2 A}{\gamma v L} - \left(\frac{s}{2}\right)^2}$$

where

a velocity of sound in inlet pipe, feet per second

γ ratio of specific heats

A effective area of inlet pipe, square feet

v volume of inlet pipe, cubic feet

x slope of throttle characteristic curve $\left(\frac{d}{dM_i}\Delta p_i\right)$

Δp_i pressure drop across inlet throttle, pounds per square foot

M_i mass flow rate across inlet throttle, slugs per second

y slope of compressor characteristic curve $\left(\frac{d}{dM_c}\Delta p_c\right)$

Δp_c pressure rise across compressor, pounds per square foot

M_c mass flow rate across compressor, slugs per second

L length of inlet pipe, feet

Because the velocity of sound is infinitely large for incompressible flow, the term $\left(\frac{\gamma v^2 xy}{a^2 L^2}\right)$ becomes infinitely small and the sign of equation (1) and the stability of operation will be determined by the denominator $\gamma v(x-y)$. When y algebraically exceeds x, the sign of s is positive and instability results.

A physical picture of the relation between the slopes of the throttle and the compressor characteristic curves and the stability of operation can be obtained by visualizing the effect of small disturbances in the flow. The throttle characteristic curves represent the variation of the total restrictions to flow in the external piping with mass flow. When the disturbance causes the pressure rise through the compressor to exceed the pressure drop through the external piping, the mass flow will increase; conversely, when the throttle pressure drop is greater than the pressure rise of the compressor, the mass flow will decrease. Figure 13(a) shows the variation of the pressure drop in the external piping and the pressure rise across the compressor with mass flow for a hypothetical unit. When the slope of the compressor characteristic curve is algebraically less than the slope of the throttle characteristic curve (assuming incompressible flow), a momentary decrease in mass flow will cause the pressure drop in the external piping to be momentarily less than the pressure rise through the compressor. Consequently, the mass flow will increase and thus restore equilibrium. A momentary increase in mass flow will likewise result in conditions that will restore equilibrium.

When the slope of the compressor characteristic curve is greater than the slope of the throttle characteristic curve at the intersection point (fig. 13(b)), the results of momentary changes in mass flow are quite different. If mass flow momentarily drops from point F, the pressure drop through the external piping will be greater than the pressure rise through the compressor at the new operating point and the mass flow will be decreased still more and thus promote unstable operation. Similarly, a momentary increase in mass flow will cause instability.

When compressibility is taken into account, however, the numerator of equation (1) cannot be neglected. The velocity of sound a becomes finite and the term $1 - \frac{\gamma v^2 xy}{a^2 L^2}$ becomes significant. Thus, the effect of the expression may make the sign of equation (1) positive with the resulting unstable operation even though the denominator remains positive $(x > y)$.

A series of tests was therefore made to obtain experimental values of the slopes of the compressor and throttle characteristic curves at or near the surge point. Complete performance data were taken when the inlet throttle alone was used to regulate the flow and the outlet throttle remained open. Data were also taken with the inlet throttle wide open and all throttling done at the outlet. If the dynamic characteristic of the compressor was to have the dimensions required by equation (1), the pressure rise developed by the compressor had to be plotted against the mass flow as shown by figure 14. The curves of the variation in pressure drop across the inlet throttle plotted against mass flow for throttle settings near the surge point are also shown.

A comparison was made between the actual values of y obtained from the curves of figure 14 and values obtained for the conditions $s = 0$ and $x - y \neq 0$, a condition satisfied by the relation

$$\frac{\gamma A^2 xy}{a^2} = 1$$

of equation (1). The foregoing expression has been reduced from the relation

$$\frac{\gamma v^2 xy}{a^2 L^2} = 1$$

by resolving the volume v into the product of an area A and length L and eliminating L in the numerator and denominator. The following values were used in the calculations:

Term	Inlet throttle only	Outlet throttle only
γ	1.3947	1.3947
A^2 (sq ft)^2	0.05	0.05
a^2 (ft/sec)^2	1,297,000	1,297,000
x	313,600	63,150
y (calculated)	59.3	294.4
y (measured)	8,750	8,710

A comparison of the actual and the calculated slopes is given by figure 14 where the dashed curves have slopes equal to the calculated slopes. Although the calculated values of y are much less than the actual values, they are both much smaller than x. Good correlation between the calculated and the actual values was not expected because the conditions of the equation and those of operation were very different. In the development of equation (1), the assumptions were made that only the flow upstream of the impeller inlet was restricted and that the volume of the compressor was negligible; whereas, in the actual case, there were flow restrictions at both the inlet and the outlet of the compressor and the volume enclosed by the compressor was appreciable. Although equation (1) cannot be used as an absolute criterion of surging, it is useful in obtaining a physical picture of the phenomenon.

Compressibility effects can be pictured with the aid of figure 13(a). If the point of operation of the compressor momentarily drops to point A from point E, the operating point of the throttles would move to point A' if the air were incompressible. Before the pressure in the pipe can drop to the pressure required for this condition, the mass of air contained by the pipe must decrease; that is, for a definite interval of time the mass flow of air leaving the pipe must be greater than that entering it. Thus a finite time is required for the static pressure to adjust itself completely to the change in flow conditions. (This time lag should not be confused with the time lapse due to the finite velocity of sound.) Because of this time lag between a change in operating conditions and the complete adjustment in static pressure, the point of operation may drop only to point B'. Compressibility may thus cause the pressure drop across the throttles to be greater than the pressure rise across the compressor and a further decrease in mass flow and unstable operation will result. Similarly, when the point of operation of the compressor momentarily moves to point C, the point of operation of the throttles would for the incompressible case be C'. Owing to compressibility, however, the pressure drop may be only D'. When the pressure rise across the compressor is larger than the pressure drop through the external piping, a further increase in mass flow and unstable operation will result. These illustrations show that compressibility reduces the limiting value of the slope of the compressor characteristic curve for the same slope of the throttle curve. Because the time lag of the pressure changes is affected by volume and the resistance offered to flow, the relative time lag of the compressor and the external system may be somewhat adjusted by altering these factors. This statement follows from the position of the v and x terms in equation (1).

Flow Conditions within a Compressor at Point of Instability

Both the experimental and the analytical results have shown that the value of y must be positive at the point of surging. If the value of y is to be positive, the pressure losses in the compressor must increase as the flow decreases (except for certain impellers with forward-swept blades), which would be indicative of the development of a breakdown in the flow at some point in the system. For example, a decrease in the flow can cause the angle of attack of the inlet blades of the impeller to become such that a large amount of additional separation occurs. Similarly, a breakdown in the flow will occur in a vaned diffuser when the flow is decreased beyond a certain limit and even in a vaneless diffuser the tendency for separation is increased as the volume flow is decreased. In the impeller itself, a decrease in the volume flow increases the blade loading near the inlet and along the radial blades; thus, additional separation in these regions can occur when the flow is sufficiently decreased. The value of volume flow at which a large-scale breakdown of flow occurs may be considered as the ultimate lower limit at which surge-free operation may be obtained with any given impeller.

Methods of Inhibiting Surging

From the previous discussion, the most effective means of delaying the occurrence of surging is to maintain a negative slope of the characteristic curve. The ideal means of accomplishing this end is to locate and eliminate the cause of the flow breakdown, which produces the positive slope of the characteristic curve. Unfortunately, attempts to correct the flow breakdown at low values of volume flow often adversely affect the upper range of operation to such an extent that the over-all performance is less satisfactory.

One method of delaying the change in slope of the characteristic curve is the use of variable-angle prerptation vanes upstream of the compressor. As throttling at the inlet is increased, the incoming air is given a rotation in the direction of impeller travel and the greater the amount of prerotation the less will be the pressure rise of the compressor. For any given setting of the vanes, the amount of the absolute prerotation will be proportional to the volume flow and, consequently, any slight decrease in volume flow will tend to increase the pressure rise of the impeller and thus delay the critical value of the positive slope.

Another means of limiting the net mass flow through a compressor system without encountering surging is to recirculate part of the total flow back through the compressor unit. The introduction of the recirculated air should be arranged to provide prerotation to the incoming air stream. Thus, the pressure ratio developed by the impeller is reduced and at the same time the volume flow through the impeller and diffuser is greater than the net flow through the system by the amount of recirculation. The main disadvantages of this system are that the outlet temperatures may be increased with a resulting decrease in over-all efficiency.

The analytical expressions derived in the appendix show that the volume of the system has a definite effect on the surge point. In general, the greater the volume of any part of the system, the slower will be the response of the related pressures to changes in the mass flow rate. Inasmuch as the volume of the compressor unit is small compared with that of the entire system, the surge-free range probably could be extended by increasing the effective capacity of the compressor unit or decreasing that of the auxiliary piping. These modifications would either make the response of the compressor unit to small variations in mass flow slower or speed up the response of the external system. This means of changing the surge point would be applicable only when the positive slope y is relatively small as compared with the slope of the throttle characteristic curve. If the slope changes abruptly, however, very little would be gained by the addition of a chamber to the compressor unit.

A device that operates on the principle of delaying the response of the compressor was developed by the General Electric Company for inhibiting surging. A sizable chamber is connected to the flow passage at the leading edges of the diffuser blades by means of very small orifices. If the positive slope of the compressor curve is due to a momentary breakdown in the flow at the diffuser tips, the resulting drop of pressure in this region will cause the chamber to discharge air into the passages and thus delay the rates of change of the decrease in the over-all pressure ratio. The effectiveness of operation of an arrangement of this type will depend on the purpose for which the compressor is to be used.

A typical compressor installation for a modern aircraft engine has a large number of components, each of which contributes its own particular resistance, capacity, and inertia effects to the totals for the system. The surge point observed during bench tests usually corresponds fairly closely to that found for the actual installation. A device for inhibiting surge by changing the response of the unit for any one system, however, cannot be expected to function properly when used in another system where the resistance, the capacity, and the inertia effects are greatly different. For this reason, investigations made to increase the surge-free range must be made on the complete installation with which the compressor unit is to be used.

SUMMARY OF RESULTS

The results of experiments with three compressor test rigs and analytical studies of surge show that:

1. As a rule, a transition region characterized by erratic pulsations of small magnitude existed between the region of stable operation and the point where definite surging begins.

2. The fact that the frequency of the pulsations throughout the system was uniform indicated that the system surged as a unit.

3. Although the pressure pulsations were periodic, their variations with respect to time were frequently nonsinusoidal.

4. Decreasing the volumetric capacity of the external pipes increased the frequency and decreased the amplitude of the pressure pulsations. Apparently the frequency and the amplitude both depended on a complex relation of the capacity and the resistance of each component of the system.

CONCLUSIONS

From the foregoing results, the following conclusions have been drawn:

1. Surging is the manifestation of an instability of flow in a compressor. If this condition is to exist, the slope of the compressor characteristic curve must be positive and a time interval must exist between a change in flow conditions in the compressor passages and the static-pressure adjustment in the external pipes.

2. The surge-free range of any compressor may possibly be extended or the magnitude of the surging pulsations be reduced by either reducing the magnitude of the positive slope of the characteristic curve or by changing the volumetric capacity of the compressor components. All such investigations should be made on the actual installation with which the compressor is to be used.

Aircraft Engine Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, April 25, 1946.

APPENDIX - EQUATION OF STABILITY

Symbols

The symbols used in the equation of stability are defined as follows:

A	cross-sectional area of pipe, sq ft
a	speed of sound, ft/sec
g	standard acceleration of gravity, 32.174 ft/sec
K_1, K_2	constants determining the relative amplitude of vibratory motion
L	length of pipe, ft
M	mass flow, slug/sec
δM	small change in mass flow, slug/sec
m	mass of air in pipe, slug
p	pressure, lb/sq ft
Δp	change of pressure, lb/sq ft
R	gas constant
s	a number the algebraic sign of which is indicative of the stability of a system
ω	number indicative of frequency of oscillation
T	temperature, $^{\circ}R$
t	time, sec
V	velocity, ft/sec
v	volume, cu ft
x	slope of throttle characteristic curve $\left(\frac{d}{dM_i}\Delta p_i\right)$
y	slope of compressor characteristic curve $\left(\frac{d}{dM_c}\Delta p_c\right)$

γ ratio of specific heats

ϕ_1, ϕ_2 constants governing phase relation of vibratory motion

ρ density of gas in pipe

Subscripts refer to conditions as follows:

1 at atmosphere

2 immediately downstream of inlet throttle

3 immediately upstream of compressor

a average in inlet pipe

c in compressor

i in inlet throttle

o at equilibrium

Derivation of Equation

A general method of determining the stability of a given motion is given in references 7 (pp. 32-35, 324-332), and 8. When a small deviation or displacement from the state of equilibrium of a system is assumed, certain vibrations will occur that may be analytically studied. If these vibrations have a tendency to die out, equilibrium is maintained; otherwise, it is destroyed. In addition to the compressor itself, a typical compressor installation contains one or more throttling units and several lengths of piping. A rigorous analysis of the stability of such a system requires a knowledge of the distribution of the pressure, the temperature, and the velocity at each point in the system.

At present, a thorough study of the stability of any specific system is not so feasible as formulating a relation between the various factors affecting stability and thus obtaining information on the fundamental causes of unstable operation. For simplicity, a compressor system (fig. 15) consisting of an inlet throttle, an inlet pipe, and the compressor unit will be assumed. Air enters the throttle directly from the atmosphere and is directly discharged from the compressor into the atmosphere.

At equilibrium, the drop in static pressure Δp_{io} across the inlet throttle is equal to the rise in static pressure Δp_{co} across the compressor (from the assumption that the difference between static and total pressure is small). If Δp_{io} is assumed to be a function of the mass flow M_i through the throttle, the occurrence of a disturbance that causes a small change δM_i in the main flow will cause the pressure drop across the throttle to become

$$\Delta p_i = \Delta p_{io} + \left(\frac{d}{dM_i} \Delta p_i\right)_o \delta M_i \qquad (2)$$

and

$$\Delta p_c = \Delta p_{co} + \left(\frac{d}{dM_c} \Delta p_c\right)_o \delta M_c \qquad (3)$$

This disturbance may or may not be equal to that through the throttle.

At any instant, the static pressure p_2 immediately downstream of the throttle must be equal to the difference between the pressure p_1 of the atmosphere and Δp_i. Similarly, the pressure p_3 immediately upstream of the compressor must equal the difference between p_1 and Δp_c. These requirements provide the relation

$$p_1 = p_2 + \Delta p_i = p_3 + \Delta p_c$$

When the values of Δp_i and Δp_c determined in equations (2) and (3) are substituted,

$$p_2 - p_3 = \left(\frac{d}{dM_c} \Delta p_c\right)_o \delta M_c - \left(\frac{d}{dM_i} \Delta p_i\right)_o \delta M_i$$

or

$$p_2 - p_3 = y\delta M_c - x\delta M_i$$

The displacements assumed for studying the stability of the system are so small that x and y may be treated as constants.

The existence of this difference in pressure at the two extremities of the inlet pipe will change the momentum of the mass of air enclosed by the inlet pipe. If friction forces are neglected, the rate of change of momentum will equal the product of the pressure difference by the area of the pipe. Thus

$$\frac{d}{dt}(mV) = A(y\delta M_c - x\delta M_i)$$

This expression may also be written

$$L\frac{d}{dt}(\rho AV) = A(y\delta M_c - x\delta M_i)$$

Because the distribution of the changes in mass flow in the inlet pipe is unknown, the mass flow rate is assumed to vary linearly along the pipe length. The quantity ρAV is the average value of the mass flow through the pipe and its value may be written as the average of M_i and M_c.

Then

$$L\frac{d}{dt}\frac{(M_i + M_c)}{2} = A(y\delta M_c - x\delta M_i)$$

but

$$\frac{d}{dt}\frac{(M_i + M_c)}{2} = 1/2\frac{d}{dt}(M_{io} + \delta M_i + M_{co} + \delta M_c)$$

therefore

$$L/2\frac{d}{dt}(\delta M_i + \delta M_c) = A(y\delta M_c - x\delta M_i) \tag{4}$$

By use of the perfect-gas law $pv = gmRT$ and the expression for the speed of sound $a^2 = \gamma gRT$,

$$m = \left(\frac{\gamma v}{a^2}\right)p_a$$

The rate of change of the mass in the pipe with respect to time may be written

$$\frac{dm}{dt} = \frac{\gamma v}{a^2}\frac{dp}{dt} \tag{5}$$

if the temperature of the air is assumed to remain unchanged.

The true variation of static pressure at the throttle and along the pipe is unknown and therefore a linear relation is assumed. When the difference between total pressure and static pressure is neglected,

$$p_a = \frac{p_2 + p_3}{2}$$

or

$$p_a = p_1 - \frac{\Delta p_{io} + \Delta p_{co}}{2} - \frac{x\delta M_i + y\delta M_c}{2}$$

Differentiating gives

$$\frac{dp}{dt} = - \frac{d}{dt} \frac{(x\delta M_i + y\delta M_c)}{2}$$

Substituting this value in equation (5) results in

$$\frac{dm}{dt} = - \frac{\gamma v}{a^2} \frac{d}{dt} \frac{(x\delta M_i + y\delta M_c)}{2}$$

The rate of change of the mass in the inlet pipe must equal the difference between the mass flow rates entering and leaving the pipe.

$$\frac{dm}{dt} = M_i - M_c = \delta M_i - \delta M_c = \frac{-\gamma v}{a^2} \frac{d}{dt} \frac{(x\delta M_i + y\delta M_c)}{2} \tag{6}$$

After the simultaneous differential equations (4) and (6) are solved the resulting expressions for δM_i and δM_c are

$$\delta M_i = K_1 e^{st} \sin (\omega t + \phi_1)$$

$$\delta M_c = K_2 e^{st} \sin (\omega t + \phi_2)$$

where K_1 and ϕ_1 are arbitrary constants that determine the values of K_2 and ϕ_2.

When expressions of this type are used to represent the dynamic characteristics of a system, the significance of the terms is as follows:

(a) The constants K_1 and K_2 determine the relative amplitude of the vibratory motion.

(b) The term e^{st} indicates the variation of the amplitude of vibration with time. If s is positive, the vibrations increase with time and unstable operation results; a negative value of s indicates stable operation.

(c) The term $\sin (\omega t + \phi_1)$ designates the frequency and phase relations. The frequency depends upon ωt and ϕ_1 governs the phase relation.

Inasmuch as the main interest of this discussion is centered on the question of stability, the term e^{st} will be considered more fully. The expressions for s and ω are

$$s = -2a^2 \left[\frac{1 - \frac{\gamma A v x y}{a^2 L}}{\gamma v (x-y)} \right] \qquad (1)$$

and

$$\omega = 2 \sqrt{\frac{a^2 A}{\gamma v L} - \left(\frac{s}{2}\right)^2}$$

As previously stated, when the sign of s is negative, the vibrations are diminished as time passes and the operation is stable. On the other hand, a positive value of s results in unstable operation. The determination of stability is therefore reduced to the determination of the sign of the variables in equation (1). All of the terms in this expression are positive with the exception of the slope y of the compressor characteristic curve, which may be either positive or negative. When $y \leqq 0$, s is negative and the motion is stable. When $y > 0$, however, the sign of the entire expression depends on whether

$$\frac{\gamma v^2 x y}{a^2 L^2} > 1$$

so long as $x > y$.

When $\frac{\gamma v^2 x y}{a^2 L^2} > 1$ and $x > y$, the sign of s is positive and unstable operation results. In the case where y is positive and $y > x$, even if the $\frac{\gamma v^2 x y}{a^2 L^2}$ term were insignificant, the denominator would become negative and thus instability would result. The coefficients of the quantity xy in the term $\frac{\gamma v^2 x y}{a^2 L^2}$ are such that the numerator of equation (1) usually becomes negative even though $x > y$.

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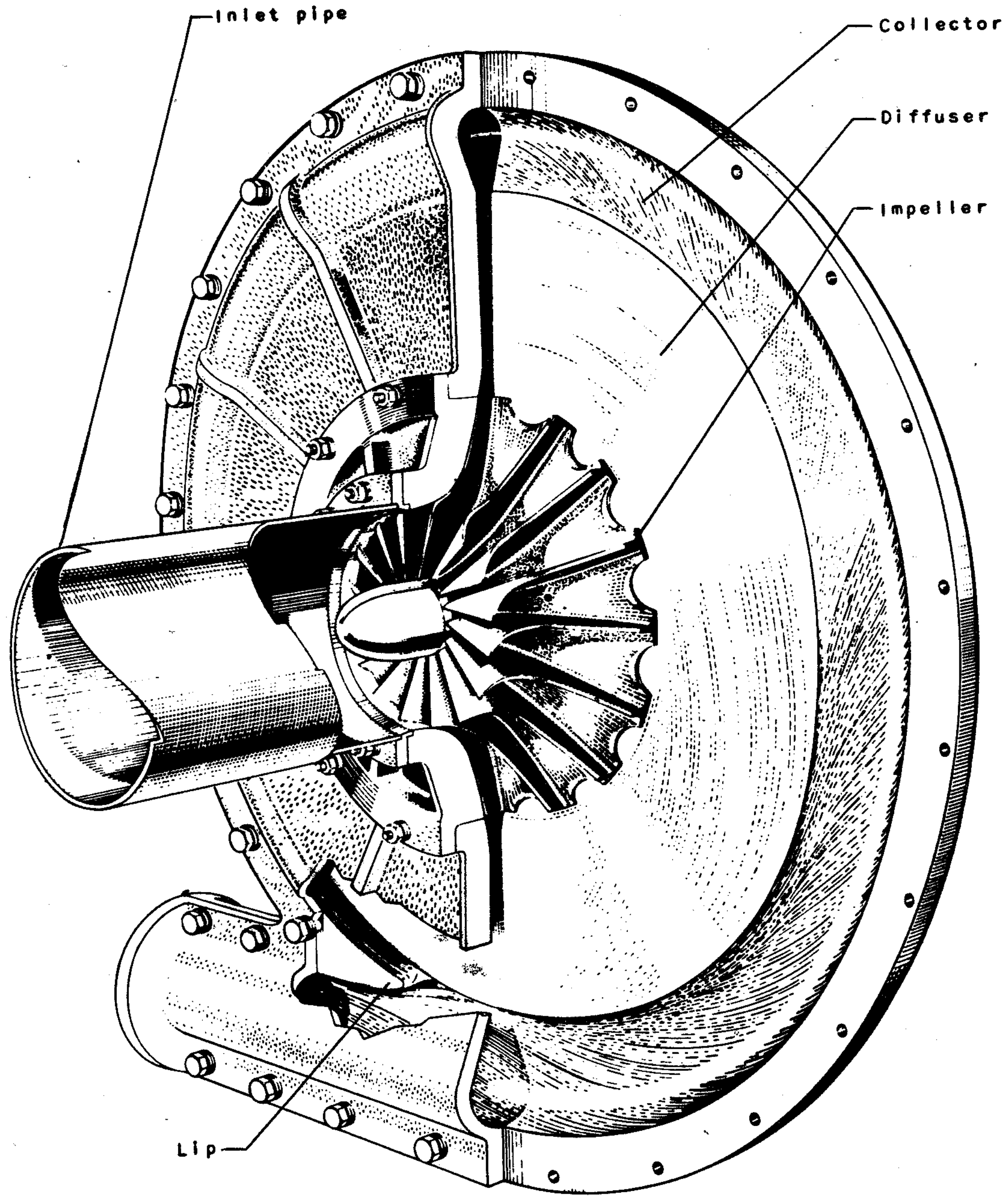

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Figure 1. - Test unit C, designed especially for surge studies.

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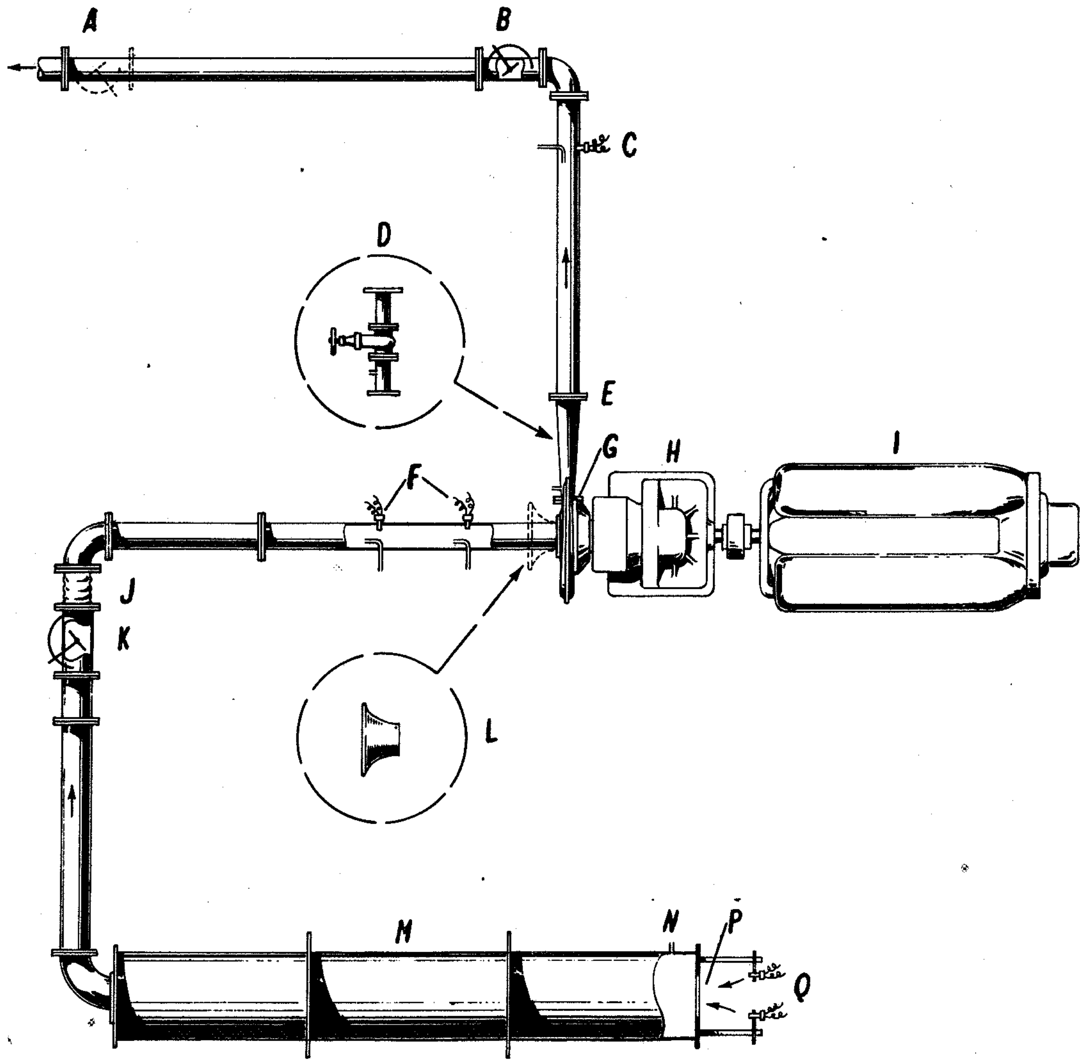

A Alternate outlet throttle position
B Normal outlet throttle position
C Outlet measuring station
D 3-inch gate valve - alternate outlet throttle, replacing E
E Conical expansion section
F Compressor-inlet measuring station
G Compressor unit
H 15:1 speed increaser
I Aircraft-engine drive
J Flexible joint
K Inlet throttle
L Alternate inlet nozzle
M Orifice tank
N Orifice static tap
P Orifice plate
Q Orifice-inlet measuring station

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Figure 2. - Schematic diagram of test rig for test unit C showing various inlet and outlet systems.

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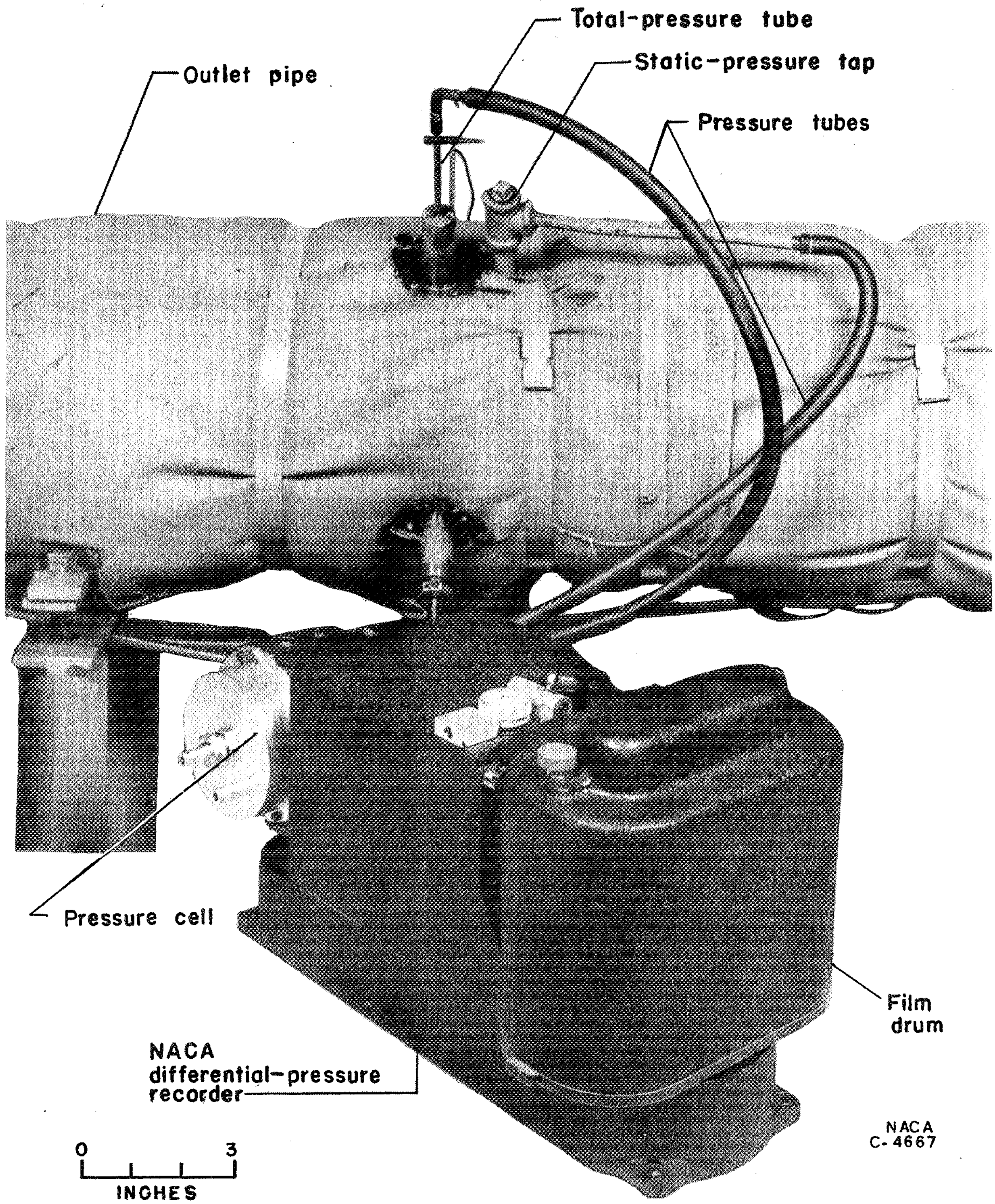


Figure 3. - Set-up used to record variation in outlet velocity pressure.

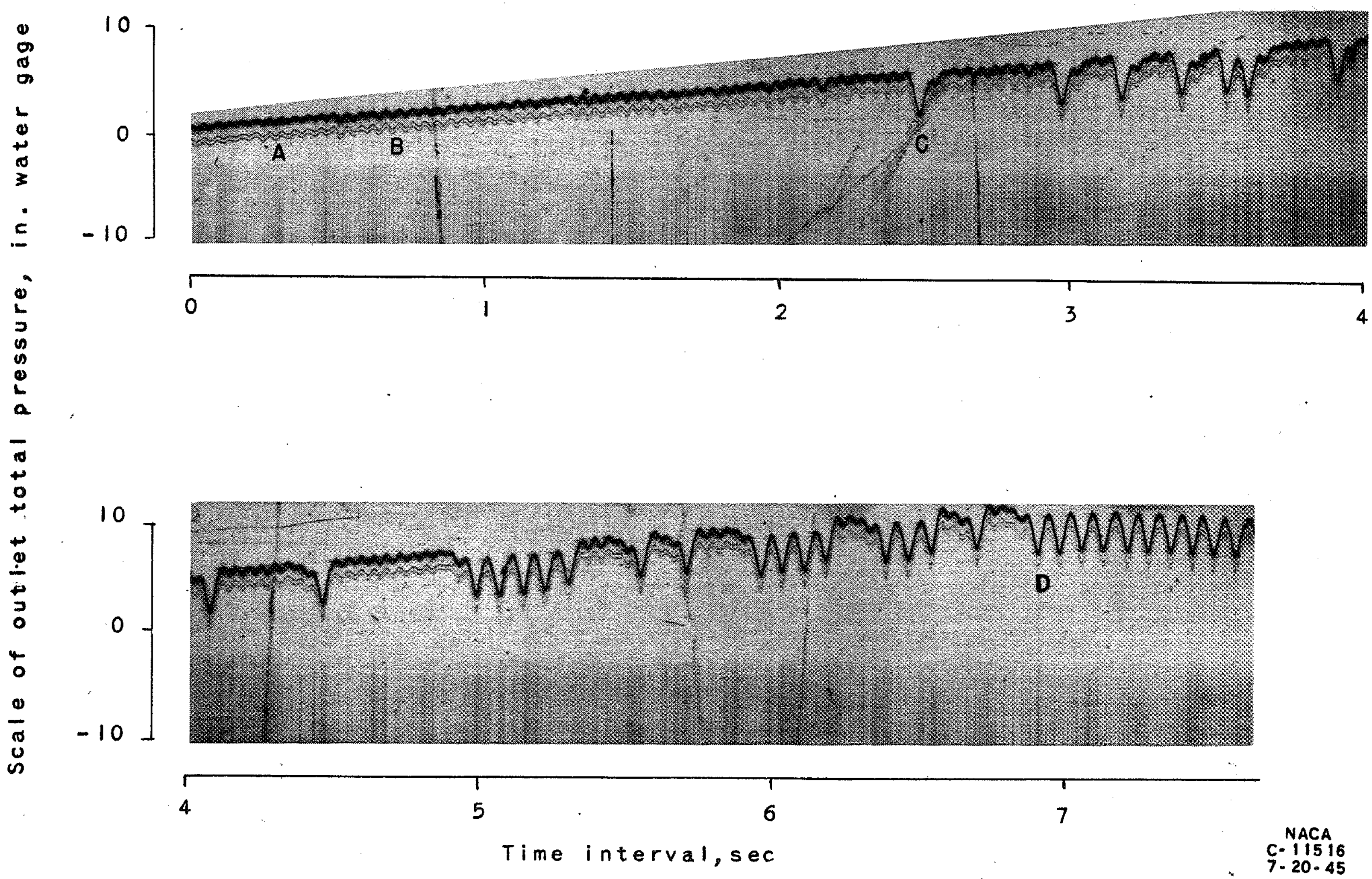


Figure 4. - Outlet total-pressure traces showing development of surge in test unit A. Outlet throttle closed to point of audible surge.

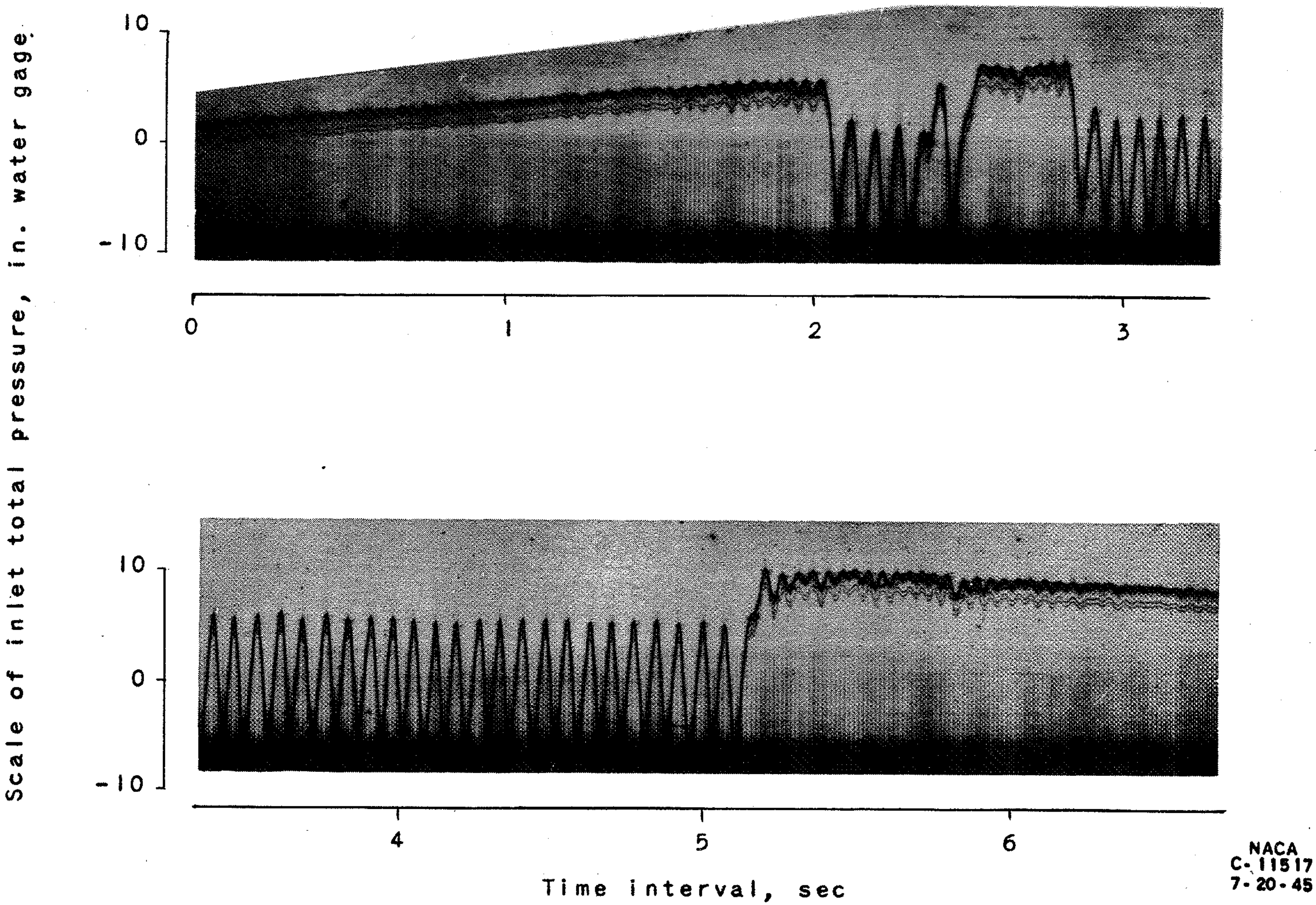
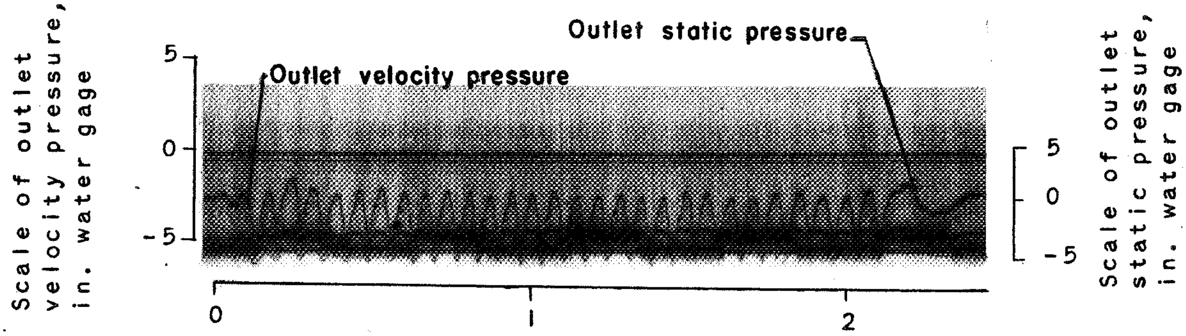
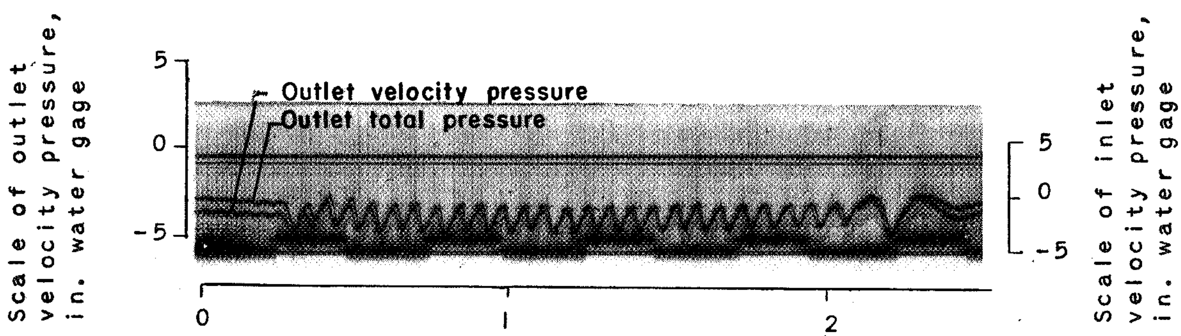


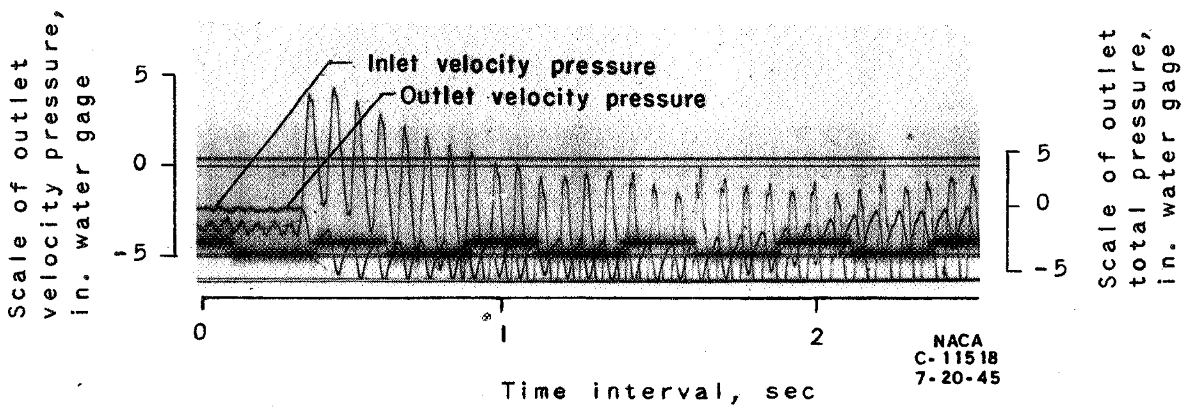
Figure 5. - Inlet total-pressure traces showing development of surge in test unit A. Inlet throttle closed to point of audible surge, then opened.



(a) Comparison of outlet velocity-pressure and outlet static-pressure traces.



(b) Comparison of outlet velocity-pressure and outlet total-pressure traces.



(c) Comparison of outlet velocity-pressure and inlet velocity-pressure traces. (Outlet velocity pressure trace inverted.)

Figure 6. - Surge traces showing phase relation of pulsations at various flow stations of test unit A.

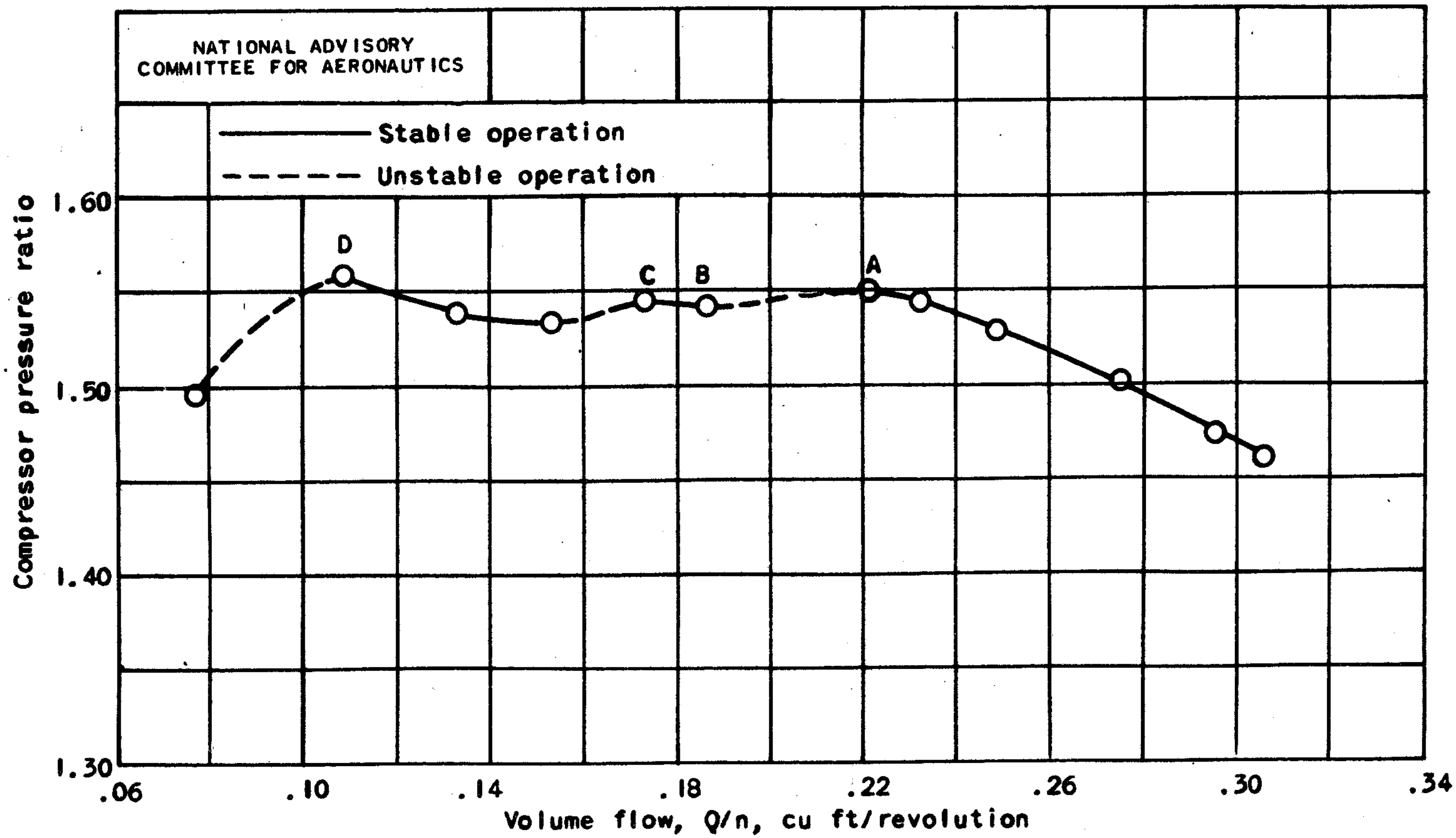


Figure 7. - Variation of compressor pressure ratio with volume flow for test unit B at impeller tip speed of 900 feet per second.

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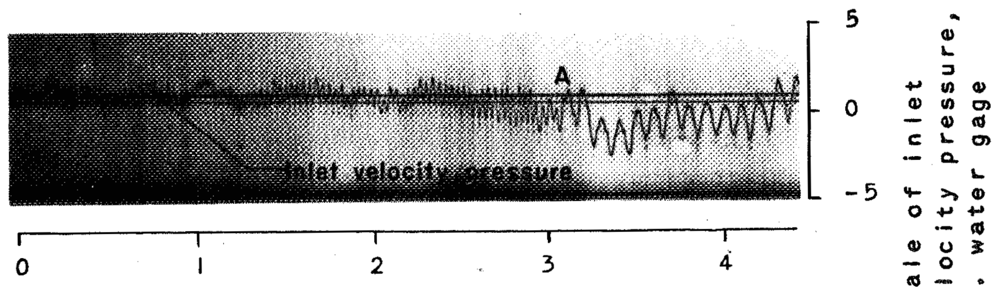
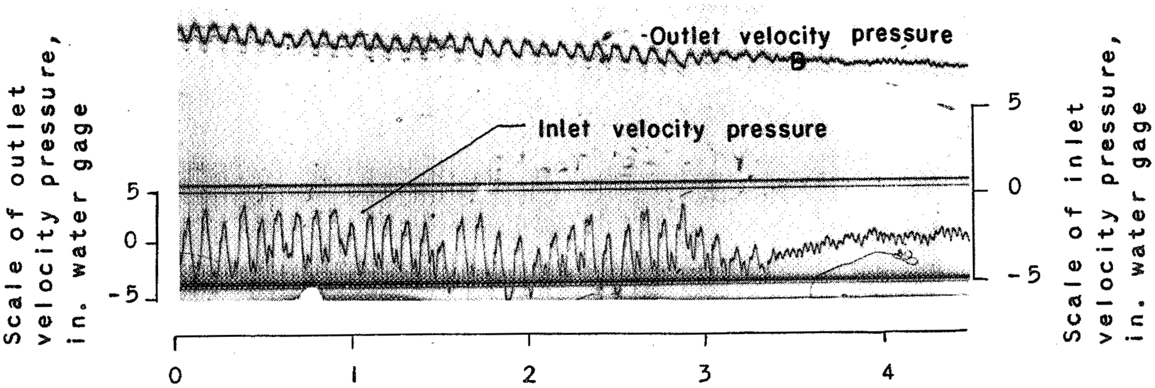
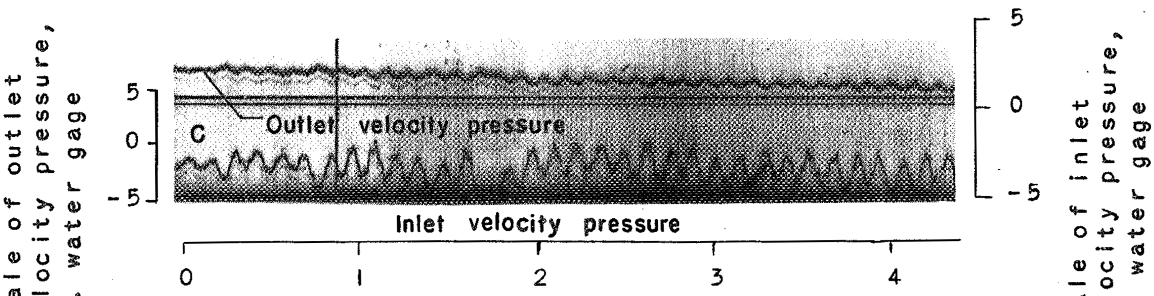


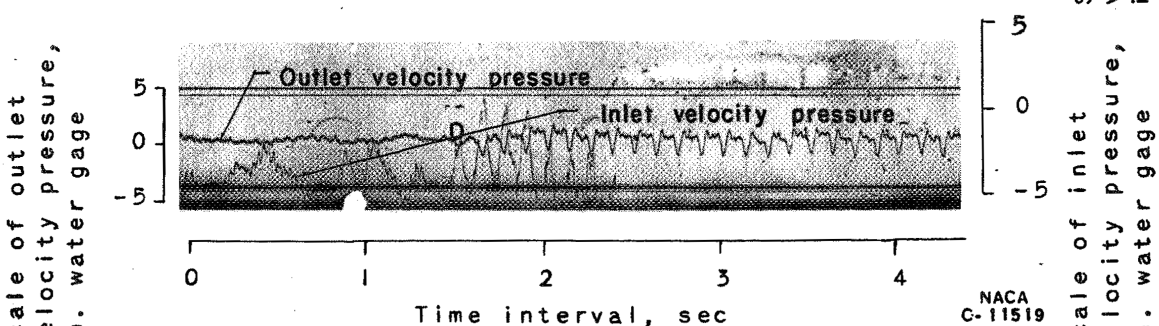
(a) Development of initial surge.



(b) End of initial surge.



(c) Beginning of second surge.



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(d) Development of final surge.

Figure 8. - Development of surge in test unit B as inlet throttle is closed.

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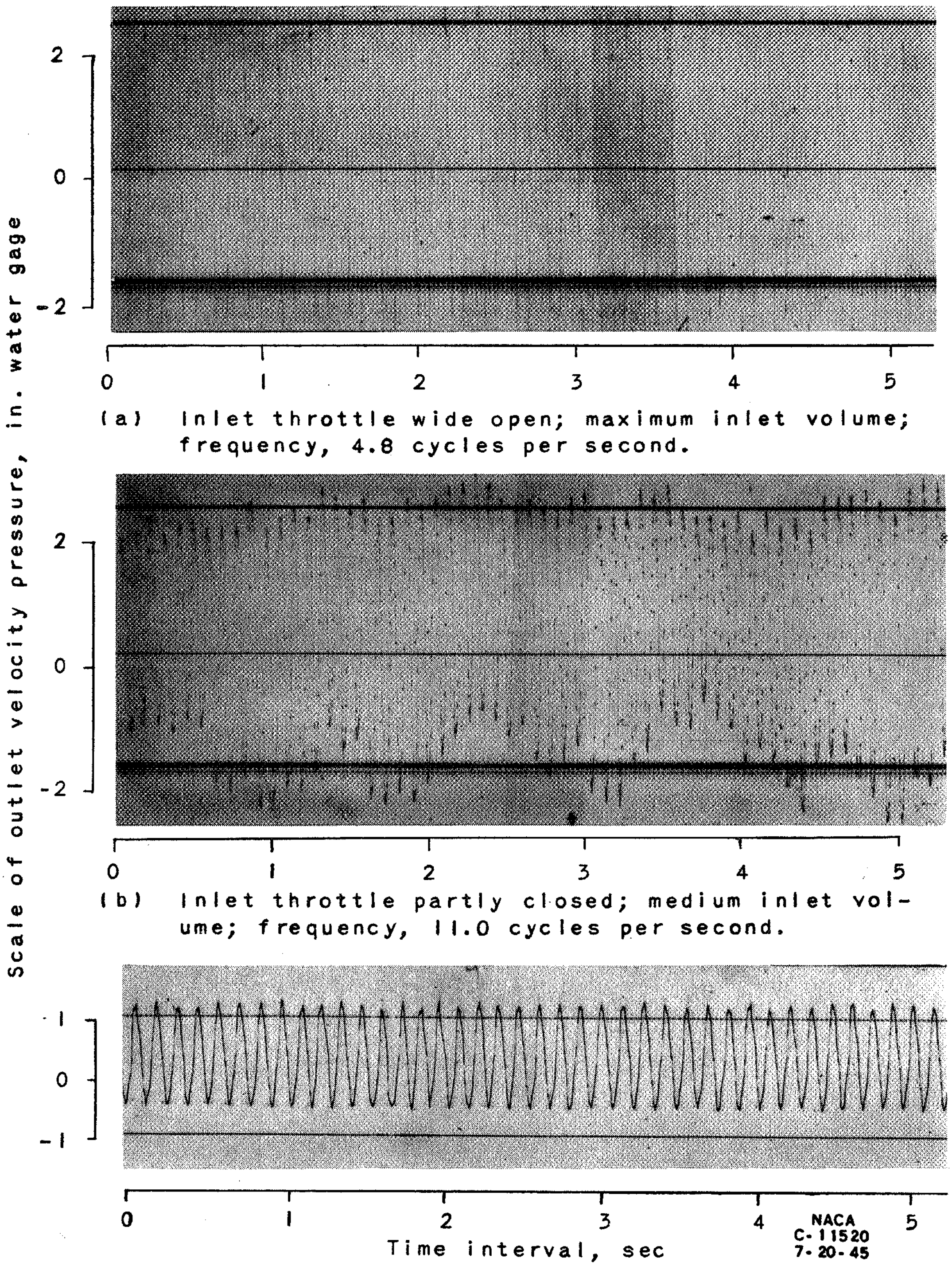

(a) Inlet throttle wide open; maximum inlet volume; frequency, 4.8 cycles per second.

(b) Inlet throttle partly closed; medium inlet volume; frequency, 11.0 cycles per second.

(c) Inlet throttle replaced by nozzle; minimum inlet volume; frequency, 8.0 cycles per second.

Figure 9. - Effect of volume capacity of inlet on frequency and amplitude of surging of test unit C. Outlet throttle, 51 diameters downstream of compressor; impeller tip speed, 960 feet per second.

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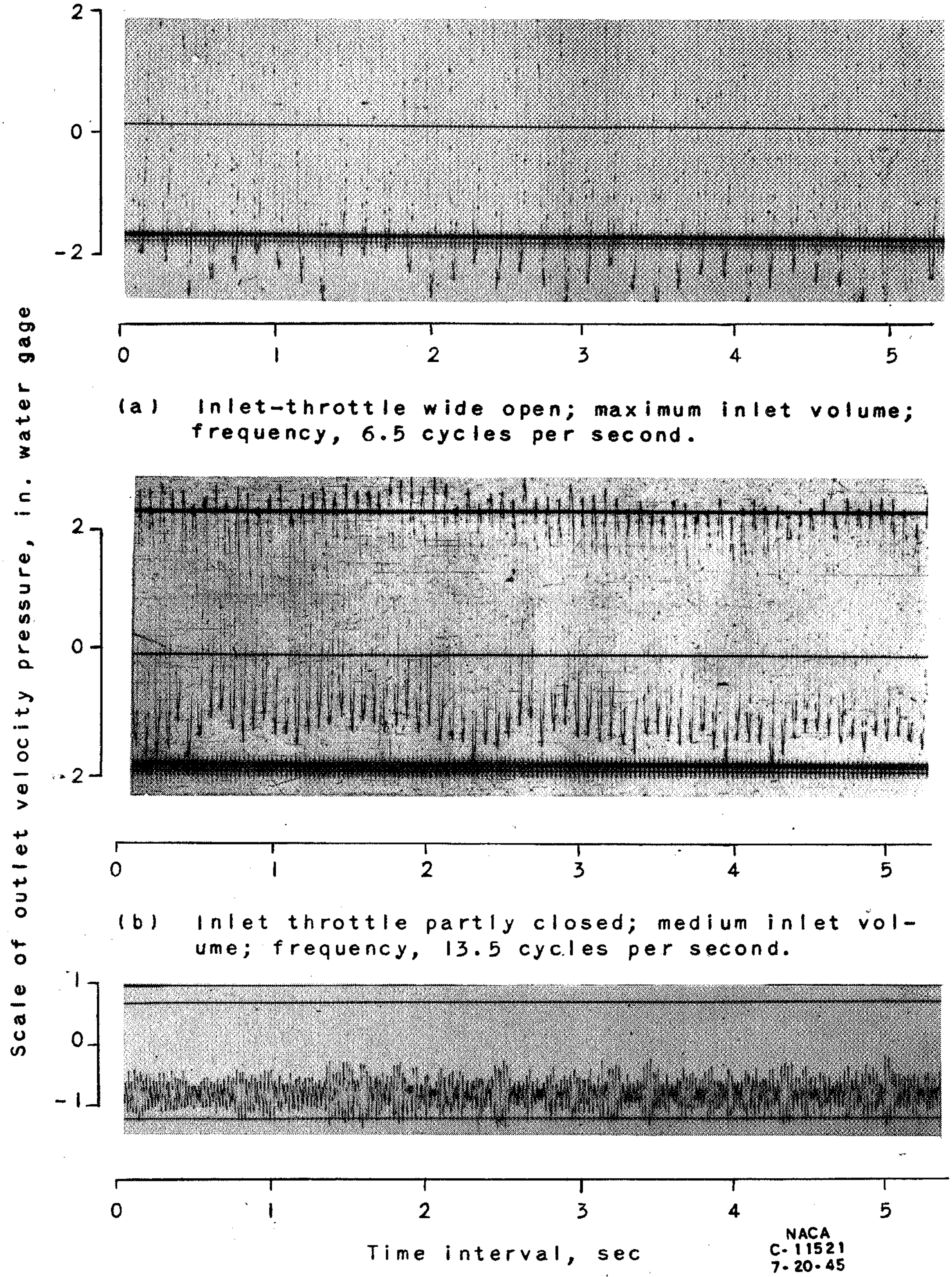
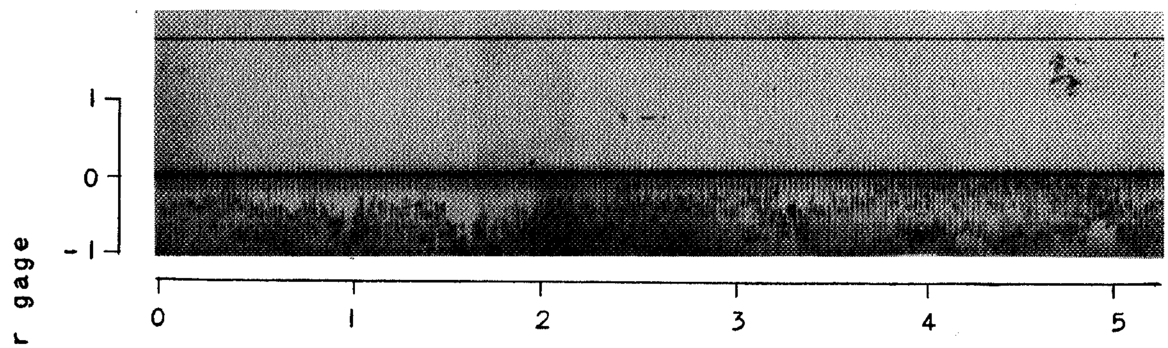


(a) Inlet-throttle wide open; maximum inlet volume; frequency, 6.5 cycles per second.

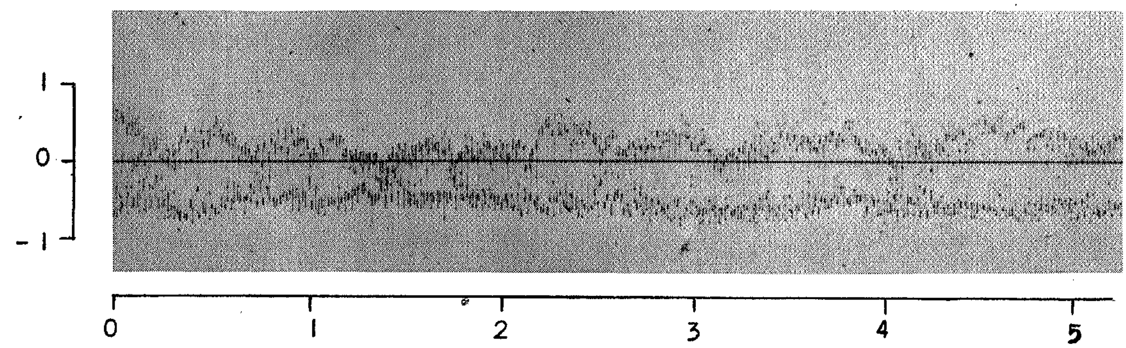
(b) Inlet throttle partly closed; medium inlet volume; frequency, 13.5 cycles per second.

(c) Inlet throttle replaced by nozzle; minimum inlet volume; frequency, 60.0 cycles per second.

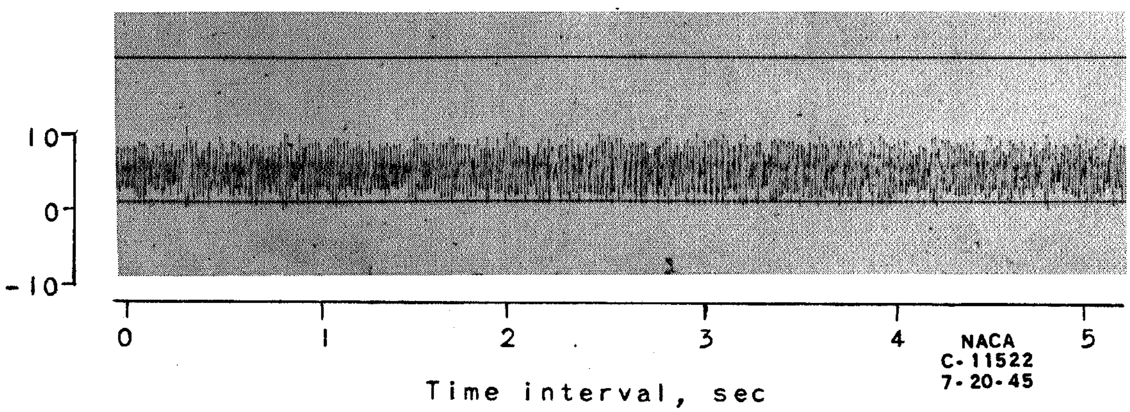
Figure 10. - Effect of volume capacity of inlet on frequency and amplitude of surging of test unit C. Outlet throttle, 30 diameters downstream of compressor; impeller tip speed. 960 feet per second.



(a) Inlet throttle wide open; maximum inlet volume; frequency, 56.0 cycles per second.

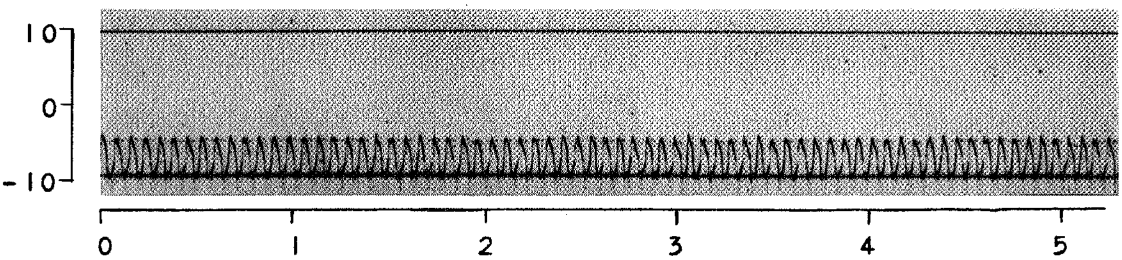


(b) Inlet throttle partly closed; medium inlet volume; frequency, 60.0 cycles per second.

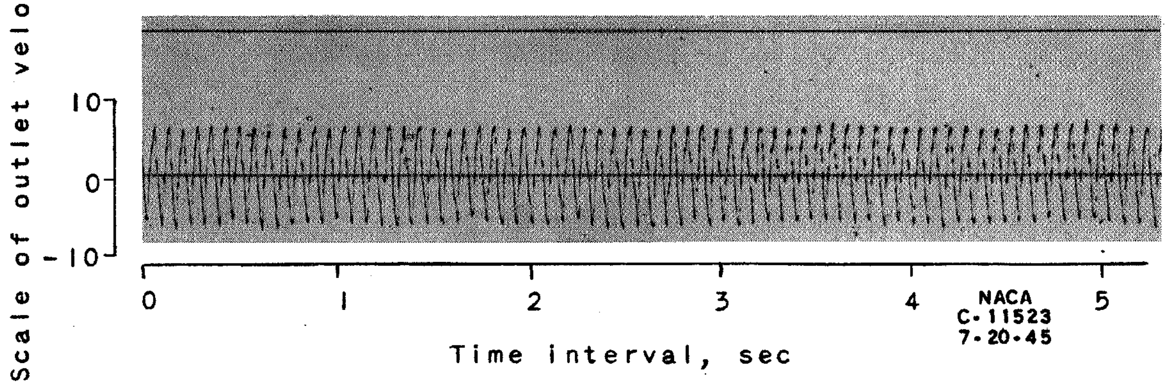


(c) Inlet throttle replaced by nozzle; minimum inlet volume; frequency, 60.0 cycles per second.

Figure 11. - Effect of volume capacity of inlet on frequency and amplitude of surging of test unit C. The 3-inch gate valve at scroll outlet; impeller tip speed, 960 feet per second.



(a) Impeller tip speed, 960 feet per second; frequency, 13.5 cycles per second.



(b) Impeller tip speed, 1200 feet per second; frequency, 13.0 cycles per second.

Figure 12. - Effect of impeller tip speed on frequency and amplitude of surging in test unit C. Outlet total pressure, 10 inches of mercury above atmospheric.

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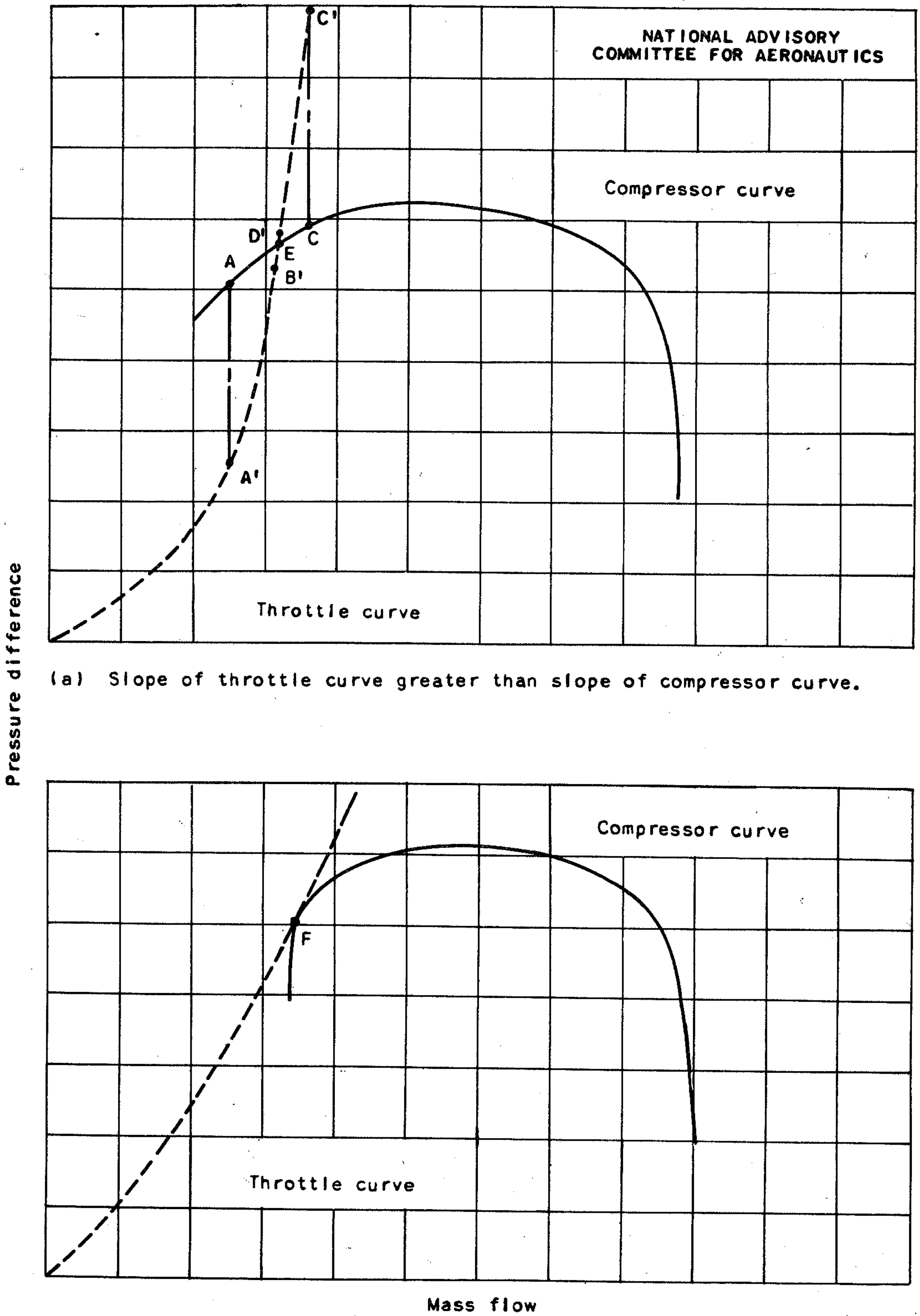

(a) Slope of throttle curve greater than slope of compressor curve.

(b) Slope of throttle curve less than slope of compressor curve.

Figure 13. – Effect of slopes of throttle characteristic curve and compressor characteristic curve on stability of operation.

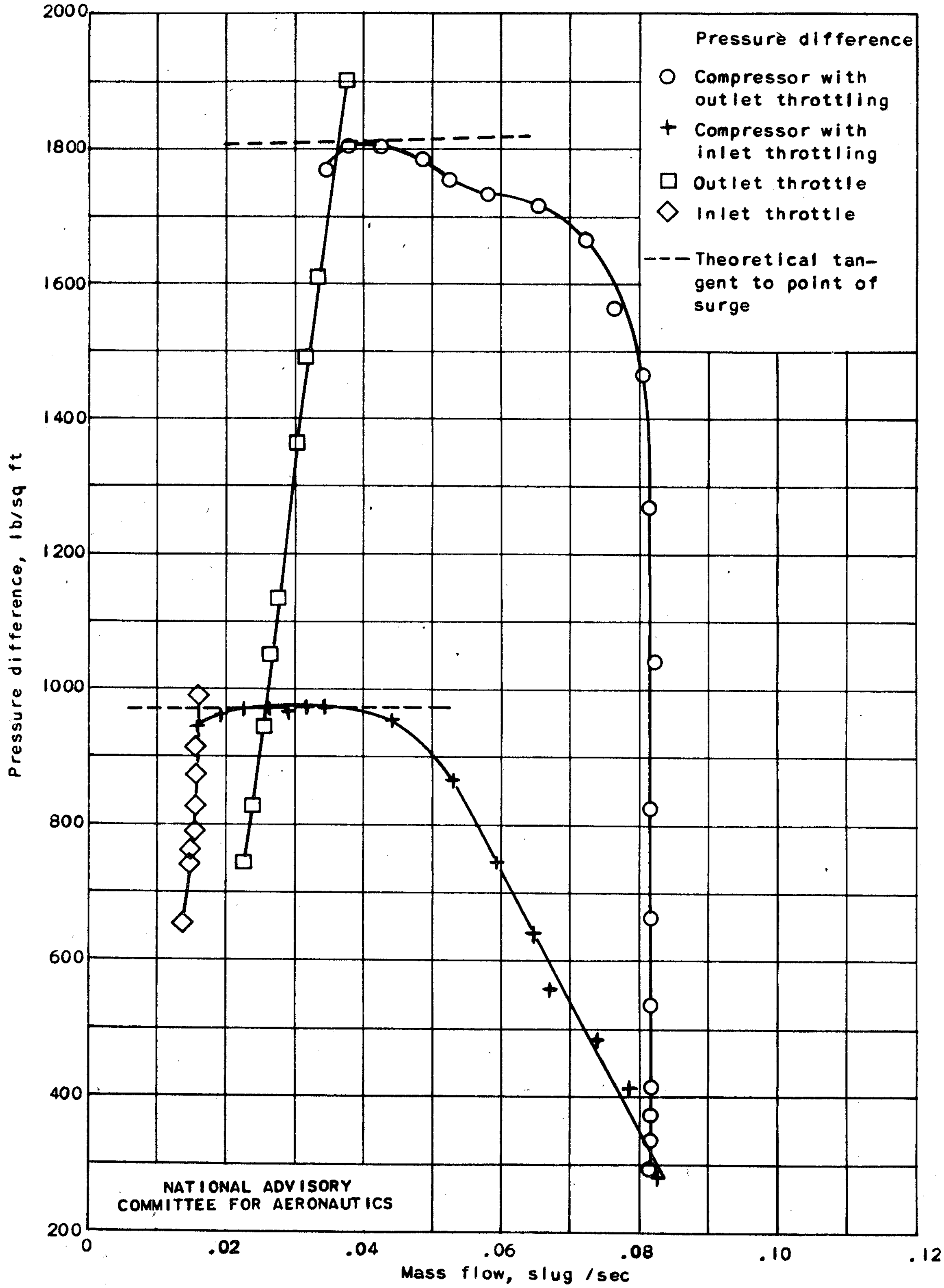

Figure 14. - Relation between throttle characteristic curves and compressor characteristic curves at impeller tip speed of 1080 feet per second.

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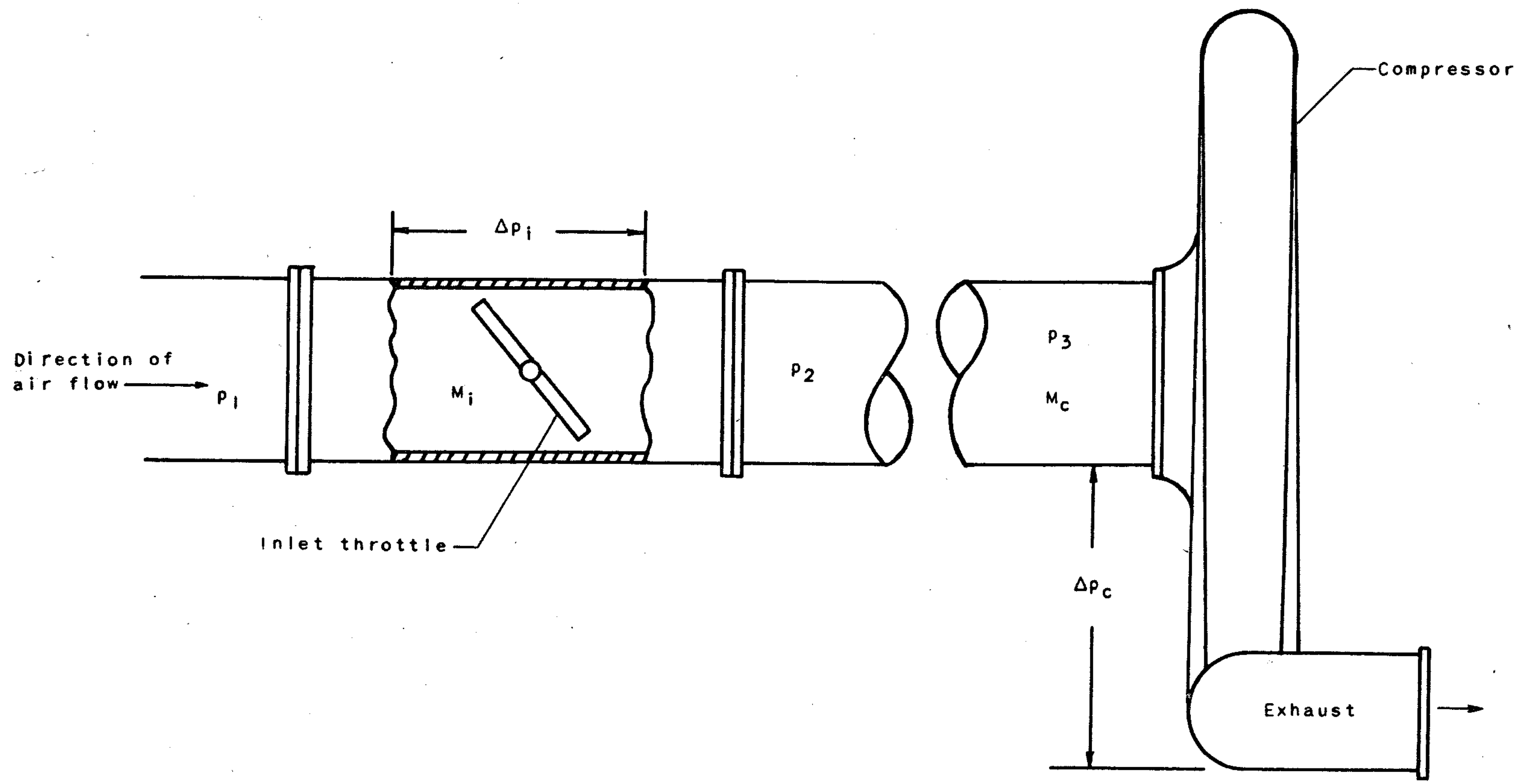

Figure 15. - Schematic diagram of simplified compressor unit.

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