NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1381

NUMERICAL EVALUATION OF MASS-FLOW COEFFICIENT AND ASSOCIATED PARAMETERS FROM WAKE-SURVEY EQUATIONS

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Langley Field, Va.

Washington
August 1947

E R R A T U M

NACA TN No. 1381

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Page 4: Equation (3) should read as follows:

$$\left(\frac{\rho}{\rho_2}\right)^{1/2} = \left(\frac{p}{p_o}\right)^{1/2\gamma}$$

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SUMMARY

A method is presented for the determination by use of charts of mass-flow coefficient and associated flow parameters from pressure surveys in internal-flow systems. For isoenergic flows the point mass-flow coefficient is shown to be an explicit function of the free-stream Mach number and of the static-pressure and total-pressure-loss coefficients at the measurement station. These parameters are easily determined from the test data; hence, their use provides a convenient method of evaluation of the point mass-flow coefficient. The charts presented cover a wide range of these parameters through the complete range of subsonic Mach numbers.

The equations have also been evaluated for flows wherein mechanical or thermal energy is added, such as flows behind radiators or propellers. The fundamental principles may be applied to the measurement of flow from jet-propulsion units; however, under these conditions the mass of the fuel and the change in the value of the ratio of specific heats must be considered.

INTRODUCTION

In the determination of the characteristics and flow quantities in aircraft internal systems, total- and static-pressure surveys have been used extensively. The precise evaluation of the internal drag, the mass-flow, and the flow parameters associated with mass flow must include consideration of the variation in air density. Because this variation in the density complicates the solution of the equations involved, a large number of steps is required for each point computed. A method for the numerical evaluation of the wake-survey equations, by means of which the values of point drag coefficient can be easily obtained from tables or charts, is presented in reference 1. The items necessary for the determination of point drag coefficient from these charts are the measured values of static-pressure coefficient, total-pressure-loss coefficient, and free-stream Mach number.

The present paper presents a method with charts by means of which the mass-flow coefficient, inlet-velocity ratio, and mass-flow rate can be quickly evaluated in a few steps by use of measured values of static-pressure coefficient, total-pressure-loss coefficient, and free-stream Mach number. The equations have been evaluated for both isoenergic flows and flows wherein mechanical or thermal energy is added. The fundamental principles may be applied to the measurement of flow from jet-propulsion units; however, under these conditions the mass of the fuel and the change in the value of the ratio of specific heats must be considered.

SYMBOLS

a	speed of sound, feet per second
V	velocity, feet per second
M	Mach number (V/a)
ρ	mass density, slugs per cubic foot
q	dynamic pressure, pounds per square foot $\left(\frac{1}{2}\rho V^2\right)$
p	static pressure, pounds per square foot
P	static-pressure coefficient $\left(\frac{p - p_o}{q_o}\right)$
H	total pressure, pounds per square foot
ΔH	total-pressure loss $\left(H_o - H\right)$
$\frac{\Delta H}{q_o}$	total-pressure-loss coefficient
A	area, square feet
F	frontal area, square feet
m	mass-flow rate, slugs per second (ρAV)
C	mass-flow coefficient $\left(\frac{m}{\rho_o F V_o}\right)$
c'	point mass-flow coefficient $\left(\frac{\rho V}{\rho_o V_o}\right)$

$\frac{V_1}{V_o}$ inlet-velocity ratio

T static temperature, ^oF absolute

T' stagnation temperature, ^oF absolute

$\Delta T'$ stagnation-temperature rise, ^oF $\left(T' - T'_o\right)$

γ ratio of specific heats; for air $\gamma = 1.40$

c_p specific heat at constant pressure; for air $c_p = 6010$ foot-pounds per slug ^oF

E energy input, foot-pounds per second

K energy-input factor $\left(\frac{E}{c_p m T_o}\right)$

F_c compressibility factor $\left(\frac{H - p}{q}\right)$

Subscripts:

o free-stream station

1 entrance station

2 station in wake where $p_2 = p_o$

K with energy added

Symbol without subscript indicates local value at measurement station.

THEORY AND METHODS

Mass-Flow Coefficient

Basic relations.- The mass-flow coefficient is defined as

$$C = \frac{m}{\rho_o F V_o}$$

$$= \frac{1}{F}\int_A \left(\frac{\rho}{\rho_o}\right) \left(\frac{V}{V_o}\right) dA$$

$$= \frac{1}{F}\int_A \left(\frac{\rho}{\rho_2}\right)^{1/2}\left(\frac{\rho_2}{\rho_o}\right)^{1/2}\left(\frac{q}{q_o}\right)^{1/2} dA \qquad (1)$$

For convenience in discussing the solution of equation (1), the integrand is defined as the point mass-flow coefficient c':

$$c' = \left(\frac{\rho}{\rho_o}\right)\left(\frac{V}{V_o}\right)$$

$$= \left(\frac{\rho}{\rho_2}\right)^{1/2}\left(\frac{\rho_2}{\rho_o}\right)^{1/2}\left(\frac{q}{q_o}\right)^{1/2} \qquad (2)$$

The numerical solution of equation (1) requires an extensive computation for direct use. The terms of this equation can be expressed by the following relations:

$$\left.\begin{aligned} p_2 &= p_o \\ H_2 &= H \end{aligned}\right\} \text{(by definition)}$$

$$\left(\frac{\rho}{\rho_2}\right)^{1/2} = \left(\frac{p}{p_o}\right)^{1/2} \qquad (3)$$

$$\left(\frac{q}{q_o}\right)^{1/2} = \left(\frac{H - p}{H_o - p_o}\right)^{1/2}\left(\frac{F_{co}}{F_c}\right)^{1/2} \qquad (4)$$

and from the general energy equation, as shown in appendix B of reference 2

$$\left(\frac{\rho_2}{\rho_o}\right)^{1/2} = \left(\frac{1 + 0.20M_o^{\,2}(q_2/q_o)}{1 + 0.20M_o^{\,2} + K}\right)^{1/2} \qquad (5)$$

where K is the energy-input factor

$$K = \frac{E}{c_p m T_o} \qquad (6)$$

Examination of equations (2) to (5) shows that for isoenergic flows, the point mass-flow coefficient is an explicit function of the free-stream Mach number M_o, the static-pressure coefficient P, and the total-pressure-loss coefficient $\Delta H/q_o$ at the measurement station. For flows wherein energy is added, the additional term K in equation (5) must be evaluated. The pressure coefficients used are merely an expression of the measured data in coefficient form. These parameters can be easily determined from the test data; hence, their use provides a convenient method for evaluation of the point mass-flow coefficient. Although the expression for the point mass-flow coefficient in terms of these parameters is complicated, the coefficient can be readily computed for given values of these parameters.

Isoenergic flows.- Values of point mass-flow coefficient have been computed for a wide range of values of P and $\Delta H/q_o$ for given values of free-stream Mach numbers and are plotted in figure 1. (See appendix B of reference 1 for details of computing procedure.) The range of total-pressure-loss coefficient has been extended into the negative region (which indicates an increase in total pressure) to permit evaluation under conditions of net low energy input approaching isoenergic flow. For conditions under which energy is added, a correction factor must be applied to values of point mass-flow coefficient read from figure 1.

The points corresponding to the attainment of sonic velocity at the measuring station have been designated by arrows on the curves for the various values of static-pressure coefficient. At values of total-pressure-loss coefficient less than those indicated by the arrows, supersonic flow exists. In order to avoid congestion the curves have not been extended into the supersonic region. The equations presented are applicable to supersonic flow; however, it should be noted that special methods may be necessary to obtain total- and static-pressure surveys in supersonic flow.

The plots of figure 1 are for values of Mach number in increments of 0.10. Figure 2 presents the variation of point mass-flow coefficient with Mach number for various values of P and $\Delta H/q$ and shows that for intermediate values of the Mach number, a linear interpolation can be used with sufficient accuracy for most purposes.

Flows wherein energy is added.- Equation (1) is correct for the evaluation of the mass-flow coefficient for flows to which energy has been added, such as flows through radiators or propellers. The evaluation of the density ratio $\left(\frac{\rho_2}{\rho_o}\right)^{1/2}$ (equation (5)) under these conditions includes a term (equation (6)) which is a function of the energy input. The remaining terms $\left(\frac{\rho}{\rho_2}\right)^{1/2}$ and $\left(\frac{q}{q_o}\right)^{1/2}$ (equations (3) and (4)) are unaffected by energy addition. The expression for the ratio between the point mass-flow coefficient with and without addition of energy is then

$$\frac{c'_K}{c'} = \frac{\left(\frac{\rho_2}{\rho_o}\right)_K^{1/2}}{\left(\frac{\rho_2}{\rho_o}\right)^{1/2}}$$

$$= \left(\frac{1 + 0.20M_o^2}{1 + 0.20M_o^2 + K}\right)^{1/2} \tag{7}$$

This ratio is thus a function only of the free-stream Mach number M_o and the energy-input factor K. A plot of this ratio is given for a range of values of K for various values of M_o in figure 3. The value of point mass-flow coefficient for the energy-added condition can be found by multiplying the value of point mass-flow coefficient obtained from figure 1 by the appropriate value of the energy-addition factor obtained from figure 3.

The energy-input parameter K can be calculated directly for flow conditions where the energy input is known; the mass flow is measureable and both are uniform across the survey plane. For flow conditions where the energy input is not known and where the energy input and elemental mass flow are not uniform, an evaluation of the energy parameter can be made experimentally with relative ease.

As is shown in reference 1, the energy parameter (equation (6)) can be written

$$K = \frac{T' - T'_o}{T_o}$$

$$= \frac{\Delta T'}{T_o}$$

where the prime refers to stagnation conditions. The energy parameter is, then, the ratio of the stagnation-temperature rise to the absolute stream-static temperature. Reference 1 discusses methods for measuring these items.

Integration techniques.- Evaluation of the total mass-flow coefficient requires the integration of the point mass-flow-coefficient profile. Inasmuch as the evaluation of c' is independent of the integration process, choice of integration technique can be made from consideration of the wake profile and the manner in which the wake is surveyed. By use of values of point mass-flow coefficient obtained from the charts presented herein, the value of total mass-flow coefficient is

$$C = \frac{1}{F}\int_A c'dA \tag{8}$$

where F is the area upon which the mass-flow coefficient is based (in this case taken as the frontal area).

Mass Flow

The mass-flow rate can be obtained simply from the mass-flow coefficient:

$$m = \left(\frac{m}{\rho_o F V_o}\right)\rho_o F V_o$$

$$= C\rho_o F V_o \tag{9}$$

The items ρ_o, F, and V_o are normally known for given test or operating conditions.

Inlet-Velocity Ratio

The inlet-velocity ratio V_1/V_o can be evaluated from the mass-flow coefficient $\frac{m}{\rho_o F V_o}$, the free-stream Mach number M_o, and the inlet area A_1. From Bernoulli's equation,

$$\frac{{V_o}^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_o}{\rho_o} = \frac{{V_1}^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_1}{\rho_1}$$

the continuity equation

$$\rho_o A_o V_o = \rho_1 A_1 V_1$$

the relation

$$a = \sqrt{\frac{\gamma p}{\rho}}$$

and the isentropic relation

$$\frac{p_o}{p_1} = \left(\frac{\rho_o}{\rho_1}\right)^{\gamma}$$

the following relation is obtained:

$$1 + \frac{2}{(\gamma - 1)M_o^{2}} = \left(\frac{V_1}{V_o}\right)^{2} + \frac{2}{(\gamma - 1)M_o^{2}}\left(\frac{A_1}{A_o}\right)^{1 - \gamma}\left(\frac{V_1}{V_o}\right)^{1 - \gamma} \qquad (10)$$

The solution of this equation is presented for fixed values of free-stream Mach number in figure 4. The ratio A_o/A_1 is the mass-flow coefficient based on the entrance area A_1 and is obtained from the mass-flow coefficient as follows:

$$\frac{A_o}{A_1} = \frac{m}{\rho_o A_1 V_o}$$

$$= C\frac{F}{A_1} \qquad (11)$$

where F and A_1 are known areas dependent upon the geometry of the installation. Using values of A_o/A_1 thus obtained permits the corresponding value of inlet-velocity ratio V_1/V_o to be read from figure 4 at any value of M_o.

It should be noted that equation (10) includes the assumption of isentropic one-dimensional flow between the two stations involved. Isentropic flow can ordinarily be expected between the free stream and the entrance of an air inlet located at the leading edge of a body, such as a nose inlet or wing inlet. However, for entrance conditions where appreciable energy losses exist (due to uncontrolled boundary layer, for example), equation (10) is not strictly applicable because the density does not vary according to the isentropic relation. Also, if under such conditions the velocity distribution at the inlet becomes nonuniform, the parameter V_1/V_o tends to lose its significance. For this case, measurement of flow conditions at the entrance may be necessary.

Duct cross-sectional areas at stations other than entrance can be used in equation (10) (and in the application of fig. 4) to obtain the velocity at such stations, provided that the flow between the free stream and the station under consideration is approximately isentropic.

NUMERICAL EXAMPLES

Isoenergic flow.- The use of the charts is illustrated herein by means of examples. The following conditions for the flow through an airplane duct are assumed:

$$M_o = 0.70$$

$$\rho_o = 0.0020 \text{ slug per cubic foot}$$

$$A_1 = 1 \text{ square foot}$$

$$V_o = 750 \text{ feet per second}$$

$$F = 3 \text{ square feet}$$

At the measuring station

$$A = 2.5 \text{ square feet}$$

$$P = 0.75$$

$$\frac{\Delta H}{q_o} = 0.25$$

From figure 1 the value of point mass-flow coefficient is obtained:

$$c' = \frac{\rho V}{\rho_o V_o} = 0.383$$

If, for the purposes of the example, this value is assumed to represent the average value of point mass-flow coefficient in the duct, the total mass-flow coefficient becomes

$$\begin{aligned} C = \frac{m}{\rho_o F V_o} &= c' \frac{A}{F} \\ &= 0.383 \times \frac{2.5}{3} \\ &= 0.319 \end{aligned}$$

The mass flow m is

$$m = 0.319\rho_o F V_o$$

$$= 0.319 \times 0.002 \times 750 \times 3$$

$$= 1.436 \text{ slugs per second}$$

The inlet-velocity ratio is obtained from the mass-flow coefficient C in two steps:

$$\frac{A_o}{A_1} = \frac{m}{\rho_o F V_o} \frac{F}{A_1}$$

$$= 0.319 \times \frac{3}{1}$$

$$= 0.957$$

and from figure 4, for the value of A_o/A_1,

$$\frac{V_1}{V_o} = 0.922$$

Flow wherein energy has been added.- Assume, in addition to the conditions in the preceding example, that

$$\Delta T' = 20^\circ \text{ F}$$

$$T_o = 500^\circ \text{ F absolute}$$

Then

$$K = \frac{20}{500}$$

$$= 0.04$$

From figure 3 for this value of K

$$\frac{c'_K}{c'} = 0.982$$

The point mass-flow coefficient is

$$c'_K = 0.383 \times 0.982$$

$$= 0.376$$

The procedure for determining the remaining items is the same as for the isoenergic case calculated previously. Then,

$$C = 0.313$$

$$m = 1.410 \text{ slugs per second}$$

$$\frac{A_o}{A_1} = 0.939$$

$$\frac{V_1}{V_o} = 0.893$$

The internal drag can be obtained from charts or tables in reference 1 by using the same initial values of the parameters.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., May 13, 1947

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2. Becker, John V., and Baals, Donald D.: Analysis of Heat and Compressibility Effects in the Internal Flow Systems and High-Speed Tests of a Ram-Jet System. NACA Rep. No. 773, 1943.

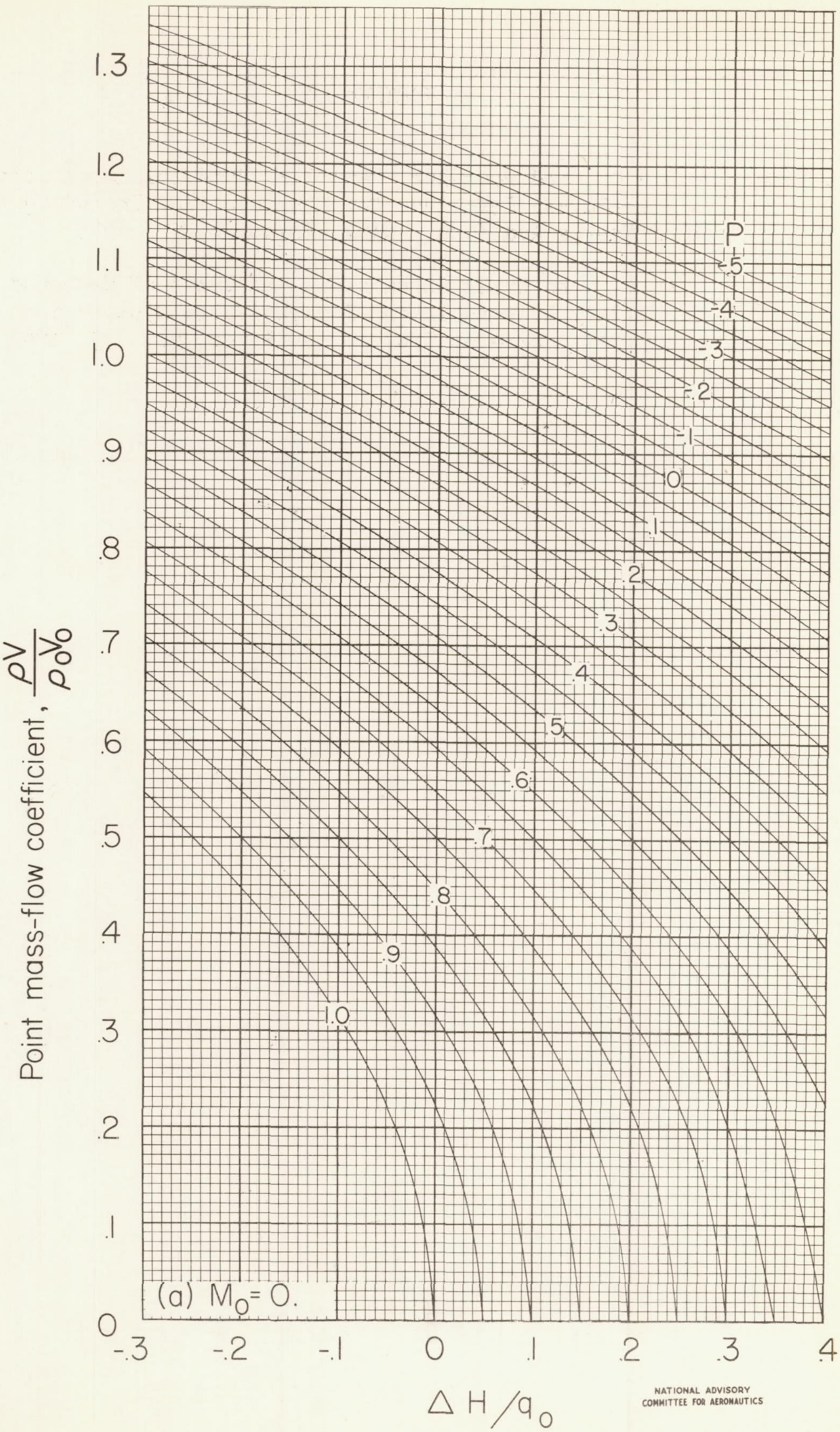
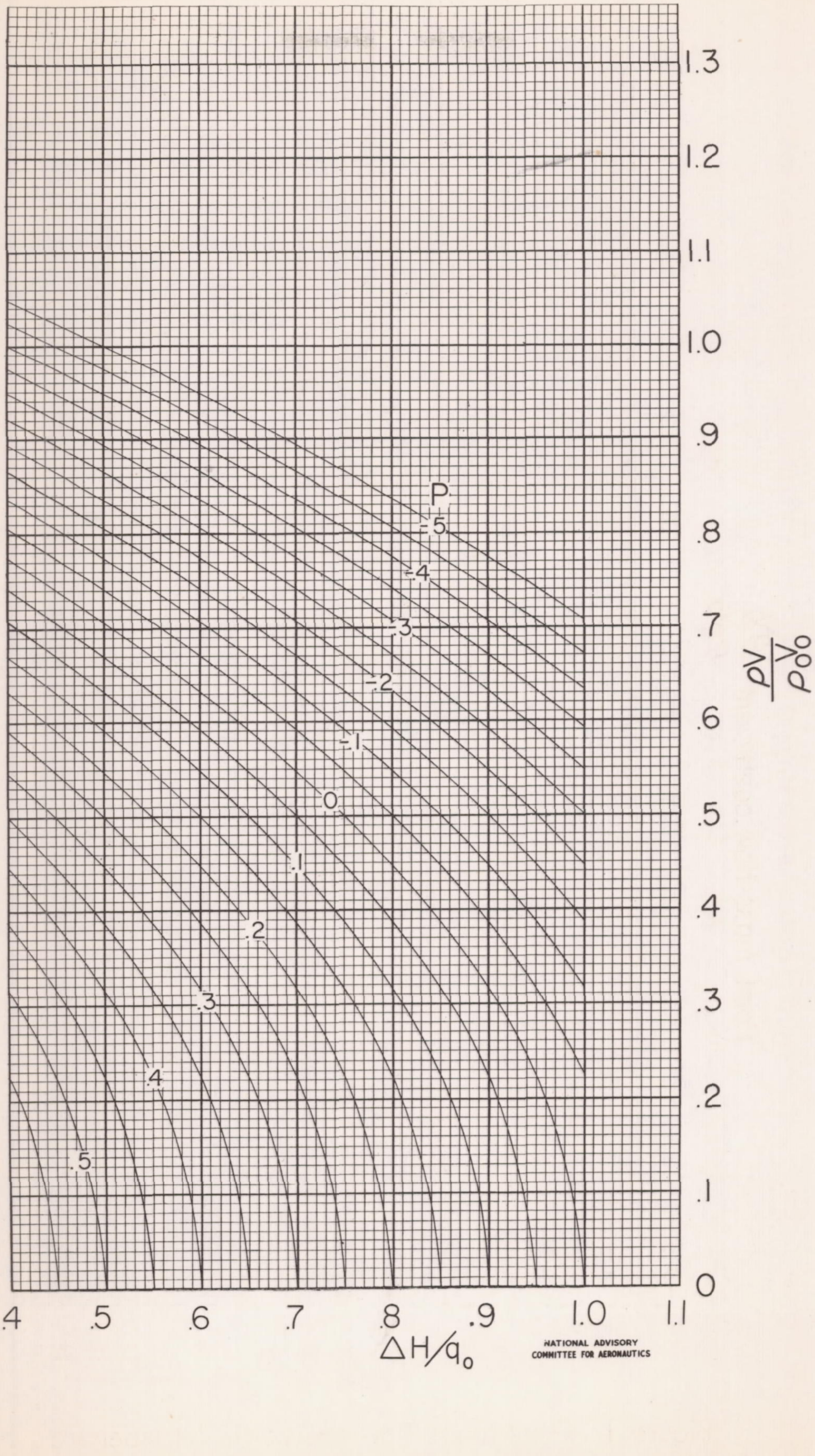


Figure 1.–Point mass-flow coefficient for isoenergic flow.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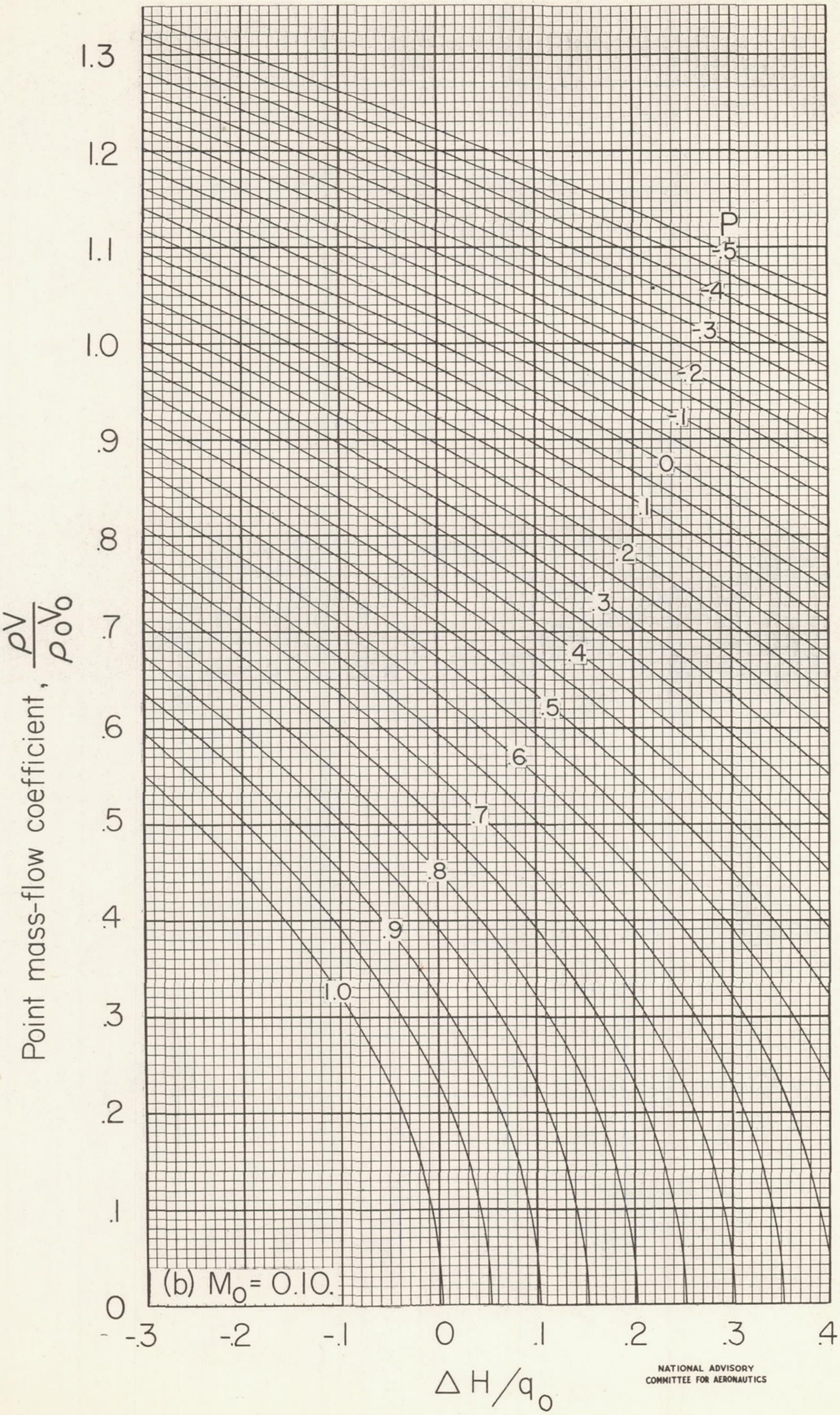

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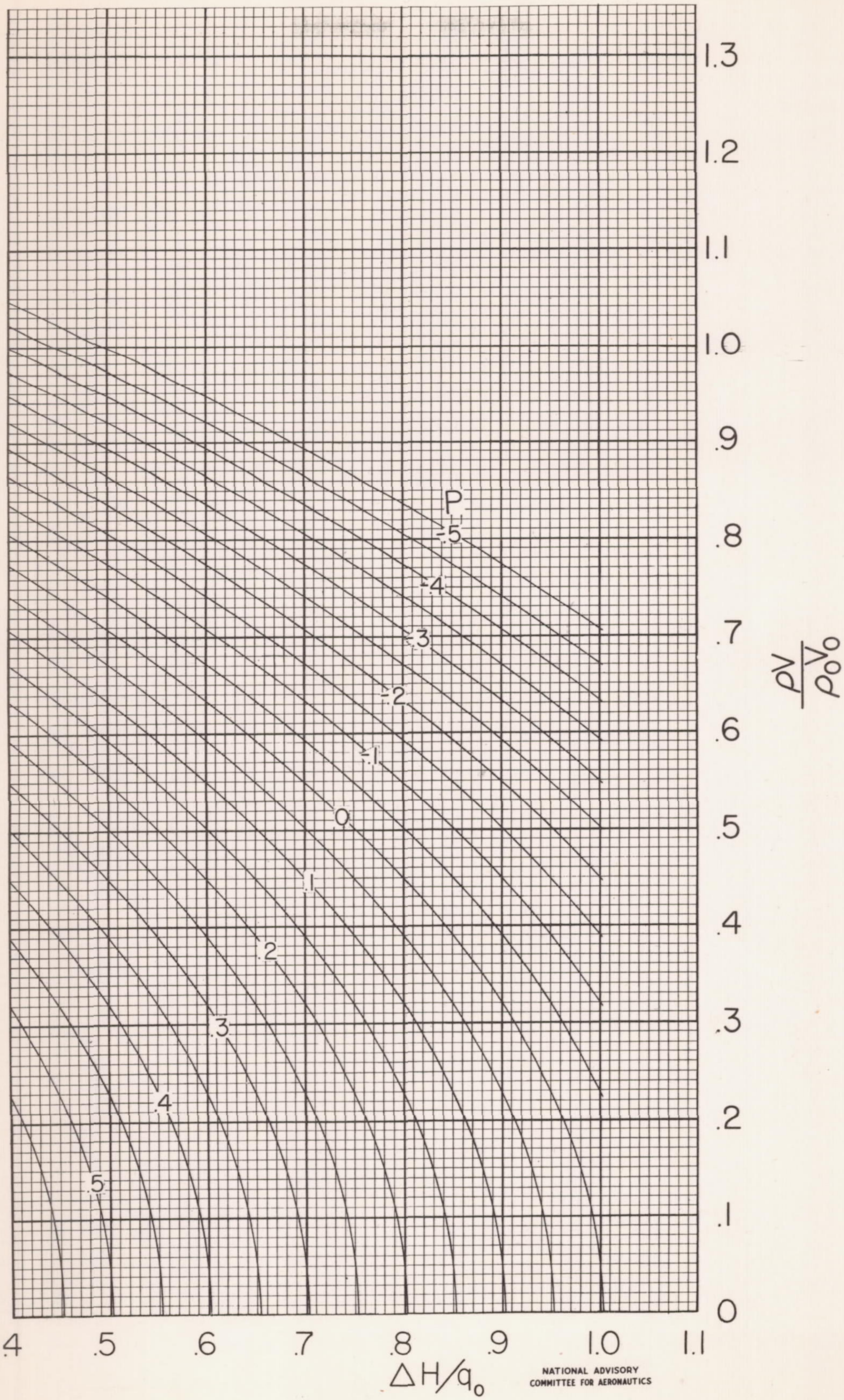

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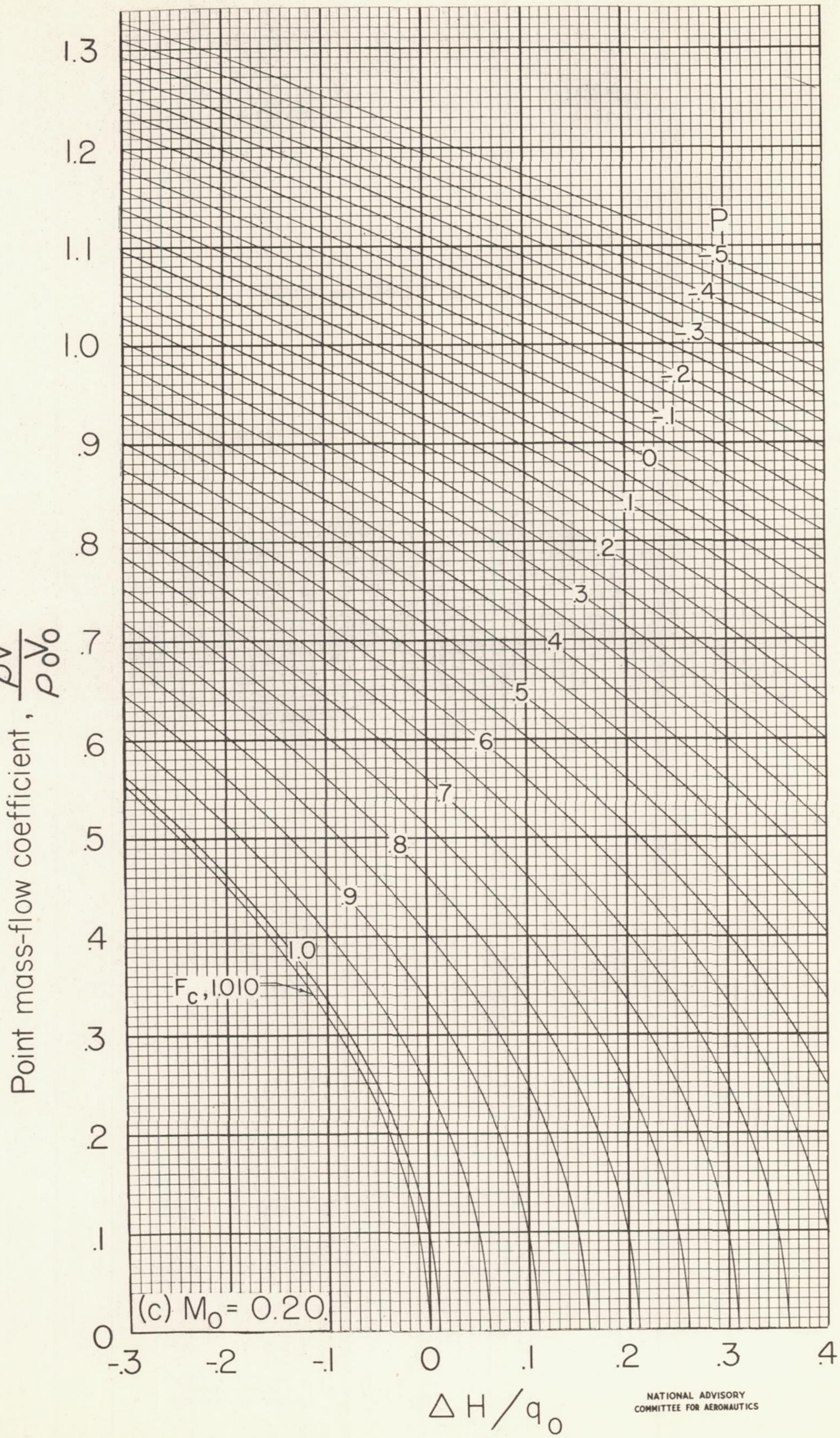

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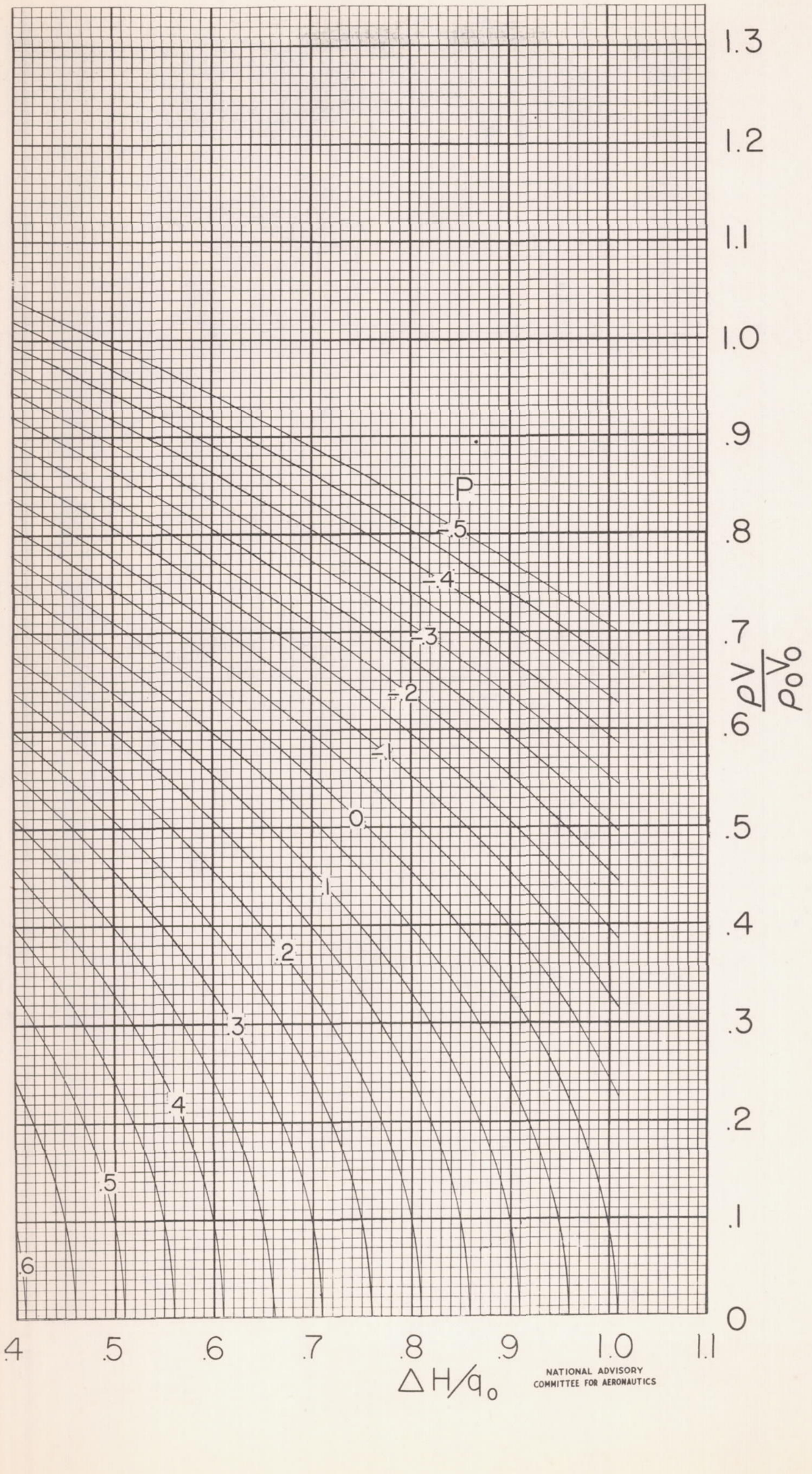

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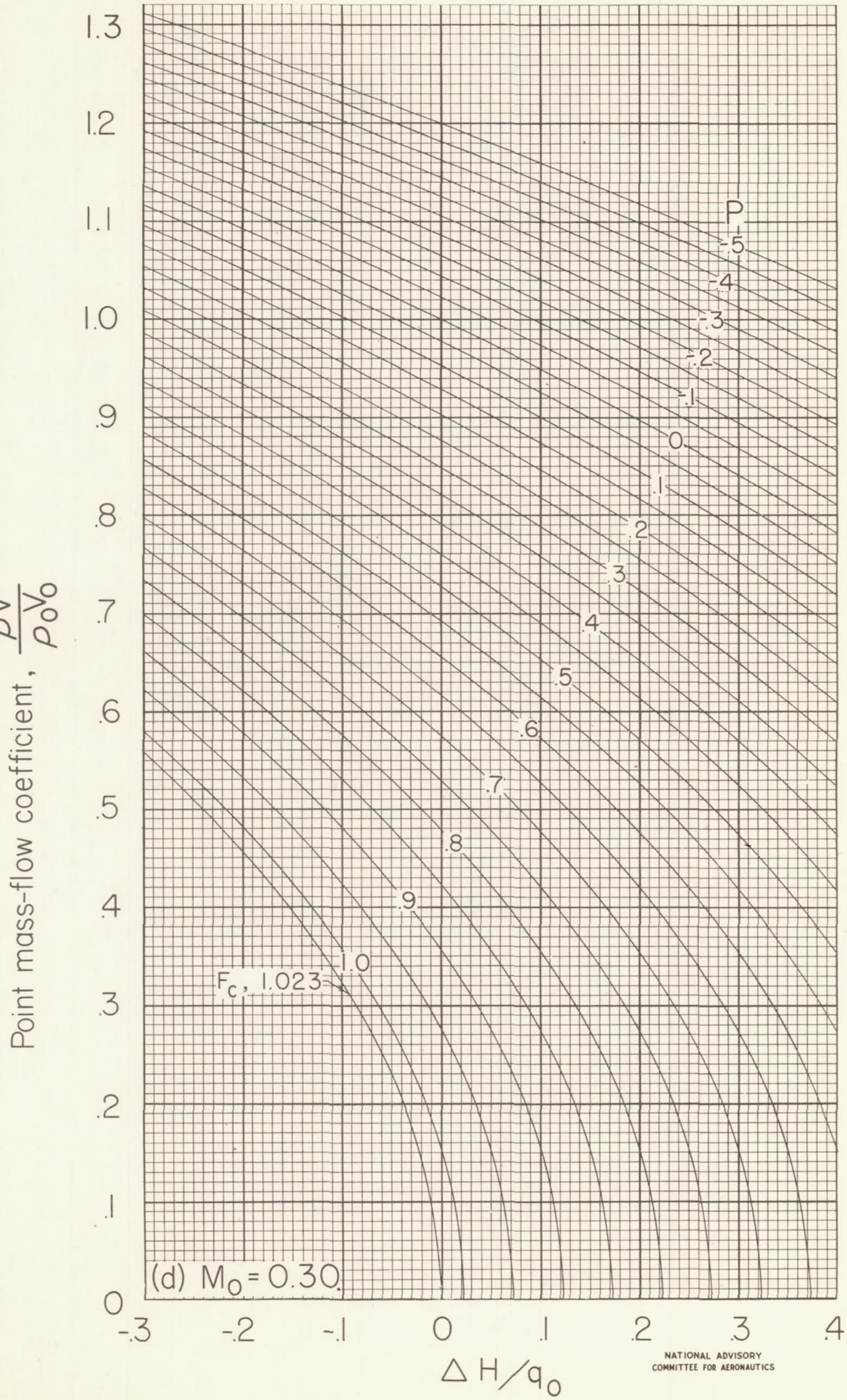

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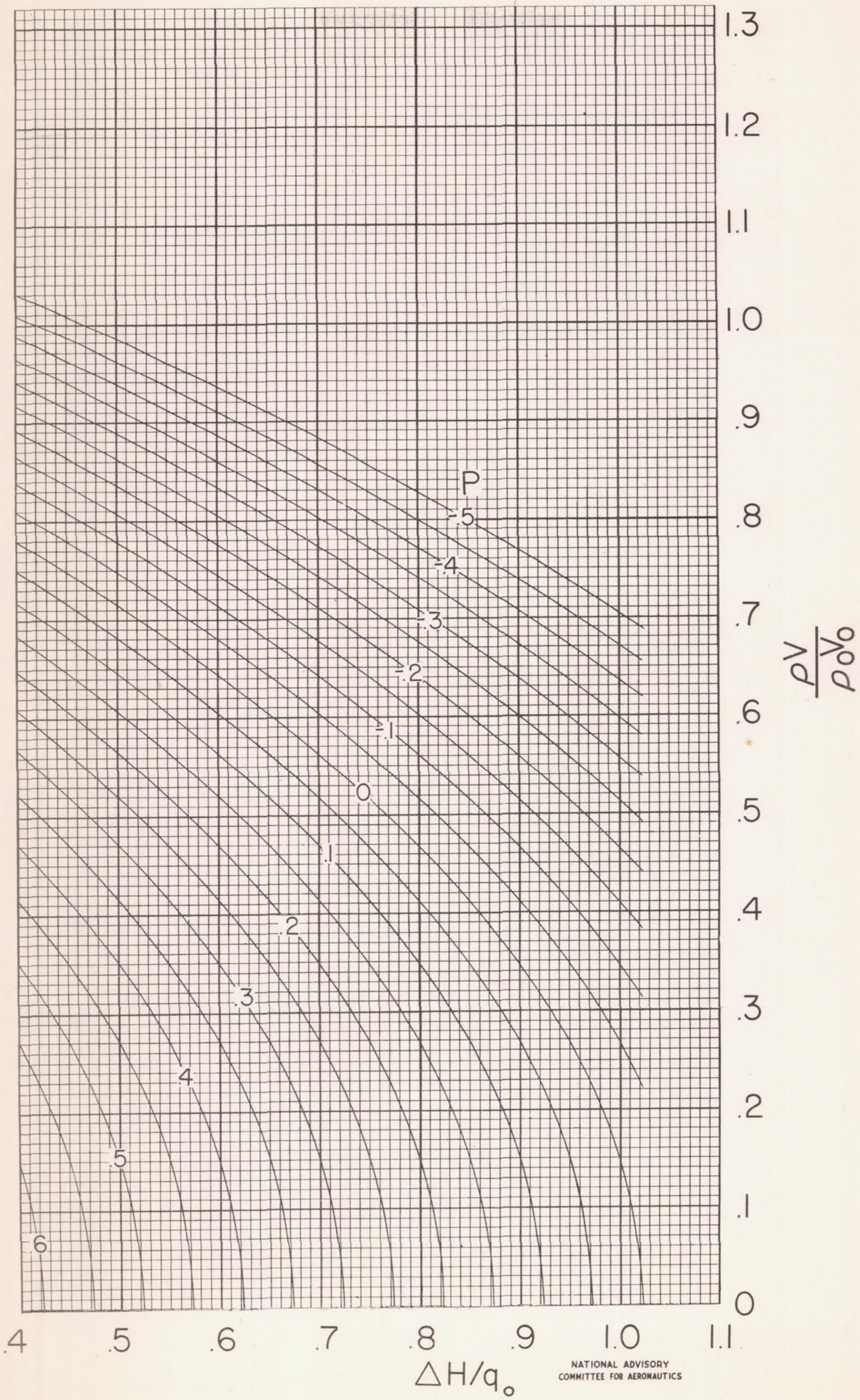

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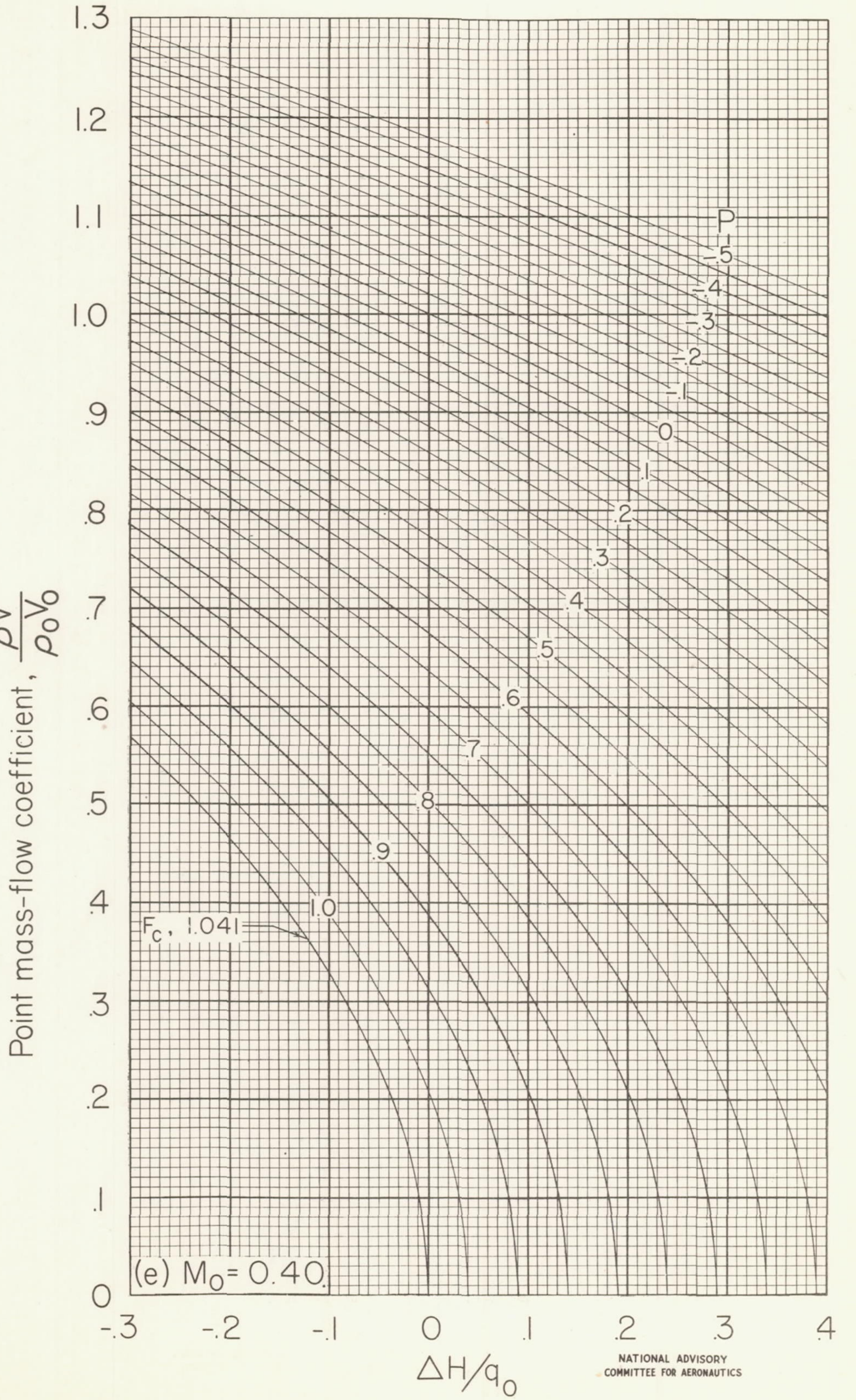

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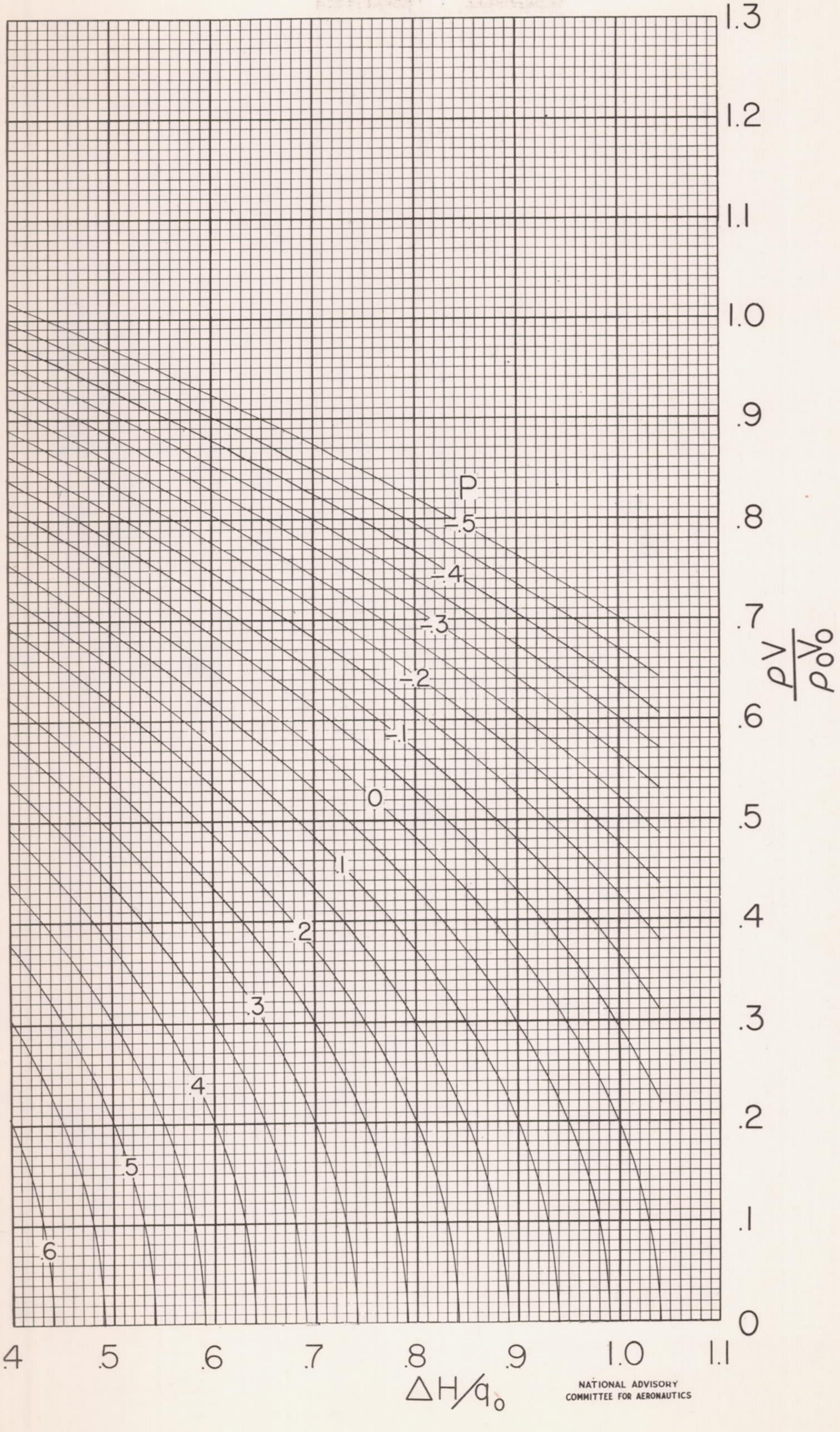

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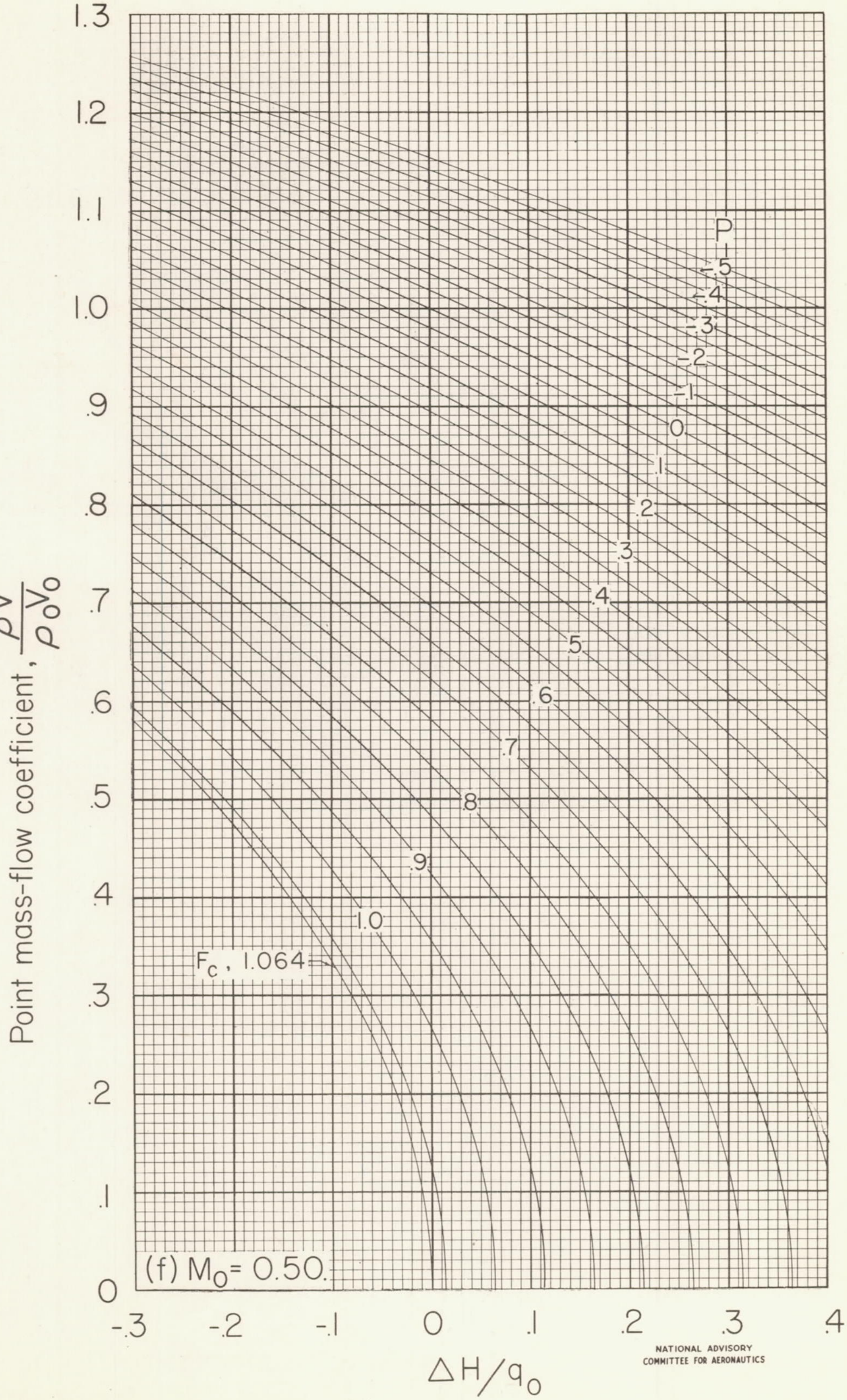

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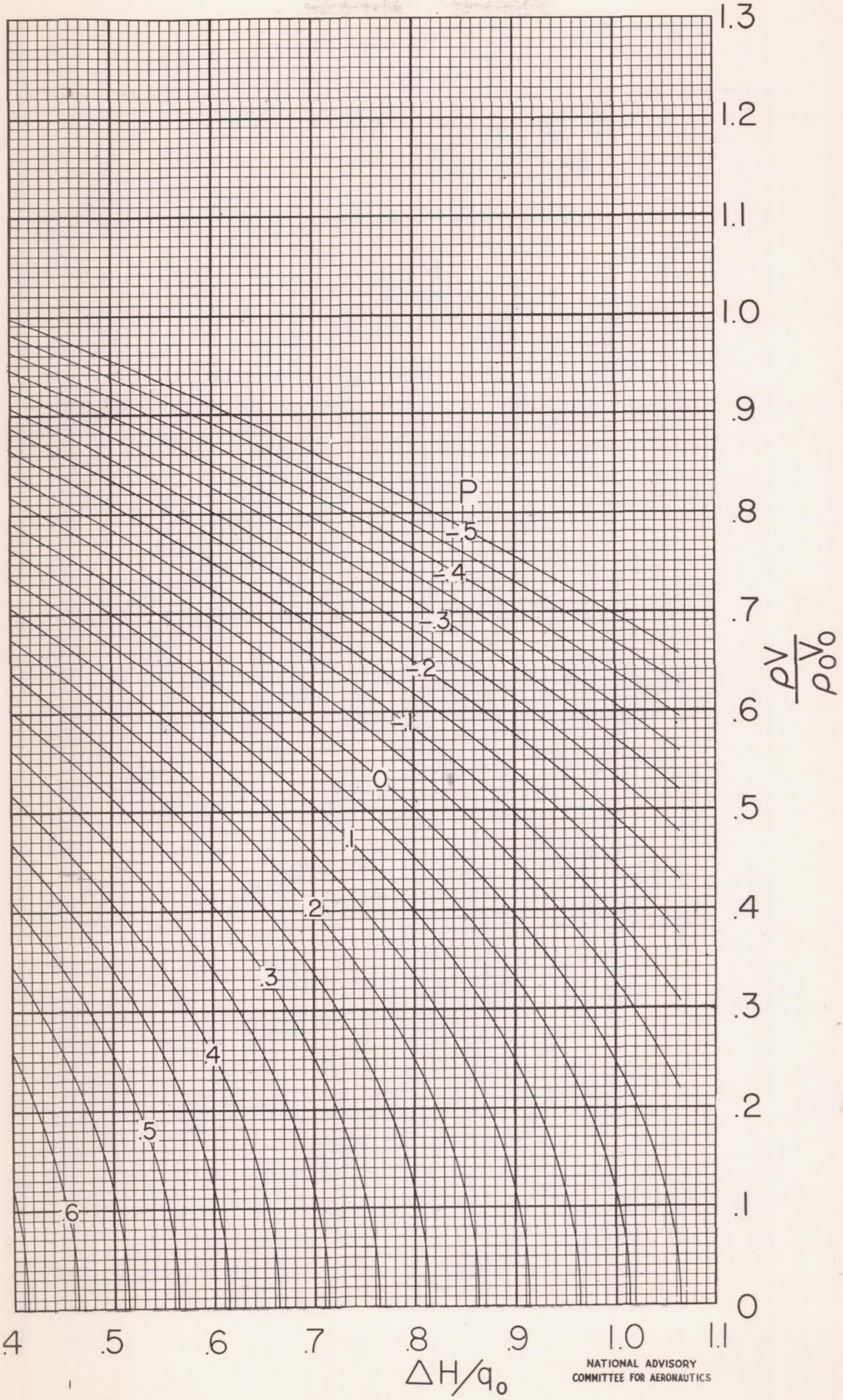
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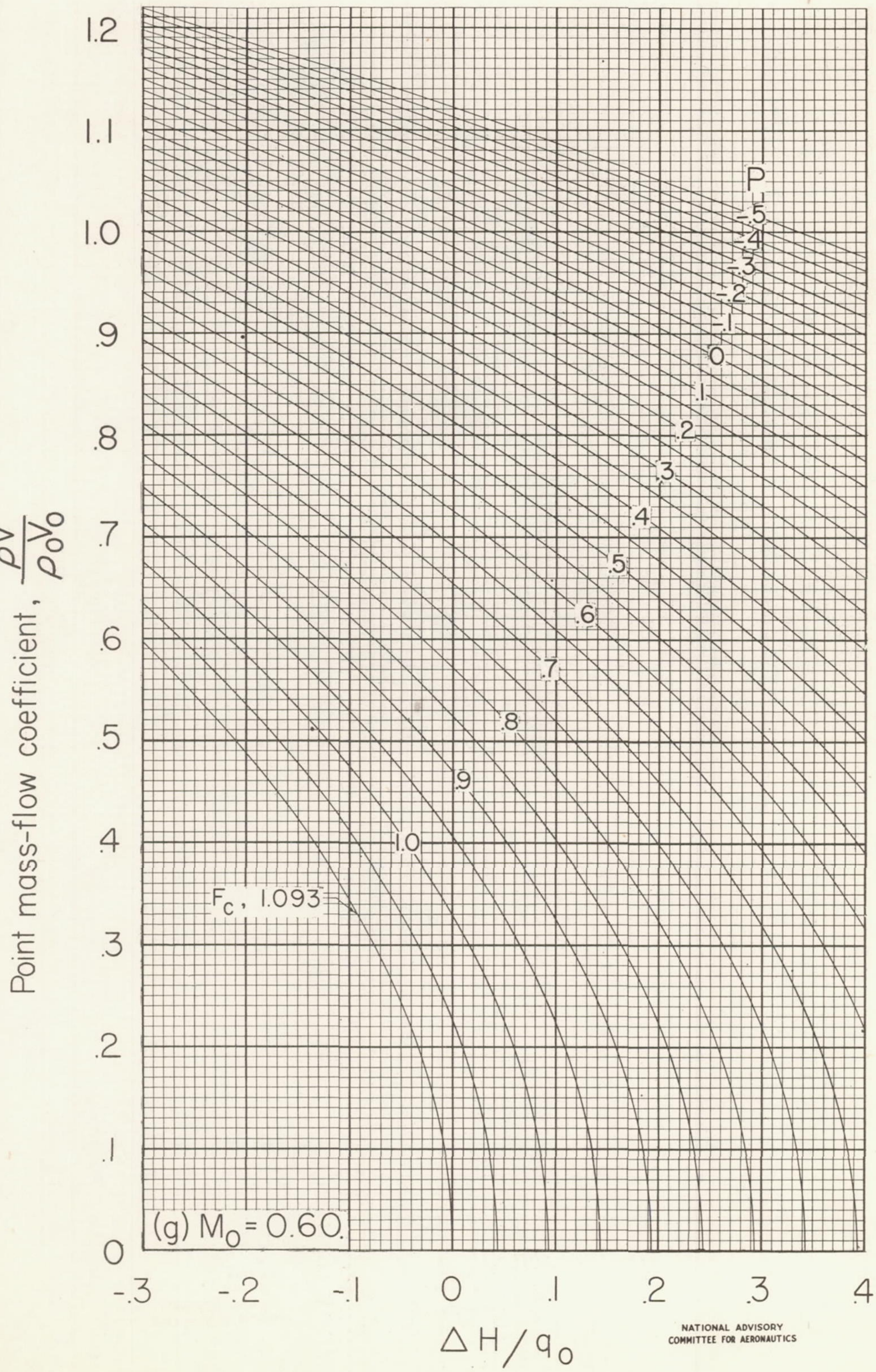

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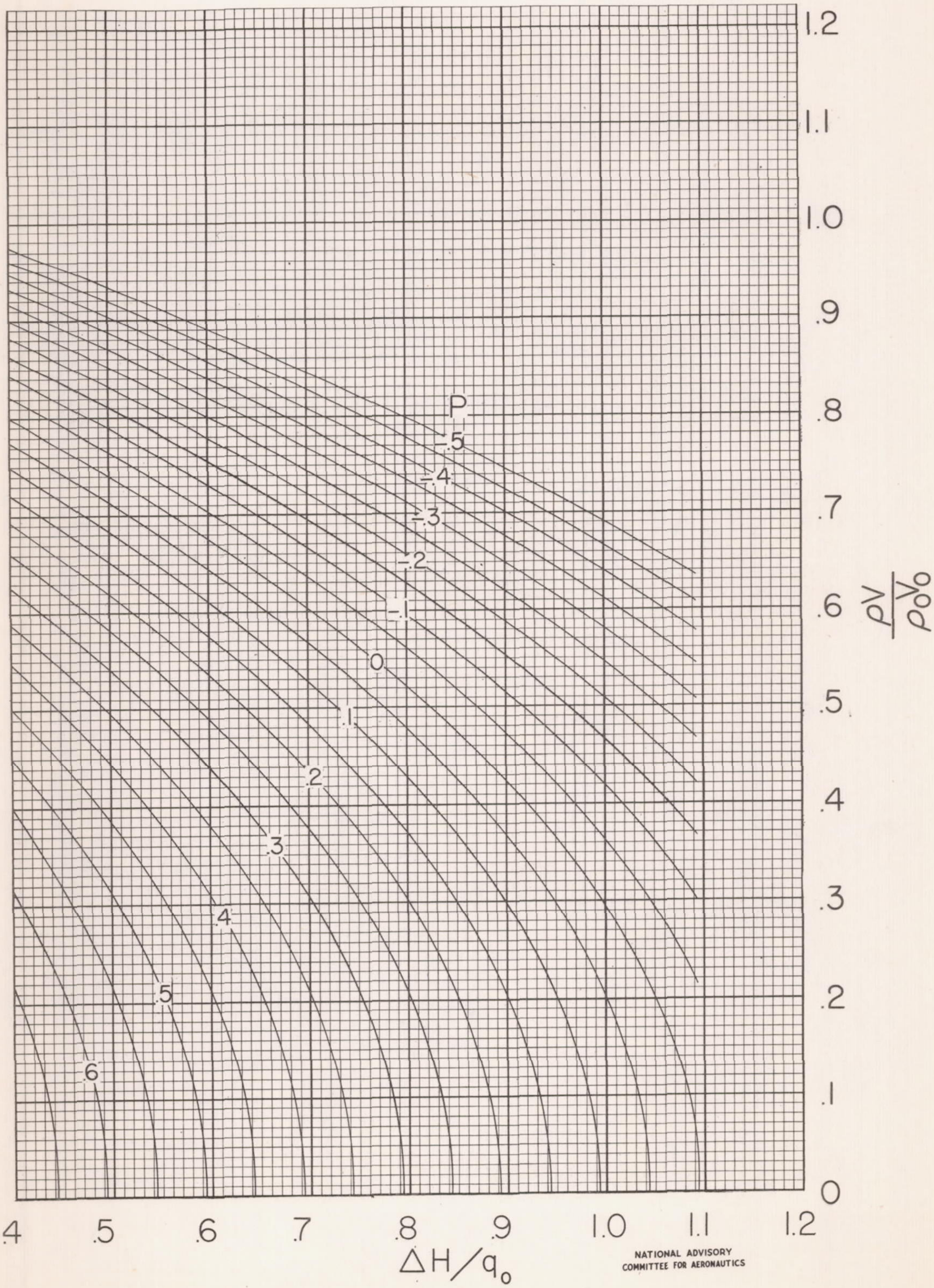

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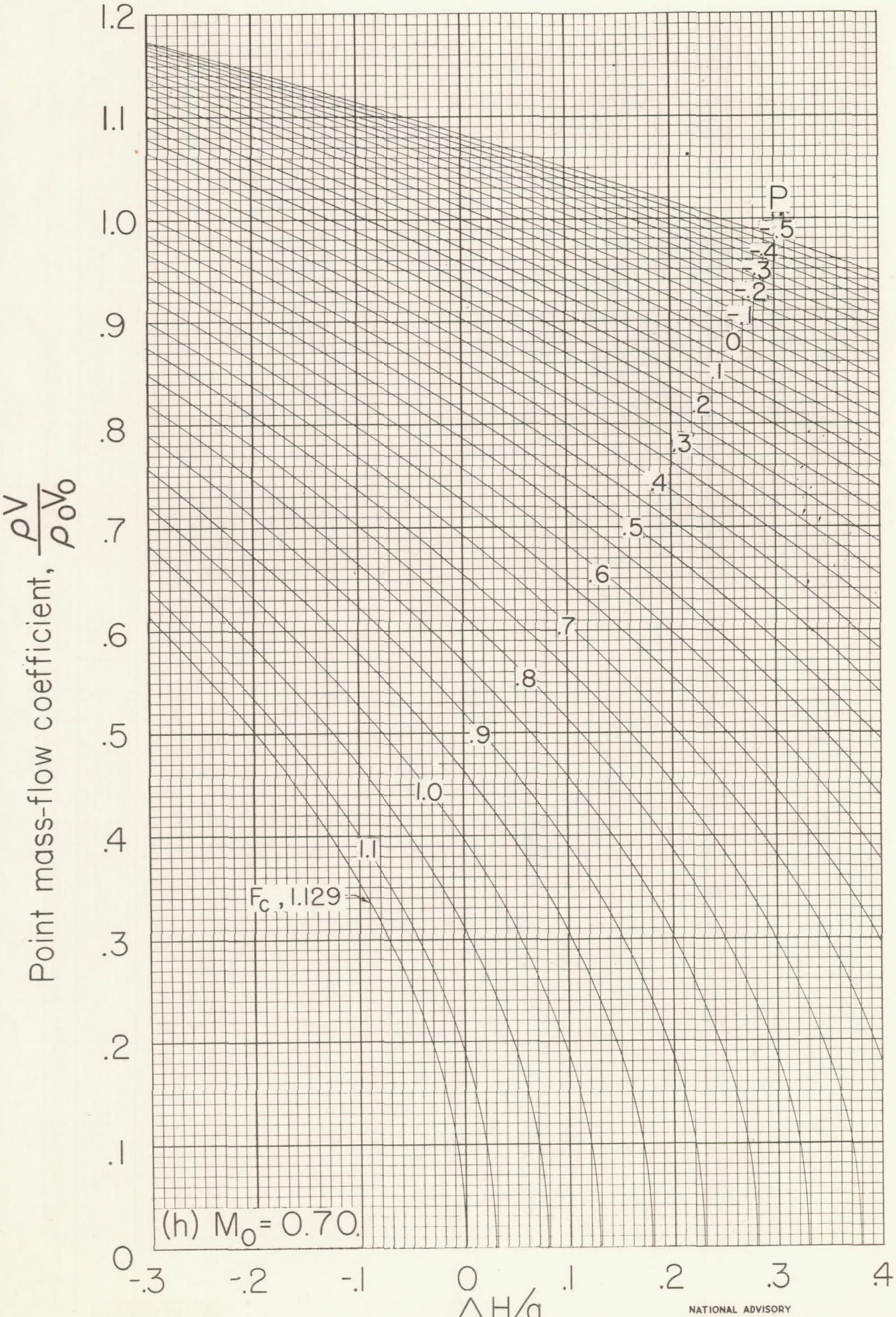

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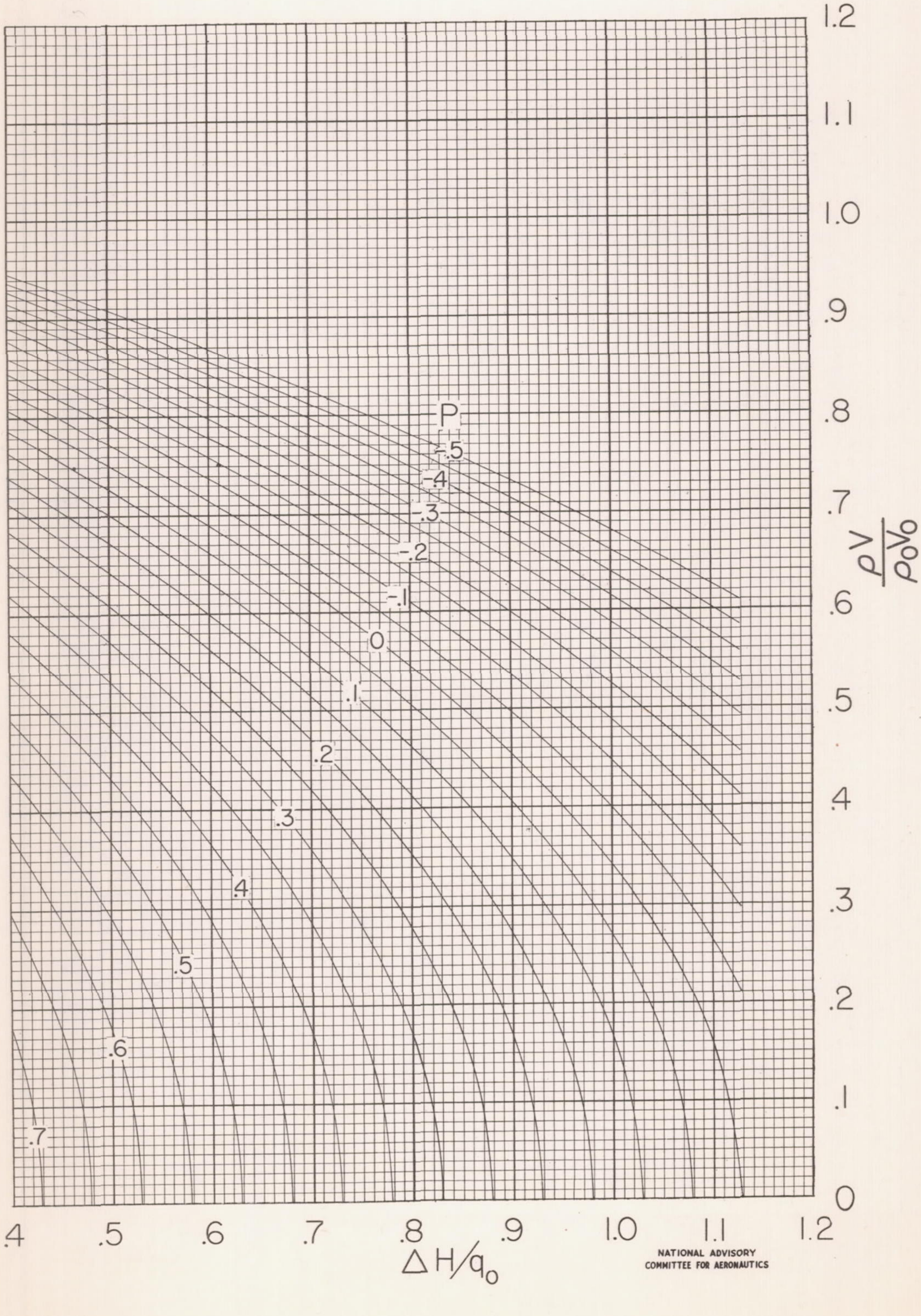
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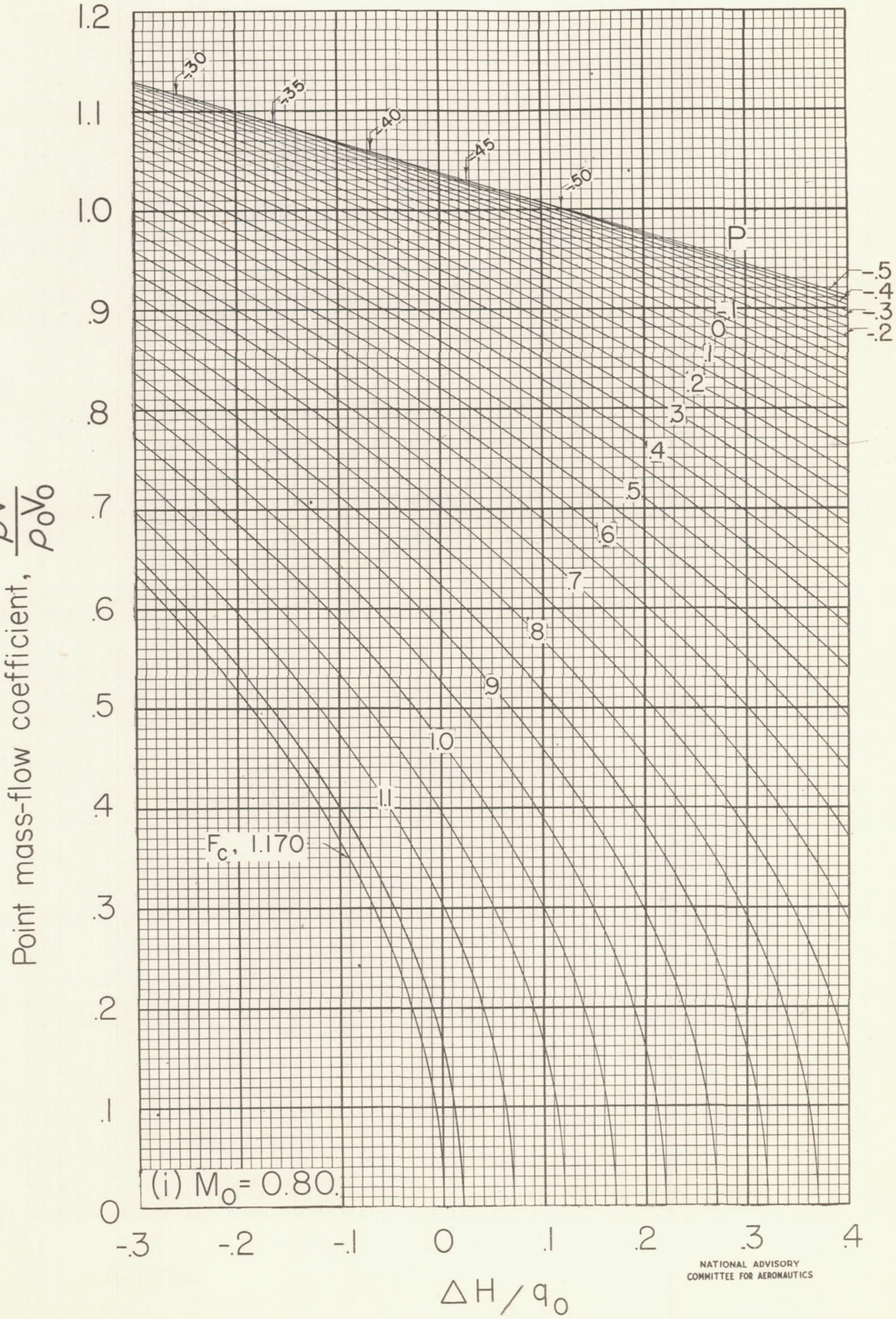

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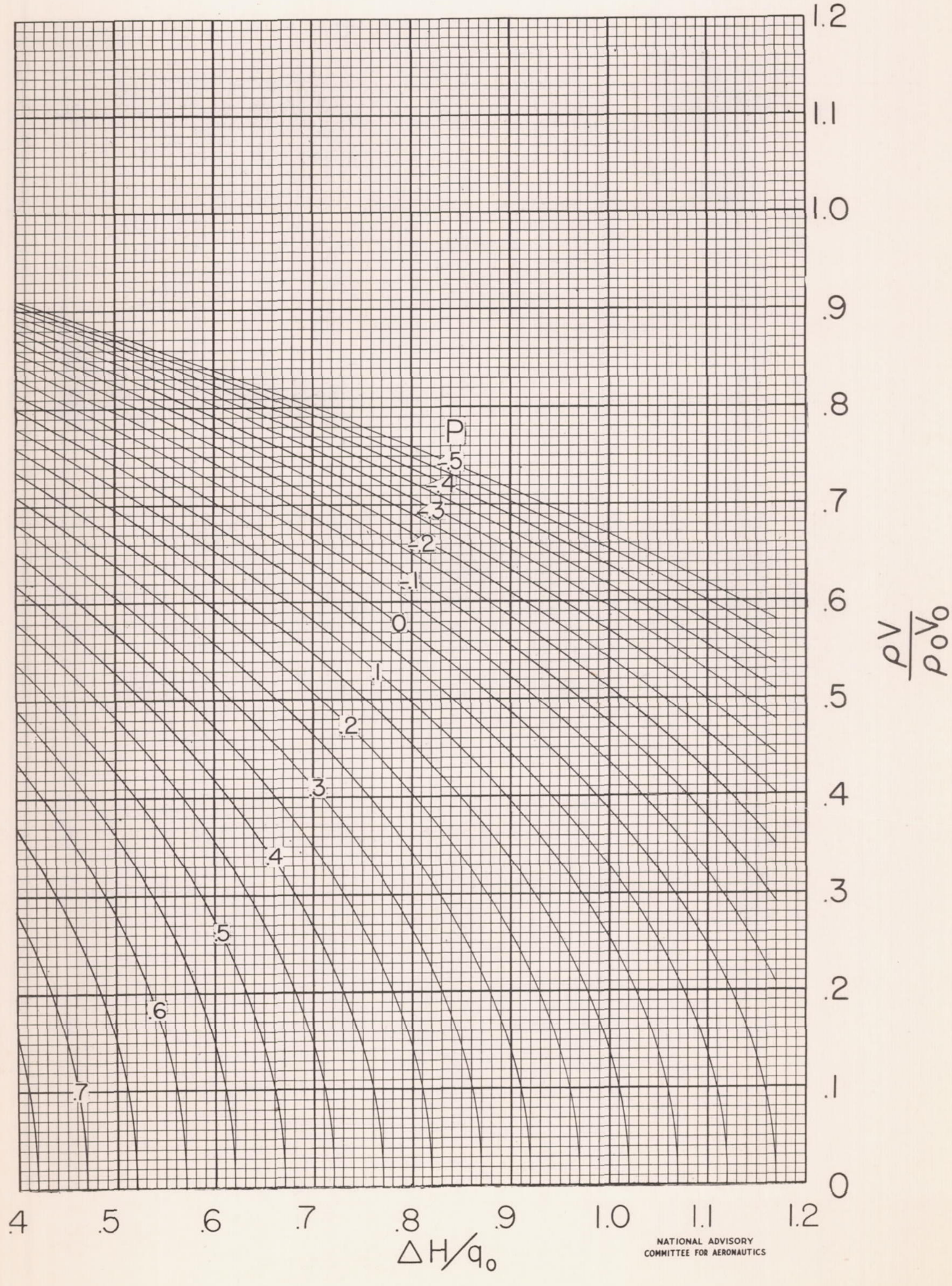

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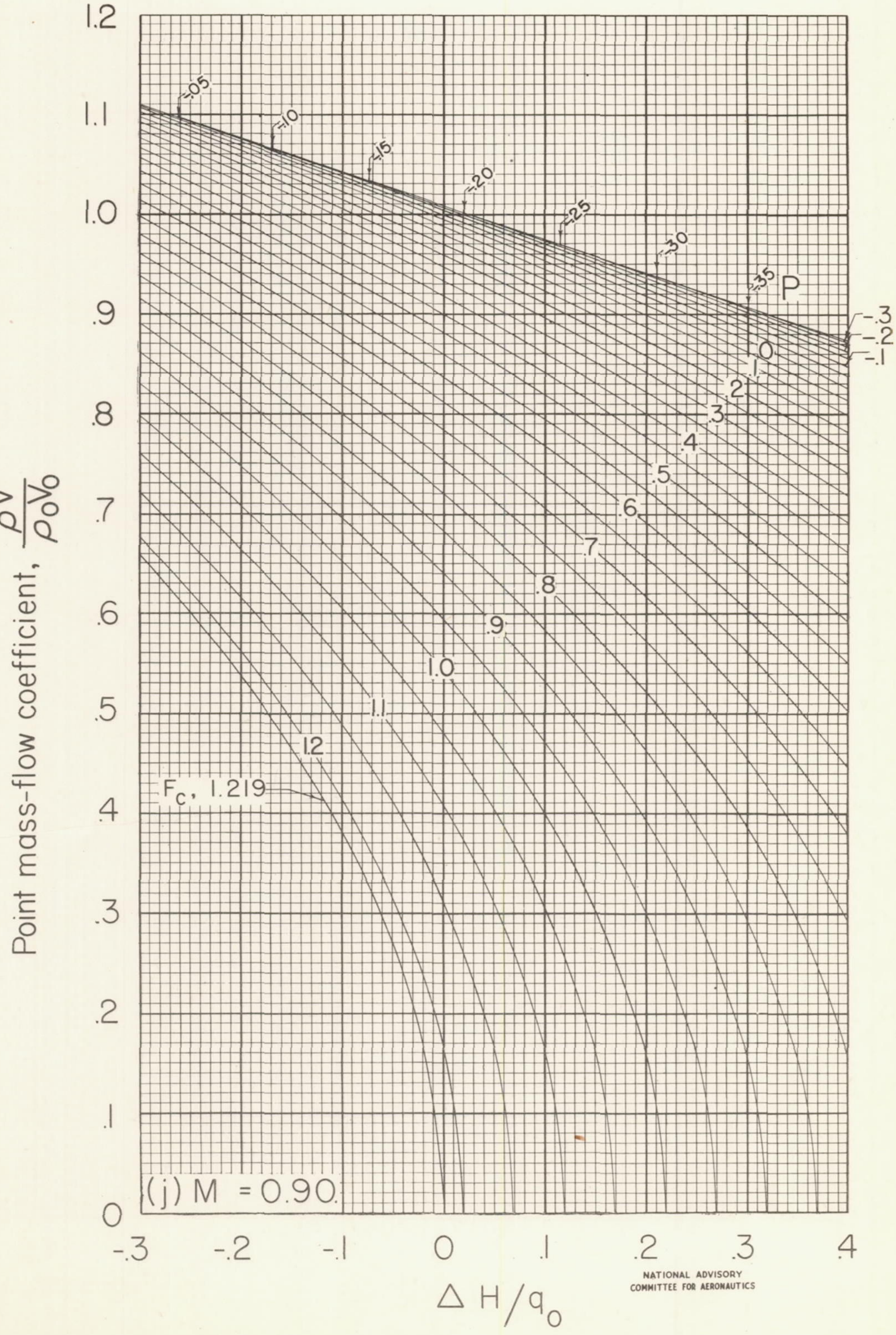

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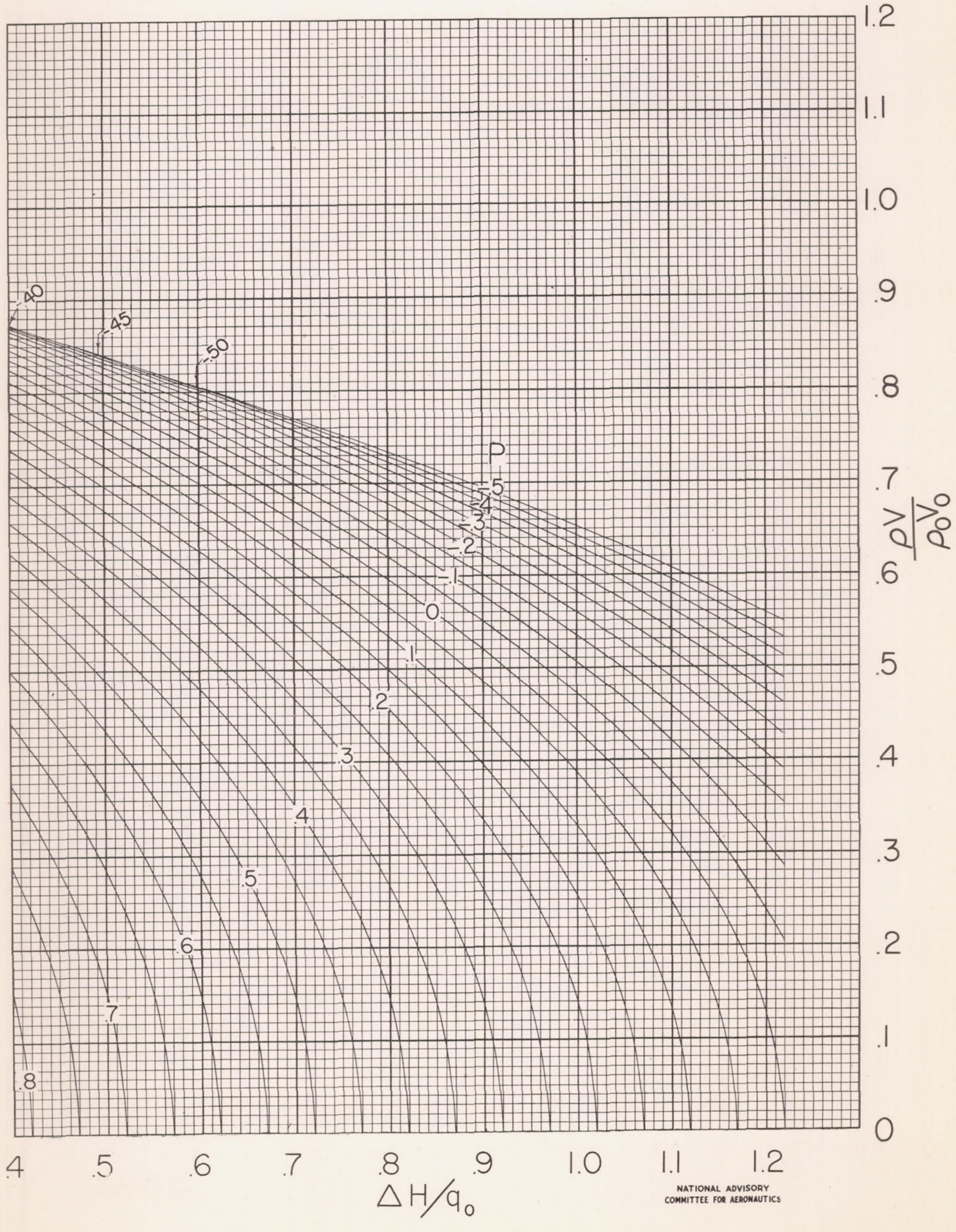
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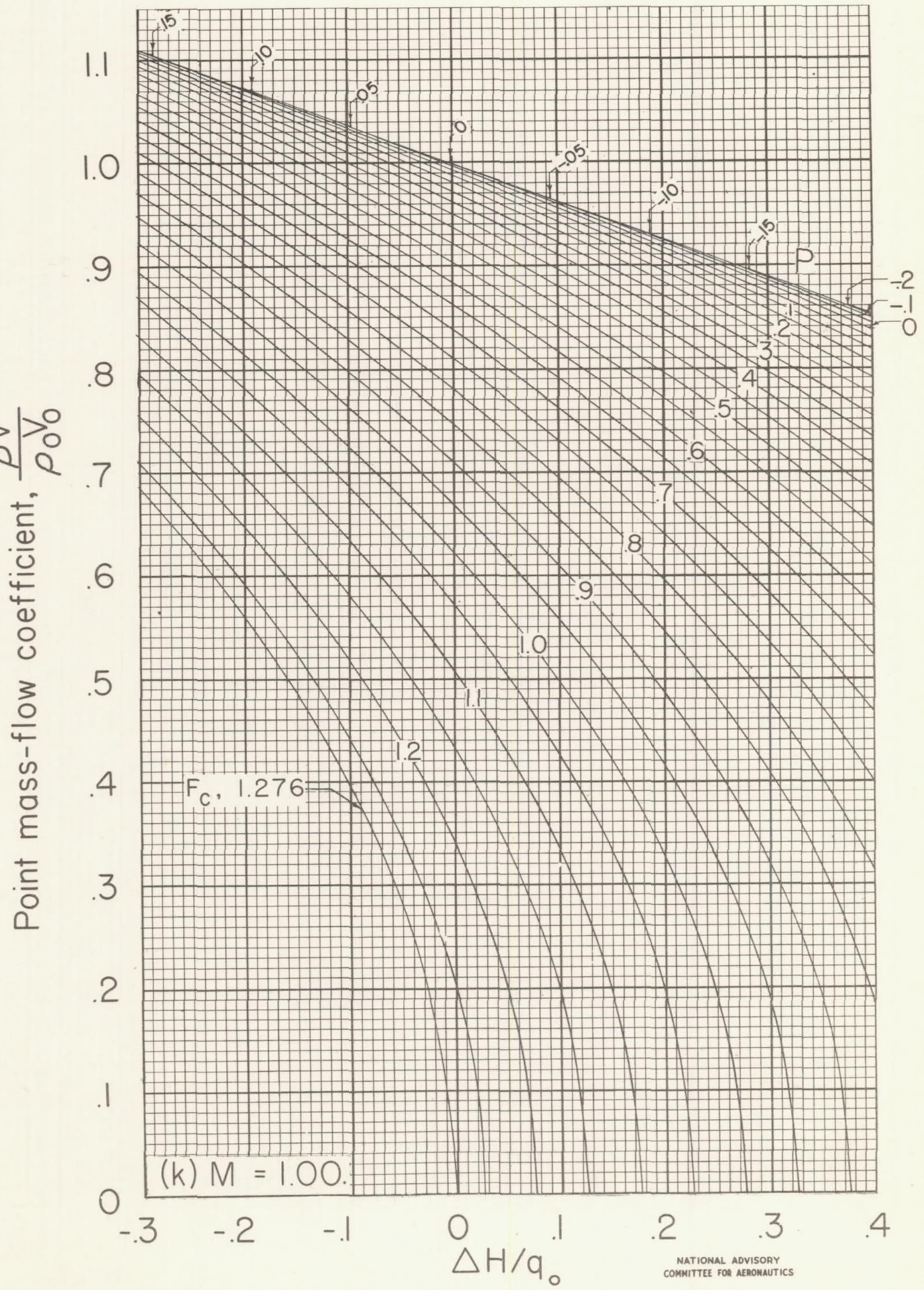

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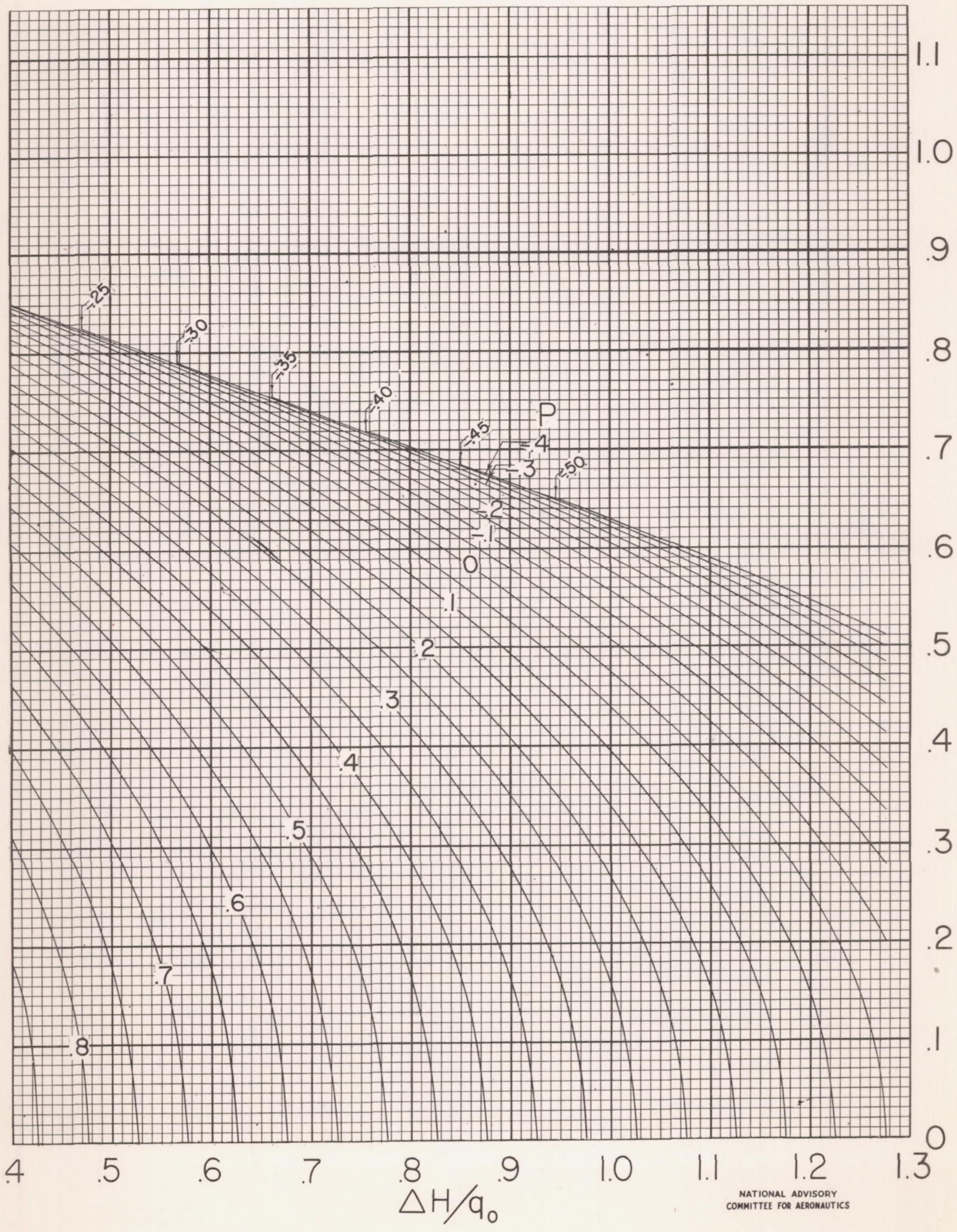
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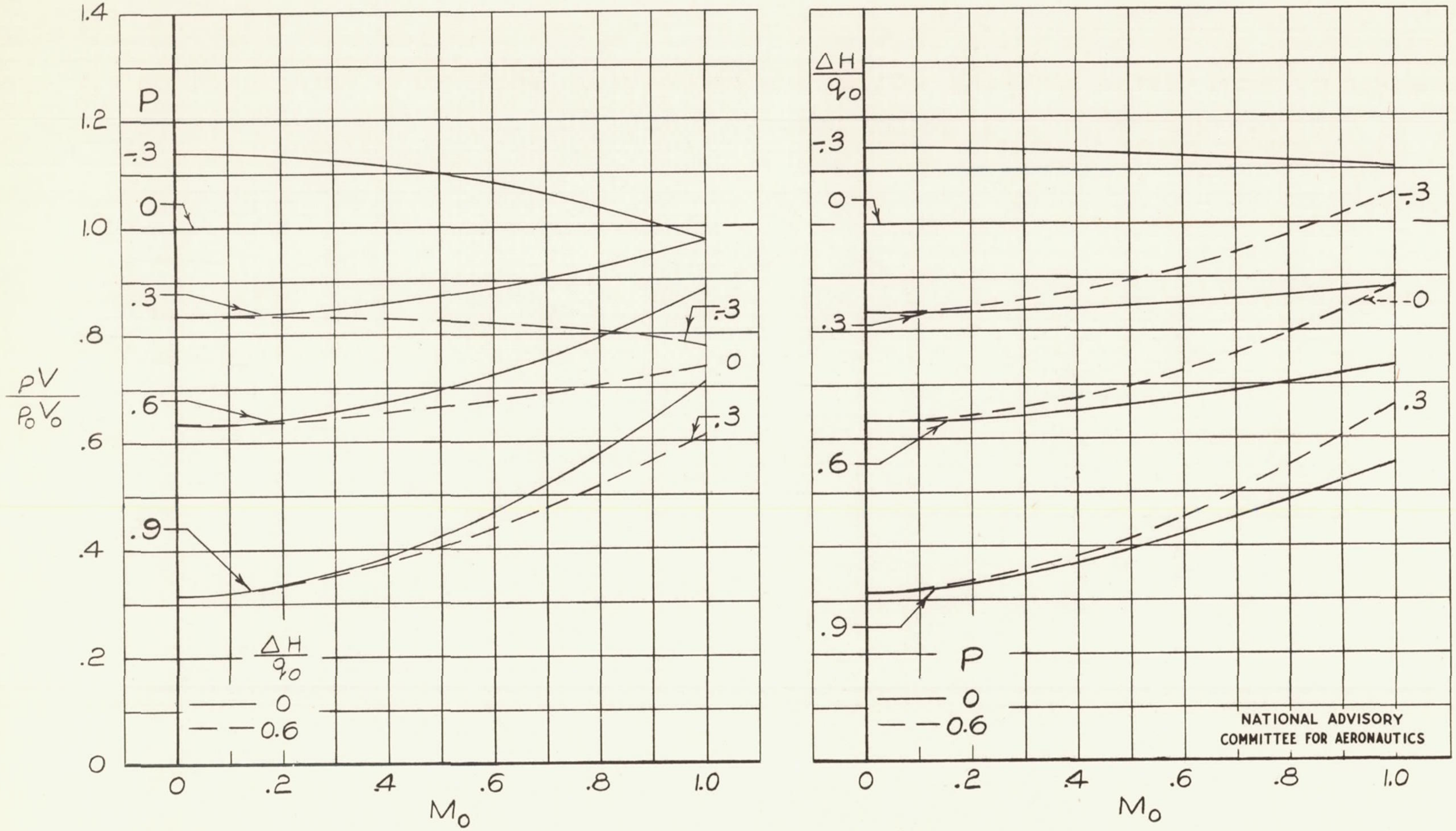

Figure 2.- Variation of point mass-flow coefficient with Mach number for various values of static-pressure and total-pressure-loss coefficients.

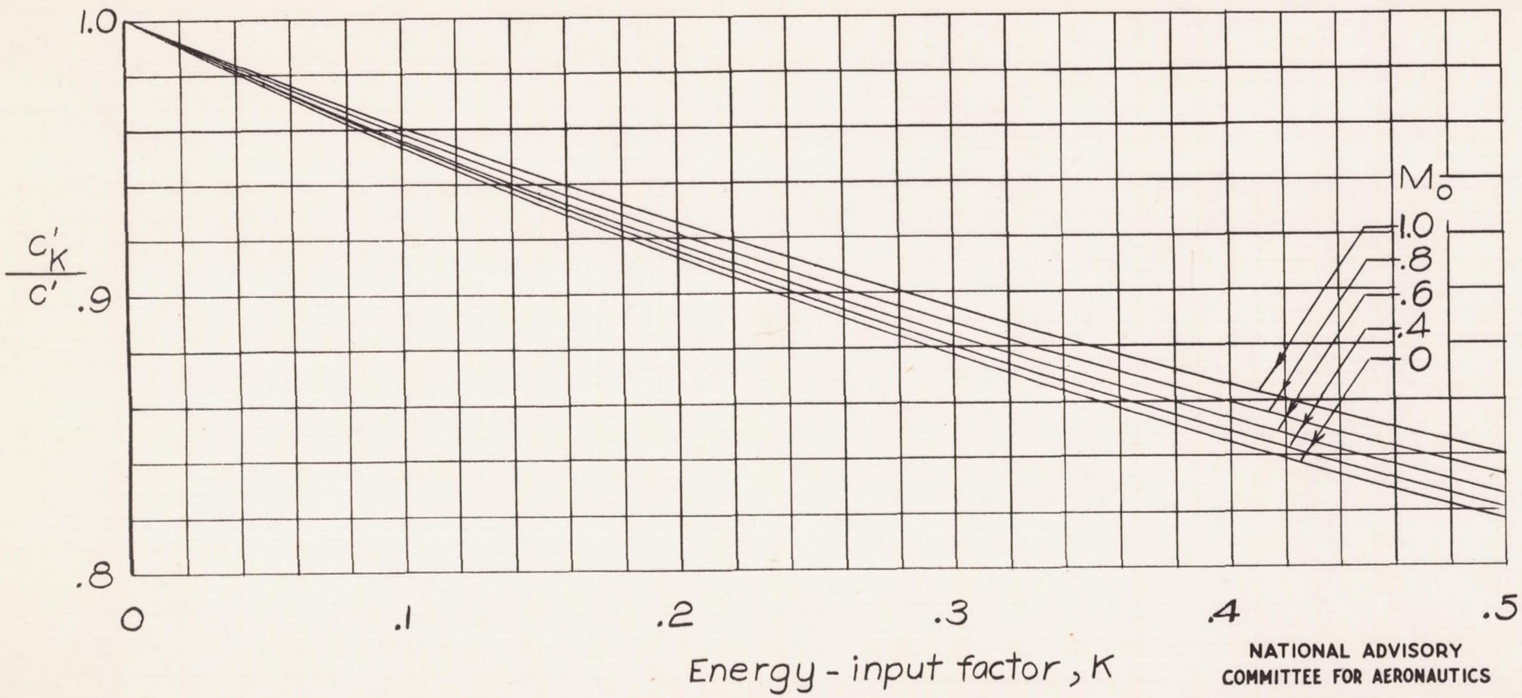

Figure 3 .— Energy-addition correction factor for point mass-flow coefficient.

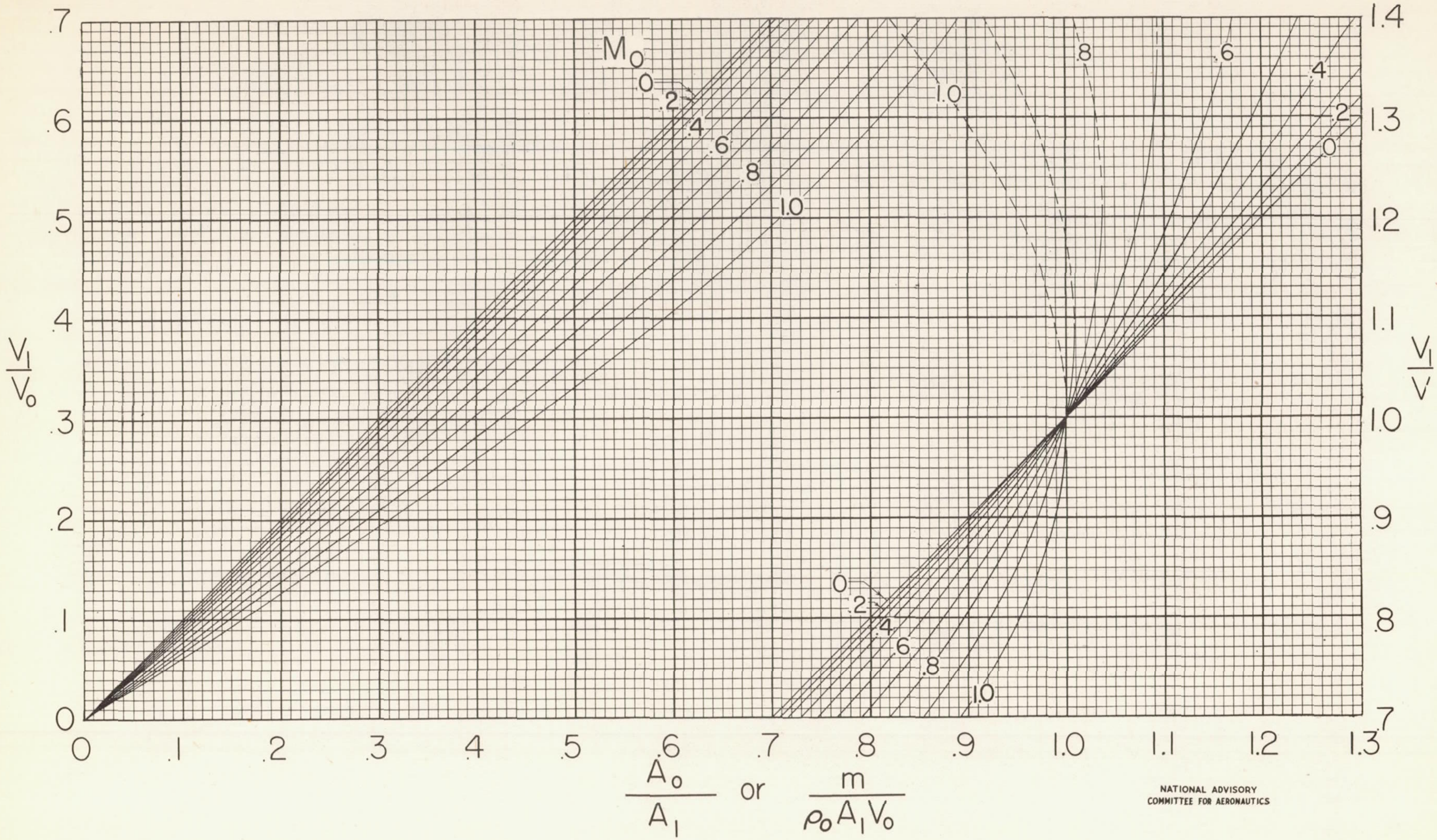

Figure 4.- Chart for converting mass-flow coefficient to inlet-velocity ratio. Broken line indicates supersonic flow at station 1.