NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1450

AN INVESTIGATION OF AIRCRAFT HEATERS

XXVI - DEVELOPMENT OF A SENSITIVE PLATED-TYPE THERMOPILE FOR MEASURING RADIATION

By L. M. K. Boelter, E. R. Dempster,
R. Bromberg, and J. T. Gier

University of California

Washington
July 1948

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 1450

AN INVESTIGATION OF AIRCRAFT HEATERS

XXVI – DEVELOPMENT OF A SENSITIVE PLATED-TYPE THERMOPILE FOR MEASURING RADIATION

By L. M. K. Boelter, E. R. Dempster,
R. Bromberg, and J. T. Gier

SUMMARY

An analysis is presented of the factors determining the power efficiency of radiation thermopiles of that type in which the receivers consist either of parts of the conductors themselves or of coatings or other electrically insulating materials in intimate contact with these conductors. On the basis of this analysis, which is applicable to thermopiles of a wide variety of constructions and materials, criterions for maximum power efficiency can be calculated. Such criterions are obtained for silver-constantan plated-type thermopiles. The design and construction of thermopiles in accordance with these criterions are described, and test data are reported, showing agreement between actual and predicted performance.

It is shown that an approach to maximum power efficiency in a thermopile can be realized independently of its resistance over a wide range. Within this range, therefore, maximum sensitivity in a thermopile to be used with a particular galvanometer or other measuring or controlling device can be obtained by designing simultaneously for maximum power efficiency and for whatever value of resistance is required to produce proper damping, to provide maximum power transference, or to satisfy some other criterion.

Calculations are presented for silver-constantan junctions which indicate that it is theoretically possible to obtain, with thermopiles of the type analyzed, about 80 percent of the voltage produced under equivalent conditions by an ideal thermopile in which the conductors lose no heat by radiation or convection.

INTRODUCTION

The preceding report of this series (reference 1) described the construction of the thermopile radiometer and its use in the study of problems involved in heating airplane cabins. The purpose of the investigation herein reported was to design and construct, for use in such radiometers, thermopiles retaining the advantages of those already available in the Spectroradiometric Laboratory of the Division of Mechanical Engineering of the University of California but possessing greater sensitivity and smaller size. The available thermopiles, which are constructed of silver-plated constantan wire as previously described (reference 2), have been found reasonably satisfactory in the applications mentioned with respect to mechanical ruggedness, drift compensation, quickness of response, and especially nonselectivity with respect to wave-length. In order to achieve this nonselectivity, the receiver has been so placed as to receive the radiation to be measured prior to any reflection and, as is also necessary, the thermopile is operated in air in order to avoid selective filtering by the envelope of an evacuated chamber.

Analysis permitting design calculations to obtain highest sensitivity has previously been carried out for but one type of thermopile (see, for example, references 3 to 6) – that type in which one or more receivers make contact with the conductors only at their junctions. The difficulties encountered in building thermopiles of this type and the great labor involved when many junctions are required have stimulated a number of workers to invent novel designs and methods of construction. In most of these designs the conductors themselves, or electrically insulating materials, usually radiation-absorbent coatings, in intimate contact with these conductors, act as receivers of the radiation to be measured. (See appendix A.)

A consideration of many possible designs and methods of construction led to the conclusion that the purposes of this investigation were most likely to be realized by thermopiles of the type mentioned last. The performance of thermopiles of this general type was therefore analyzed in order to determine the design relations required to obtain maximum sensitivity. No such analysis has hitherto been published, although interesting calculations have been presented with respect to thermocouples of a less general type subjected to periodic irradiation (reference 7).

This program of research in the Spectroradiometric Laboratory of the University of California was conducted under the sponsorship and with the financial assistance of the National Advisory Committee for Aeronautics.

The authors wish to express their appreciation to Mr. R. W. Lockhart for making many of the computations, and to Mr. E. H. Morrin for his criticisms and suggestions.

ANALYSIS, DESIGN, AND TEST PROCEDURE

A thermopile, when used to convert radiant energy into electrical energy for purposes of measurement or control, is shielded, generally by a housing (see appendix F), so that the radiation intended to be effective in producing a temperature difference between alternate junctions falls on a definite area, the front surface of the receiver. If the cold junctions are in thermal contact with structures of relatively large thermal capacitance, the thermopile is said to be uncompensated; if, on the other hand, both hot and cold junctions are similarly constructed and disposed, except that the equivalent of the receiver area at the cold junction is shielded from the radiation mentioned, the thermopile is said to be compensated. In a compensated thermopile, both sets of junctions are similarly affected by, and hence the thermopile response remains relatively independent of, changes in air temperature as well as, to some degree, uncontrolled radiation.

Analysis of System

Figures 1 and 2 illustrate a single thermocouple of an uncompensated and a compensated thermopile, respectively, of the type considered, in which parts of the conductors, or of the coating or binding materials which may be associated with the conductors, serve as receivers. The system analyzed is further defined by the 14 postulates listed in appendix B. These postulates, or in some instances their more simple consequences, may be summarized as follows: The thermopile is uniformly irradiated on all its surfaces (ideal radiation at housing temperature in the case of the uncompensated thermopile) except that the front surface of the receiver is subjected, in addition, to irradiation G'; there is no temperature gradient in a conductor, or in the coating or binding materials associated with it, in a direction perpendicular to its length, and all conduction of heat in a direction parallel to its length is through the metallic conductor itself; a linear relation exists between thermopile surface temperature and the power loss due to convection and radiant emission; this linear relation for the average power loss per unit area around, or on both sides of, a conductor is uniform over the entire thermopile; and a steady state of heat flow exists.

Derivation of Equations to Determine Junction Temperatures

A necessary step in the analysis of the relations between the electrical output of a thermopile and its design is the expression of the temperature difference between the hot and cold junctions of a single thermocouple in terms of the many factors on which it depends. Such an expression is derived in appendix B for an uncompensated thermocouple in the following manner: Appropriate substitutions are made in a general equation for heat flow and temperature distribution along a uniformly irradiated fin (strip or rod), the ends of which are at a specified temperature, in order to obtain separate equations for the portions of the conductors of figure 1 between cross sections 1 and x, x and h, h and y, y and 2. Upon equating the rates of heat flow at cross sections x, h, and y for the two portions of conductor which meet, in each case, at these cross sections, solving the resulting simultaneous equations, and setting $t_1 = t_2 = 0$, the temperature of the hot junction is obtained as follows:

$$t_h = H\left\{\frac{\sqrt{R}\left[\sinh py - (\coth p)(\cosh py - 1)\right] + \left[\sinh qx - (\coth q)(\cosh qx - 1)\right]}{\sqrt{R}\coth p + \coth q}\right\} \qquad (1)$$

For definitions of the terms of this expression, see appendix G.

It is shown in appendix B that $\left(t_h - t_c\right)$ for the compensated thermocouple (fig. 2) is independent of air temperature and is given by the same expression (equation (1)). The temperature difference between junctions is thus the same for a compensated and an uncompensated thermocouple in which the conductor lengths of the former are twice that of the latter. (See figs. 1 and 2.) It therefore follows that for every uncompensated thermopile a compensated one can be designed having the same receiver area and producing the same electromotive force but having double the electrical resistance.

Power Efficiency and Its Relation to Maximum Sensitivity

Power efficiency is defined as power output divided by power input. The power output of a thermopile, in watts, is expressed as follows:

$$(\text{Current})^2(\text{Resistance of load})$$

The following substitution may be made:

$$\text{Current} = \frac{\text{Electromotive force}}{\text{Resistance of load} + \text{Resistance of thermopile}}$$

Further, let

$$z = \frac{\text{Resistance of thermopile}}{\text{Resistance of load}}$$

The power output then becomes:

$$\frac{(\text{Electromotive force})^2}{\text{Thermopile resistance}} \times \frac{z}{(1 + z)^2}$$

or, for a given value of z, the power output is proportional to the term:

$$\frac{(\text{Electromotive force})^2}{\text{Thermopile resistance}}$$

The electromotive force is a linear function (see next section) of G', which is the difference between that fraction of G_0 (the radiation entering the housing opening and incident per unit area of receiver surface) absorbed by the receiver surface and that fraction of G_0 which must be absorbed to produce equality of hot and cold junction temperatures. A reasonable expression for power input is then:

$$\frac{G'}{\alpha'} \times \text{Receiver area}$$

where α' is the emissivity (absorptivity) of the receiver surface to radiation from the source of interest, and $\frac{G'}{\alpha'}$ is, therefore, the power from this source that must fall on the unit area of the receiver surface in order that the amount G' may be absorbed. An expression proportional to thermopile efficiency on these bases is then:[1]

$$\pi = \frac{(\text{Electromotive force})^2/\text{Thermopile resistance}}{\frac{G'}{\alpha'} \times \text{Receiver area}} \qquad (2)$$

The value of π is the same for a complete thermopile of the type considered as for one of its thermocouples considered separately, since n thermocouples in a series connection will have n times the electromotive force, resistance, and receiver area of a single thermocouple.

It will be shown in the next section that π is composed of two factors, the first of which is a function only of the thermal, electrical, and thermoelectric properties of the materials used, the emissivity of the receiver, the values of f' and β, (see symbols in appendix G), and the magnitude of the irradiation G', whereas the second factor Γ depends on the geometry of the design. If Γ is independent of thermopile resistance it must be maximized, through proper geometrical design, in order to attain optimum thermopile sensitivity in any application whatsoever. It will be shown that maximum Γ may be approached independently of certain variables, such as wire size, and that these

---

[1]The emissivity α' is not necessarily equal to α (see appendix G) as, for example, when the source occupies only a small part of the area "viewed" by the receiver surface through the housing opening or when the radiation from the source is projected through filtering materials which themselves act as subsidiary sources. When α' and α are not equal to ϵ, an estimate of the intensity of the radiation of interest may be obtained from the difference between thermopile response when the source is present (reading 1) and the response when the source is absent, but the other components of radiation G_0 are altered as little as possible (reading 2). The input of interest is then $\left[\frac{1}{\alpha'}(G' \text{ from reading 1}) - (G' \text{ from reading 2})\right]$, and the electromotive force of significance is, similarly, the difference between the electromotive forces of the two readings. Since G' and the electromotive force are linearly related (see appendix B), it is merely necessary in such cases to substitute the difference between the two values of G' in place of G' in equation (2); the electromotive force remains that produced by the absorption, per unit area of receiver, of power equal to this difference.

variables can be chosen so as to obtain, within certain limits of practicality, any desired resistance. A thermopile of the type under consideration and of given receiver area may therefore be designed to have maximum sensitivity in a given application by:

1. Choice of materials possessing the most favorable thermoelectric and radiation-absorbing properties

2. Consideration of the possibility of minimizing the values of f' and β in order to increase the temperature difference between hot and cold junctions

3. Maintenance of the relations for maximizing Γ

4. Designing for optimum thermopile resistance

The optimum resistance, when Γ is independent of resistance, is a function of only the electrical measuring instrument employed and, in some instances, the conditions of use. Thus if a galvanometer of the Thomson type is to be employed, the optimum resistance is that of the galvanometer coil; if a particular potentiometer is to be used, the best resistance may be that for which the smallest unbalance that can be read on the voltage scale produces a just-perceptible deflection of the balancing galvanometer; and if a galvanometer of the D'Arsonval type is to be used, the best resistance may be that providing critical damping although, in some applications, the use of a lower resistance giving greater deflections can be justified.

Power Efficiency in Terms of Properties of Materials and Thermocouple Dimensions

The following substitutions can be made in equation (2) as applied to a single compensated thermocouple (fig. 2):

Receiver area can be replaced by the expression $w(xa + yb)$ which, in turn, equals $w\left(\frac{xq}{m_a} + \frac{yp}{m_b}\right) = \frac{w}{m_a}\left(xq + \sqrt{R}\ yp\right)$

Electromotive force can be replaced by the expression:

$$ce\left(t_{h,1} - t_{h,2}\right)$$

where e is the thermoelectric power for the two materials and c is a factor which allows for reduction in voltage (in conductors of the plated type) due to circulating currents in the plated conductor. It is shown in appendix C that:

$$c = \frac{R - D}{R - D + Ds}$$

where D is the ratio of the cross-sectional area of the core of the plated conductor to that of the unplated conductor, and R is the ratio of the thermal conductance per unit length of the plated conductor to that of the unplated conductor.[2]

The term $\left(t_{h,1} - t_{h,2}\right)$ in the expression for electromotive force may be replaced by the right-hand side of equation (1).

An expression to replace thermopile resistance in equation (2), applied to a single compensated thermocouple, may be derived as follows:

Resistance of compensated thermocouple

= Resistance of conductor a + Resistance of conductor b

$$= \frac{2a}{C_a A_a} + \frac{2b}{C_b A_b}$$

but

$$\frac{C_b A_b}{C_a A_a} = \frac{C_s A_s}{C_a A_a} + \frac{DC_a A_a}{C_a A_a} = \frac{k_s A_s}{sk_a A_a} + D = \frac{1}{s}(R - D + sD)$$

Therefore this expression becomes:

Resistance of compensated thermocouple

$$= \frac{2}{C_a A_a}\left[a + \frac{b}{\frac{1}{s}(R - D) + D}\right]$$

$$= \frac{2}{C_a A_a m_a}\left(q + \frac{s\sqrt{R}\,p}{R - D + sD}\right)$$

---

[2]The ratio of cross-sectional area of the core of the plated conductor to the cross-sectional area of the unplated conductor D ordinarily is equal to 1 (for plated-type construction) or 0 (for other types of construction). The writers are indebted to Mr. J. E. Gullberg for the suggestion that the portions of conductor to be plated might first be reduced in cross section electrolytically. When this is done, $0 < D < 1$.

If the various substitutions just described are made in equation (2) and if, in addition, m_a^2 is replaced by $\frac{\beta w f'}{k_a A_a}$ and H is replaced by $\frac{G'}{f'\beta}$, the following is obtained:

$$\pi = \left(\frac{\alpha' G' e^2 C_a}{2k_a f'\beta}\right)\left\{\frac{\sqrt{R}\left[\sinh py - (\coth p)(\cosh py - 1)\right] + \left[(\sinh qx) - (\coth q)(\cosh qx - 1)\right]}{\sqrt{R}\coth p + \coth q}\right\}^2$$

$$\times \left[\frac{\left(\frac{R - D}{R - D + sD}\right)^2}{\left(q + \frac{s\sqrt{R}\,p}{R - D + sD}\right)\left(qx + \sqrt{R}\,py\right)}\right] \qquad (3a)$$

$$\pi = \left(\frac{\alpha' G' e^2 C_a}{2k_a f'\beta}\right)(\Gamma) \qquad (3b)$$

For an uncompensated thermocouple the resistance is halved, and π is therefore greater by a factor of 2.

Design Relations for Maximum Power Efficiency

The first term in parentheses in equation (3a) depends on the materials used and on the magnitude of the irradiation; it indicates that power efficiency varies directly with the irradiation, with the ratio of electrical to thermal conductivity of one of the metals (that of the other metal entering in the term s), with the receiver absorptivity and the square of the thermoelectric power (Peltier effect being neglected) and inversely with β and with the average overall unit thermal conductance. The remainder of the expression, Γ, which is dimensionless, can be maximized for a given value of s in terms of the six variables contained. From physical considerations it is clear that the smaller D is, the greater the efficiency becomes; hence, the degree of core reduction preparatory to plating should be restricted only by constructional considerations.

It appeared to be too cumbersome to maximize Γ by the method of solving simultaneously the five equations obtained when the partial derivatives with respect to p, q, x, y, and R are set equal to zero. Resort was had, therefore, to trial-and-error solutions, s being taken as 0.62 (a value appropriate for silver and constantan) and D as 1 (corresponding to plated construction with no reduction in cross-sectional area of the core). If metals having a different value of s are to be used, or if core reduction or unplated construction is to be employed, new trial-and-error solutions are required.

For the values of s and D selected, it was found that Γ, and hence the efficiency, continuously increases with R (the ratio of the thermal conductance per unit length of the plated conductors to that of the unplated conductors), but that the gain from increasing R above a certain value is slight. The thickness of plating corresponding to large values of R may be constructionally undesirable or difficult to achieve; consequently, optimum values of the other four variables are given for a number of values of R. These results are presented in the curves of figure 3.

The term Γ is the same for different thermopiles having the same values of p, q, x, y, s, D, and R, and a consideration of the definitions of these terms leads to the conclusion that the optimum length of a conductor for given values of s, β, D, and R varies directly as the square root of its thickness and of its thermal conductivity, and inversely as the square root of the average unit thermal conductance for heat loss by convection and radiation. Conductors may be made very thin in order to increase the speed of response and, provided they are also shortened so that the values of p and q are unaltered, the efficiency will not be lowered.

In a thermopile of given receiver area, thermoelectric materials, and value of f', the resistance is not fixed by the relations which maximize Γ. For example, if the conductor thickness is decreased the required decrease in conductor length is proportionately less; the consequently greater resistance per thermocouple and the greater number of thermocouples required for a given receiver area both act to increase total thermopile resistance. If the conductors are flat strips, the resistance may be changed without alteration of receiver size or shape by simultaneous alteration of the conductor width and number. If the space between conductors is occupied by electrically insulating material of low thermal conductivity (as in the thermopiles of different design to be discussed in a later section) an increase in spacing will require a decrease in conductor length, and the net effect, for given receiver area, will be a decrease in resistance. Adjacent wires, however, must be relatively close or the efficiency will decrease because of departure from postulates 1 and 2 (appendix B). In plated-type construction the resistance and conductor length can be considerably modified, with only a slight change in efficiency, by alteration of the thickness of plating and thus of the term R. The electrical connection of groups of thermocouples in series-parallel arrangement also permits control of resistance, with efficiency constant, in relatively large steps.

The maximum value of resistance for given receiver area obtainable without loss of efficiency by these various procedures is limited only by constructional considerations or by the deterioration of thermoelectric properties of metals to be expected in very thin films. There is, however, a theoretical minimum of resistance for given thermoelectric materials and values of f' and R – that of a single thermocouple designed to provide the total given receiver area.

Comparison of Power Efficiency of Thermopiles Analyzed Herein with That of Conventional Thermopiles

Probably the most efficient radiation thermopile is the idealized one analyzed by Johansen (references 3 and 4), in which it is postulated that the receiver is at a uniform temperature and that the conductors lose no heat by radiation or convection. The ratio of the power output of a thermopile of the type treated in the present report to that of the idealized Johansen type provides, therefore, an index of the degree to which the efficiency of the former approaches the theoretical maximum.

This index will be greatest for unplated construction $(D = 0)$, but since design optima have been worked out for plated construction, the ratio has been computed for this slightly unfavorable condition at a value of R (a measure of the thickness of plating) of 25 and a value of β of 2. The index can then be determined on assuming equality of the irradiation, receiver area, average unit thermal conductance, and thermal and electrical conductance of both metals, except for a slight and complex dependence on the value of s (the ratio of the thermal conductivities of the two metals divided by the ratio of their electrical conductivities). Since design optima have been determined for silver and constantan for which $s = 0.62$, computations of the index are made for this value. This is not very different from the ratio s for other commonly used metals, as, for example, iron and constantan (0.73), or bismuth and 95 – 5 percent bismuth-tin (approx. 0.64).

The computations presented in appendix D show that the index in these conditions is 0.63. This result indicates that thermopiles of the type treated in this report can be expected, under the conditions mentioned, to produce 0.63 times as much electrical power output, and 0.80 times as much voltage for equal resistance, as those of the Johansen type.

If a thermopile of the Johansen type is operated in air, the heat lost by convection from the conductors is not negligible; in fact, Cartwright (reference 6) estimates that the loss by convection from conductors may be greater than that from receivers directly. The highest obtainable efficiency of thermopiles of the Johansen type may therefore be no greater or actually be less than that of thermopiles of the type considered in this report when both are operated in air, as is ordinarily necessary if the selective filtering of an evacuated envelope is to be avoided.

Speed of Response

No calculations are presented in regard to speed of response since the immediate requirements which led to this investigation were not very critical in this respect. Experience with thermopiles constructed (as described in appendix E) of silver-plated No. 40 constantan wire and mounted in fairly heavy, high-conducting housings indicates that satisfactory voltage equilibrium is generally achieved in about 60 seconds. In general, the attainment of quick response in radiation-measuring thermopiles requires a low ratio of the thermal capacitances per unit area of receiver and conductors to the value of f'; however, in some thermopiles the heat capacity of supporting or surrounding structures is more serious than that of receiver or conductors. In order to approach or exceed, with thermopiles of the type analyzed in this report, the quickness of thermopiles of conventional type with receivers of gold leaf 10^{-5} centimeter in thickness (reference 8), conductors of the same order of thickness must be used. Conductor thicknesses of a micron (10^{-4}cm) or less have been achieved by rolling (references 9 and 10), by electroplating (references 11 and 12), and by sputtering or evaporating in vacuum (references 13 to 15). The procedures presented in the present report for maximizing Γ are also applicable to thermopiles constructed by these techniques.

Application of Formulas

It was desired to produce a compensated, plated silver-constantan thermopile to have high output and to:

1. Be mounted in a tubular housing of $\frac{7}{8}$-inch inside diameter with clearances of approximately $\frac{1}{32}$ inch.

2. Be operated in air so as to avoid the selective filtering of an evacuated envelope.

3. Have a resistance suitable for use with a Leeds and Northrup type 8662 potentiometer. (Since it provides critical damping and an easily perceptible galvanometer deflection for the smallest voltage unbalance that can be read on the dial, 80 ohms is a preferred value.)

The construction adopted is that of a spaced coil made self-supporting by a plastic filler between the turns of wire. The method of fabrication is described in appendix E. By making the coil, in end view, in the form of an isoceles trapezoid (fig. 4), a maximum receiver area can be accommodated in a given tube, and no shield is required within the housing to restrict irradiation to the proper length of conductor. It is also an advantage that the inside diameter of the housing is limited only by the dimensions of the receiver area.

The length of each conductor exposed to irradiation is $(xa + yb)$, where:

a (half length of each unplated conductor in a compensated thermopile) $= \dfrac{q}{\sqrt{\beta w f'/k_a A_a}}$

b (half length of each plated conductor in a compensated thermopile) $= \dfrac{p\sqrt{R}}{\sqrt{\beta w f'/k_a A_a}}$

The values used in the computations are as follows:

s for constantan and silver is taken as 0.62.

f' is estimated to be about 2 Btu/(hr)(sq ft)($^{\circ}$F) for surfaces of the dimensions and characteristics concerned. This value is a little more than twice that due to radiation losses alone.

β is taken as 2, a value appropriate for a strip which does not lose heat at its edges.

k_a for constantan is taken as 13.1 Btu/(hr)(sq ft)($^{\circ}$F/ft).

w was taken tentatively as 0.000603 foot, corresponding to 138 turns per inch.

p, q, x, and y are given for a range of values of R in figure 3.

A for No. 40 B. & S. wire is 5.39×10^{-8} square foot.

The optimum length of conductor $(xa + yb)$ to be exposed to irradiation was next computed for four values of R, 12, 10, 9, and 8, and found to be 0.84, 0.78, 0.75, and 0.63 inch, respectively. All of these lengths are greater than the length of a side of the largest square (0.57 in.) that will fit into a circle of $\frac{13}{16}$-inch diameter.

The following alternatives are possible:

1. No. 40 constantan wire and an R value of 8 may be used if the receiver surface forms a rectangle of 0.64 by 0.50 inch. A rectangle of these dimensions will fit into a circle of $\frac{13}{16}$-inch diameter and has an area about 3 percent less than that of the 0.57-inch square.

2. Wire of smaller diameter could be used. This would permit the use of a square receiver having 3 percent greater area than that of the rectangle, also a higher value of R, and a consequently slightly increased value of Γ. The output would thus be somewhat greater than that resulting from alternative (1), and the resistance would be greater.

3. No. 40 wire could be used, and by reducing the value of R the exposed conductor length would be decreased, thus permitting use of the square receiver surface. Change in resistance would be very slight.

4. No. 40 wire could be used, and by increasing wire spacing the conductor length would be decreased. This would permit the use of the square receiver surface and an equal or higher value of R. Resistance would be decreased.

Alternative (2) was rejected tentatively because smaller wire was not immediately available. Alternative (3) was rejected because the output would be less than that resulting from the adoption of alternative (1). Alternative (4) was rejected because of the possible loss in efficiency due to departure from postulates (1) and (2) and because resistance would be less than optimum.

A coil constructed approximately according to alternative (1) and mounted in the tubular housing of $\frac{7}{8}$-inch inside diameter had a measured resistance of 80 ohms. (The close agreement between this value and that required for critical damping is purely coincidental.) It is worth noting that this resistance is close to the maximum obtainable, without sacrifice of efficiency, with silver-plated No. 40 constantan wire and the given receiver area. The use of finer wire together with an equal or larger value of R and equal or closer spacing would result in considerable increase of resistance and equal or greater efficiency; wire much finer than No. 44 or No. 46 would, however, be much more troublesome to handle during fabrication. A resistance one-fourth as great could be obtained by connecting two halves of the thermopile in parallel, and intermediate values could be obtained by the use of this connection together with finer wire and a larger value of R.

The new thermopile had an irradiation-voltage constant of 7.9 [Btu/(sq ft)(hr)]/(mv) when mounted in the copper tube. This value is comparable with a constant of 6.7 for the best of the previous silver-constantan plated-type thermopiles used in the Spectroradiometric Laboratory, which has a resistance of 267 ohms and requires a housing of at least 3-inch inside diameter. Thus the new thermopile, by comparison, produces 0.85 times the voltage at less than one-third the resistance (hence 2.4 times the power) and will fit into a housing of one-fourth the diameter.

It is interesting to estimate the extent to which sensitivity might be increased by designing this thermopile for operation in vacuum. The value of f' due to radiation alone, would then be about 1; or, if the inside of the coil could be surfaced so as to have a very low emissivity, this value might possibly be reduced to 1/2. Since π varies inversely with f', the power output might be increased by some factor between 2.0 and 4.0, and the voltage, for fixed resistance, by the square roots of these factors, or a value between 1.4 and 2.0. These values are much lower than those usually given in the literature (see, for example, reference 5) because of the shapes and dimensions of the surfaces involved. Very small receivers (of the order of 1 sq mm in area), and particularly the very fine wires generally used in thermopiles of conventional type, have a very large ratio of power loss by convection to power loss by radiation.

Comparison of Actual and Predicted Performance

A comparison of irradiation-voltage constants, obtained (1) by measurement of the electromotive forces resulting from exposure of thermopiles to known irradiation and (2) by calculations for these same thermopiles based on the expression derived for junction temperature, will serve as a check of the applicability of the expression for junction temperature and of the postulates on which its derivation is based.

The irradiation-voltage constant K is given by the following expression:

$$K = \frac{G'/\alpha'}{nce\left(t_{h,1} - t_{h,2}\right)} \quad \left[\text{Btu/(hr)(sq ft)}(^{\circ}\text{F/ft})\right]/(\text{mv})$$

where

n number of thermocouples

c factor allowing for voltage reduction due to circulating currents in the plated conductor

e thermoelectric power of the two metals

$\left(t_{h,1} - t_{h,2}\right)$ temperature difference between hot and cold junctions due to irradiation of intensity G'

When c is replaced by $\frac{R - D}{R - D + Ds}$ (see appendix C), $\left(t_{h,1} - t_{h,2}\right)$ by the right-hand side of equation (1), and H in the latter expression by $G'/\beta f'$, the following expression is obtained:

$$\frac{1}{K} = \frac{\alpha' en}{\beta f'} \; \frac{\sqrt{R}\left[\sinh py - (\cosh py - 1)\coth p\right] + \left[\sinh qx - (\cosh qx - 1)\coth q\right]}{\sqrt{R}\coth p + \coth q} \; \frac{R - D}{R - D + Ds}$$

In applying this expression to particular thermopiles constructed of silver-plated No. 40 constantan wire, the following values were used:

$\alpha' = 0.98$ (estimated)

$k_c = 13.1$ Btu/(hr)(sq ft)($^{\circ}$F/ft)

$f' = 2$ (estimated)

$k_s = 242$ Btu/(hr)(sq ft)($^{\circ}$F/ft)

$s = 0.62$

$D = 1.0$

$\beta = 2$

$e = 0.023$ mv/$^{\circ}$F (experimentally determined with plated wire)

Values of R were computed from the ratios of actual thermopile resistances to those expected in the absence of plating and the ratios of the measured lengths of plated and unplated conductor. Values of w were determined by the setting of the coil-winding machine, and values of a, b, x, and y were obtained from careful measurements of completed thermopiles.

Four thermopiles, constructed as described in appendix E, were subjected to the procedure outlined with the following results:

Thermo-pile	n	w (ft)	R	a (ft)	b (ft)	x	y	Com-puted K	Observed K	Ratio com-puted to observed K
A	164	0.000603	4.0	0.0125	0.099	0.63	0.45	2.64	2.55	1.04
B	134	.000603	9.1	.0125	.099	.63	.45	3.30	3.53	.93
C	234	.000703	8.8	.0087	.121	.75	.50	1.94	1.65	1.17
D	236	.000703	7.4	.0087	.121	.75	.50	1.81	1.81	1.00

The rather good agreement between actual and predicted response constitutes evidence of the applicability of the formulas developed, the postulates made, and the approximate correctness of the estimated value of f'.

CONCLUSIONS

The derivation and application of design criterions for efficiency in thermopiles of the type in which parts of the conductors themselves act as receivers have resulted in the construction of thermopile radiometers which are more suitable for the study of problems involved in airplane cabin heating than those previously employed in the Spectroradiometric Laboratory of the University of California. The advantages obtained from smaller size and greater efficiency are illustrated by a silver-constantan thermopile mounted in a housing having one-fourth the diameter of that required for the older type and providing nearly the same voltage output at one-third the resistance.

It is possible to obtain about 0.8 times the voltage at the same resistance from a thermopile of the type analyzed in the present work as from an ideal thermopile, composed of the same thermoelectric materials having receivers in contact with conductors at their junctions. The advantage in efficiency of the ideal type can be expected to diminish or to disappear in nonvacuum operation because of loss of heat from conductors by convection.

The formula derived in this report for efficiency is applicable to a wide variety of constructions and should be of particular value in cases in which, because of the number of junctions required or for other reasons, the fabrication of thermopiles of the Johansen type is difficult or impossible.

Department of Engineering
University of California
Berkeley, Calif., November 29, 1944

APPENDIX A

REVIEW OF LITERATURE ON RADIATION-MEASURING THERMOPILES

In view of the voluminous literature bearing on the design of radiation-measuring thermopiles it is impossible for a short review pertinent to the present investigation to be at all exhaustive; in general, an attempt has been made to select contributions which appear important with respect to soundness and clarity of analysis or to basic innovation in theory or design. The literature first discussed in this review deals with general considerations and with thermopiles herein referred to as of the conventional type, that is, those in which the receivers of the radiation to be measured are distinct from the thermoelectric leads and make thermal contact with them at only the hot junctions (uncompensated thermopiles) or at the hot and cold junctions (compensated thermopiles). The last part of the review deals with radiation-measuring thermopiles of other types.

Johansen's early contributions (references 3 and 4) to the design of vacuum-operated thermopiles are the basis of much later work. His significant conclusions with respect to obtaining maximum galvanometer deflection may be stated as follows:

1. The conductors in uncompensated thermopiles should be designed to extract the same quantity of heat from the receiver as is lost by convection and radiation.

2. The resistance of conductors between junctions in compensated thermopiles should be twice that in uncompensated thermopiles having the same receiver area per hot junction.

3. The load resistance (Thomson galvanometer) must equal that of the thermopile.

4. The conductors between hot and cold junctions should all have the same ratio of thermal conductance to electrical resistance.[1]

In arriving at these conclusions Johansen assumed that the "Beschaffenheit" "c" of the Thomson galvanometer was fixed but that the resistance could be varied; in his nomenclature $c = \frac{1}{{J_1}^2 G}$, J_1 being

[1]Conditions (3) and (4) were actually set forth much earlier by Lord Rayleigh (reference 16), who was concerned, however, with the possible use of thermopiles to obtain electric power from a source of heat and not for the measurement of radiation.

the current required to produce 1-millimeter deflection, G the galvanometer resistance, and hence J_1^2G the power supplied to the galvanometer. This assumption of a fixed value of c is, then, equivalent to stating that the value or measure of sensitivity of a thermopile is the power that can be obtained from it. It is clear, from Johansen's formulas, that the maximum power available from a thermopile of the type that he analyzed and of fixed receiver area does not depend on the number of junctions or on the corresponding values of thermopile resistance.

The concept of thermopile output in terms of power took on additional significance when it was discovered, many years after the publication of Johansen's work, that there is a theoretical minimum of direct current detectable by a D'Arsonval galvanometer (reference 17) or by any other means (reference 18) and that this limit, for a given temperature and time of observation, depends only on the virtual power of the thermopile or generative circuit, that is, (Electromotive force)^2/Resistance, a quantity directly proportional to the obtainable power output.[2] In order to approach the limit of detection in practice, it is necessary, first, that all perturbations be reduced to the same order of magnitude as those due to Brownian motion alone and second, that the very small deflections obtained be magnified, optically or otherwise, so as to be observable. Ising (reference 17) showed that Moll and Burger (reference 19) had attained these very difficult conditions, and later workers have reduced Brownian perturbations as low or lower. (See, for example, reference 20.) The very small deflections of the primary galvanometers in these examples were magnified by either a thermoelectric relay (reference 19) or photoelectric relay (reference 20). Both types of relay are similar in principle to the thermoelectric relay proposed by Wilson and Epps (reference 21) in that a spot of light from the mirror of the primary galvanometer falls on two adjacent and symmetrically disposed heat-sensitive surfaces (or light-sensitive surfaces in the photoelectric relays) the electrical circuits of which are connected, voltages in opposition, to a secondary galvanometer.

It is desirable to consider a system devised by Pfund called resonance radiometry (reference 22), since its use requires special properties in a thermopile – namely, rapid response and, in the case of electronic amplification, power output at high thermopile resistance.

---

[2]When this expression is applied to the case of a D'Arsonval galvanometer, the term "resistance" should be taken to mean the resistance in the entire circuit (thermopile plus galvanometer). There appears, however, to be no theoretical limit to the extent to which coil resistance in a good galvanometer of long period can be made small in comparison with the external resistance required for critical damping; for any given ratio of coil to thermopile resistance, moreover, the expression is still directly proportional to obtainable power output.

In these resonance systems the thermopile is periodically shielded from the radiation to be measured so that it produces an alternating voltage.[3] The alternating voltage is used to build up resonance in a tuned system consisting, in Pfund's case, of an underdamped D'Arsonval galvanometer connected by means of a photoelectric relay to another galvanometer, also underdamped. As Czerny (reference 24), Van Lear (reference 25), and Hardy (reference 20) have shown, this system in theory is less efficient than that of a critically damped galvanometer connected to a thermopile subjected to uninterrupted irradiation, but in practice is actually more efficient when a relatively long period – of the order of 1 minute or more – is available for observation. Harris and Scholp (reference 15) have applied the principle of resonance radiometry to electronic amplification and were able to detect 10^{-8} volt at 2 cycles per second. It must be emphasized that electronic amplification does not offer the possibility of lowering the theoretical limit of power detectable (see references 18, 26, and 27), but it may permit improvement in stability, portability, ease of operation, and cost of apparatus.

Several contributions have dealt with the design of thermopiles to be used with particular galvanometers, particularly of the D'Arsonval type. Firestone (reference 5) showed that the conductor and receiver relations developed by Johansen (references 3 and 4) are applicable to the case of the thermopile connected to a D'Arsonval galvanometer, and he developed formulas for computing optimum conductor resistances and numbers of junctions for conventional thermopiles of given total receiver area for the following cases: (1) a Thomson galvanometer of given resistance; (2) a D'Arsonval galvanometer requiring a given resistance for perfect damping; and (3) a D'Arsonval galvanometer with adjustable magnetic shunt. Cartwright (references 6, 8, and 28) covers much the same ground and considers carefully the minimum radiant energies detectable by various instruments. He also pays considerable attention to the degree to which optimum efficiency is approached with thermopiles having less than the optimum number of junctions. Notice should be taken of the valuable detailed information given by Cartwright and Strong (reference 29) concerning methods of design and construction of sensitive and quick-acting thermopiles and of properties of materials available for use in thermopiles and auxiliary apparatus. The performance curves given toward the end of that article may be misleading in some instances since there must be a limitation, not mentioned, with respect to galvanometer resistance (presumably to 15 ohms as in the curves of Cartwright, reference 6).

Neither Johansen nor Firestone considers the effect on design of the Peltier, Thomson, and Joule effects (see reference 30 for definition of these terms). Altenkirch (reference 31) treats of these matters in

---

[3] It is worth noting in this connection that direct-current voltages can be converted to alternating-current voltages by the use of a commutator and with the introduction of extraneous voltages no greater than 5×10^{-10} according to Gunn (reference 23).

detail with respect to power production. The Thomson and Joule effects are considered by all workers to be negligible when temperature differences are small, as is the case with thermopiles used to measure radiation. Coblenz (reference 32) studies the changes in the electromotive force of certain thermopiles caused by the different Peltier cooling when used on galvanometers having different resistances; he found these changes to be negligible in the examples considered. Cartwright allowed for the Peltier effect in equations descriptive of the performance of conventional thermopiles and concluded that the voltage correction was generally less than 10 percent in the case of a properly constructed thermopile having bismuth-tin and bismuth conductors with a thermoelectric power of 125 microvolts per degree centigrade (reference 28) and about 1 percent for an iron-constantan thermopile with thermoelectric power of about 56 microvolts per degree centigrade (reference 6). In reference 6 a cut-and-try method for determining design optima is proposed in which allowance is made for the Peltier effect. Such allowance, however, can result in only a very slight increase in sensitivity unless new thermoelectric materials are discovered having considerably improved properties.

Altenkirch (reference 31) showed clearly, as might also have been inferred from Rayleigh's equations, that the suitability of a pair of thermoelectric materials, as far as their thermal and electrical properties are concerned, is only a function of the thermoelectric power of the combination and of the ratio of the thermal and electrical conductivities of each. High resistivities are, then, disadvantageous in theory only because they are generally accompanied by marked deviations from the Franz-Wiedemann law. Such deviations, as well as high resistivities, are both characteristic of materials giving very high thermoelectric power. (See references 33 and 34.)

Considerable discussion has appeared in regard to receiver coatings. Soot from burning oil and camphor is frequently used (see, for example, reference 10), easily applied, and generally considered to be satisfactory. Pfund (references 35 and 36) found that greater absorption over a wider range could be obtained by the use of bismuth black evaporated from a hot filament in a partial vacuum. Other metals may be used in a similar manner, as, for example, antimony (reference 13). The heat capacity of the coatings mentioned may be greater than that of a very thin uncoated receiver; thus Firestone (reference 5) found that quicker response was obtained when his receivers were blackened with "aquadag" instead of soot, and Moll and Burger (reference 9) likewise found soot inferior to an aqueous suspension of colloidal carbon, an effect which has been attributed in part, however, to poor conductivity of soot in vacuum (reference 10). Cartwright has pointed out that very substantial increases in sensitivity can be obtained by the use of coatings having high absorptivity for the radiations to be measured and low emissivity at the wavelengths characteristic of radiations at receiver temperature. Platinum black, for example, which is highly absorbent in the visible range, is stated to have an emissivity of only 0.25 or 0.20 (references 6 and 29) at long wavelengths.

Thermopiles operated in air present a somewhat different problem from those operated in vacuum. Firestone (reference 5) suggests the use of the same formulas with the introduction of a much larger value of unit thermal conductance to allow for power lost from the receiver surface by convection. Cartwright (reference 6), however, points out that the power loss from the leads by convection may be as great as that from the receiver so that the postulate on which the formulas for vacuum thermopiles are based — that no power is lost from the conductor surfaces — is not even approximately realized in practice. This author introduces in his formulas empirical expressions for unit thermal conductance obtained from Warburg (reference 37) but makes the somewhat dubious postulate that the temperature drop along the conductors is linear with respect to distance. Collins (reference 38, pp. 37–44), in his treatment of power loss by convection and radiation from conductors in a thermopile of other than the conventional type, postulates that the temperature drop with distance from the hot junction is the same as would occur in a conductor of infinite length; the actual drop in a thermopile of good design must be considerably greater. The literature on heat loss from conductors in air leaves a good deal to be desired.

The efficiency, with respect to electrical energy produced by irradiation of a given intensity on a given receiver area, of well designed and constructed conventional thermopiles operated in vacuum is not likely to be equaled or exceeded by thermopiles of radically different type. Nevertheless, much attention has been devoted to unconventional designs with, generally, one or more of the following aims: to increase the number of junctions possible and thus increase the voltage response (at the expense of higher internal resistance); to decrease the time and skill required for construction; or to increase the rapidity of response. It is also possible that the efficiency of unconventional thermopiles operated in air might equal or exceed that of conventional thermopiles under equivalent conditions. In practically all thermopiles of unconventional type (but not that of Gier and Boelter, reference 2) portions of the conductors act also as receivers.

Wilson and Epps (reference 21) in an early important contribution of this nature, describe methods of constructing thermopiles by the electrodeposition of one of the thermoelectric metals on the other. By winding fine constantan wire in spaced coils of one layer and many turns, partly immersing such coils in an electrolyte, and plating on silver or copper, thermopiles were produced that have many junctions in series, the hot and cold junctions being connected alternately by unplated and plated constantan wire. Since the thermal and electrical conductivities of the plating were much greater than those of the core, the plated wire responded very much as though the core were absent. The authors made many experiments to determine the optimum thicknesses of plating and lengths of plated and unplated conductors, but the values obtained appear to be not at all general in application.

The deposition technique, as applied by Wilson and Epps, has certain limitations. For example, (1) some metals and alloys otherwise desirable from a thermoelectric standpoint cannot be readily plated; (2) in order to avoid loss of voltage due to currents in the core of the plated conductors, the sheath must have relatively high conductance and hence be relatively thick; and (3) for equal lengths of conductors the ratios of thermal to electrical resistance cannot be made equal. Among methods devised by later workers for removing one or more of these disadvantages is that of Kersten and Schaffert (reference 12). The two thermoelectric metals were plated on different, but overlapping, portions of a stainless-steel form and the plated metal stripped off. In one instance a polished steel rod was plated on opposite, but overlapping, sides with different metals, the rod put in a lathe, and a spiral cut made through the deposited metals. The long spiral ribbon consisting of alternate lengths of the two metals was then stripped off and wound on a form of appropriate dimensions. In various trials cobalt, nickel, copper, cadmium, iron, silver, and brass were all plated and stripped successfully; when very thin strips were produced, they were strengthened with varnish before being stripped and the varnish later removed with turpentine. Müller (reference 11) produced thermocouples with alloys of relatively high thermoelectric power by plating very thin strips of electrolytically produced nickel foil with copper on one end and chromium on the other and then holding them at red heat until the plated metals diffused into the nickel core. It seems possible that this method could be applied to the production of multijunction thermopiles although this apparently has not been done.

A result somewhat similar to Müller's is obtainable by a different method devised by Moll as early, apparently, as 1913. (See references 9 and 39.) A strip of manganin and one of constantan are soldered together with silver, edge to edge, and the joined strips then rolled out in the direction of the silver seam. By cutting perpendicular to the seam, a large number of thin thermocouples of any desired width can be produced. An etching technique was used in some cases to produce very minute junctions of desired shape.

Very thin thermocouples and thermopiles can be produced by sputtering or by evaporating in vacuum; for details, see references 40 to 42. The metals are usually precipitated on a thin nonmetallic film the durability and heat capacity of which are important because its removal, as discovered by Harris and Johnson (reference 43), is sometimes impracticable. These authors devised a method for making films composed partly of cellulose nitrate which are exceedingly thin and at the same time surprisingly strong. Films 0.1 micron thick survived prolonged sputtering, whereas films 1 micron in thickness were capable, according to the authors, of holding a vacuum over an area of approximately 12 square millimeters. Improved specifications are given by Harris and Scholp (reference 15).

Burger and Van Cittert (reference 13) constructed a thermoelectric relay by evaporation of bismuth and antimony in films of 1 and 0.5 micron thickness, respectively. The thermoelectric power was only 75 microvolts per degree centigrade as compared with an expected maximum of 150; the low value was shown to be due to the orientation of the bismuth crystals. Johnson and Harris (reference 44) using bismuth films obtained by sputtering, found that the thermoelectric power (against antimony) depends on the thickness of the bismuth, amounting to 73 microvolts per degree centigrade at 1 micron or thicker but dropping to 30 microvolts at a thickness of 0.05 micron. These limitations are unfortunate since other pure metals are not so favorable as bismuth would be in the absence of the disadvantages mentioned, and alloys are difficult to control in sputtering or evaporating. The diffusion method of Müller (reference 11) might perhaps be applicable for films which can be self-supporting.

Harris (reference 7) worked out equations for temperature of sputtered thermopiles exposed to fluctuating radiation as a function of distance from junction and time. Harris and Scholp (reference 15) measured radiations with such thermopiles by interposing a shutter between the source and the thermopile, connecting the thermopile to an alternating-current electronic amplifier, and measuring the amplifier output by a direct-current microammeter in connection with a copper rectifier or commutator. In the sputtering of the metal films in these thermopiles, the regions over which deposit was desired were masked, and the circuit was made zig-zag in shape, so that all cold junctions were in one line and hot junctions in another. The distance between junctions was much greater than that which would give maximum power output for a given area irradiated, as evidenced by the experimental determination of the areas for which radiation was effective. It would appear that many more junctions could have been obtained in the same area, and with greatly reduced resistance, by placing all junctions in one line and loading the cold junctions with relatively thick deposits to minimize their temperature variation when exposed to the fluctuating radiations.

Most of the workers on unconventional thermopiles have made calculations or experimental determinations with respect to speed of response. Cartwright (reference 28) claimed a response of about 0.1 second in vacuum for a conventional thermopile, Moll and Burger (reference 9) of 2 to 3 seconds in vacuum with thermocouples made of thermofoil, and Jones (reference 10) claimed 99-percent response in 0.08 second with thermocouples made of rolled and plated constantan and operated in air. In general, a quick response requires that the ratio of the heat capacity of the system, the temperature of which is raised by the irradiation, to the rate of heat loss from this system by conduction, convection, or radiation be small (see, for example, Cartwright, reference 6); a consequence of this condition is that all thermopiles and thermocouples respond more quickly when operated in air than in vacuum. A rough idea of the relative rapidity of response of different thermopiles can be gained by comparison of receiver thicknesses. Cartwright (reference 8) constructed conventional thermopile receivers of gold leaf 0.1 micron in thickness, although he and Strong later recommend 0.5 micron (reference 29); Moll and Burger (reference 9) made thermofoil 1 micron in thickness; Müller (reference 11) claims to have reached the value of 0.1 micron when the base metal was flattened Wollaston wire; Kersten and Schaffert (reference 12) were able to produce and wind coils of flattened constantan 5 microns thick; Jones (reference 10) rolled constantan wire to about 0.6 micron; and Johnson and Harris (reference 44) made metal films at least as thin as 0.05 micron and supporting films less than 0.1 micron, although the films of the thermopiles they later used for making irradiation measurements were thicker. As previously suggested, it is difficult to apply efficient radiation-absorbing coatings with a heat capacity not greater than that of metallic films of the thicknesses mentioned.

The work on radiation-measuring thermopiles may be briefly summarized as follows: The theory has been well developed for operation in vacuum of the conventional thermopiles in which receivers make thermal contact with the conductors at their junctions. The theory has not been very well developed for thermopiles of this type operated in air. Although many unconventional thermopiles have been designed and constructed, their theory of operation has been largely neglected (except for the excellent work of Harris, reference 7), and apparently no design criterions for obtaining high power efficiency are found in the literature.

APPENDIX B

DERIVATION OF EXPRESSIONS FOR COMPUTING JUNCTION TEMPERATURES

Definition of the System

The system analyzed is defined by figures 1 and 2 and the following postulates:

1. There is no temperature gradient within a conductor, or its associated coating or binding materials, in a plane perpendicular to the direction of length of this conductor.

2. Thermal conduction parallel to the direction of length of the conductors is solely through these conductors and not through associated binding or coating materials.

3. The individual thermocouples are placed or coiled side by side so that similar regions of the different thermocouples (and also regions 2 and 2' of fig. 2) are adjacent to each other and exist at the same temperature.

4. The Peltier, Thomson, and Joule effects are negligible (for definitions of these effects, see reference 30).

5. The thermopile is disposed in a housing (see fig. 4) the temperature of which is uniform and the inside surfaces of which have an emissivity of 1.00 (ideal radiator).

6. The temperature of the air surrounding the thermopile is uniform.

7. In the case of the uncompensated thermopile, the air temperature, thermocouple ends 1 and 2 (fig. 1), and the housing are all at the same temperature.

8. The temperature difference $T - T_1$ between any differential area of thermopile surface dA and the housing is so small that the substitution of $4T_1^3(T - T_1)$ in place of $T^4 - T_1^4$ in the expression for net power exchange between the two by radiation introduces only a negligible error.

9. The average values of ϵ and of f_c for the surfaces of a differential length of conductor, considered together with its associated coating and binding materials, are uniform over the entire thermopile and are independent of the temperatures[1] of the conductors and of the ambient air.

10. All parts of the thermopile surface, other than the front surface of the receiver, exchange power by radiation with only the inside surfaces of the housing; except that, in the compensated thermopile, there may be a uniform excess or deficiency of irradiation resulting in a corresponding uniform excess or deficiency of power absorption per unit length of conductor over the entire thermopile.[2]

11. The front surface of the receiver exchanges power by radiation only with the inside surfaces of the housing and the housing opening.

12. The front surface of the receiver is completely diffusing, and the same fraction $F_{0 \leftarrow R}$ of the radiant energy leaving any part of it passes through the housing opening prior to any reflection.

13. All parts of the front surface of the receiver are irradiated at equal intensity by radiant energy entering the housing opening.

14. A steady state of heat flow exists throughout the system; that is, the temperatures are not a function of time.

---

[1]The direct proportionality found to exist between measured irradiation and thermopile voltage is in agreement with the postulate that the unit thermal conductance, as well as emissivity, is independent of temperature over the ranges ordinarily encountered in practice.

[2]The last condition is approximated in a compensated thermopile constructed according to the method described in appendix E in that there is mutual exchange of radiant power between different parts of the inside surface of the trapezoidal coil which are not necessarily at housing temperature. A deviation from the condition occurs in that the self-irradiation of the inside coil surface is not quite uniform, being greater on the cooler parts of the conductors and less on the warmer parts. The effects of this nonuniformity are almost the same as those which would result from a slightly increased value of f'.

Net Power Exchange from Radiometer Surfaces

by Radiation and Convection

The power lost by radiation by a differential length of conductor may be expressed as follows:

$$\left[\epsilon\sigma w\beta T_1^{4} + 4\epsilon\sigma w\beta T_1^{3}\left(T - T_1\right)\right] dx = \left(\epsilon\sigma w\beta T_1^{4} + 4\epsilon\sigma w\beta T_1^{3}t\right) dx \qquad (B1)$$

(See postulates (5), (8), and (10).) The power lost by convection is

$$w\beta f_c\left(t - t_a\right) dx \qquad (B2)$$

The power gained by radiation by a differential length of conductor not directly associated with receiver surface is

$$\left(B + \epsilon\sigma w\beta T_1^{4}\right) dx \qquad (B3)$$

The power gained by radiation by a differential length of conductor that is directly associated with receiver surface may be expressed as follows:

$$B\ dx + \epsilon\sigma T_1^{4} F_{R\leftarrow h} \times \text{Area housing} \times \frac{w\beta\ dx}{\text{Area receiver}}$$

$$+ \alpha G_0 w\ dx + \epsilon\sigma T_1^{4} w(1 - \beta)\ dx \qquad (B4)$$

(See postulates (11), (12), and (13).) But, by reciprocity law,

$$F_{R\leftarrow h} \times \text{Area housing} = F_{h\leftarrow R} \times \text{Area receiver}$$

and

$$F_{h\leftarrow R} = 1 - F_{0\leftarrow R}$$

(See postulate (11).) On making these substitutions, expression (B4) becomes:

$$B\,dx + \epsilon\sigma T_1^{\,4}\left(1 - F_{0 \leftarrow R}\right)w\beta\,dx + \epsilon\sigma T_1^{\,4} w(1 - \beta)\,dx + \alpha G_0 w\,dx \qquad (B5)$$

The net power loss by radiation and convection by a differential length of conductor not directly associated with receiver surface is then:

$$\text{Expression (B1)} + \text{Expression (B2)} - \text{Expression (B3)}$$

$$= 4\epsilon\sigma w\beta T_1^{\,3} t\,dx + w\beta f_c\left(t - t_a\right)dx - B\,dx$$

$$= \left(4\epsilon\sigma T_1^{\,3} + f_c\right)w\beta t\,dx - f_c w\beta t_a\,dx - B\,dx \qquad (B6)$$

Similarly, net power loss by radiation and convection by a differential length of conductor directly associated with receiver surface is given by:

$$\text{Expression (B1)} + \text{Expression (B2)} - \text{Expression (B5)}$$

$$= \epsilon\sigma w\beta T_1^{\,4} F_{0 \leftarrow R}\,dx - \epsilon\sigma T_1^{\,4} w(1 - \beta)\,dx - \alpha G_0 w\,dx$$

$$+ \left(4\epsilon\sigma w\beta T_1^{\,3} t\right)dx + w\beta f_c\left(t - t_a\right)dx - B\,dx$$

$$= -\left(\alpha G_0 - \epsilon\sigma\beta T_1^{\,4} F_{0 \leftarrow R}\right)w\,dx + \left(4\epsilon\sigma T_1^{\,3} + f_c\right)w\beta t\,dx - f_c w\beta t_a\,dx - B\,dx$$

$$= -G' w\,dx + \left(4\epsilon\sigma T_1^{\,3} + f_c\right)w\beta t\,dx - f_c w\beta t_a\,dx - B\,dx \qquad (B7)$$

Expression (B6) is the same as expression (B7) with G' set equal to zero. Let t_e be the temperature at which, for $G' = 0$, no net exchange by radiation or convection occurs. Then:

$$\left(4\epsilon\sigma T_1^{\,3} + f_c\right)w\beta t_e\,dx = f_c w\beta t_a\,dx + B\,dx$$

or

$$t_e = \frac{f_c t_a + \frac{B}{w\beta}}{4\epsilon\sigma T_1^{\,3} + f_c}$$

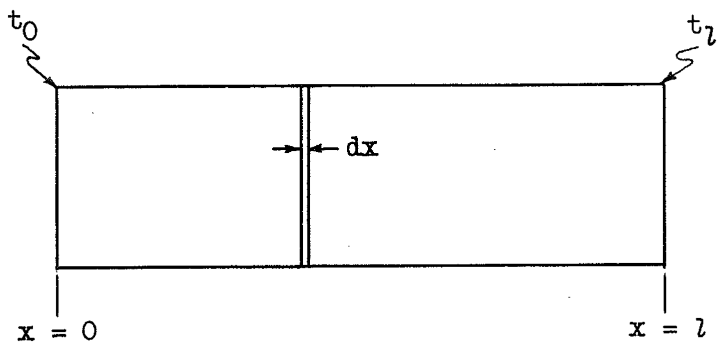
Expressions (B6) and (B7) may now be written:

Net loss by radiation and convection from a differential length of conductor

$$= -G'w\,dx + \left(t - t_e\right)\left(4\epsilon\sigma T_1^{\,3} + f_c\right)w\beta\,dx$$

$$= -G'w\,dx + \left(t - t_e\right)w\beta f'\,dx \qquad \text{(B8)}$$

where $G' = 0$ for portions of the conductor not directly associated with receiver surface. The values of t_a and B are zero, and hence also that of t_e, for the uncompensated thermopile. (See postulates (7) and (10).) Consider now a differential length of uniformly irradiated conductor, as shown in the following sketch:



From equation (B8),

$$\frac{dq}{dx} = G'w - \left(t - t_e\right)w\beta f'$$

but

$$\frac{dt}{dx} = -\frac{q}{kA} \qquad \frac{dq}{dx} = -kA\frac{d^2t}{dx^2}$$

Therefore

$$\frac{d^2t}{dx^2}(kA) = \left[-wG' + \left(t - t_e\right)w\beta f'\right]$$

$$\frac{d^2t}{dx^2} = \left[-\frac{G'w}{kA} + \frac{\beta f'w}{kA}\left(t - t_e\right)\right]$$

$$= -m^2H + m^2\left(t - t_e\right)$$

$$= -m^2\left(H + t_e\right) + m^2t$$

where $H = \frac{G'}{\beta f'}$ and $m^2 = \frac{\beta f'w}{kA}$. This equation has the general solution:

$$t = C_1e^{mx} + C_2e^{-mx} + \left(H + t_e\right) \tag{B9}$$

When $x = 0$,

$$t = t_0$$

and when $x = l$,

$$t = t_l$$

therefore

$$t_0 = C_1 + C_2 + \left(H + t_e\right)$$

$$t_l = C_1 e^{ml} + C_2 e^{-ml} + \left(H + t_e\right)$$

From these two equations can be obtained:

$$C_1 = \frac{t_l - t_0 e^{-ml} - \left(H + t_e\right)\left(1 - e^{-ml}\right)}{e^{ml} - e^{-ml}}$$

$$C_2 = \frac{t_0 e^{ml} - t_l - \left(H + t_e\right)\left(e^{ml} - 1\right)}{e^{ml} - e^{-ml}}$$

From equation (B9),

$$q = -kA\frac{dt}{dx} = kAm\left(C_2 e^{-mx} - C_1 e^{mx}\right)$$

then,

$$q_{x=0} = kAm\left(C_2 - C_1\right)$$

$$= kAm\left[\frac{t_0 \cosh ml - t_l - \left(H + t_e\right)(\cosh ml - 1)}{\sinh ml}\right] \qquad \text{(B10)}$$

and

$$q_{x=l} = kAm\left\{\frac{e^{-ml}\left[t_0 e^{ml} - t_l - \left(H + t_e\right)\left(e^{ml} - 1\right)\right] - e^{ml}\left[t_l - t_0 e^{-ml} - \left(H + t_e\right)\left(1 - e^{-ml}\right)\right]}{e^{ml} - e^{-ml}}\right\}$$

$$= \frac{kAm\left[2t_0 - 2t_l \cosh ml + \left(H + t_e\right)(2 \cosh ml - 2)\right]}{2 \sinh ml}$$

$$= \frac{kAm}{\sinh ml}\left[t_0 - t_l \cosh ml + \left(H + t_e\right)(\cosh ml - 1)\right] \qquad \text{(B11)}$$

Equations (B10) and (B11) may be used to express the heat flow at the two ends of each of the four conductor lengths of figure 1 (uncompensated thermocouple) or of the right-hand half of figure 2 (compensated thermocouple) by substituting t with appropriate subscripts in place of t_0 and t_l and by setting $H = 0$ for the lengths at the extreme right and left. The flow at the right end of the length from 1 to x must equal the flow at the left-hand end of the next length; likewise the flow at the right-hand end of the length from x to h must equal the flow at the left-hand end of the length from h to y, and similarly for the two lengths meeting at y. By equating the flows at these three cross sections, three simultaneous equations can be obtained for solving t_x, t_h, and t_y in terms of the following: w, m_a, m_b, x, and y, values of which are fixed by the unit thermal conductance and the conductivities and dimensions of the metals; H, which involves the intensity of irradiation of the receiver; and t_e, t_1, and t_2. It is convenient to introduce the following new variables:

$$R = \frac{k_b A_b}{k_a A_a} \qquad \text{from which can be obtained} \qquad m_b = \frac{m_a}{\sqrt{R}}$$

$$p = m_b b$$

$$q = m_a a$$

The three simultaneous equations are, then:

(1)

$$\frac{t_1 - t_x \cosh q(l - x) + t_e\left[\cosh q(l - x) - 1\right]}{\sinh q(l - x)}$$

$$= \frac{t_x \cosh xq - t_h - \left(H + t_e\right)(\cosh xq - 1)}{\sinh qx}$$

(2)

$$\frac{t_x - t_h \cosh qx + \left(H + t_e\right)(\cosh qx - 1)}{\sinh qx}$$

$$= \frac{\sqrt{R}\left[t_h \cosh py - t_y - \left(H + t_e\right)(\cosh py - 1)\right]}{\sinh py}$$

(3)

$$\frac{t_h - t_y \cosh py + \left(H + t_e\right)(\cosh py - 1)}{\sinh py}$$

$$= \frac{t_y \cosh p(l - y) - t_2 - t_e\left[\cosh p(l - y) - 1\right]}{\sinh p(l - y)}$$

Solving these three equations simultaneously gives the following result for the hot junction temperature t_h:

$$t_h = \frac{(H + t_e)\sqrt{R}\left[\sinh py - (\coth p)(\cosh py - 1)\right] + (H + t_e)\left[\sinh qx - (\coth q)(\cosh qx - 1)\right]}{\sqrt{R}\coth p + \coth q} + C_3 t_1 + C_4 t_2$$

$$= \omega H + t_e + C_3(t_1 - t_e) + C_4(t_2 - t_e) \tag{B12}$$

where

$$\omega = \frac{\sqrt{R}\left[\sinh py - (\coth p)(\cosh py - 1)\right] + \left[\sinh qx - (\coth q)(\cosh qx - 1)\right]}{\sqrt{R}\coth p + \coth q}$$

$$C_3 = \frac{1}{\sinh q\left(\sqrt{R}\coth p + \coth q\right)}$$

and

$$C_4 = \frac{\sqrt{R}}{\sinh p\left(\sqrt{R}\coth p + \coth q\right)}$$

The value of t_h for the uncompensated thermocouple is obtained from equation (B12) by setting:

$$t_1 = 0, \quad t_2 = 0, \quad \text{and} \quad t_e = 0$$

(see postulate (7), appendix B) which gives:

$$t_h = \omega H \quad \left(\begin{array}{l}\text{Temperature difference be-}\\ \text{tween junctions for uncom-}\\ \text{pensated thermopile}\end{array}\right)$$

Equation (B12) also applies to the right-hand half of figure 2 for the compensated thermopile, and also to the left half of this figure by symmetry, provided H is set equal to zero. The term t_2 can be used instead of t_2' (expression (3)). This gives, then, the following two simultaneous equations:

$$t_h = \omega H + t_e + C_3\left(t_1 - t_e\right) + C_4\left(t_2 - t_e\right)$$

$$t_c = t_e + C_3\left(t_1 - t_e\right) + C_4\left(t_2 - t_e\right)$$

By subtracting, the following is obtained:

$$t_h - t_c = \omega H \quad \left(\begin{array}{l}\text{Temperature difference be-}\\ \text{tween junctions for compen-}\\ \text{sated thermopile}\end{array}\right)$$

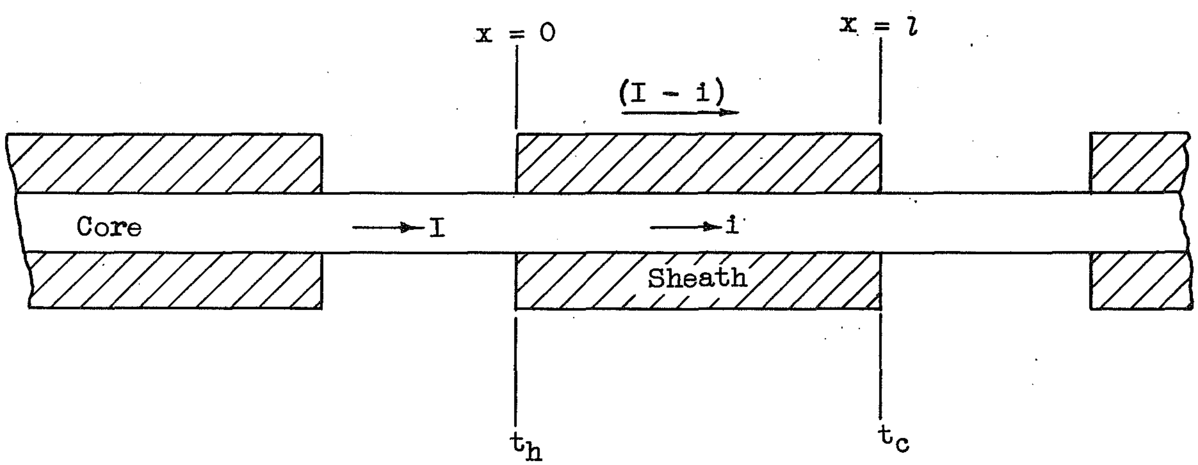
The temperature difference between junctions is therefore the same for the compensated thermocouple of figure 2 as for the uncompensated thermocouple of figure 1. In the case of the compensated thermocouple, however, the temperature difference is independent of t_e and hence of the temperature of the ambient air and of any radiation resulting in a power absorption that is uniform, per unit length of conductor, over the entire thermopile.

APPENDIX C

DERIVATION OF EXPRESSION FOR VOLTAGE LOSS DUE TO CURRENTS CIRCULATING IN THE SHEATH AND CORE OF PLATED CONDUCTORS

The system analyzed is indicated by the following postulates and sketch:

1. The thermoelectric power is uniform over the temperature range involved
2. The sheath and core are uniform in cross section
3. The contact resistance between unit area of the core and sheath interface is zero

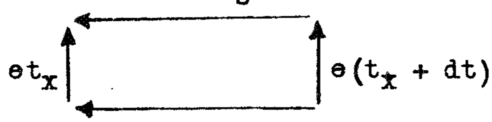


Cross section of part length of a plated-type thermopile conductor.

The voltage difference between the ends of an unplated section can be only the resistance drop (Current × Resistance). Therefore, an expression is obtained for the voltage difference between the ends of a plated section. This difference is written as follows:

$$-R_c \int_{x=0}^{x=l} i \, dx \tag{C1}$$

Considering a differential length of conductor, the sum of voltages in the following complete circuit must equal zero: core to sheath; along sheath distance dx; sheath to core; back along core distance dx. This circuit is expressed diagrammatically and algebraically as follows:



$$et_x - (I - i)R_s\,dx - e(t_x + dt) + iR_c\,dx = 0$$

from which is obtained:

$$e\,dt + \left[(I - i)R_s - iR_c\right] dx = 0$$

$$+\, i\left(R_c + R_s\right) dx - IR_s\,dx = e\,dt$$

$$-R_c \int_{x=0}^{x=l} i\,dx = \frac{R_c}{R_c + R_s}\left[-\int_{x=0}^{x=l} IR_s\,dx - \int_{t_h}^{t_c} e\,dt\right]$$

$$= -\frac{R_c R_s l I}{R_c + R_s} + \frac{R_c}{R_c + R_s}\left(t_h - t_c\right)e \qquad \text{(C2)}$$

The required voltage (expression (C1)) is thus given by two terms on the right-hand side of equation (C2). The first of these is the drop which would result from current I flowing through the resistance obtained by connecting the core and sheath in parallel – hence the actual resistance of the plated section – and the second term is the thermoelectric power multiplied by the temperature difference (as in an ordinary thermocouple), multiplied in turn by the factor given below:

$$\text{Factor allowing for reduction in voltage due to currents circulating in the sheath and core of a plated conductor} = \frac{R_c}{R_c + R_s} \qquad \text{(C3)}$$

This factor may be expressed in terms of R, D, and s as follows:

$$\text{Thermal conductance of sheath} = (R - D)\begin{pmatrix}\text{Thermal conductance of} \\ \text{the unreduced core}\end{pmatrix}$$

But:

$$s = \frac{\text{Thermal conductance of sheath/Electrical conductance of sheath}}{\text{Thermal conductance of core/Electrical conductance of core}}$$

and:

$$\text{Thermal conductance of sheath} = \begin{pmatrix}\text{Thermal conductance} \\ \text{of unreduced core}\end{pmatrix}(R - D)$$

$$\text{Electrical conductance of sheath} = \left(\frac{\begin{matrix}\text{Electrical conductance} \\ \text{of unreduced core}\end{matrix}}{s}\right)(R - D)$$

therefore:

$$\text{Electrical resistance of sheath} = \frac{s\begin{pmatrix}\text{Electrical resistance} \\ \text{of unreduced core}\end{pmatrix}}{R - D}$$

Substituting in equation (C3) gives the following:

$$\text{Factor allowing for reduction in voltage due to currents circulating in the sheath and core of a plated conductor} = \frac{R - D}{R - D + Ds}$$

It should be noted that this expression does not include the voltage loss due to reduction of the temperature difference between junctions caused by the Peltier effect of the circulating current. This temperature reduction is, however, exceedingly small compared with that due to heat conduction along the plated conductor where R is reasonably large.

APPENDIX D

COMPARATIVE EFFICIENCY OF THERMOPILES ANALYZED IN THIS REPORT AND THERMOPILES OF THE JOHANSEN TYPE

The purpose of this appendix is to ascertain the degree to which the power efficiency of thermopiles of the type analyzed in this report approaches the theoretical maximum. This maximum, it is postulated, is that attained in a thermopile of the type analyzed by Johansen (references 3 and 4), in which perfectly conducting receivers make contact with the thermoelectric wires only at their junctions and in which the wires lose no power by radiation or convection. It should be noted that in both systems the Peltier effect, as well as the Thomson and Joule effects, is neglected. The comparison is made for a single thermocouple of each type, and is based on the following conditions:

1. The thermocouples compared are either both uncompensated or compensated

2. The areas of the front receiver surfaces are the same

3. The properties of the metals of one thermocouple are the same as those of the other

4. The average unit thermal conductance for heat loss by radiation and convection for both sides of the receiver of the Johansen thermopile is equal to f' for the type analyzed herein

5. A value of $\beta = 2$ is taken, which corresponds to the flat receiver of the Johansen thermocouple

6. The power absorbed by the front surface of the receiver in excess of that required to produce equality of junction temperatures, G' is the same for the two thermocouples compared

7. The absorptivity α' is the same for the receiving surfaces of the two thermocouples

Equation (2) is used as a measure of efficiency for the reasons previously discussed. It is shown that equation (2), for a compensated thermopile of the type treated in this report, can be expressed as in equation (3b). The efficiency for an uncompensated couple is twice this value since the resistance is half as great and the electromotive force the same; multiplying, therefore, the right-hand side of equation (3b) by 2, and setting $\beta = 2$ (see condition (5)) gives:

$$\pi\begin{pmatrix}\text{for an uncompensated thermocouple}\\ \text{of the type treated in this report}\end{pmatrix} = \frac{\alpha' G' e^2 C_a}{2k_a f'}\Gamma \qquad \text{(D1)}$$

In the Johansen thermocouple:

$$\text{Temperature of receiver} = \frac{G'}{2(2f')}$$

(See condition (1), appendix A.) The electromotive force is e times this value, or:

$$\text{Electromotive force} = \frac{G'e}{4f'}$$

The thermocouple resistance can be expressed as follows:

$$\text{Thermocouple resistance} = \frac{a}{C_a A_a} + \frac{b}{C_s A_s}$$

where a and b are the lengths of the two conductors, C_a and C_s are the electrical conductivities of the two metals, and A_a and A_s are the cross-sectional areas of the two conductors.

Making these substitutions in equation (2) and letting M represent the area of the front surface of the receiver gives:

$$\pi\begin{pmatrix}\text{for an uncompensated}\\ \text{Johansen thermocouple}\end{pmatrix} = \frac{G'e^2\alpha'}{16f'^2 M\left(\frac{a}{C_a A_a} + \frac{b}{C_s A_s}\right)} \qquad \text{(D2)}$$

But C_s can be replaced by $\frac{k_s C_a}{s k_a}$

and

$$2Mf' = \frac{A_a k_a}{a} + \frac{A_s k_s}{b}$$

(see condition (1), appendix A); therefore M can be replaced by $1/2f'$ multiplied by the right-hand side of this expression. The following also holds:

$$\frac{A_a^2 k_a C_a}{a^2} = \frac{A_s^2 k_s C_s}{b^2}$$

(see condition (4), appendix A); therefore

$$\frac{a}{b} = \frac{A_a\sqrt{k_a C_s}}{A_s\sqrt{k_s C_s}}$$

On making these successive substitutions in equation (D2), and replacing $\frac{k_s C_a}{k_a C_s}$ by s and its reciprocal by 1/s, equation (D2) becomes:

$$\pi\begin{pmatrix}\text{for an uncompensated}\\ \text{Johansen thermocouple}\end{pmatrix} = \frac{G' e^2 \alpha' C_a}{8f' k_a\left(1 + \sqrt{s}\right)^2} \qquad \text{(D3)}$$

The relative efficiency of the type of thermopile analyzed in this report is then obtained by dividing the right-hand side of equation (D1) by the right-hand side of equation (D3):

$$\begin{matrix}\text{Relative efficiency, compared}\\ \text{with Johansen thermopiles}\end{matrix} = 4\Gamma\left(1 + \sqrt{s}\right)^2 \qquad \text{(D4)}$$

Since the computation of maximum Γ involves time-consuming trial-and-error calculations, expression (D4) is evaluated for the conditions ($s = 0.62$, $D = 1$) for which such computation has already been made. (See fig. 3.) After the appropriate substitutions are made, the following is obtained for the case in which $R = 25$:

$$\text{Relative efficiency} = 4(0.049)\left(1 + \sqrt{0.62}\right)^2 = 0.63$$

$$\text{Relative voltage, for fixed resistance} = \sqrt{0.63} = 0.80$$

Operation of thermopiles in air, instead of in vacuum, results in an increase in the value of f'. Since f' does not appear, directly or indirectly, in equation (D4), the comparison is applicable for either mode of operation, provided the postulates on which the derivations of the efficiency expressions are based can still be approximated in practice. There is, however, one postulate which is far from realized in air-operated thermopiles of the Johansen type – namely, that of zero power loss by convection and radiation from the conductors. Cartwright states (see reference 6) that more power may be lost from the conductors by convection than from the receiver itself. When this circumstance is taken into account, it seems reasonable to conclude that with proper design there should be little or no advantage in efficiency, for operation in air, of thermopiles of the conventional type as compared with thermopiles in which parts of the conductors act as receivers.

APPENDIX E

METHOD USED FOR CONSTRUCTING SOME PLATED-SILVER-CONSTANTAN THERMOPILES

Thermopiles of the type diagramed in figure 4 and described in the text have been constructed in the following way:

Constantan wire (No. 40 B. & S. gage has generally been used in this investigation) is wound closely spaced on two round rods supported in a demountable frame, as illustrated in figure 5. These rods are vitreous enameled, except near their ends, so that in the subsequent plating operation the wires will not become attached to the rods. Previous to the winding, a long, narrow strip of metal foil is lightly cemented to one of the rods; the wire is wound over this foil which thus serves to short-circuit all the turns. This short circuiting permits uniform plating of the different turns even though the plating current is introduced through the ends of the constantan wires.

The optimum length of the unplated regions of the conductors is relatively short for reasonable values of R. A strip of tissue paper of width equal to this length is cemented over the wires where they are in contact with one of the rods and in such a position that it also covers the strip of metal foil. It is worth while lacquering unprotected metal surfaces of the frame before plating so as to limit the silver deposit to the wires. The whole assembly is now washed in a hot potassium-cyanide solution and then immersed in the plating bath and the desired thickness of silver is applied; this thickness is controlled by fixing current and time.

After being washed and dried the assembly is soaked in acetone and the strip of tissue paper removed. The wires are now to be cemented together into a solid sheet except where they are near, or in contact with, the supporting rods. Lacquer of phenol-formaldehyde cement, applied with a brush, has been found suitable for this purpose. It may be brushed on in a current of warm air, or the lacquer may be thickened first by leaving it for many days in an unstoppered bottle. Additional strength is obtained by laying a few thin glass threads perpendicular to the wires when the lacquer is applied. The assembly, after drying in air, is placed in an oven at 275° F for about 7 minutes; this brief baking partly sets the lacquer but leaves it very flexible. The enameled rods are now rotated, by means of pins inserted in the holes at their ends, until the unlacquered portions of wire are free and the lacquered portions are in contact with the rods. The strip of foil, which generally stays with the wires rather than the rod, is now removed, and lacquer is applied to the free wires as before. The coil is placed in the oven again for 5 minutes.

At this stage the frame is demounted and the coil, which becomes approximately elliptical in cross section, is removed. Since the lacquer is very flexible, the coil can easily be pressed around metal forms of appropriate dimensions into the desired trapezoidal shape. The coil is next placed in an oven for at least 20 minutes at 275° F. The entire external surface is subsequently blackened by any of the usual methods; for example, the coil is usually painted with a suspension of lampblack in turpentine and subsequently sooted over a flame. The completed thermopiles are attached to supports by human hairs lightly cemented to the coils; these hairs should not, of course, make contact with the thermopiles near the conductor junctions.

APPENDIX F

GENERAL COMMENTS ON THE DESIGN OF THERMOPILE RADIOMETER HOUSINGS

Some of the problems involved in housing design are presented in this section, and some relations and methods basic to their solution are discussed and illustrated.

An Ideal Housing

The conditions in the following list are proposed as characterizing an ideal housing. Although these conditions cannot be attained in practice, the degrees to which they are approached - evaluated with respect to their desirability or necessity in the particular applications intended - constitute criterions of success in housing design.

1. An ideal housing shields the thermopile from radiation originating outside the housing except for the exposure of the receiver to the radiation that it is desired to measure; the latter, however, can be selected by the housing only with respect to direction of propagation and location in space or to criterions of equivalent meaning.[1]

2. The radiation entering the opening of an ideal housing and incident upon the receiver during measurement is the same as that existing in the absence of the radiometer and is similarly located with respect to direction of propagation and position in space.

3. An ideal housing irradiates the thermopile uniformly from all directions except for the irradiation of the receiver surface from the direction of the housing opening or from the direction of a lens or reflector that may be mounted within the housing.

4. The irradiation of the thermopile from the inside surface of an ideal housing is constant with respect to time.

5. An ideal housing of a thermopile operated in air protects the thermopile from movements of air originating outside the housing.

6. An ideal housing of a thermopile operated in air maintains the air surrounding the thermopile at a constant and uniform temperature.

---

[1]The limits of receiver sensitivity also place certain restrictions on the specification of radiation than can be measured.

Materials of Construction

Conditions (3), (4), and (6) require for their realization a uniform and constant temperature of the housing, or at least of that part of it immediately surrounding the thermopile.

In a housing exposed to nonuniformity of air temperature or of irradiation, as is often necessary under conditions of use, there will be at equilibrium a transfer of power by conduction from regions of greater absorption to regions of lesser absorption. The magnitudes of the consequent temperature differences will be inversely related, for given conditions and housing dimensions, to the thermal conductivity k of the housing material. Such conductivity should therefore be high. It is possible, however, to compensate within limits for a deficiency of conductivity by increase of housing-wall thickness, provided the thickness is small with respect to housing radius so that such increase will have relatively little effect on over-all outside housing dimensions.

Alteration of thermal environment, often unadvoidable in practice, may result in a net exchange of power between a housing and its surroundings and a consequent shift in average temperature of the housing as a whole. The time required for the attainment of near equilibrium in this respect for housings of substantial construction is likely to be very great, and it is usually important, therefore, that the thermopile be usable during a period of such shift. Such use is facilitated if the rate of shift is low, and since this rate, for given conditions and housing dimensions, is inversely related to the heat capacity per unit volume $c_p\gamma$ of the housing material, it is generally desirable that such capacity be large. It is possible, however, to compensate for deficiency in heat capacity by increase of wall thickness under the conditions described in the preceding paragraph.

When measurement of irradiation is based on thermopile output and housing temperature, any nonuniformity of the temperature will result in errors of unknown magnitude. It is therefore desirable in such cases that a state of uniformity be approached as rapidly as possible on removal of circumstances producing a temperature difference. The rate of approach to uniformity, for given conditions and housing dimensions, increases with increased diffusivity $k/c_p\gamma$ of the housing material.

It appears, then, that high conductivity, high heat capacity, and high diffusivity are generally advantageous, although the relative desirability of each depends on other features of radiometer design and on the manner of use. The values of these properties of some metals which are, or might be considered to be, suitable for housings are listed below:

	γ $\left(\frac{\text{lb}}{\text{cu ft}}\right)$	k $\left(\frac{\text{Btu}}{(\text{hr})(\text{sq ft})(^{\circ}\text{F/ft})}\right)$	$c_p\gamma$ $\left(\frac{\text{Btu}}{(\text{cu ft})(^{\circ}\text{F})}\right)$	$\frac{k}{c_p\gamma}$ $\left(\frac{\text{sq ft}}{\text{hr}}\right)$
Silver	658	242	37	6.6
Copper	556	224	52	4.3
Aluminum	170	117	35	3.3
Magnesium	109	92	27	3.4
Brass (65-percent copper, 35-percent zinc)	546	70	46	1.5
Iron	495	39	66	.59

It will be noted that silver is best with respect to diffusivity and conductivity; its use is often to be preferred aside from the matter of cost. Copper has a higher heat capacity and a nearly equal conductivity, and its diffusivity is about two-thirds that of silver; copper may be rated, therefore, from nearly as good as silver to, in some applications, possibly better. Aluminum, however, is much lighter and has a diffusivity nearly equal to, and a conductivity not much less than, that of copper. The heat capacity of aluminum is somewhat less, but this may frequently be compensated for by the use of thicker walls without eliminating its advantage in weight. Magnesium, in turn, is slightly superior to aluminum in diffusivity and will weigh less for a cross-sectional area providing equal conductance; its heat capacity for a given weight is also slightly greater. Therefore, in those cases in which the necessary thickness of wall does not contribute too large a proportion of the over-all outside housing dimensions or is not otherwise objectionable, aluminum may be considered superior to copper, and magnesium to aluminum. The use of brass cannot be justified, and the use of iron or steel could be considered desirable only from the standpoint of cost.

Properties of Housing Surfaces

The external surfaces of a housing should have a high reflectivity for any irradiation the temporal or spacial nonuniformity of which might cause local or extensive temperature changes in conflict with condition (3), (4), or (6). Polished gold plate has excellent reflectivity for the longer wavelengths, such as are characteristic of thermally produced radiation emitted by materials at temperatures below that of incandescence; the most commonly used polished metals, such as chromium and nickel plate, aluminum, and freshly polished copper or brass, are also quite highly reflective for radiation of this character. When variable heating effects of radiation of shorter wavelength are likely to be of importance, as in cases of exposure to direct sunlight, a careful comparison should be made of the reflectivities to such radiation of the various metallic, as well as of nonmetallic, coatings available.

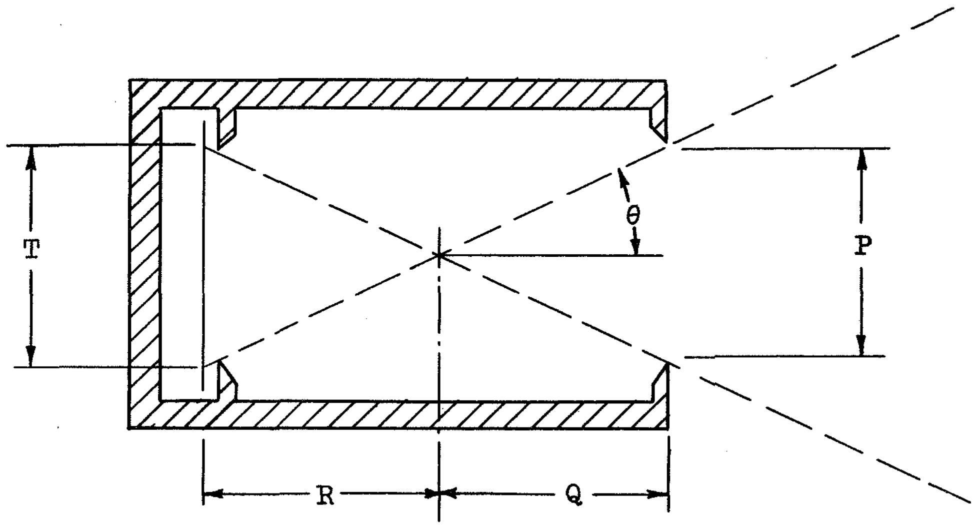
A reflective coating on the front surfaces of any baffles that may be provided in a housing between the opening and the receiver surface will tend to divert incident radiation and consequent heating toward the front part of the housing, if not actually through the housing opening, thus providing partial protection from uneven heating for that part of the housing immediately surrounding the thermopile.

An approach to conditions (1), (2), (3), and (4) is facilitated by a high absorptivity of all inside surfaces of the housing except the front surface of the baffles and, of course, also any reflectors that may be used to concentrate the energy to be measured onto the receiver surface. The application of a flat black paint followed by sooting over a flame of Diesel oil results in a highly absorbent surface. A previous scoring of the inside surfaces, as by the cutting of threads if the inside is circular in cross section, helps to ensure an absorptivity closely approaching 1.00.

Geometry of Housing

The problems of size and shape are exceedingly various, and depend largely on the applications for which a given radiometer is intended. In general, small over-all dimensions greatly facilitate an approach to uniformity of housing temperature, and a small inside diameter and the use of baffles should assist in an approach to conditions (5) and (6). The possibility exists of using spaced double or triple walls in order to approach condition (3) more closely with a lighter housing.

The following simple geometrical problem is presented as illustrative of the many that arise in particular applications: A simple cylindrical housing, having an opening of which the diameter P is fixed by other considerations, is to be used with a thermopile having a flat receiver of fixed shape and of perfect diffusivity. The radiation measured is to be limited to that the direction of which (postulating straight-line propagation) forms an angle of θ or less with any intersecting line parallel to the housing axis. The receiver is postulated to "view" through the opening a diffuse source of uniform intensity. This situation is diagramed in the following sketch of the radiometer:



P previously determined diameter of housing opening

T twice the greatest distance from the center of the receiver to its periphery – the diagonal in case the receiver is square or rectangular in shape

Q distance from plane of housing opening to intersection of construction lines making angle θ with housing axis

Q + R distance from receiver to plane of housing opening

Given these conditions the following question may be asked: What value of R + Q will permit maximum thermopile output?

The receiver must be at least Q distance from the opening, or it will intercept radiation having directions of propagation forming angles with lines parallel to the axis of the radiometer greater than θ. For the same reason the receiver area must be included within the construction lines shown making angle θ with the housing. Since the power input available for producing thermopile output is equal to receiver area multiplied by the irradiation G' (see appendix G) and efficiency is proportional to G' (see equation (3a)), the output is proportional to (Receiver area) $\times (G')^2$. The irradiation G' is proportional to the shape modulus of the opening to the receiver. If $R + Q$ is large compared with P, as will be the case if θ is fairly small, the shape modulus of the opening to all parts of the receiver will be almost equal to that at the center of the receiver. This factor (see reference 38, p. 15) is $\dfrac{P^2/4}{\frac{P^2}{4} + (R + Q)^2}$. The irradiation G' may then be taken as approximately proportional to:

$$\frac{P^2}{\frac{P^2}{4} + (R + Q)^2}$$

Since receiver area for fixed shape is proportional to the square of any linear dimension, it is proportional to T^2. The thermopile output is therefore proportional, to a first approximation, to:

$$\frac{T^2P^4}{\left[\frac{P^2}{4} + (R + Q)^2\right]^2}$$

but

$$T = P\frac{R}{Q}$$

Therefore, output is approximately proportional to:

$$\frac{P^6 \times R^2}{Q^2\left[\frac{P^2}{4} + (R + Q)^2\right]^2}$$

with R the variable. In order that this expression may be a maximum, $\dfrac{R}{\frac{P^2}{4} + (R + Q)^2}$ must be a maximum. When its derivative, with respect to R, is set equal to zero, the following is obtained:

$$\frac{P^2}{4} + R^2 + 2RQ + Q^2 - 2R(R + Q) = 0$$

from which:

$$\frac{P^2}{4} - R^2 + Q^2 = 0$$

Therefore:

$$R^2 = Q^2 + \frac{P^2}{4}$$

and hence:

$$T = P\sqrt{1 + \frac{P^2}{4Q^2}}$$

Since $P^2/4$ will be small compared with Q^2 when θ is small, it may be concluded that, under the conditions mentioned, the maximum radius of the receiver should be just slightly greater than the radius of the housing opening, and the distance from the receiver to the plane of the housing opening must then be slightly more than twice the distance from the latter to a point on the housing axis for which the radius of the opening subtends the limiting angle θ. If the maximum opening possible in a simple hollow cylinder is to be realized, then the thermopile receiver should be just as large as can be accommodated inside the cylinder and the distance from the plane of the opening should be $\left(1 + \frac{T}{P}\right)\left(\frac{P}{2} \cot \theta\right)$.

APPENDIX G

SYMBOLS

a	total length (uncompensated thermocouple) or half length (compensated thermocouple) of one conductor – the unplated conductor in plated-type construction, ft; as subscript, refers to one of the thermoelectric metals – the metal of the unplated conductor in plated-type construction – or to the conductor composed of this metal
A	cross-sectional area of wire, sq ft
b	total length (uncompensated thermocouple) or half length (compensated thermocouple) of one conductor – the plated conductor in plated-type construction, ft; as subscript, refers to one of the conductors, the plated conductor in plated-type construction
B	difference between power actually absorbed per unit length of conductor of a compensated thermopile and power that would be absorbed if all parts of thermopile surface, except front surface of receiver, exchanged power by radiation with only thermopile housing (see appendix B, postulate (10)), Btu/(hr)(ft)
c	proportion to which thermopile voltage is reduced in plated construction because of circulating currents in core and plate of plated conductor; as subscript, refers to core of plated conductor
c_p	specific heat at constant pressure, Btu/(lb/^{o}F)
C	electrical conductivity, ohms/(sq ft/ft)
D	ratio of cross-sectional area of core of plated conductor to cross-sectional area of unplated conductor; $D = 0$ expresses the situation corresponding to unplated construction
e	thermoelectric power, mv/^{o}F
f_c	unit thermal conductance due to heat transfer by convection averaged around or on both sides of a conductor (together with associated binding or coating material) (see appendix B, postulate (9))

f' — $\left(f_c + 4\epsilon\sigma T_1^3\right)$; increase in power loss by convection and radiant emission, per unit area of thermopile surface, resulting from unit rise of temperature, Btu/(hr)(sq ft)(°F)

F — shape modulus, used with subscripts. (The shape modulus of one surface to another is the fraction the denominator of which is the radiant power that would be emitted by the first surface were it at uniform temperature, of uniform emissivity, and perfect diffusivity, and the numerator of which is the amount of this power reaching the second surface prior to reflection and in the absence of any intervening absorbing medium.)

$F_{h \leftarrow R}$ — shape modulus, front surface of receiver to inside surface of housing (see definition of F)

$F_{R \leftarrow h}$ — shape modulus, inside surface of housing to front surface of receiver (see definition of F)

$F_{0 \leftarrow R}$ — shape modulus, front surface of receiver to opening in housing (see definition of F)

G_1 — irradiation of back side of receiver element, Btu/(hr)(sq ft)

G_2 — irradiation of front side of element, Btu/(hr)(sq ft)

G_0 — part of G_2 coming through the opening in housing, Btu/(hr)(sq ft)

G' — $\left(\alpha G_0 - \epsilon F_{0 \leftarrow R} T_1^4\right)$; difference between irradiation absorbed per unit area of receiver surface and that absorbed per unit area of all other thermopile surfaces; it is, therefore, the irradiation effective in producing a temperature difference between hot and cold junctions, Btu/(hr)(sq ft)

H — $G'/\beta f'$; rise in temperature that would result from irradiation G' in an irradiated length of conductor if the latter were thermally nonconducting in the direction of its length. The actual rise due to G' is less and, at the junction, is ω times H, °F

I — total current

i — current through core at any distance x from end of plated section

k — thermal conductivity, Btu/(hr)(sq ft)(°F/ft)

K constant in equation $G_0 = K(mv) + F_{0 \leftarrow R} T_1^{\,4}$, Btu/(hr)(sq ft)(°F/ft)

l length of uniformly irradiated strip

m $\sqrt{\frac{\beta w f'}{kA}}$, 1/ft (Subscripts of m, k, and A are to be the same.)

mv electromotive force, millivolts

M area of front surface of receiver in thermocouple of Johansen type, sq ft

n number of junctions in a thermopile

p $m_b b$

q $m_a a$; with subscript, heat flow, Btu/hr

R $\frac{k_b A_b}{k_a A_a}$; ratio of thermal conductance per unit length of conductors of one type (the plated conductor in plated-type thermopiles) to that of conductors of other type

R_c resistance of core of plated conductor per unit length, ohms/ft

R_s resistance of deposited metal of plated conductor per unit length, ohms/ft

s $\frac{k_s/k_a}{C_s/C_a}$; ratio of quotient of thermal conductivities of the two thermoelectric metals to quotient of their electrical conductivities. As subscript, refers to one of the thermoelectric metals, the deposited metal of the plated conductor in plated-type construction

$\dot{t}_e$ $\frac{f_c t_a + \left(\frac{B}{w\beta}\right)}{4\epsilon\sigma T_1^{\,3} + f_c}$; temperature difference between differential length of conductor and housing at which there is no net power exchange by radiation and convection, combined, between the surface of this length of conductor and the surroundings except, in the case of the receiver surface only, for the absorption of irradiation G'. For the uncompensated thermopile $t_a = 0$, $B = 0$, and hence $t_e = 0$, °F.

t_a difference between temperature of air surrounding thermopile and that of housing, $^{\circ}F$

t difference between temperature of differential length of conductor or fin at regions which may be indicated by subscripts and that of housing and (in the case of the uncompensated thermopile) ambient air, $^{\circ}F$

T absolute temperature of differential length of conductor, $^{\circ}R$

T_1 absolute temperature of housing

w width of receiver area associated with a single conductor, ft

x fraction of length a irradiated (also used to represent distance in ft)

y fraction of length b irradiated

ω t_h/H or $\left(t_{h,1} - t_{h,2}\right)/H$

β total distance, in plane perpendicular to length of a conductor, along surfaces of this conductor (together with its associated binding or coating material) from which heat is lost by convection and radiation, all divided by term w

α absorptivity of thermopile receiver to that part of the radiation falling on it which comes in the housing opening

ϵ average emissivity of different sides of a conductor with respect to radiation produced by an ideal radiator at receiver temperature (see appendix B, postulate (9))

α' absorptivity of the thermopile receiver to radiation from the source of interest

σ Stefan-Boltzmann radiation constant, Btu/(hr)(sq ft)$(^{\circ}R)^4$

π term directly proportional to efficiency of thermopile (See equations (2) and (3a).)

Γ term directly proportional to efficiency of thermopile (See equations (3a) and (3b).)

γ specific weight, lb/cu ft

REFERENCES

1. Boelter, L. M. K., Bromberg, R., Gier, J. T., and Dempster, E. R.: An Investigation of Aircraft Heaters. XXV – Use of the Thermopile Radiometer. NACA ARR No. 5A13, 1945.

2. Gier, J. T., and Boelter, L. M. K.: The Silver-Constantan Plated Thermopile. Temperature, Its Measurement and Control in Science and Industry. Am. Inst. of Phys., Reinhold Pub. Co. (New York), 1941, pp. 1284-1292.

3. Johansen, E. S.: Über die Vakuumthermosäule als Strahlungsmesser. Ann. der Phys. Bd. 33, Heft 3, Sept. 1910, pp. 517-536.

4. Johansen, E. S.: Notiz über die Konstrucktion empfindlicher Thermosäullen. Physikalische Zeitschr. Bd. 14, Heft 15, Oct. 1913, pp. 998-1000.

5. Firestone, F. A.: Radiation Thermopile Design. Rev. Sci. Instr., vol. 1, no. 11, Nov. 1930, pp. 630-649.

6. Cartwright, C. H.: Über die Empfindlichkeit von Thermosäulen, Mikroradiometern und Bolometern. Zeitschr. f. Phys. Bd. 92, Heft 3 and 4, 1934, pp. 153-171.

7. Harris, L.: Thermocouples for the Measurement of Small Intensities of Radiation. Phys. Rev., ser. 2, vol. 45, no. 9, May 1934, pp. 635-640.

8. Cartwright, C. H.: General Theory, Design, and Construction of Sensitive Thermopiles. Rev. Sci. Instr., vol. 1, no. 10, Oct. 1930, pp. 592-604.

9. Moll, W. J. H., and Burger, H. C.: A New Vacuum Thermoelement. Phil. Mag., vol. 50, no. 297, Sept. 1925, pp. 618-622.

10. Jones, R. V.: Radiation Thermopiles of Quick Response. Jour. Sci. Instr., vol. 14, no. 3, March 1937, pp. 83-89.

11. Müller, C.: Neue Strahlungsmessgeräte Thermoelemente von gesteigerter Empfindlichkeit und Einstellschnelligkeit. Naturwiss. Bd. 19, Heft 20, May 1931, pp. 416-419.

12. Kersten, H., and Schaffert, Roland: Construction of Thermoelements by Electro-deposition. Rev. Sci. Instr., vol. 3, no. 4, April 1932, pp. 189-195.

13. Burger, H. C., and van Cittert, P. H.: Die Herstellung von Wismut-Antimon Vakuumthermoelementen durch Verdampfung. Zeitschr. f. Phys. Bd. 66, Heft 3 and 4, Nov. 1930, pp. 210-217.

14. Johnson, Ellis A., and Harris, Louis: The Technique of Sputtering Sensitive Thermocouples. Rev. Sci. Instr., vol. 5, no. 4, April 1934, pp. 153-158.

15. Harris, Louis, and Scholp, Alvin C.: The Response of Sputtered Thermocouples to Interrupted Radiation. Jour. Optical Soc. Am., vol. 30, no. 11, Nov. 1940, pp. 519-522.

16. Rayleigh, Lord: On the Thermodynamic Efficiency of the Thermopile. Phil. Mag., ser. 5, vol. 20, no. 125, Oct. 1885, pp. 361-363.

17. Ising, Gustaf: A Natural Limit for the Sensibility of Galvanometers. Phil. Mag., ser. 7, vol. 1, no. 4, April 1926, pp. 827-834.

18. Zernike, F.: Die natürliche Beobachtungsgrenze der Stromstärke. Zeitschr. f. Phys. Bd. 40, Heft 8, Dec. 1926, pp. 636-638.

19. Moll, W. J. H., and Burger, H. C.: The Sensitivity of a Galvanometer and Its Amplification. Phil. Mag., ser. 6, vol. 50, no. 297, Sept. 1925, pp. 626-631.

20. Hardy, J. D.: Note on the Resonance Radiometer. Rev. Sci. Instr., vol. 5, no. 3, March 1934, pp. 120-124.

21. Wilson, Wm. Hamilton, and Epps, Miss T. D.: The Construction of Thermocouples by Deposition. Phys. Soc. London, Proc., vol. 32, 1920, pp. 326-340.

22. Pfund, A. H.: Resonance Radiometry. Sci., vol. 69, no. 1777, Jan. 1929, pp. 71-72.

23. Gunn, Ross: Some Experiments on the Amplification of Thermocouple Electromotive Forces. Rev. Sci. Instr., vol. 9, no. 9, Sept. 1938, pp. 267-269.

24. Czerny, M.: Über die Brownshe Molekularbewegung also Grenze der Messteknik. Ann. d. Phys., Folge 5, Bd. 12, Heft 8, Mar. 1932, pp. 993-1010.

25. Van Lear, G. A.: The Brownian Motion of the Resonance Radiometer. Rev. Sci. Instr., vol. 4, no. 1, Jan. 1933, pp. 21-27.

26. Johnson, J. B.: Thermal Agitation of Electricity in Conductors. Phys. Rev., vol. 32, no. 1, July 1928, pp. 97-109.

27. Nyquist, H.: Thermal Agitation of Electric Charge in Conductors. Phys. Rev., vol. 32, no. 1, July 1928, pp. 110–113.

28. Cartwright, C. H.: Natural Observation Limit of Radiometric Measurements. Phys., vol. 1, no. 4, Oct. 1931, pp. 211–229.

29. Cartwright, C. Hawley, and Strong, John: Vacuum Thermopiles and the Measurement of Radiant Energy. Ch. VIII, pp. 305–340 of Procedures in Experimental Physics by John Strong et al. Prentice-Hall, Inc. (New York), 1939.

30. Hume-Rothery, W.: The Metallic State. Oxford Univ. Press (London), 1931, pp. 95–98.

31. Altenkirch, von Edmund: Über den Nutzeffect der Thermosäule. Phys. Zeitschr. Bd. 14, Heft 16, Aug. 1909, pp. 560–568.

32. Coblentz, W. W.: Instruments and Methods Used in Radiometry. U. S. Bur. Standards, Bull. 4, 1908, pp. 391–460.

33. Cartwright, C. H.: Wiedemann-Franzsche Zahl Wärmeleitfähigkeit und thermoelectrische Kraft von Tellur. Ann. d. Phys., Folge 5, Bd. 6, Heft 6, Nov. 1933, pp. 656–678.

34. Cartwright, C. H., and Haberfeld-Schwartz, M.: On Some Electron Properties of Tellurium and Wilson's Mechanism of Semi-Conductivity. Roy. Soc. London, Proc., ser. A, vol. 148, no. 865, Feb. 1935, pp. 648–664.

35. Pfund, A. H.: Bismuth Black and Its Applications. Rev. Sci. Instr., vol. 1, no. 7, July 1930, pp. 397–399.

36. Pfund, A. H.: The Blackening of Radiometers. Chap. VI, pp. 210–214, of Measurement of Radiant Energy, W. E. Forsythe, ed., McGraw-Hill Book Co., Inc., 1937.

37. Warburg, Emil Gabriel: Über Wärmeleitung und andere ausgleichende. J. Springer (Berlin), 1924, pp. 106.

38. Boelter, L. M. K., Cherry, V. H., and Johnson, H. A.: Heat Transfer, Supplementary Notes. Third ed., Univ. of Calif. Press (Berkeley, Calif.), 1942. Contribution by A. R. Collins: Steady State Thermopile Performance, ch. XVIII.

39. Moll, W. J. H.: A Thermopile for Measuring Radiation. London Phys. Soc., Proc. vol. 35, 1923, pp. 257–260.

40. Cartwright, C. Hawley: Notes on the Evaporation of Ag, Be, Cr, and Si. Rev. Sci. Instr., vol. 3, no. 6, June, 1932, pp. 298–304.

41. O'Bryan, H. M.: Evaporation Technique for Highly Refractory Substances. Rev. Sci. Instr., vol. 5, no. 3, March 1934, pp. 125-126.

42. Caldwell, Wallace C.: The Evaporation of Molten Metals from Hot Filaments. Jour. Appl. Phys., vol. 12, no. 11, Nov. 1941, pp. 779-781.

43. Harris, L., and Johnson, E. A.: The Production of Strong Cellulose Acetate Films. Rev. Sci. Instr., vol. 4, no. 8, Aug. 1933, pp. 454-455.

44. Johnson, E. A., and Harris, Louis: Thermoelectric Force on Thin Films. Phys. Rev., ser. 2, vol. 44, no. 11, Dec. 1933, pp. 944-945.

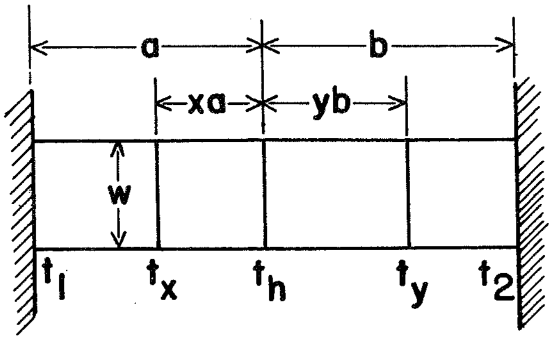


Figure 1.- Uncompensated thermocouple.

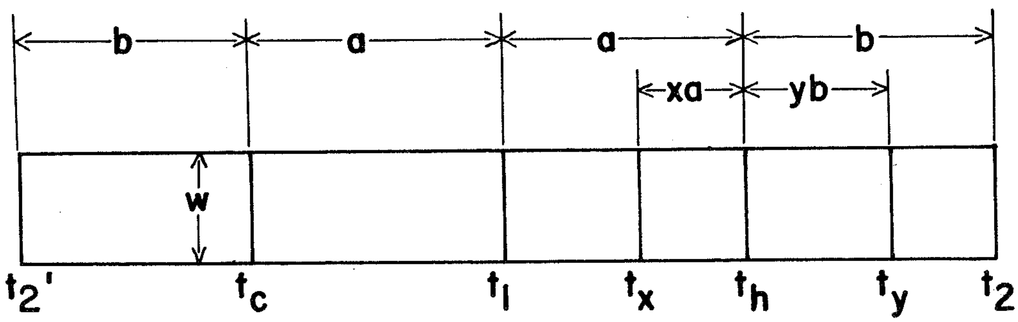


Figure 2.- Compensated thermocouple.

a and b — Length (uncompensated thermocouple) or half lengths (compensated thermocouple) of the thermoelectric conductors. If conductor b is of plated construction, its core and conductor a are composed of the same metal.

xa or yb — Portion of length of conductor, or of coating or binding materials intimately associated with it, that is exposed to the radiation to be measured.

w — Width or portion of perimeter of each conductor, or of coating or binding materials intimately associated with it, that is exposed to the radiation to be measured. Then w(xa + yb) is receiver area per couple.

t — Temperature difference between conductors and housing at the cross sections indicated by the subscripts. In the case of the uncompensated thermocouple it is also the temperature difference between the conductors and ambient air (see appendix B, postulate (7)); $t_2 = t_2'$ (see appendix B, postulate (3)).

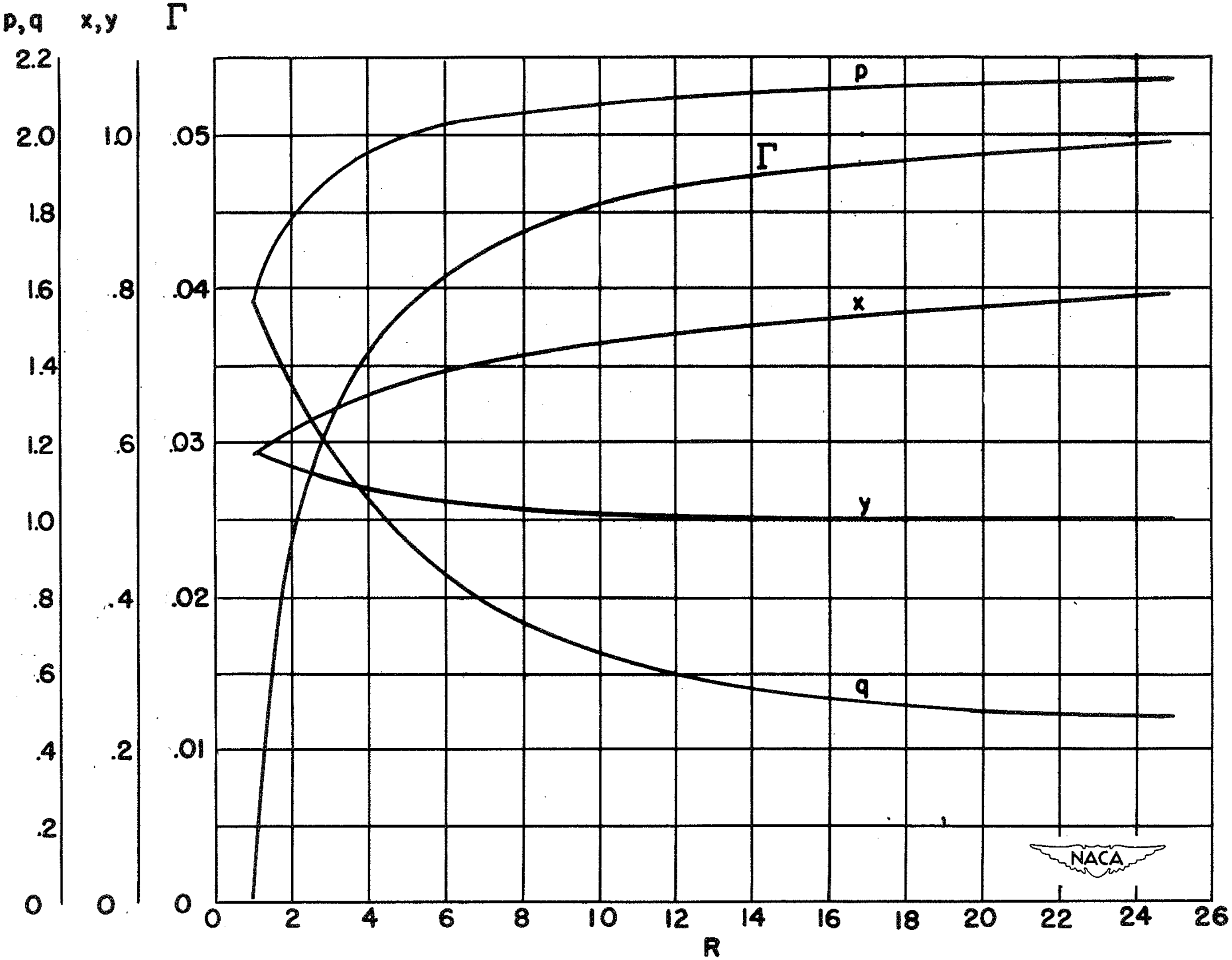
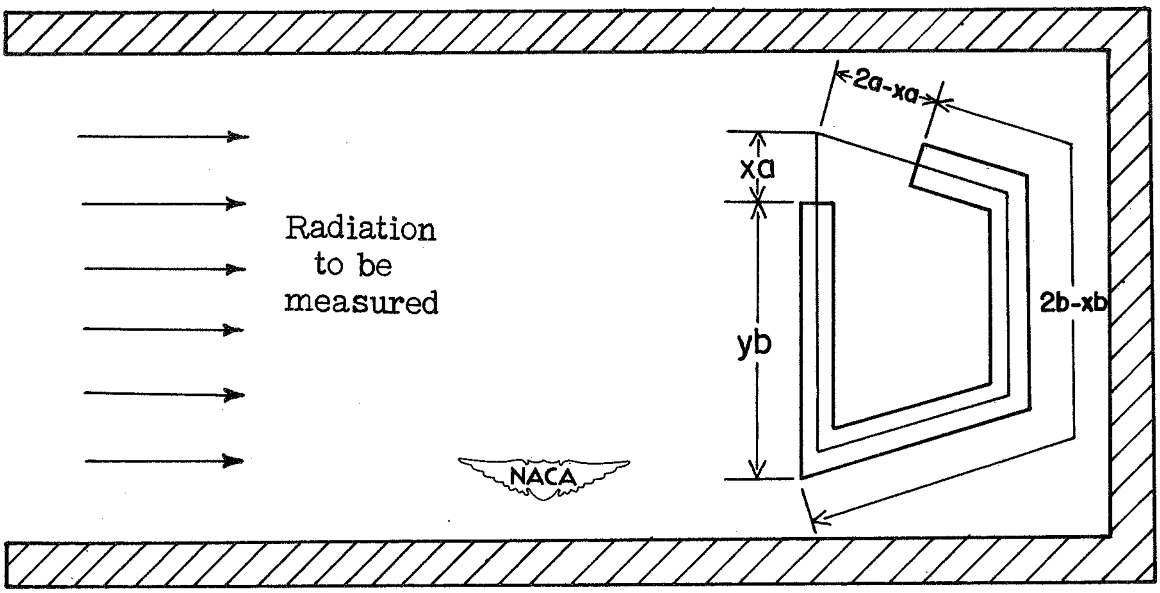


Figure 3.- Optimum values of x,y,p,q, and Γ as functions of R for design of thermopiles. (For definition of symbols, see appendix G.)



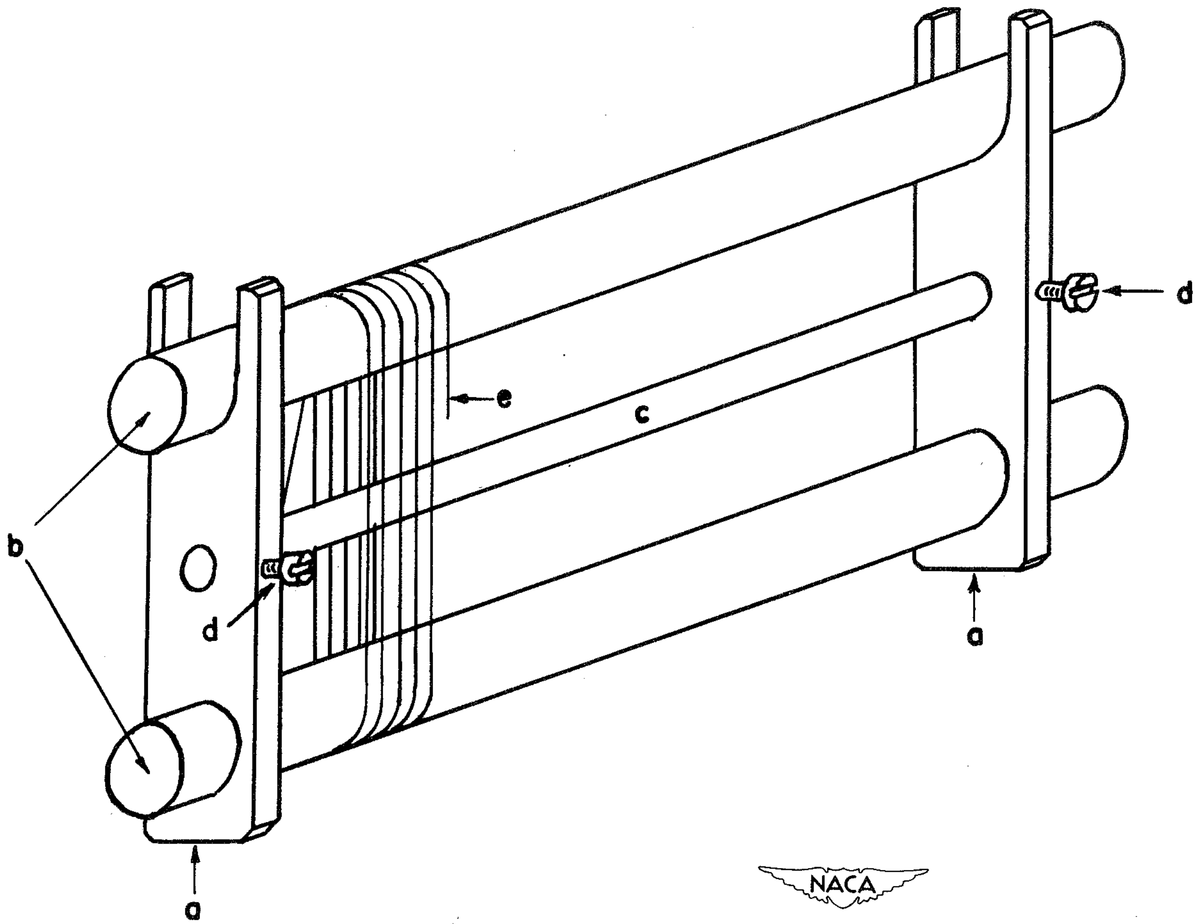
a half length of unplated conductor

b half length of plated conductor

xa length of unplated conductor, and associated plastic and absorbent coating, exposed to radiation from outside the housing

yb length of plated conductor, and associated plastic and absorbent coating, exposed to radiation from outside the housing

Figure 4.- Cross section of housing and end view of a compensated thermopile.



a End pieces

b Rotatable rods, vitreous enameled except at ends and bearing surfaces

c Rod holding end pieces in position

d Screws clamping rod c to end pieces

e Start of coil of constantan wire - to continue to near end piece

Figure 5.- Sketch of fixture used in constructing thermopile.