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IMPACT THEORY FOR SEAPLANE LANDINGS

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SUMMARY

A step landing of a seaplane on smooth water is analyzed by solving Newton's differential equation of motion. Various limiting assumptions are made for convenience in the analysis when they appear reasonable. Formulas are developed for the maximum acceleration, the maximum draft, and the draft at the instant of maximum acceleration. The solution is dependent upon the definition of the apparent water mass and indicates the need for further basic research in regard to this water mass. Graphical representations of approximate solutions for design use are presented. Comparisons of theory with experimental values obtained in the National Advisory Committee for Aeronautics impact basin are also illustrated.

INTRODUCTION

In the analysis to be presented it will be assumed that the seaplane is landing on smooth water. If the reader has ever "skipped" a flat rock off the surface of a smooth lake or pond, he has witnessed a physical action which is similar to that of a seaplane landing. If the rock is thrown horizontally, close to the water surface, it will "skip," or rebound, from the water surface several times before final submerging. Similarly, in a seaplane landing, if the trim were held constant, several rebounds, or impact periods would occur before the seaplane finally settled onto the water. During each of these impact periods the reaction force exerted by the water on the hull increases from zero to a maximum and then decreases again to zero. In each consecutive impact period the maximum value of the reaction which is developed is less than in the previous period. Thus the maximum value of the reaction force will occur during the first impact period. Consequently a mathematical analysis may be limited to a study of the motion and associated forces during the first impact. During this period the lift force on the wing is approximately equal to the weight of the seaplane. As a matter of convenience, it will be assumed throughout the analysis that the wing lift and seaplane weight remain equal through the impact period.

SYMBOLS

A	loaded area
C_l	lift coefficient
c	half width of a unit loaded strip
F	total reaction force
f	force on a unit strip of loaded area
g	acceleration due to gravity
L	length of loaded area
M	mass of seaplane
m	additional mass of water
r	ratio of $\tan \gamma$ to $\tan \tau$
s	step draft normal to keel
t	time
V	velocity
x	displacement parallel to keel
z	displacement normal to keel
α	factor occurring in the definition of ϵ
β	dead-rise angle
γ	flight-path angle
ϵ	coefficient which defines m
λ	length parameter equal to $\sqrt[3]{M/\rho}$
μ	ratio of m to M
ρ	mass density of water
τ	trim angle
ψ	convenient universal function

ξ space coordinate parallel to the X-axis

ζ space coordinate parallel to the Z-axis

Subscripts:

h horizontal component

v vertical component

m value of quantity at instant of maximum acceleration

n value of quantity at instant of maximum draft

o value of quantity at instant of entry $(t = 0)$

Prime: A prime is used to indicate differentiation with respect to a space variable.

Dot: A dot over a letter is used to indicate differentiation with respect to time.

Bar: A bar over a letter is used to indicate that the quantity is of two-dimensional nature.

DISCUSSION OF MOTION

At the instant of entry of the seaplane into the water there are certain initial conditions of the motion. These are illustrated in figure 1(a). The trim angle τ, made by the keel with the water surface, is assumed to remain constant during the impact. This assumption is approximately true in a step landing. The initial velocity is V_o at the instant of entry and the direction of V_o is given by the initial flight-path angle γ_o. The enlarged view of figure 1(b) shows the seaplane entering the water at point O on the water surface. The point O is to be chosen as the origin of a fixed reference coordinate system. The point A on the seaplane is defined as the intersection of the keel and the step. At the instant of entry $(t = 0)$ the point A of the seaplane enters the water at point O. The motion of point A may be regarded as the motion of the seaplane..

In figure 2(a) the shape of the path of motion of point A is indicated. The motion of point A along this path will be referred to as the motion of the seaplane throughout the analysis. The seaplane enters the water at point O and emerges at point P.

At some intermediate position the draft passes through a maximum. If the differential equation of motion of the seaplane can be properly developed and solved, the relationships between time, displacements, velocities, and accelerations during the impact can be determined. The bottom reaction will also vary during the motion somewhat as illustrated in figure 2(b). This reaction is proportional to the vertical acceleration of the seaplane when its wing lift is equal to its weight. It is found, both from theory and experiment, that the maximum value of the upward acceleration (maximum deceleration) occurs before the instant of maximum draft. In experimental research work on float models it is possible to measure the maximum draft and the draft at the instant of maximum acceleration, as well as the maximum acceleration. A correct theory should predict all three of these quantities in agreement with experiment.

The motion will not only be affected by the initial conditions but also by certain physical properties of the seaplane. The properties which are of primary importance are the mass of the seaplane and the shape of the hull bottom. The elasticity of the seaplane structure is of secondary importance in most cases and will be neglected in this analysis. In the special case of a large flying boat, with engines in the wing, the elasticity of the wing may have an appreciable effect upon the motion. The bottom of the hull is of wedge shape with some curvature, or flare, of the bottom. As a matter of convenience it is assumed in the analysis that the bottom is a flat-sided wedge without flare. The bottom shape is then defined by the dead-rise angle. The two physical properties of the seaplane which affect the motion are, then, the mass M and the dead-rise angle β.

In a landing with or without power the initial conditions of the motion may vary over a range of values depending upon the pilot's skill and landing technique. Consideration of reasonable landings for design purposes would require a considerable amount of statistical evidence regarding the initial conditions of motion as obtained in actual flight operations. Such evidence is not available at present. Flight research in this field is seriously needed.

If the friction of the water on the hull bottom is neglected, the component of velocity of the seaplane parallel to the keel will remain constant during the impact period. In experimental research work on model floats the float is launched from a heavy carriage which rolls on a horizontal track, usually overhead. This method of launching the model allows it to have freedom of vertical motion but forces it to maintain almost a constant horizontal velocity because of the large inertia of the carriage to which it remains attached. In the present analysis it will be assumed that the horizontal component of velocity remains constant during the impact in order that the resulting formulas can be compared directly with experimental evidence. Since the trim angle in a step landing must be small, the maximum reaction given by a solution involving constant horizontal velocity will differ only slightly from a solution for constant velocity along the keel and, hence, should be satisfactory for design purposes.

DISCUSSION OF REACTION DISTRIBUTION

When a hull is immersed a small amount at a particular instant during the impact, the loaded area is of approximately triangular shape. The base of the triangle is at the step. Due to the effect of "piled-up" water the triangular loaded area is larger than would be determined from the geometrical intersection of the hull with the plane of the original water surface. For design purposes it is necessary to know the transverse distribution of pressure on this area and the longitudinal, or fore-and-aft, distribution. The longitudinal distribution must be rounded off in some manner at the step as indicated in figure 3(a). This three-dimensional effect can only be brought into the analysis, at present, by means of an aspect-ratio factor applied to the pressure at all points of the loaded area.

In figure 3(b) a unit transverse strip of the loaded area is shown with the load acting on this strip. The reaction force on a unit strip at a particular station is dependent upon the draft of the keel at this station as well as the components of velocity and acceleration of the seaplane normal to the keel. The component of velocity parallel to the keel does not affect the force on the unit strip. In order to determine the total reaction, the reaction per unit of length must be determined and then an integration performed in the fore-and-aft direction over the loaded area. The reaction force on a unit strip can be determined by considering the two-dimensional case. For this reason the two-dimensional case will be analyzed first. This analysis will be carried out more completely than is necessary for the purposes of this paper in order to provide formulas which may be useful for comparison with results from future experimental research studies. The transverse pressure distribution on a unit strip will be somewhat as shown in figure 3(c). No attempt will be made in this paper to determine this distribution.

TWO-DIMENSIONAL CASE

In this case a wedge-shaped float is assumed to enter the water vertically as shown in figure 4. The wedge is assumed to be of infinite length and infinite width. The action of a unit slice of the wedge and a unit slice of the fluid is considered in the analysis. It is assumed that the momentum of the fluid is equal to the product of an "additional mass" $\bar{m}$ and the instantaneous velocity $\dot{z}$ of the float. The displacements, velocities, and accelerations are considered positive downward. The additional mass is a half cylinder of water as shown in figure 4. This elementary concept of the additional mass and its use in the calculation of the reaction on the float were first given by Th. von Kármán (reference 1). The assumed additional mass is in agreement with the additional mass which is known (reference 2) to be associated with a flat plate moving through an ideal fluid of infinite extent normal to the plane of the plate.

Due to the effect of piled-up water which would actually occur and the fact that the float has a wedge bottom rather than a flat bottom, it is necessary to multiply the additional water mass as shown in figure 4 by correction factors. Such factors were first introduced by H. Wagner (reference 3). In any case, however, the water mass $\bar{m}$ must be proportional to the square of the displacement and may be written as

$$\bar{m} = \bar{\epsilon} z^2 \tag{1}$$

The bar over a quantity is used to indicate that the quantity is of two-dimensional nature. The derivatives of $\bar{m}$ with respect to displacement and time are required in the analysis and may be written in the following forms:

$$\frac{d\bar{m}}{dz} = \bar{m}' = 2\bar{\epsilon}z = \frac{2\bar{m}}{z} \tag{2}$$

$$\frac{d\bar{m}}{dt} = \dot{\bar{m}} = 2\bar{\epsilon}z\dot{z} = \frac{2\bar{m}\dot{z}}{z} = \bar{m}'\dot{z} \tag{3}$$

The reaction force is due to the rate of change of momentum of the fluid. The change of momentum is due both to the change of velocity of the float and the change of size of the additional mass caused by a change of the loaded width. All effects of viscosity and buoyancy are omitted from the analysis. The force may be expressed in the following form:

$$f = \frac{d}{dt}(\bar{m}\dot{z}) = \bar{m}\ddot{z} + \dot{\bar{m}}\dot{z} \tag{4}$$

Using equation (3) this may be written,

$$f = \bar{m}\ddot{z} + \bar{m}'\dot{z}^2 \tag{5}$$

This force acts upward and thus it must be preceded by a negative sign in writing Newton's law of motion for the float. Newton's law, when applied to the float, is expressed by,

$$\bar{M}\ddot{z} = -f \tag{6}$$

The solution of this equation will be that which was originally given by von Kármán. Substitution of equation (5) gives,

$$\bar{M}\ddot{z} + \bar{m}\ddot{z} + \bar{m}'\dot{z}^2 = 0 \tag{7}$$

or

$$(\bar{M} + \bar{m})\ddot{z} + \bar{m}'\dot{z}^2 = 0 \tag{8}$$

It becomes convenient in the analysis to introduce the quantity $\bar{\mu}$ as the ratio of the water mass to the float mass.

$$\bar{\mu} = \frac{\bar{m}}{\bar{M}} \tag{9}$$

Equation (8) becomes,

$$(1 + \bar{\mu})\ddot{z} + \bar{\mu}'\dot{z}^2 = 0 \tag{10}$$

From equations (3) and (9) it is seen that,

$$\dot{\bar{\mu}} = \bar{\mu}'\dot{z} \tag{11}$$

Hence equation (10) may be written,

$$(1 + \bar{\mu})\frac{d\dot{z}}{dt} + \dot{z}\frac{d\bar{\mu}}{dt} = 0 \tag{12}$$

or

$$(1 + \bar{\mu})\,d\dot{z} + \dot{z}\,d\bar{\mu} = 0 \tag{13}$$

or

$$\frac{d\dot{z}}{\dot{z}} + \frac{d\bar{\mu}}{1 + \bar{\mu}} = 0 \tag{14}$$

Integration gives,

$$\log \dot{z} + \log (1 + \bar{\mu}) = \log C \qquad (15)$$

The constant of integration is determined from the initial conditions. When $t = 0$ the displacement z is zero. Hence the mass $\bar{m}$ is zero and also the ratio $\bar{\mu}$. The initial velocity at the instant of entry may be indicated as $\dot{z}_o$. Then $C = \dot{z}_o$ and equation (15) becomes,

$$\log \dot{z} + \log (1 + \bar{\mu}) = \log \dot{z}_o \qquad (16)$$

or

$$\dot{z} = \frac{\dot{z}_o}{(1 + \bar{\mu})} \qquad (17)$$

This equation was first given by von Kármán (reference 1). This differential equation may be regarded as a formula for the velocity in terms of the displacement. Correspondingly, equation (10) may be regarded as a formula for the acceleration in terms of the displacement and velocity. Solving equation (10) for the acceleration gives,

$$\ddot{z} = \frac{-\bar{\mu}' \dot{z}^2}{(1 + \bar{\mu})} \qquad (18)$$

From equations (2) and (9) it is seen that,

$$\bar{\mu}' = \frac{2\bar{\mu}}{z} \qquad (19)$$

Equation (18) becomes,

$$\ddot{z} = \frac{-2\bar{\mu}\dot{z}^2}{z(1 + \bar{\mu})} \qquad (20)$$

Equation (17) may be substituted to obtain the acceleration as a function of the displacement

$$\ddot{z} = \frac{-2\bar{\mu}\dot{z}_o{}^2}{z(1 + \bar{\mu})^3} \qquad (21)$$

This formula was also given by von Kármán (reference 1). By assuming various values of z the velocities and accelerations can be computed from equations (17) and (21).

In order to illustrate the variation of velocity and acceleration with displacement a simple numerical example has been calculated for the following physical constants.

$\bar{\epsilon} = 0.05$ slug per cubic inch

$\bar{M} = 1$ slug per inch

$\dot{z}_0 = 6$ feet per second .

A plot of $\dot{z}$ and $\ddot{z}$ against z is shown in figure 5. It is seen that the acceleration passes through a maximum at a displacement of 2 inches. The accelerations are plotted in terms of the acceleration g due to gravity and the ordinates to this curve may be multiplied by the weight of the float to obtain the reaction force. It may be noted that the velocity remains positive indefinitely. This means that the float moves continuously downward without reversing its direction of motion. This is due to the omission of the buoyancy force from the equation of motion. At the displacement for maximum acceleration the buoyancy force is very small and, hence, it appears reasonable to omit the buoyancy force in determining maximum acceleration.

For purposes of structural design it is not necessary to know the value of the acceleration at various instants but only the maximum value. This may be obtained by differentiating the formula for the acceleration. Differentiating equation (21) with respect to z gives,

$$\frac{d\ddot{z}}{dz} = \frac{\ddot{z}}{z}\left(1 - \frac{6\bar{\mu}}{1 + \bar{\mu}}\right) \qquad (22)$$

Setting the quantity within the brace equal to zero and solving for $\bar{\mu}$ gives,

$$\bar{\mu}_m = \frac{1}{5} \qquad (23)$$

The subscript m is used to indicate the value of a quantity when the acceleration is a maximum. A universal constant value of 1/5 is obtained for $\bar{\mu}_m$ which holds true for all values of $\bar{M}$ and β. This constant was first published by J. Sydow (reference 4).

From equations (1) and (9) it is seen that the displacement at maximum acceleration is given by,

$$z_m = \sqrt{\frac{\bar{\mu}_m \bar{M}}{\bar{\epsilon}}} \tag{24}$$

or

$$z_m = \sqrt{\frac{\bar{M}}{5\bar{\epsilon}}} \tag{25}$$

Substitution of equation (23) into equation (21) gives the formula for maximum acceleration.

$$\ddot{z}_m = -\left(\frac{25}{108}\right)\frac{\dot{z}_o^2}{z_m} \tag{26}$$

Equations (25) and (26) give the maximum acceleration in terms of the physical properties of the float and the initial sinking speed.

In order to plan proper instrumentation in experimental research work it is desirable to know the time which elapses from the instant of entry to the instant of maximum acceleration. The relationship of displacement, velocity, and acceleration to time may be determined by integrating equation (17).

$$(1 + \bar{\mu})\dot{z} = \dot{z}_o \tag{27}$$

or

$$\left(1 + \frac{\bar{\epsilon}z^2}{\bar{M}}\right) dz = \dot{z}_o \, dt \tag{28}$$

Direct integration gives,

$$z + \frac{\bar{\epsilon}z^3}{3\bar{M}} = \dot{z}_o t \tag{29}$$

or

$$t = \frac{z}{\dot{z}_o}\left(1 + \frac{\bar{\mu}}{3}\right) \tag{30}$$

The constant of integration is zero. The time for maximum acceleration is obtained by substituting equation (23)

$$t_m = \frac{16}{15}\left(\frac{z_m}{\dot{z}_o}\right) \tag{31}$$

Equation (30) is a cubic equation in z which could be solved for z in terms of t. However, it is more convenient for calculation purposes to assume various values of z and compute the associated values of t. The associated values of $\dot{z}$ and $\ddot{z}$ can also be computed. Displacement, velocity, and acceleration can then be plotted against time. Graphs of this nature have been calculated for the numerical example previously considered and are shown in figure 6.

The two-dimensional case has been given considerable attention because the steps which have been taken suggest similar steps which can be taken in the three-dimensional case.

THREE-DIMENSIONAL CASE

The three-dimensional analysis will be made for an ideal prismatic float of semi-infinite length and infinite width with a flat-sided wedge bottom. The float begins at the step and extends indefinitely forward. It is assumed to have no afterbody. The float is assumed to maintain a constant trim angle throughout the impact period.

In figure 7 an ideal float is shown immersed in the water after having travelled along a path of motion indicated by the dotted line. The origin of coordinates is chosen as the point on the water surface where the point A of the float enters the water. The X-axis is chosen to be parallel to the keel and the Z-axis is normal to the keel. The positive directions of the axes are as shown. In the position of the float as shown the flight-path angle has decreased from the initial value γ_o to the instantaneous value γ.

The fluid beneath the float is shown divided into unit slices by planes which are perpendicular to the keel. Due to the lack of a satisfactory three-dimensional hydrodynamic theory it is necessary to assume that the reaction on the float is due to the motion in these slices of fluid which have a combined width equal to the wetted keel length. It is necessary to assume that the motion of the fluid in each slice is independent of the motion in adjacent slices and that the displacements, velocities, and accelerations in each slice are in a plane normal to the keel.

A particular unit slice has been indicated by cross hatching in figure 7. At the center line of this slice the keel is at a distance ζ from the water surface. Corresponding to the two-dimensional theory previously developed there may be associated with a particular slice of fluid the additional water mass $\overline{m}$ given by

$$\bar{m} = \bar{\epsilon}\,\zeta^2 \tag{32}$$

It is understood that the coefficient $\bar{\epsilon}$ now contains a reduction factor for aspect-ratio effect.

The center line of the cross-hatched slice in figure 7 is at a distance ξ from the leading edge of the loaded area. The quantities ξ and ζ are instantaneous values. The fluid in the cross-hatched slice exerts, on the float, a force f given by the following formula:

$$f = \frac{d}{dt}\left(\bar{m}\dot{\zeta}\right) = \bar{m}\ddot{\zeta} + \bar{m}'\dot{\zeta}^2 \tag{33}$$

In order to obtain the total force on the entire bottom of the float, the above formula may be integrated over the wetted keel length L.

$$F = \int_0^L f\,d\xi = \int_0^L \left(\bar{m}\ddot{\zeta} + \bar{m}'\dot{\zeta}^2\right) d\xi \tag{34}$$

Substitution of equation (32) and its derivative gives,

$$F = \int_0^L \bar{\epsilon}\zeta^2\ddot{\zeta}\,d\xi + \int_0^L 2\bar{\epsilon}\zeta\dot{\zeta}^2\,d\xi \tag{35}$$

The distances ζ and ξ are related through the trim angle.

$$\zeta = \xi \tan\tau \tag{36}$$

Substituting this formula into equation (35) and bringing the constant terms outside of the integrals gives,

$$F = \bar{\epsilon}\ddot{\zeta}\tan^2\tau \int_0^L \xi^2\,d\xi + 2\bar{\epsilon}\dot{\zeta}^2 \tan\tau \int_0^L \xi\,d\xi \tag{37}$$

$$= \bar{\epsilon}\ddot{\zeta}\tan^2\tau\left(\frac{L^3}{3}\right) + 2\bar{\epsilon}\dot{\zeta}^2\tan\tau\left(\frac{L^2}{2}\right) \tag{38}$$

It may be seen from the definitions of ζ and z that, although their magnitudes are different, their first and second derivatives with respect to time are equal.

$$\dot{\zeta} = \dot{z}, \qquad \ddot{\zeta} = \ddot{z} \tag{39}$$

Substitution in equation (38) gives,

$$F = \left(\frac{\bar{\epsilon} L^3 \tan^2 \tau}{3}\right) \ddot{z} + \left(\bar{\epsilon} L^2 \tan \tau\right) \dot{z}^2 \qquad (40)$$

If s is the value of ζ at the step, it will be related to the length L through the trim angle.

$$s = L \tan \tau \qquad (41)$$

Thus equation (40) may be written,

$$F = \left(\frac{\bar{\epsilon} L s^2}{3}\right) \ddot{z} + \left(\bar{\epsilon} L s\right) \dot{z}^2 \qquad (42)$$

The quantity which is within the first parenthesis may be recognized as the mass of a half cone of water. This three-dimensional additional water mass is illustrated in figure 8. This mass may be written as,

$$m = \frac{\bar{\epsilon} L s^2}{3} = \left(\frac{\bar{\epsilon}}{3 \tan \tau}\right) s^3 = \epsilon s^3 \qquad (43)$$

The derivative of m with respect to s may be written,

$$\frac{dm}{ds} = m' = \frac{\bar{\epsilon} s^2}{\tan \tau} = 3 \epsilon s^2 \qquad (44)$$

The quantity which is contained in the second parenthesis of equation (42) may be recognized as equal to the above derivative. Hence equation (42) may be written,

$$F = m \ddot{z} + m' \dot{z}^2 \qquad (45)$$

This formula was originally developed in approximately the same form by W. L. Mayo (reference 5).

In the above equation it is of particular importance to note that the derivative m' is taken with respect to s rather than z as in the two-dimensional case. The importance of this distinction arises from certain new features of the three-dimensional case. Although the reaction of a particular unit slice on the float is not affected by the component of velocity parallel to the keel, the number of unit slices and the rate of change of this number are affected by this component of velocity. Or, from another viewpoint, the additional water mass is determined by the step distance s which varies due to $\dot{x}$ as well as $\dot{z}$. This matter has been given rather extensive discussion by Mayo (reference 5).

The analysis will be based on the assumption that the horizontal velocity V_h remains constant. This means that the resultant acceleration is vertical. Hence the equation of motion must be written for forces in the vertical direction. The resultant value of the acceleration, which is vertical, is given by $\ddot{z}/\cos\tau$. The vertical component of reaction force is given by $F\cos\tau$. This assumes that the resultant reaction force is normal to the keel. Newton's equation of motion may be written,

$$\frac{M\ddot{z}}{\cos\tau} + F\cos\tau = 0 \tag{46}$$

Substitution of equation (45) gives,

$$\frac{\ddot{z}}{\cos\tau} + m\ddot{z}\cos\tau + m'\dot{z}^2\cos\tau = 0 \tag{47}$$

An equation of this type, showing considerable variation, however, was originally published by Mayo (reference 6). A single integration of his equation was performed by Mayo after which relations between displacement, velocity, acceleration, and time were obtained by numerical methods.

It is convenient to introduce the dimensionless constant μ as,

$$\mu = \frac{m\cos^2\tau}{M} \tag{48}$$

Equation (47) may be written as

$$\ddot{z} + \left(\frac{m\cos^2\tau}{M}\right)\ddot{z} + \left(\frac{m'\cos^2\tau}{M}\right)\dot{z}^2 = 0 \tag{49}$$

or

$$(1 + \mu)\ddot{z} + \dot{z}^2\frac{d\mu}{ds} = 0 \tag{50}$$

This equation may be written in the form,

$$(1 + \mu)\ddot{z} + \left(\frac{\dot{z}^2}{\dot{s}}\right) \frac{d\mu}{dt} = 0 \tag{51}$$

It is now necessary to develop expressions for $\dot{z}$ and $\dot{s}$ in terms of the constant velocity V_h and a convenient variable. The following geometric relations are apparent from figure 8.

$$\dot{z} = V \sin (\gamma + \tau) \tag{52}$$

$$= V \sin \gamma \cos \tau + V \cos \gamma \sin \tau \tag{53}$$

$$= V_v \cos \tau + V_h \sin \tau \tag{54}$$

$$(\dot{z})^2 = (V_v \cos \tau + V_h \sin \tau)^2 \tag{55}$$

$$= \left(\frac{V_v \cos \tau}{V_h \sin \tau} + 1\right)^2 V_h^2 \sin^2 \tau \tag{56}$$

The ratio V_v/V_h is the tangent of the flight-path angle. It is convenient to introduce the dimensionless variable r given by the ratio of $\tan \gamma$ to $\tan \tau$.

$$r = \frac{\tan \gamma}{\tan \tau} \tag{57}$$

The flight-path angle may be regarded as a dynamic trim angle and the ratio r is the ratio of the dynamic to the static trim angle. Equation (56) becomes,

$$(\dot{z})^2 = (r + 1)^2 V_h^2 \sin^2 \tau \tag{58}$$

The depth of the point A on the float below the water surface at any instant is equal to $s \cos \tau$. The vertical velocity of A may be obtained by differentiating this quantity with respect to time. Hence,

$$V_v = \dot{s} \cos \tau \tag{59}$$

or

$$\dot{s} = \frac{V_v}{\cos \tau} = \left(\frac{V_v \cos \tau}{V_h \sin \tau}\right) \left(\frac{V_h \sin \tau}{\cos^2 \tau}\right) \tag{60}$$

$$= \frac{r V_h \sin \tau}{\text{[illegible]}} \tag{61}$$

It is desirable to determine the derivative of r with respect to time.

$$\frac{dr}{dt} = \frac{1}{\tan \tau} \frac{d}{dt}\left(\frac{V_v}{V_h}\right) = \left(\frac{1}{V_h \tan \tau}\right)\left(\frac{\ddot{z}}{\cos \tau}\right) \tag{62}$$

$$= \frac{\ddot{z}}{V_h \sin \tau} \tag{63}$$

Substituting equations (58), (61), and (63) into equation (51) gives,

$$(1 + \mu)\ V_h \sin \tau \frac{dr}{dt} + \frac{(r + 1)^2}{r} V_h \sin \tau \cos^2\tau \frac{d\mu}{dt} = 0 \tag{64}$$

or

$$(1 + \mu)\ dr + \frac{(r + 1)^2}{r} \cos^2\tau\ d\mu = 0 \tag{65}$$

Separating the variables and integrating gives,

$$\log (1 + r) + \frac{1}{1 + r} + \cos^2\tau \ \log (1 + \mu) = C \tag{66}$$

The constant of integration C must be determined from the initial conditions. At $t = 0$, the distance s is zero, the water mass m is zero and, hence, $\mu = 0$. The value of r at $t = 0$ may be indicated as r_o. This value will be determined by the initial flight-path angle γ_o.

$$r_o = \frac{\tan \gamma_o}{\tan \tau} \tag{67}$$

Substituting the initial conditions in equation (66) gives the value of C.

$$C = \log (1 + r_o) + \frac{1}{1 + r_o} \tag{68}$$

Equation (66) becomes,

$$\log (1 + r) + \frac{1}{1 + r} + \cos^2\tau \ \log (1 + \mu) = \log (1 + r_o) + \frac{1}{1 + r_o}$$

This equation expresses a relation between instantaneous values of μ and r for a particular trim angle and a given initial flight-path angle. By assuming various values of r from r_o to zero the associated values of μ can readily be computed. This provides a direct relationship between the vertical velocity and the draft of the float.

At this point in the analysis it is necessary to return to the original differential equation of motion, just as in the two-dimensional case, and to regard this equation as a formula for the acceleration in terms of the displacement and velocity. Solving equation (50) for $\ddot{z}$ gives,

$$\ddot{z} = -\frac{\dot{z}^2}{1 + \mu}\frac{d\mu}{ds} \tag{70}$$

From equations (44) and (48) it is seen that,

$$\frac{d\mu}{ds} = \frac{3\mu}{s} \tag{71}$$

Substitution of equations (58) and (71) into equation (70) gives,

$$\ddot{z} = -\frac{1}{s}\left(\frac{3\mu}{1 + \mu}\right)(1 + r)^2 V_h^{\,2} \sin^2\tau \tag{72}$$

A relationship between μ and r which holds true at the instant of maximum acceleration can be obtained by differentiating the formula for $\ddot{z}$ with respect to s and equating the result to zero. In order to carry out this differentiation it is necessary to establish the derivative of r with respect to s. Dividing equation (63) by $\dot{s}$ gives,

$$\frac{dr}{dt}\frac{dt}{ds} = \frac{\ddot{z}}{\dot{s}\, V_h \sin\tau} \tag{73}$$

Substituting equation (61) on the right-hand side gives

$$\frac{dr}{ds} = \frac{\ddot{z}}{V_h \sin\tau}\left(\frac{\cos^2\tau}{rV_h \sin\tau}\right) = \frac{\ddot{z}}{rV_h^{\,2} \tan^2\tau} \tag{74}$$

Differentiating equation (72), and using the above formula, results in the following equation,

$$\frac{d\dddot{z}}{ds} = \frac{\ddot{z}}{s}\left[\frac{2 - \mu}{1 + \mu} - \frac{6\mu}{1 + \mu}\left(\frac{1 + r}{r}\right)\cos^2\tau\right] \tag{75}$$

The quantity within the brace may be equated to zero to obtain a relation between μ and r which must hold true at the instant of maximum acceleration. These particular values may be indicated as μ_m and r_m.

$$\frac{2 - \mu_m}{1 + \mu_m} - \frac{6\mu_m}{1 + \mu_m}\left(\frac{1 + r_m}{r_m}\right)\cos^2\tau = 0 \tag{76}$$

This equation may be solved for μ_m or r_m to obtain the formulas.

$$\mu_m = \frac{2r_m}{r_m(1 + 6\cos^2\tau) + 6\cos^2\tau} \tag{77}$$

$$r_m = \frac{6\mu_m\cos^2\tau}{2 - (1 + 6\cos^2\tau)\mu_m} \tag{78}$$

By integrating the original differential equation of motion a relation between μ and r was established as given by equation (69). This relation must hold true at all instants of the impact period. Equation (76) gives a relation which must hold true at the instant of maximum acceleration. Equations (69) and (76) may be solved simultaneously to obtain solutions for μ_m and r_m in terms of the known initial quantity r_o. The resulting values may then be substituted into equation (72) to determine the maximum acceleration.

Since equation (69) is of a transcendental nature, no direct analytical solution can be obtained for the maximum acceleration. If s_m and $\ddot{z}_m$ are introduced to indicate the values of these quantities when the acceleration is a maximum, equation (72) gives,

$$\ddot{z}_m = \frac{-1}{s_m}\left(\frac{3\mu_m}{1 + \mu_m}\right)(1 + r_m)^2 V_h^2 \sin^2\tau \tag{79}$$

From equations (43) and (48) the distance s_m is given by,

$$s_m = \sqrt[3]{\frac{\mu_m M}{\epsilon \cos^2 \tau}} \tag{80}$$

It is now apparent that, by using numerical and graphical methods for determining μ_m and r_m, it would be possible to calculate the maximum acceleration if the parameter ϵ were known. This parameter determines the apparent mass of the water. No attempt will be made in this paper to discuss the various theoretical and experimental studies which have been made to determine this parameter. These developments have been discussed by Mayo (reference 5). A considerable amount of basic research work in this field is still needed.

The maximum acceleration may be expressed in terms of various components of the initial velocity. That component which seems most rational to use is the component normal to the keel, which is indicated as $\dot{z}_o$. From equation (58) it is seen that,

$$V_h^2 \sin^2 \tau = \frac{\dot{z}_o^2}{(1 + r_o)^2} \tag{81}$$

Substitution into equation (79) gives,

$$\ddot{z}_m = -\frac{1}{s_m}\left(\frac{3\mu_m}{1 + \mu_m}\right)\left(\frac{1 + r_m}{1 + r_o}\right)^2 \dot{z}_o^2 \tag{82}$$

For design purposes it is most convenient to express the acceleration in terms of the resultant initial velocity V_o. Using the definition of r as given by equation (57) it is readily shown that,

$$V_h^2 = \frac{V_o^2}{1 + r_o^2 \tan^2 \tau} \tag{83}$$

Substitution into equation (79) gives,

$$\ddot{z}_m = -\frac{1}{s_m}\left(\frac{3\mu_m}{1 + \mu_m}\right)\frac{(1 + r_m)^2 V_o^2 \sin^2 \tau}{(1 + r_o^2 \tan^2 \tau)} \tag{84}$$

At the instant of maximum draft the flight-path angle is zero. Substituting $r = 0$ in equation (69) gives,

$$\cos^2\tau \log (1 + \mu_n) = \log (1 + r_o) - \frac{r_o}{1 + r_o} \qquad (85)$$

The subscript n is used to indicate the value of functions at the instant of maximum draft. For any given value of r_o, the corresponding value of μ_n can be computed. The maximum draft may then be determined from equation (80) by changing the subscript m to n.

APPROXIMATE FORMULAS FOR DESIGN

Since the trim angle is a small angle in normal landings it appears reasonable to assume $\cos \tau = 1$ wherever this proves to be convenient. Under this assumption the approximate form of equations (69) and (77) becomes,

$$\log (1 + r) + \frac{1}{1 + r} + \log (1 + \mu) = \log (1 + r_o) + \frac{1}{1 + r_o} \qquad (86)$$

$$\mu_m = \frac{2r_m}{7r_m + 6} \qquad (87)$$

A special case is worthy of note for research purposes. In the case of a vertical drop, with guides to maintain $V_h = 0$, the value of r will be infinite at all instants. Dividing numerator and denominator of equation (87) by r_m and setting $r_m = \infty$ gives the universal constant $\mu_m = 2/7$ for the three-dimensional case. This constant was first arrived at by I. W. McCaig of Great Britain.

Equations (86) and (87) must be solved simultaneously for various values of r_o. A graphical method has been used for this purpose. An illustration of a particular solution is given in figure 9. Curve number 1 is a graphical representation of equation (86) for $r_o = 1$. Curve number 2 is a representation of equation (87). The intersection of the two curves gives the values of μ_m and r_m for $r_o = 1$. In order to show the small error involved in neglecting the effect of trim angle, the dotted curves have been plotted in figure 9 from equations (69) and (77) for $\tau = 8^\circ$.

Using the method illustrated in figure 9, the functions μ_m and r_m were computed to give the graphs shown in figure 10. These universal curves are independent of the physical properties of the float or the coefficient which defines the additional water mass.

The only factor remaining to be determined in the formula for acceleration is the step draft s_m. This quantity is affected by the float properties and the water mass coefficient. The additional water mass may be written in the following form:

$$m = \epsilon s^3 = (\alpha_1 \alpha_2 s)^3 \rho \tag{88}$$

The factor $\alpha_1{}^3$ is introduced to account for the primary effects of τ and β upon m. This factor is determined from the intersection of the float bottom with the original water surface. It defines a half cone of water which is the three-dimensional analog of the original two-dimensional concept of von Kármán. Thus, from the geometry of figure 8 it is seen that,

$$\alpha_1{}^3 = \frac{\pi}{6 \tan \tau \tan^2\beta} \tag{89}$$

The factor $\alpha_2{}^3$ is introduced into equation (88) to account for secondary effects of τ and β upon m. Some of these effects are:

(1) Effect of piled-up water.

(2) Dead-rise angle correction to flat-plate theory.

(3) Aspect-ratio effect.

There is a serious need for both theoretical and experimental research studies to establish these factors accurately. Using the best evidence available, Mayo (reference 5) has suggested the following formula:

$$\alpha_2{}^3 = 0.82 \tan^2\beta \left(\frac{\pi}{2\beta} - 1\right)^2 \left(1 - \frac{\tan \tau}{2 \tan \beta}\right) \tag{90}$$

For a practical range of τ from 3° to 12° and β from 20° to 25°, equation (90) gives $\alpha_2{}^3$ in the range $0.9 < \alpha_2{}^3 < 1.3$.

In order to obtain dimensionless quantities for plotting purposes, it is convenient to introduce the fundamental length parameter λ.

$$\lambda = \sqrt[3]{\frac{M}{\rho}} \tag{91}$$

Dividing through equation (88) by M, and using equation (91) gives the dimensionless variable μ as,

$$\mu = \left(\alpha_1\alpha_2 \frac{s}{\lambda}\right)^3 \tag{92}$$

or

$$\frac{\alpha_1 s}{\lambda} = \frac{1}{\alpha_2} \sqrt[3]{\mu} \tag{93}$$

By using figure 10 for μ_m the right-hand side of this equation has been computed for $\alpha_2{}^3 = 0.9$, 1.1, and 1.3. These theoretical curves are shown in figure 11. For a given float, landing at a given trim angle and flight-path angle, the step draft s_m may be determined to a practical degree of accuracy from the solid line of figure 11.

For plotting purposes, equation (84) may be written in the following form by using equation (93).

$$\frac{\ddot{z}_m\lambda}{\alpha_1 V_o{}^2} = -\frac{\alpha_2}{\sqrt[3]{\mu_m}}\left(\frac{3\mu_m}{1 + \mu_m}\right)\frac{(1 + r_m)^2 \sin^2\tau}{(1 + r_o{}^2 \tan^2\tau)} \tag{94}$$

The right-hand side of this equation is a function of r_o which may be computed and plotted for various trim angles. Using an average value of $\alpha_2{}^3 = 1.1$ this function is plotted in figure 12 for $\tau = 3^\circ$ and in figure 13 for $\tau = 12^\circ$. From such curves the maximum acceleration may be readily determined for any float. The reaction force is then obtained by multiplying the acceleration by the mass of the float.

The longitudinal distribution of the reaction may be determined. The ordinate to the load distribution diagram is given by f. Referring to equation (33) it is seen that f consists of two parts. The first part is proportional to $\overline{m}$ which varies as ξ^2.

Hence the first part has a quadratic variation. The second part has a linear variation since $\bar{m}'$ varies as ξ. Integration of the first part of f gives the first part of the total reaction F. The magnitude of this first part of F is seen to be $m\ddot{z}$ or $\mu M\ddot{z}$. Consequently the second part of F must be given by $-(1 + \mu)M\ddot{z}$. The two parts of the reaction and their distribution are indicated by the diagrams of figure 14.

COMPARISON OF THEORY WITH EXPERIMENT

Some excellent experimental results have been recently published by the National Advisory Committee for Aeronautics for landing impacts on smooth water. These tests were run with a float weighing 1100 pounds and having a dead-rise angle of 22.5°. The floats were launched at various trim angles and various flight-path angles. Experimental values will be taken from two reports by S. A. Batterson (references 7 and 8) for $\tau = 3^{\circ}$ and $\tau = 12^{\circ}$. No attempt will be made to explain the discrepancies between theory and experiment. These discrepancies may be due to any of a number of causes which are not sufficiently well understood at present.

In figure 11 experimental values of the relative step draft at maximum acceleration have been plotted. The scattering of the points indicates the necessity for accurate displacement measurements and the need for effective statistical methods of interpreting hydrodynamic test data. The experimental spread of the points is greater than the theoretical spread predicted by the Mayo formula for α_2. The general trend of the points for $\tau = 3^{\circ}$ is higher than that for $\tau = 12^{\circ}$. This is in direct opposition to theoretical prediction and there is no apparent explanation of the contradiction.

In figures 12 and 13 experimental values of relative acceleration have been plotted. The agreement between theory and experiment is very good. The curves are plotted for an average value of α_2.

The value of μ_n at the instant of maximum draft may be computed from the following approximate form of equation (85).

$$\log (1 + \mu_n) = \log (1 + r_o) - \frac{r_o}{1 + r_o} \qquad (95)$$

The right-hand side of equation (93) has been computed for three values of α_2 to give the curves shown in figure 15. Experimental values of the relative draft are also shown. In general, the theory predicts larger draft values than are found from experiment. The same contradiction between theory and experiment is found for the maximum draft values of figure 15 as was noted in connection with the draft values shown in figure 11.

Corresponding to equation (94) other dimensionless acceleration formulas may be derived from equations (79) and (82),

$$\frac{\ddot{z}_m \lambda}{\alpha_1 V_h^2 \sin^2 \tau} = -\frac{\alpha_2}{\sqrt[3]{\mu_m}} \left(\frac{3\mu_m}{1 + \mu_m}\right)(1 + r_m)^2 \qquad (96)$$

$$\frac{\ddot{z}_m \lambda}{\alpha_1 \dot{z}_o^2} = -\frac{\alpha_2}{\sqrt[3]{\mu_m}} \left(\frac{3\mu_m}{1 + \mu_m}\right)\left(\frac{1 + r_m}{1 + r_o}\right)^2 \qquad (97)$$

It may be noticed that the right-hand side of each of these equations is affected by variations in τ and β only through the minor variations of the secondary factor α_2. Using an average value of α_2, a theoretical curve has been computed for each of these equations and compared with experimental values in figures 16 and 17. The general trend of the experimental points for $\tau = 3^\circ$ is higher than that for $\tau = 12^\circ$. This is in agreement with theoretical prediction. Either of these curves may be used for design purposes.

For research purposes equations (79) and (82) may be written in the following forms:

$$\psi_1 = -\frac{\ddot{z}_m s_m}{\dot{z}_o^2} = \left(\frac{3\mu_m}{1 + \mu_m}\right)\left(\frac{1 + r_m}{1 + r_o}\right)^2 \qquad (98)$$

$$\psi_2 = -\frac{\ddot{z}_m s_m}{V_h^2 \sin^2 \tau} = \left(\frac{3\mu_m}{1 + \mu_m}\right)(1 + r_m)^2 \qquad (99)$$

In each case the first definition of ψ contains quantities which may be measured experimentally. The second definition is a function of r_o only. The ψ functions are universal functions of r_o being in no way affected by variations in τ or β. These ψ functions have been plotted and compared with experiment in figures 18 and 19. These functions are not dependent upon the properties of the float or the definition of the additional water mass.

CONCLUSIONS

The landing impact of a seaplane may be analyzed by means of Newton's law of motion and elementary hydrodynamic theory, providing a number of restrictive assumptions are made. Some of these assumptions are as follows:

1. The water surface is smooth.

2. The seaplane is a rigid body.

3. The trim angle remains constant during the impact.

4. The wedge bottom of the hull is not flared.

5. The finite width of the hull has no effect on the motion.

6. The wing lift equals the weight of the seaplane.

7. Buoyancy and viscosity forces may be neglected.

The three-dimensional apparent mass of the water may be taken as a direct analog of the two-dimensional concept of elementary hydrodynamics. On this basis a solution for maximum acceleration may be obtained which shows good agreement with experimental results. Agreement between theory and experiment in regard to displacements is more difficult to obtain and calls for further improvements in the analysis through theoretical and experimental research. The maximum acceleration is found to be proportional to the square of the initial velocity and inversely proportional to the draft at the instant of maximum acceleration. The constant of proportionality is given by the theoretical analysis and is found to be independent of the float properties or the definition of the additional water mass.

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Structural Engineer

ESY

APPENDIX

In the preparation of the graphical illustrations of the paper a number of tables of functions were computed. Some of the tables are included in this appendix in order to provide assistance in further research studies and for design use of the graphs of accelerations. The following function is given in table I.

$$\phi = \log (1 + r) + \frac{1}{1 + r} \tag{100}$$

In table II are given the values of r_m and μ_m as obtained by simultaneous solution of equations (86) and (87). The values of r_m were read from the intersections of graphs and cannot be considered as accurate in the last significant figure. The values of μ_m were not read from the graphical intersections but were computed from equation (87) using the values of r_m.

In table III the values of μ_n are given as obtained by solving equation (95). In table IV values of α_1 are given as defined by equation (89). Table V gives values of the functions ψ_1 and ψ_2 as defined by equations (98) and (99).

REFERENCES

1. von Kármán, Th.: The Impact on Seaplane Floats During Landing. NACA TN No. 321, 1929.

2. Lamb, H.: Hydrodynamics. The MacMillan Co., N. Y., 1932, p. 85.

3. Wagner, H.: Über Stoss - und Gleitvorgänge an der Oberfläche von Flüssigkeiten. ZAMM, Vol. 12, No. 4, August 1932, pp. 193-214.

4. Sydow, J.: The Effect of Spring Support and Keeling on Landing Impact. British Air Ministry Translation No. 61. (originally published in Jahr dent. Luft.), Vol. 1, 1938, pp. 329-338.

5. Mayo, W. L.: Analysis and Modification of Theory for Impact of Seaplanes on Water. NACA TN No. 1008, 1945.

6. Mayo, W. L.: Theoretical and Experimental Dynamic Loads for a Prismatic Float Having an Angle of Dead Rise of $22\frac{1}{2}^{\circ}$. NACA RB No. L5F15, 1945.

7. Batterson, S. A.: Variation of Hydrodynamic Impact Loads with Flight-Path Angle for a Prismatic Float at 3° Trim and with a $22\frac{1}{2}^{\circ}$ Angle of Dead Rise. NACA RB No. L5A24, 1944.

8. Batterson, S. A.: Variation of Hydrodynamic Impact Loads with Flight-Path Angle for a Prismatic Float at 12° Trim and with a $22\frac{1}{2}^{\circ}$ Angle of Dead Rise. NACA RB No. L5K21a, 1945.

TABLE I

r	ϕ	r	ϕ
0	1.00000	2.1	1.45398
.1	1.00440	2.2	1.47565
.2	1.01565	2.3	1.49695
.3	1.03159	2.4	1.51790
.4	1.05076	2.5	1.53847
.5	1.07214	2.6	1.55871
.6	1.09500	2.7	1.57860
.7	1.11887	2.8	1.59816
.8	1.14334	2.9	1.61739
.9	1.16817	3.0	1.63629
1.0	1.19315	3.1	1.65489
1.1	1.21813	3.2	1.67318
1.2	1.24301	3.3	1.69118
1.3	1.26769	3.4	1.70887
1.4	1.29214	3.5	1.72630
1.5	1.31629	3.6	1.74345
1.6	1.34013	3.7	1.76033
1.7	1.36362	3.8	1.77695
1.8	1.38676	3.9	1.79332
1.9	1.40954	4.0	1.80944
2.0	1.43194		

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TABLE II

r_o	r_m	μ_m
0	0	0
.2	.050	.0157
.4	.149	.0423
.6	.277	.0698
.8	.421	.0941
1.0	.561	.1130
1.2	.710	.1294
1.4	.860	.1431
1.6	1.012	.1547
1.8	1.166	.1647
2.0	1.318	.1731
2.2	1.471	.1805
2.4	1.626	.1871
2.6	1.782	.1929
2.8	1.937	.1981
3.0	2.090	.2026
3.2	2.244	.2067
3.4	2.400	.2105
3.6	2.554	.2139
3.8	2.710	.2171
4.0	2.865	.2199

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TABLE III

r_o	μ_n
0.2	0.0158
.4	.0521
.6	.0997
.8	.1541
1.0	.2131
1.2	.2751
1.4	.3393
1.6	.4051
1.8	.4722
2.0	.5402
2.2	.6091
2.4	.6785
2.6	.7484
2.8	.8188
3.0	.8895
3.2	.9605
3.4	1.0317
3.6	1.1032
3.8	1.1748
4.0	1.2467

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TABLE IV

VALUES OF α_1

τ (deg) \ β (deg)	20	22.5	25
1	6.10	5.59	5.17
2	4.84	4.44	4.10
3	4.22	3.88	3.58
4	3.84	3.52	3.25
5	3.56	3.27	3.02
6	3.35	3.07	2.84
7	3.18	2.92	2.70
8	3.04	2.79	2.58
9	2.92	2.68	2.48
10	2.82	2.59	2.39
11	2.73	2.50	2.31
12	2.65	2.43	2.25
13	2.58	2.36	2.18
14	2.51	2.30	2.13
15	2.45	2.25	2.08

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TABLE V

r_o	ψ_1	ψ_2
0	0	0
.2	.0355	.051
.4	.0820	.161
.6	.1247	.319
.8	.1608	.521
1.0	.1856	.742
1.2	.2077	1.007
1.4	.2256	1.300
1.6	.2407	1.627
1.8	.2539	1.990
2.0	.2643	2.38
2.2	.2735	2.80
2.4	.2821	3.26
2.6	.2897	3.76
2.8	.2963	4.28
3.0	.3016	4.83
3.2	.3066	5.41
3.4	.3115	6.03
3.6	.3155	6.68
3.8	.3197	7.37
4.0	.3231	8.08

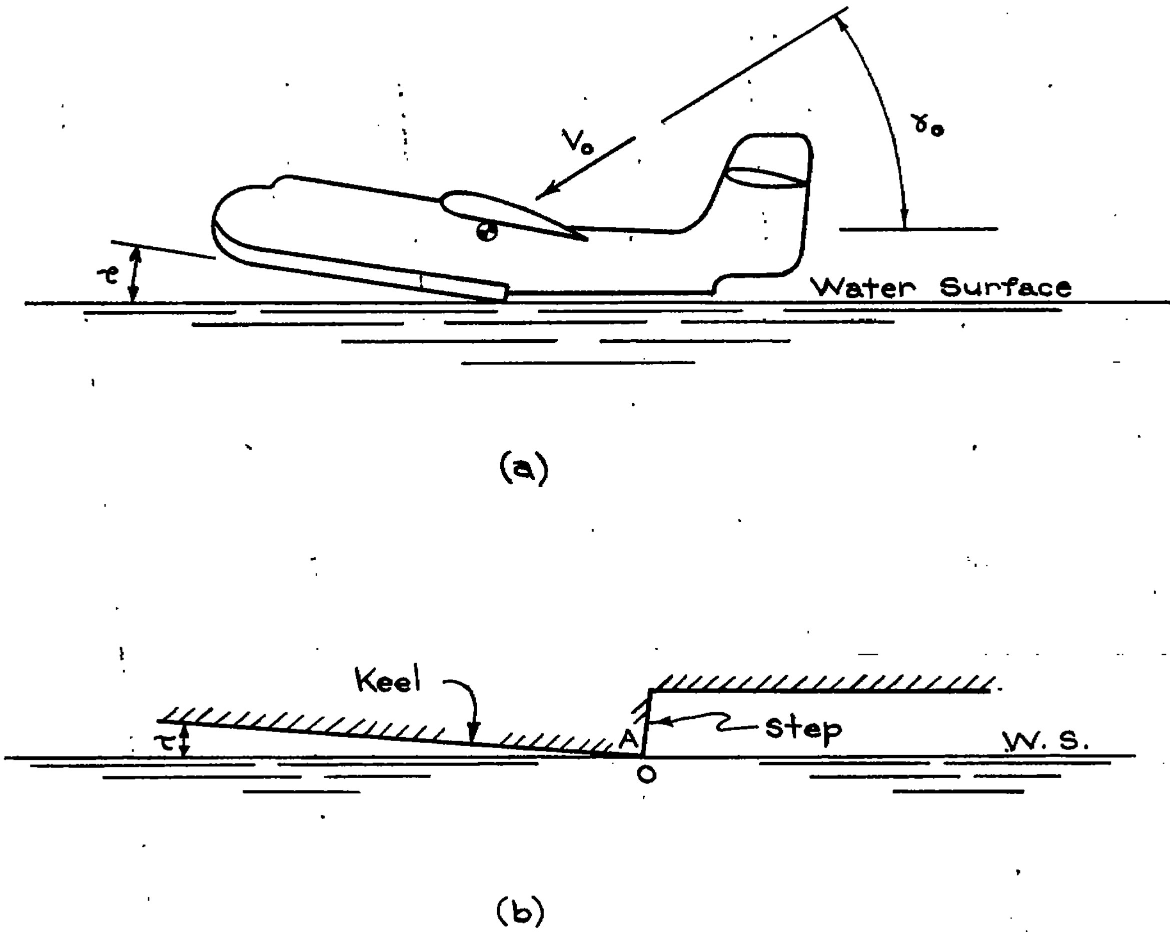

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Figure 1.—Seaplane at Instant of Entry

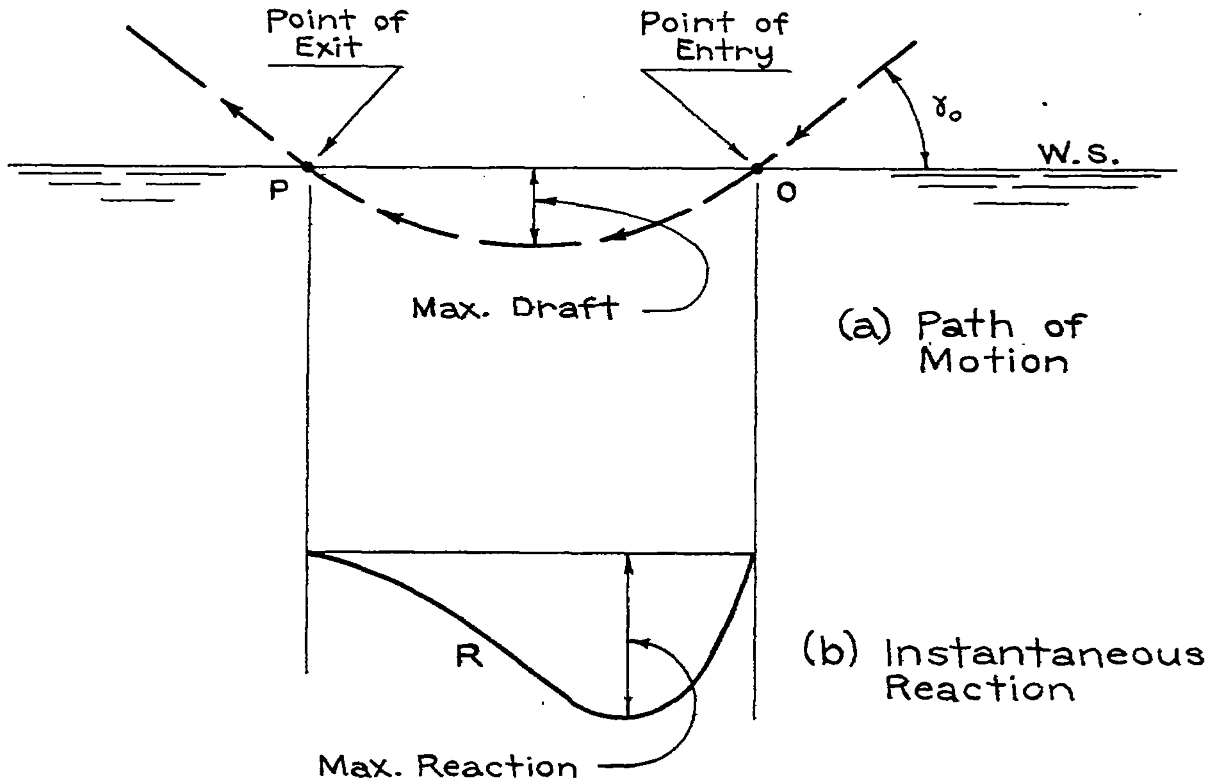

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Figure 2.– Displacements and Reaction During Impact Period

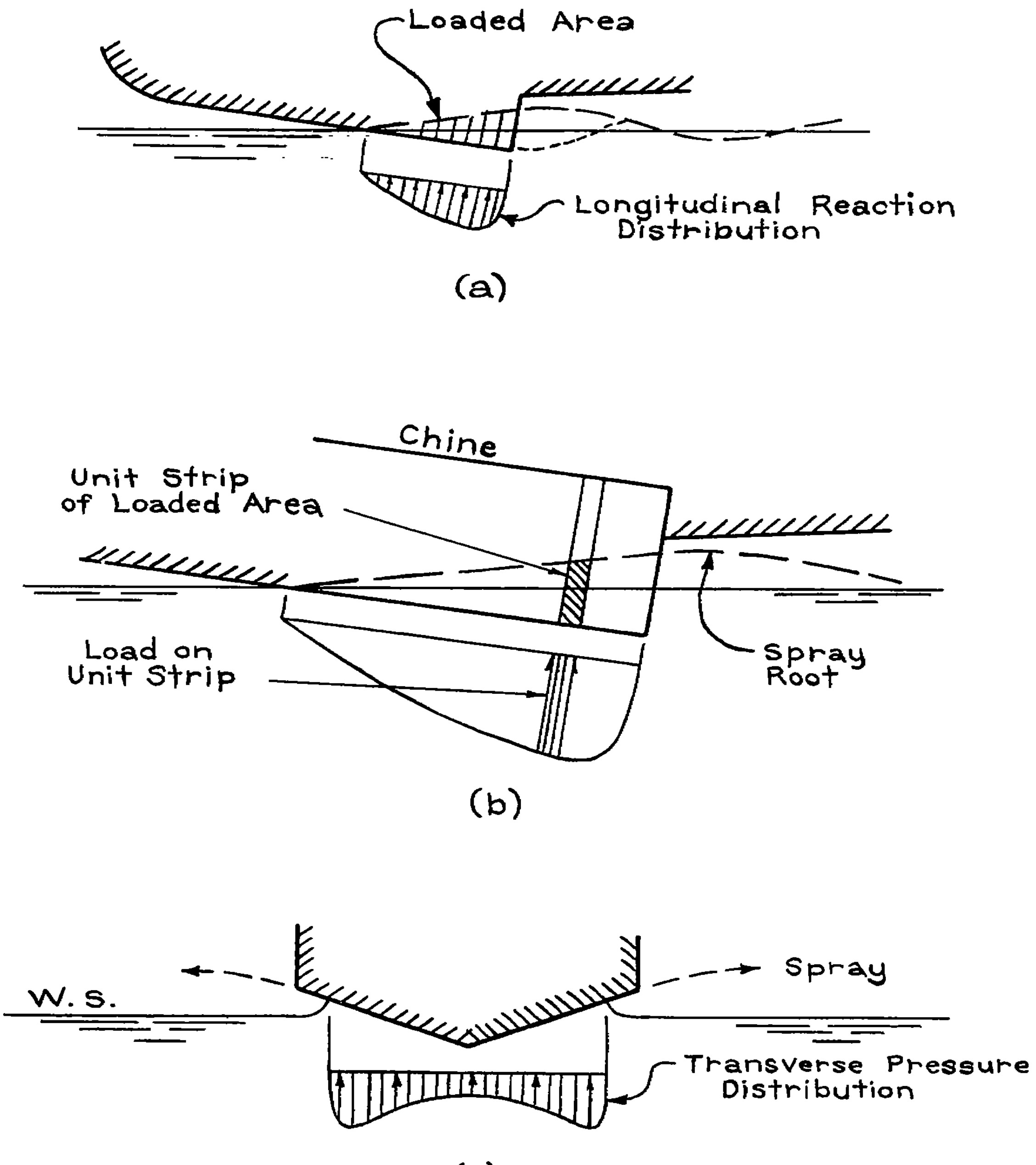

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Figure 3—Reaction Distribution

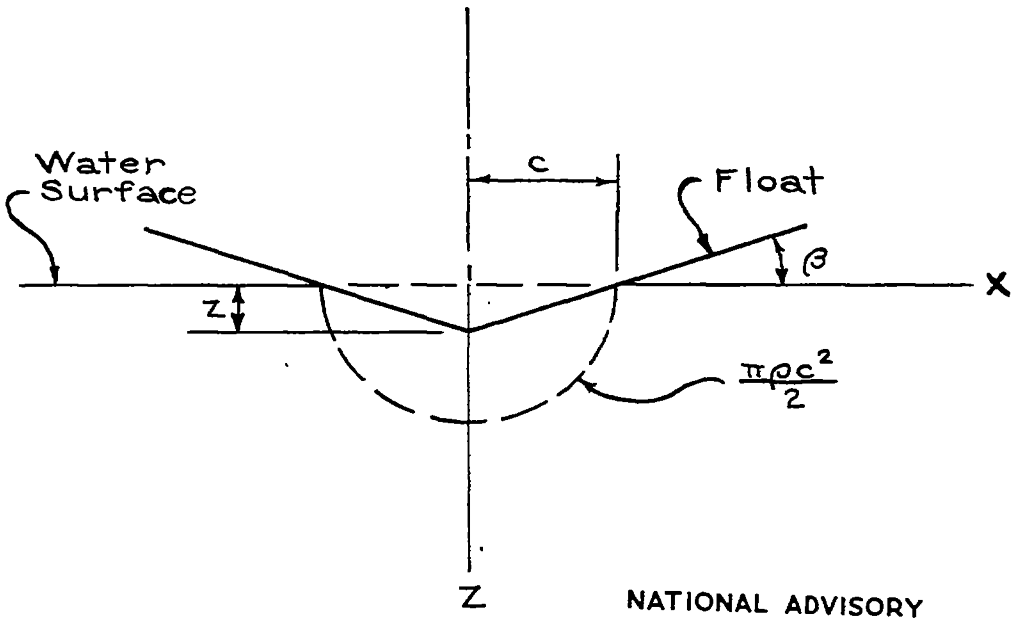

Figure 4.—Elementary Concept of Additional Mass

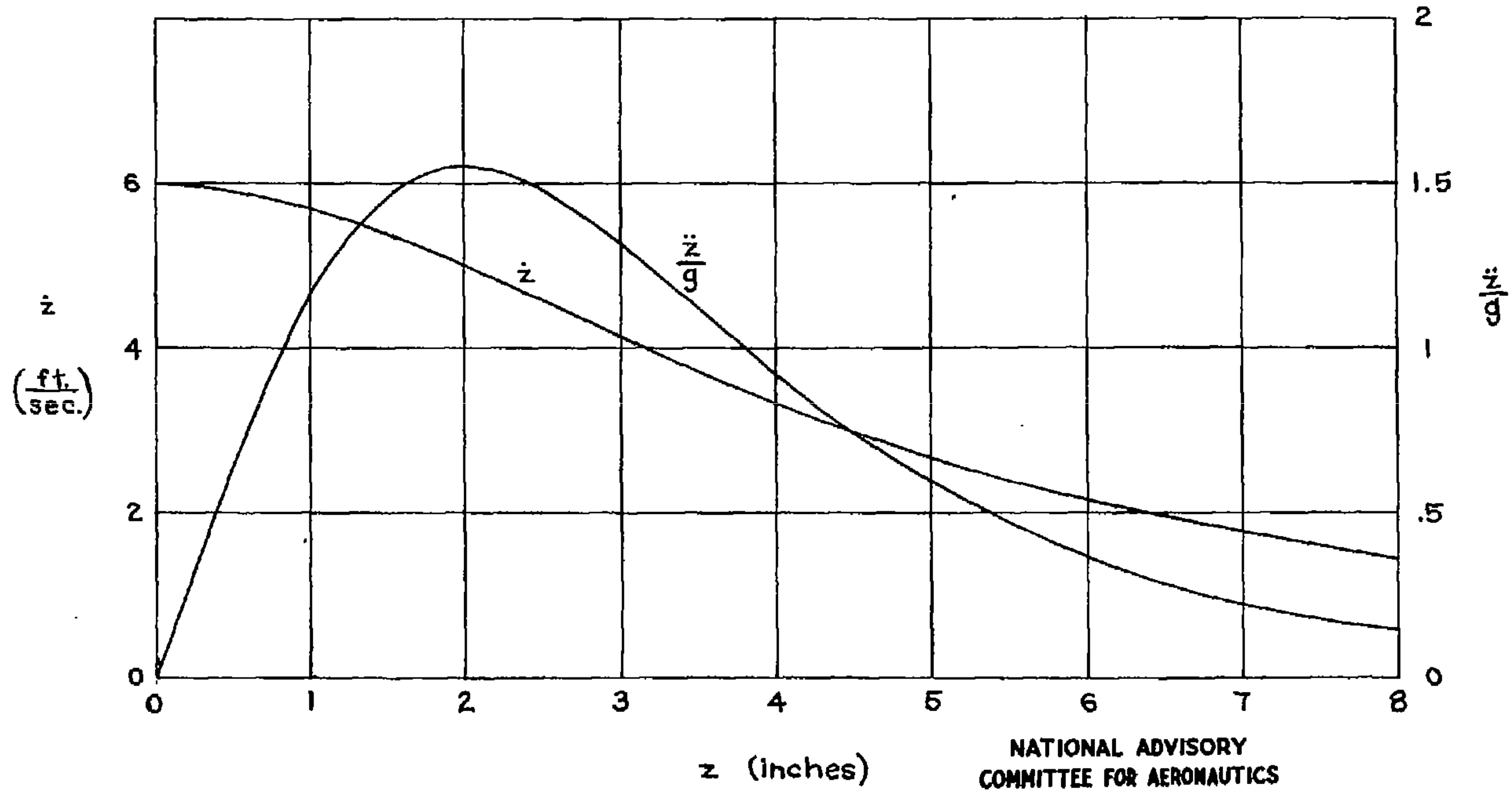

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Figure 5.—Variations with Displacement

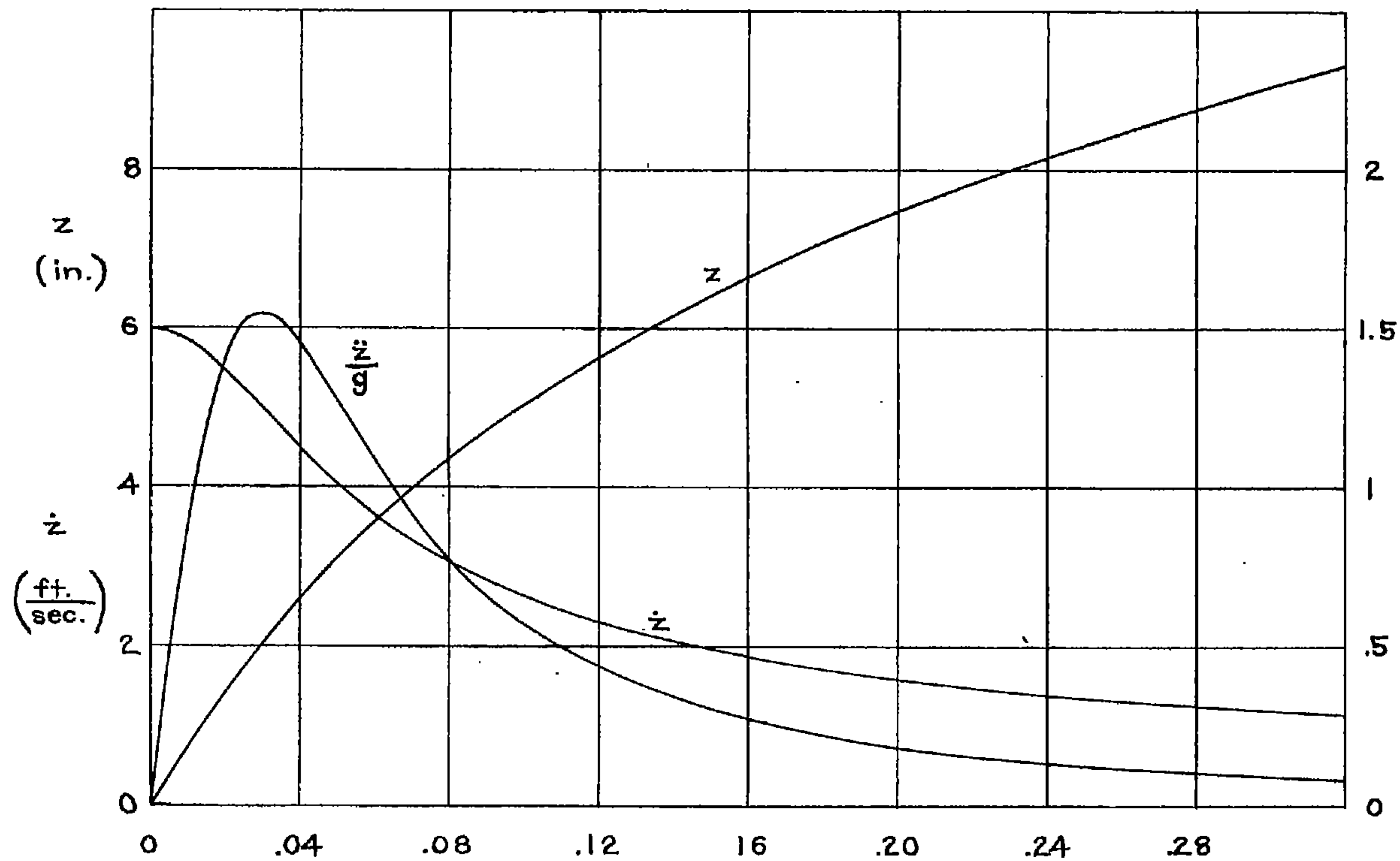

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Figure 6.– Variations with Time

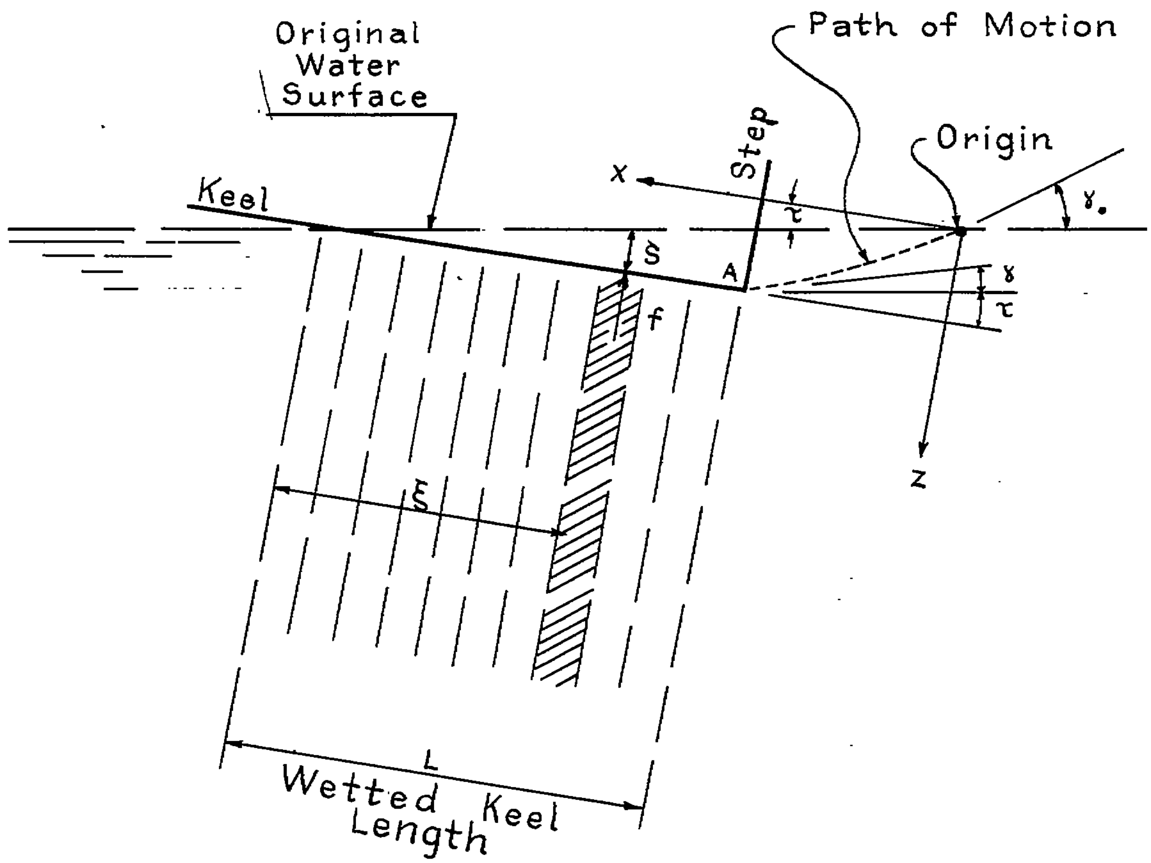

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Figure 7.- Instantaneous Position of Ideal Float.

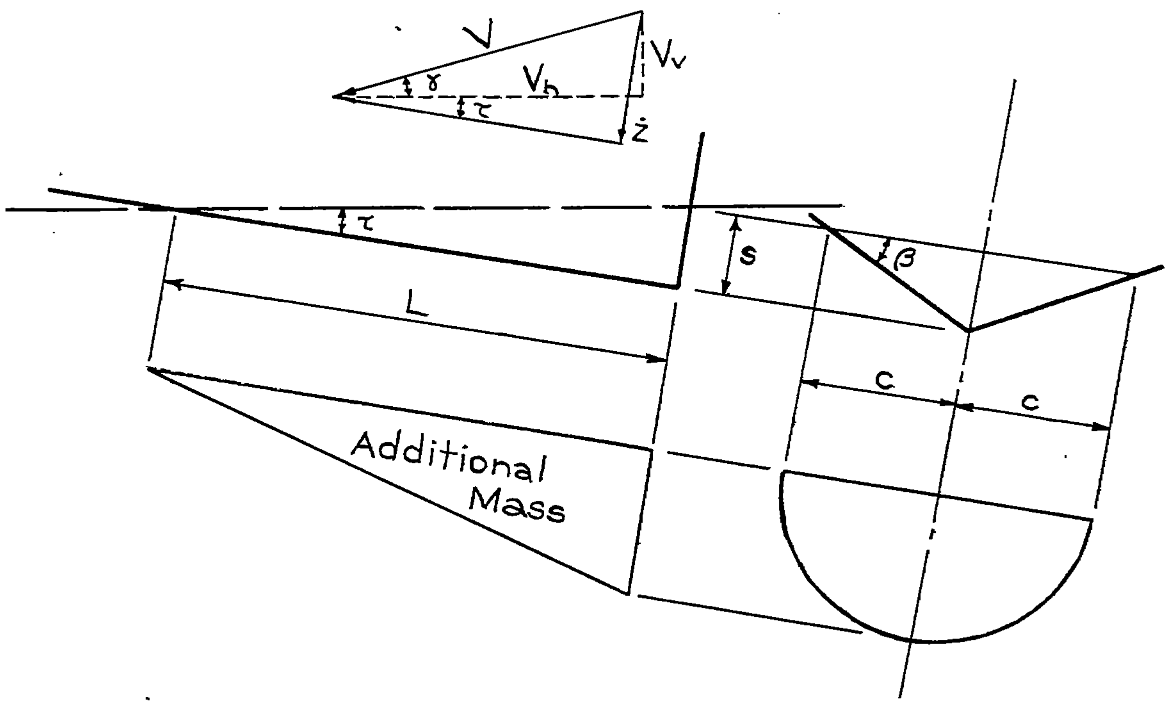

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Figure 8.—Three Dimensional Additional Water Mass

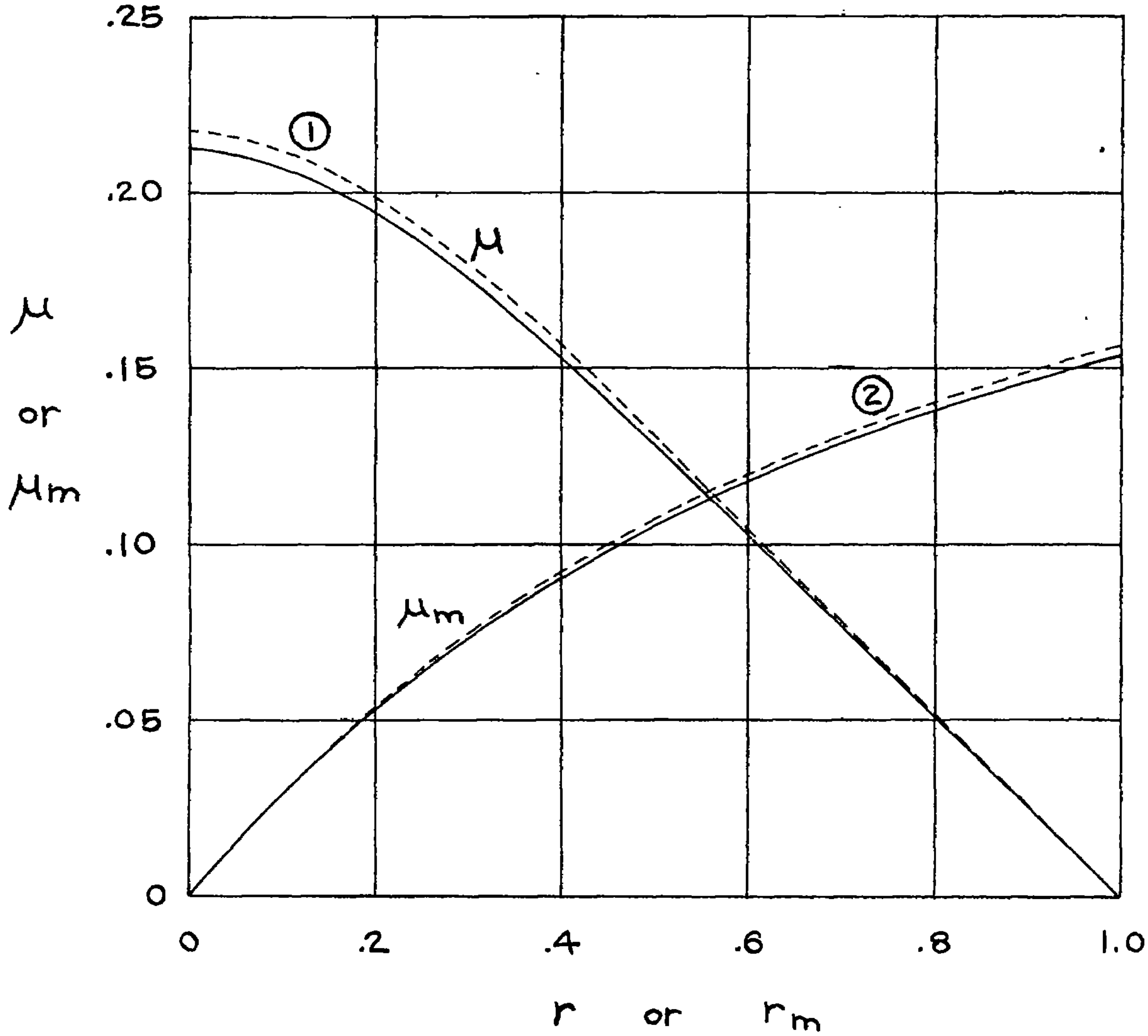

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Figure 9.- Typical Solution for μ_m and r_m

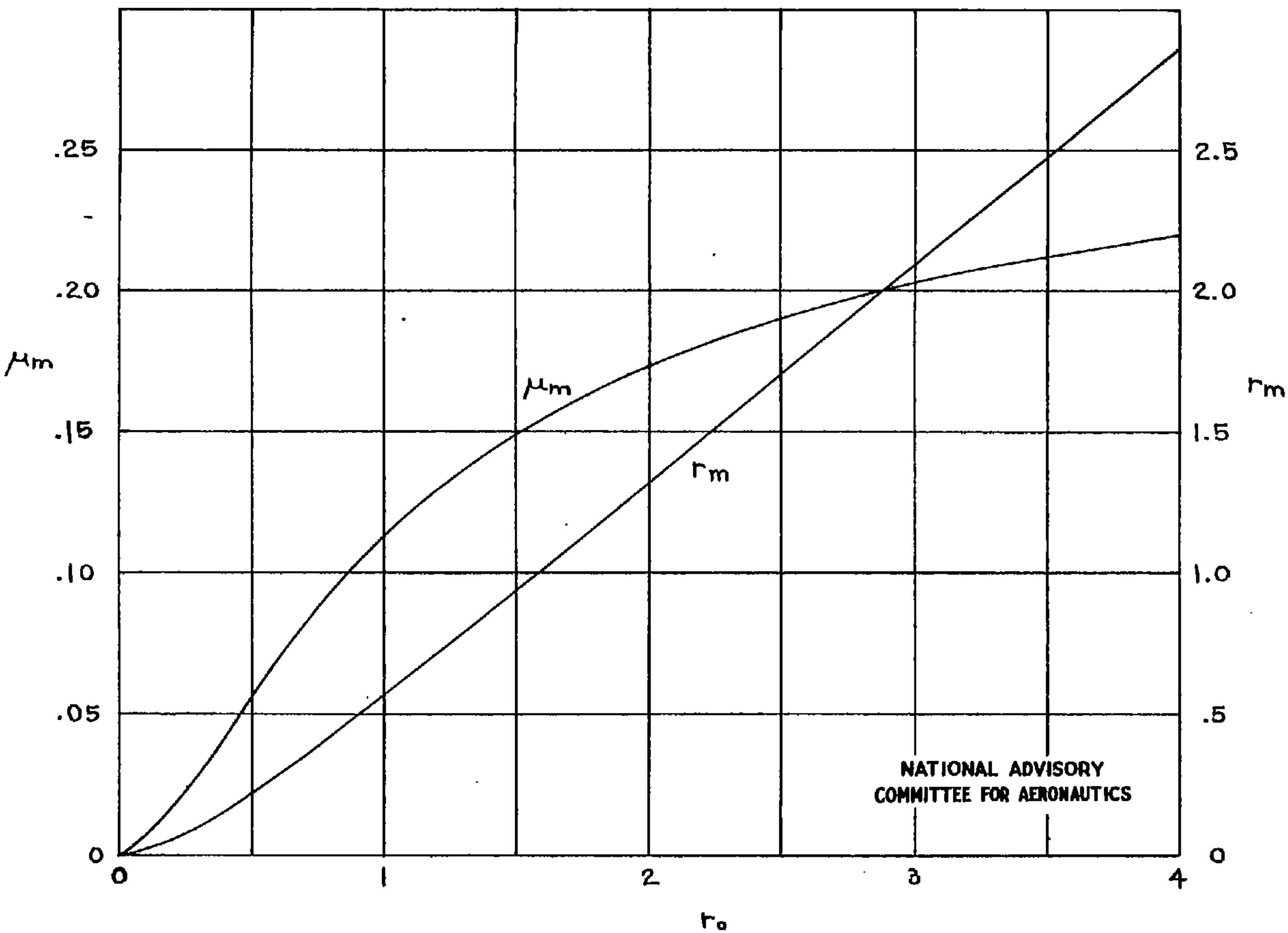

Figure 10.—Graphs of Functions μ_m and r_m

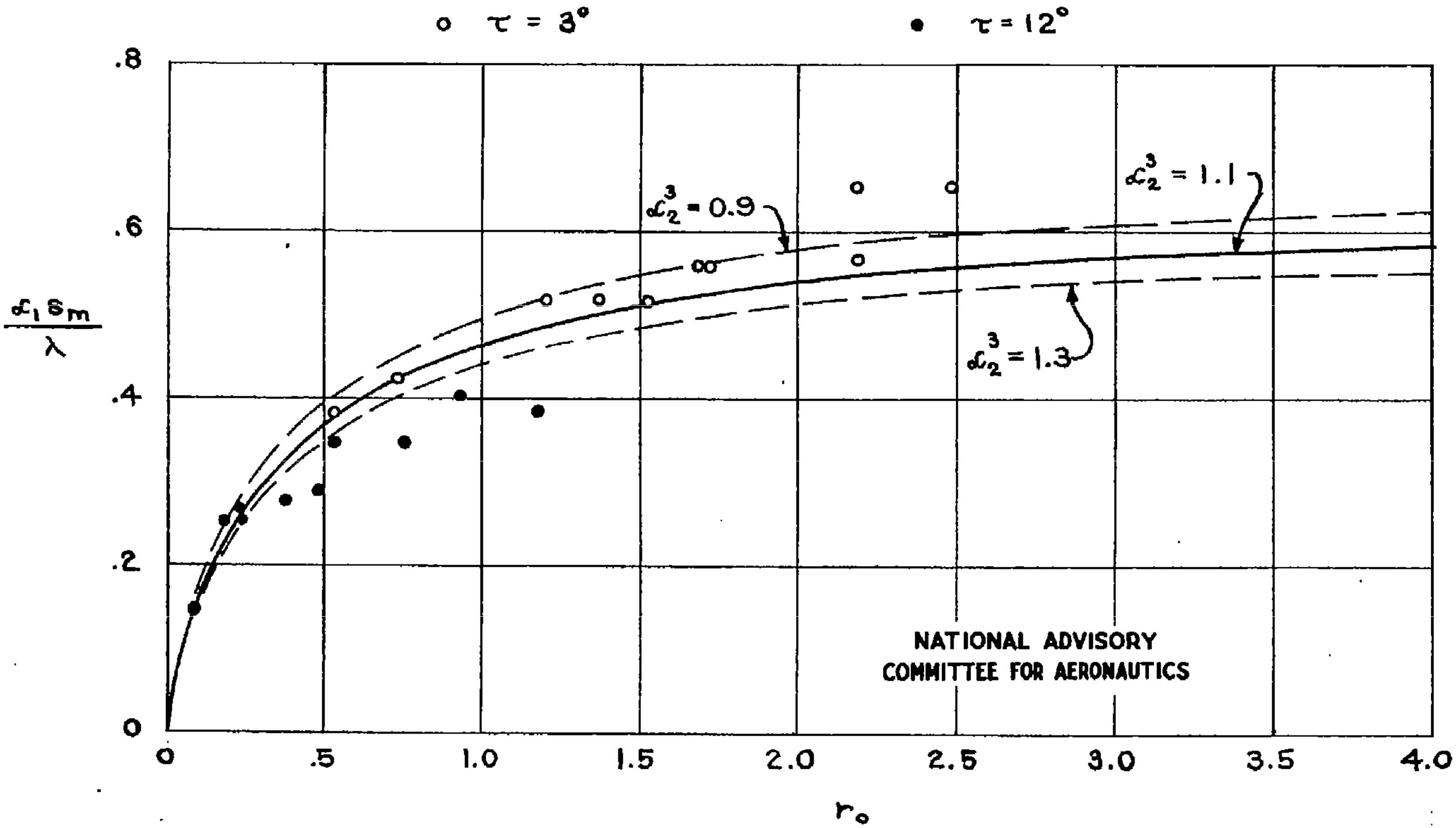

Figure 11.– Displacement at Instant of Maximum Acceleration

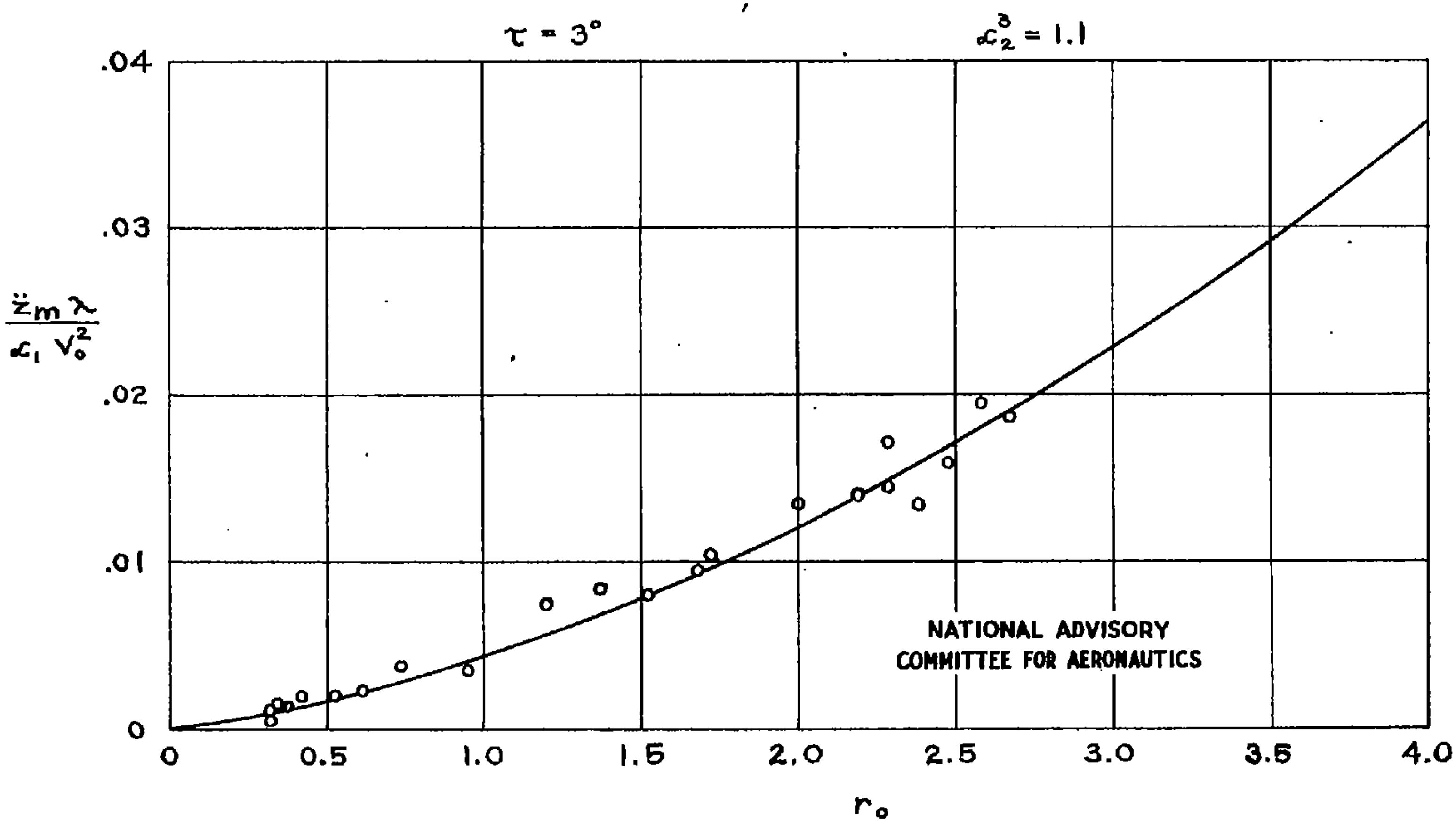

Figure 12.- Ratio of $\ddot{z}_m$ to V_o^2 for $\tau = 3°$

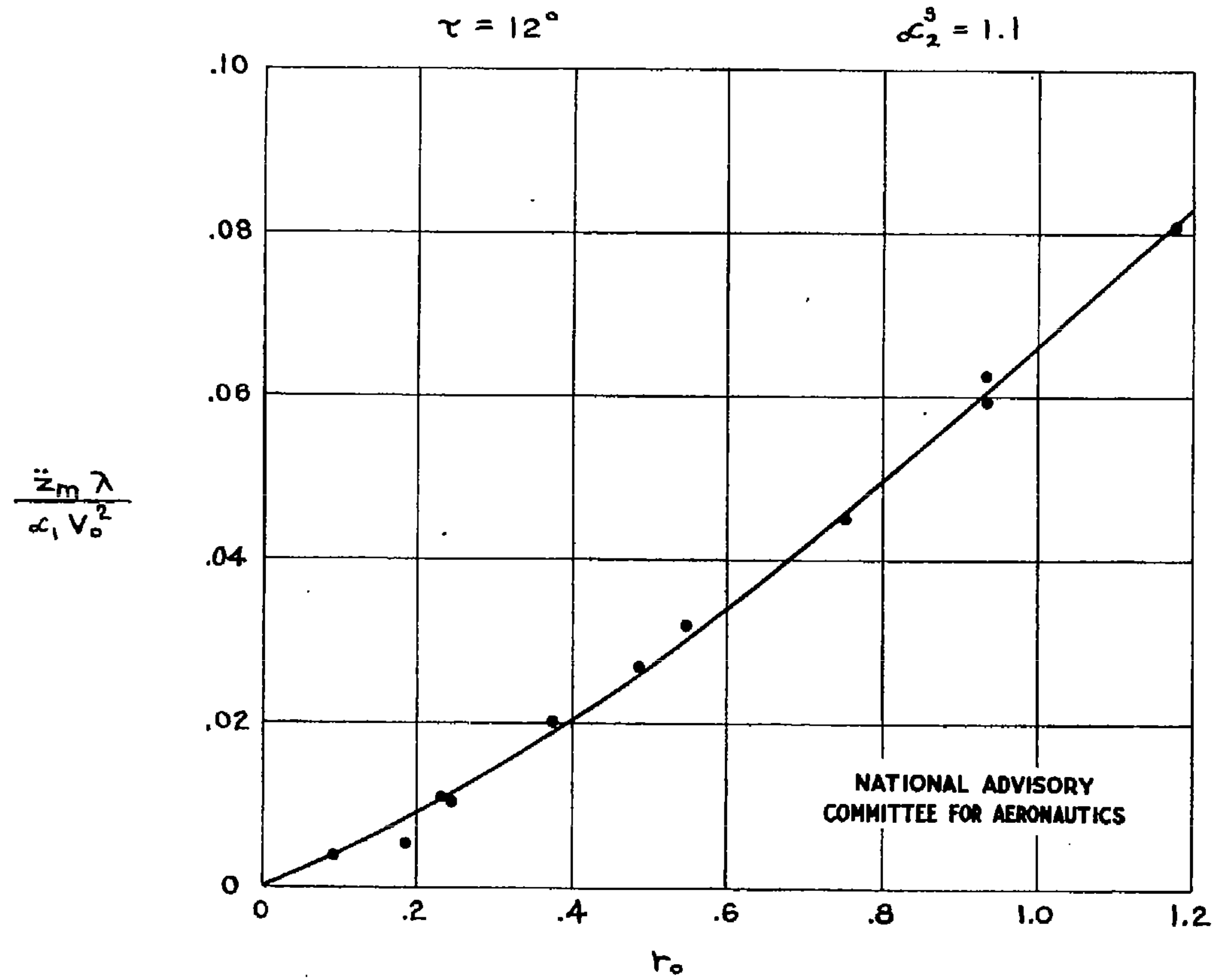

Figure 13.-Ratio of $\ddot{z}_m$ to V_o^2 for $\tau = 12°$

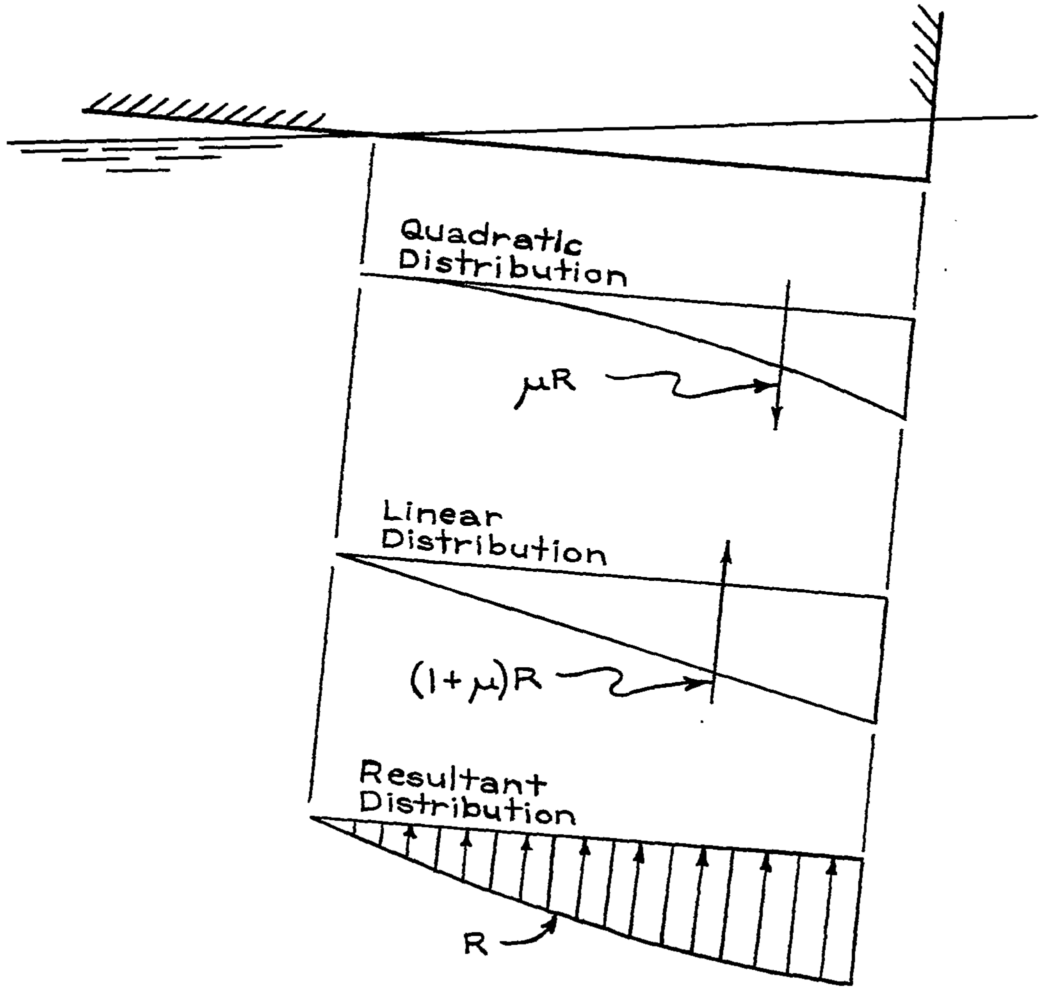

Figure 14.—Longitudinal Distribution of Reaction

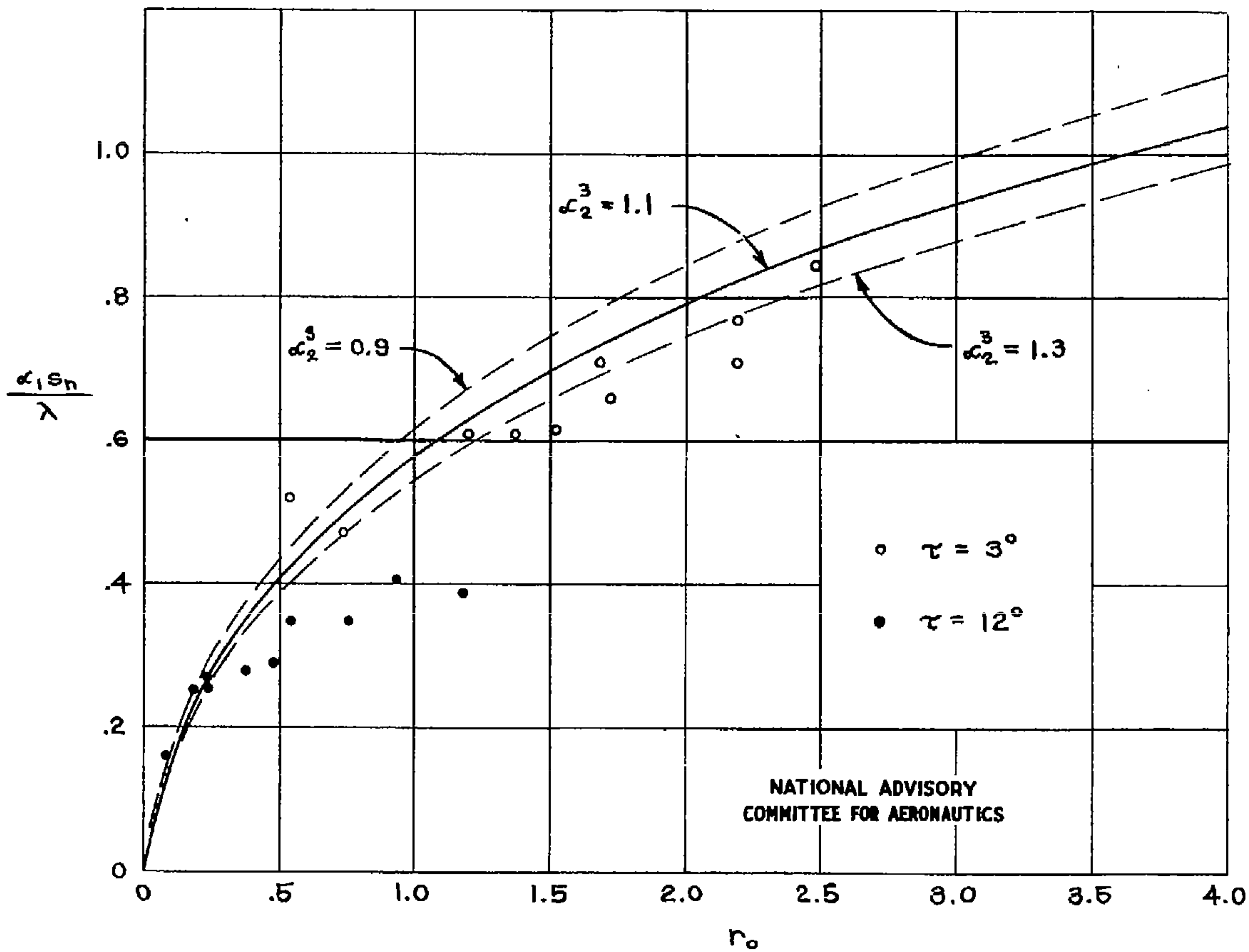

Figure 15.— Maximum Draft

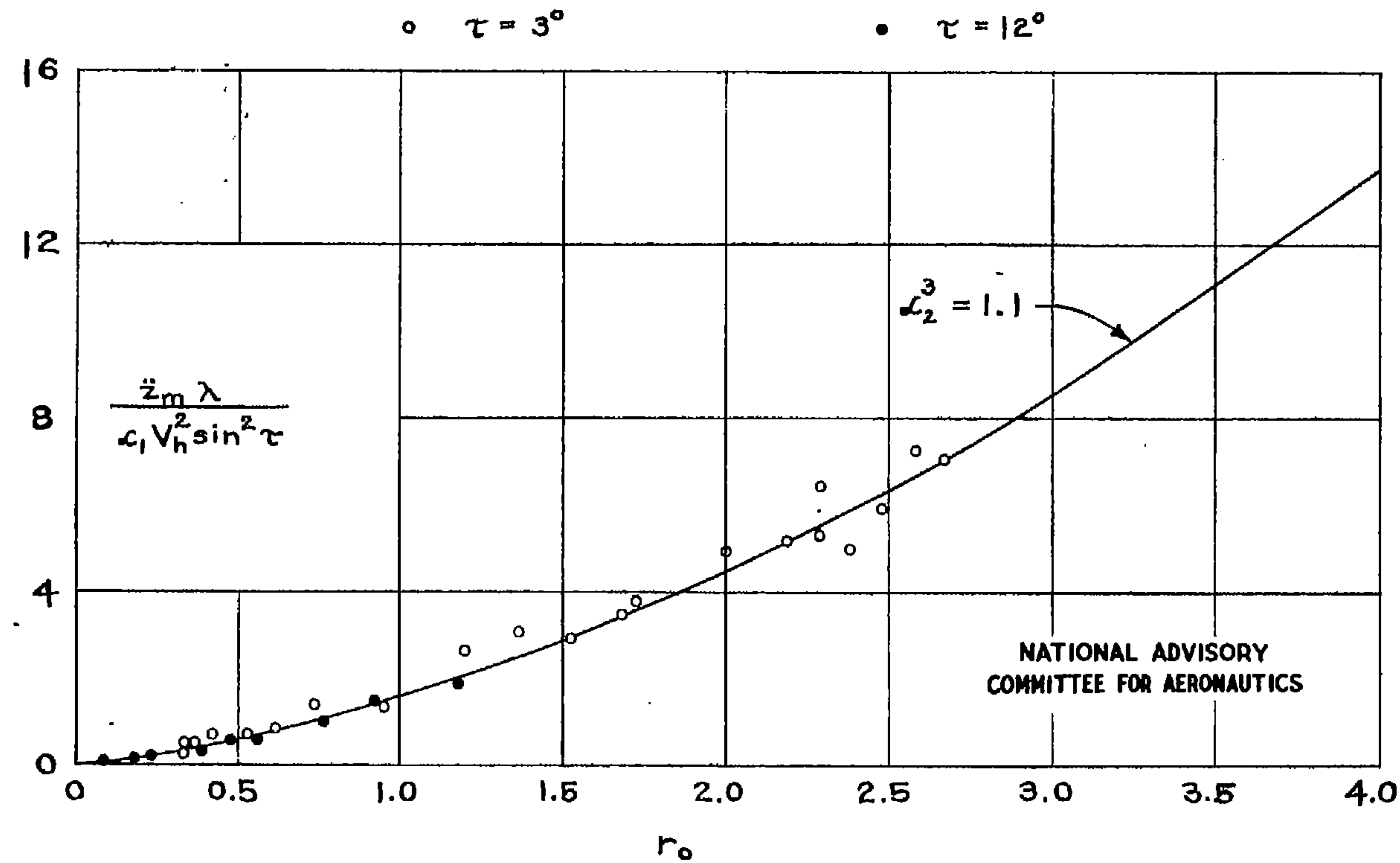

Figure 16.— Ratio of $\ddot{z}_m$ to V_h^2

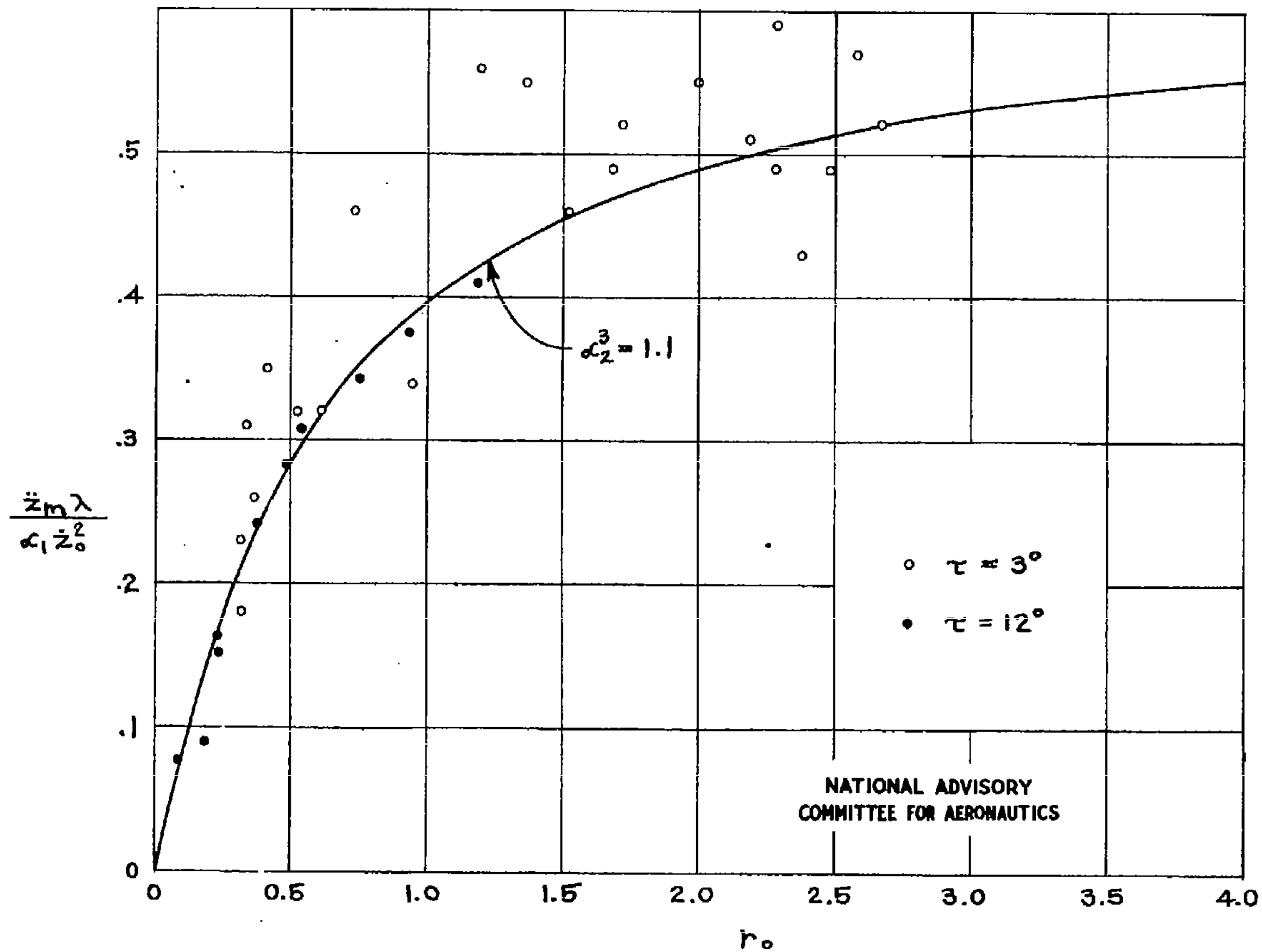

Figure 17.— Ratio of $\ddot{z}_m$ to $\dot{z}_o^2$

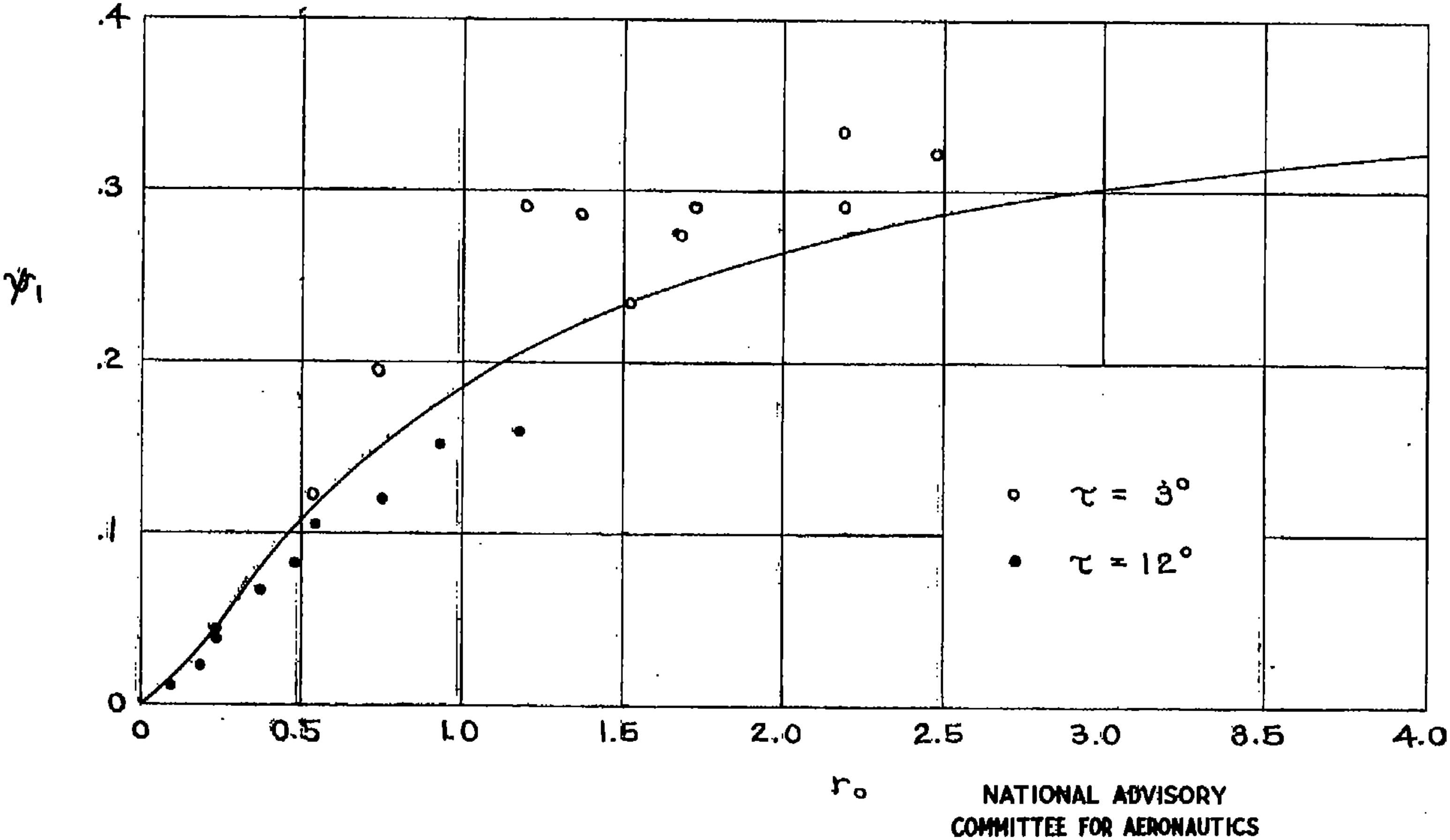

Figure 18.—Graph of Function ψ_1

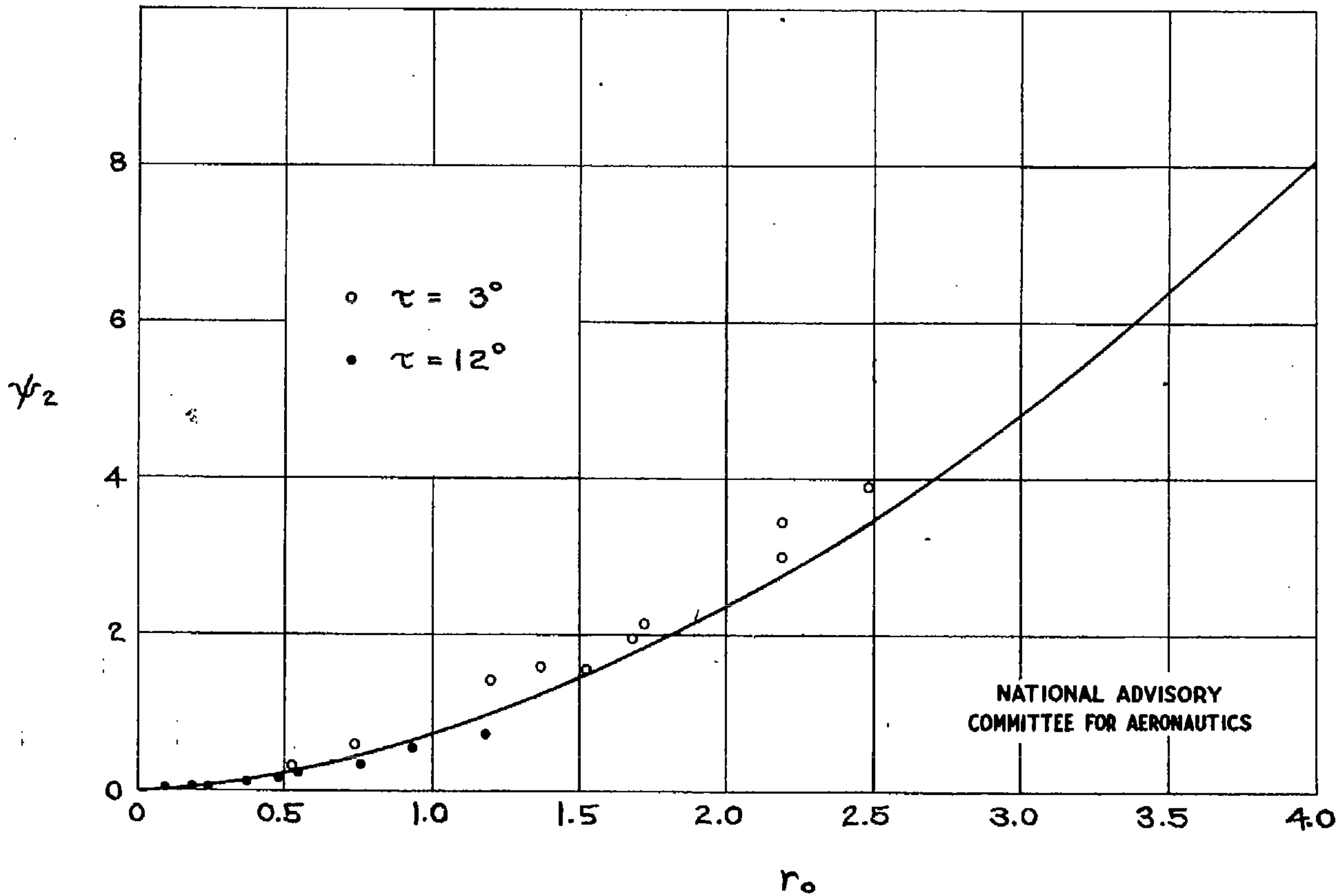

Figure 19.—Graph of Function ψ_2