NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1435

STRESSES IN AND GENERAL INSTABILITY OF MONOCOQUE CYLINDERS WITH CUTOUTS

V - CALCULATION OF THE STRESSES IN CYLINDERS WITH SIDE CUTOUT

By N. J. Hoff and Bertram Klein

Polytechnic Institute of Brooklyn

Washington
January 1948

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SUMMARY

Stresses were calculated by a numerical method in three reinforced monocoque cylinders subjected to pure bending. The cylinders were of circular cross section and were reinforced with 8 rings and either 8 or 16 stringers. There was a cutout on one side of each cylinder located symmetrically to the neutral plane and extending over 45°, 90°, or 135°. Satisfactory agreement was found between stresses calculated and those measured in part IV in the present series of investigations.

INTRODUCTION

In analytical investigations the reinforced monocoque cylinder is almost invariably assumed to be of constant section and reinforced with evenly spaced stringers and rings of constant cross-sectional properties. In reality, actual airplane structures often have openings for doors, windows, and so forth, and are reinforced locally near points of application of concentrated loads. It is believed that the stress problem of such nonuniform structures is best approached by numerical methods.

In a series of investigations carried out at the Polytechnic Institute of Brooklyn Aeronautical Laboratories an effort was made to apply Southwell's relaxation method (reference 1) to the calculation of the stresses in reinforced monocoque structures. Procedures were developed for reinforced flat and curved sheets (references 2 and 3) as well as for fuselage frames (references 4 and 5). Finally, numerical methods were used to determine the stresses in a reinforced monocoque cylinder having a symmetric cutout on the compression side (reference 6). The results obtained were in satisfactory agreement with experiments carried out earlier, which are described in reference 7. The present report deals with the problem of the stress distribution in a reinforced monocoque cylinder having a side cutout and subjected to pure bending. The results of the calculations are compared with the experiments described in reference 8.

In the first step of the procedure the structure is divided into elements, and the elastic properties of the elements are determined. In the present problem a sheet panel with its bordering segments of stringers and rings was chosen as the element of the reinforced monocoque cylinder. When the loads are applied to the cylinder, the corners of the panels undergo, in general, displacements in arbitrary directions. For the purposes of this calculation the displacements are resolved into axial (in the direction of the axis of the cylinder), tangential (in the direction of the tangent to the ring), and radial components (in the direction of the radius of the ring). At the same time, the corners are, in general, rotated about axes of arbitrary direction, and this rotation is resolved into rotations about the axial, tangential, and radial directions.

In the so-called unit problem it is assumed that the four corners of the panel are rigidly clamped to some imaginary rigid body to prevent both displacement and rotation. Then the clamps are released at one corner to permit displacement or rotation in one direction only, and a displacement (or rotation) of unit magnitude is undertaken in that particular direction. Next the reaction forces and moments caused by the displacement (or rotation) undertaken are calculated for all the four corners.

After all the unit problems of the structure are solved, the results are combined in what are termed the "operations tables." These tables are a systematic presentation of the reactions at all the corner points corresponding to unit displacements of the corner points. It is then required to find a combination of all the displacement (and rotation) components corresponding to zero resultant force and moment at each corner point at which no external load is applied and to force and moment resultants equal and opposite to the loads at the points of application of the external loads. According to Southwell's suggestion, this combination of displacements is found by systematic step-by-step approximations. At the Polytechnic Institute of Brooklyn Aeronautical Laboratories such solutions by step-by-step approximations have been established for reinforced panel problems (references 2 and 3), but when the same approach was tried for the case of monocoque cylinders having symmetric cut-outs and subjected to pure bending, the number of steps needed became almost prohibitive. On the other hand, the solution by matrix methods of the system of linear equations represented by the operations tables together with the applied loads was possible with a reasonable expenditure of work.

In the present report, the displacements are calculated from the operations tables by means of a slightly modified version of Crout's method of solving matrix equations. (See reference 9.) The number of unknowns is 34, 36, and 30 in the case of the cylinders having 45°, 90°, and 135° cutouts, respectively. The numerical part of the work was carried out on semiautomatic electric calculating machines, and 10 digits were kept throughout the calculations. As an approximate rule, it may be stated that matrices of the kind encountered in this work can be solved by an experienced calculator at the rate of 2 hours for each unknown quantity. This estimate does not allow for mistakes.

Once the displacements are known, the stresses can be easily calculated with the aid of the solutions of the unit problems and elementary considerations. Complete numerical calculations were carried out for three cylinders of the experimental series described in reference 8. Satisfactory agreement was found between theory and experiment, as may be seen from the comparison shown in the figures of the present report.

The authors acknowledge their indebtedness to Mr. Bruno A. Boley for his help in the theoretical aspects of the problem and to Mr. John G. Pulos, who took part in the calculations. The work was carried out under the sponsorship and with the financial assistance of the National Advisory Committee for Aeronautics.

SYMBOLS

a	distance between adjacent rings
A	cross-sectional area of stringer augmented by its effective width of curved sheet
A, B, C, A', B', C'	rings of cylinder or quadrants of operations table
E	Young's modulus
G	shear modulus
L	length of ring segment between adjacent stringers
M	externally applied bending moment acting on cylinder
$\overset{\frown}{nn}$	ring influence coefficient
N	bending moment acting in plane of ring
Q	torque acting on rigid end ring
r	radius of monocoque cylinder
$\overset{\frown}{rn}$, $\overset{\frown}{rr}$	ring influence coefficients
R	radial shear force acting in plane of ring
t	thickness of sheet covering
$\overset{\frown}{tt}$, $\overset{\frown}{tr}$, $\overset{\frown}{tn}$	ring influence coefficients

T	tangential force acting in plane of ring
u	tangential displacement
v	radial displacement
w	rotation
X	force acting in axial direction
Y	vertical shear force acting on rigid ring
α_t, α_r, α_n	coefficients used in calculation of forces and moments caused by shear flow existing in panel
$\Gamma = Gt/2$	
η	vertical downward translation of rigid ring
θ	rotation of rigid ring in its own plane
ξ	axial displacement
ω	rotation of rigid ring about its horizontal diameter
$\Omega = Gta/4L$	

STATEMENT OF PROBLEM AND ASSUMPTIONS

The three cylinders for which calculations were carried out are shown in figure 1. They are Cylinders 35, 39, and 40 of the present series and are described in reference 8. A number of structural changes were assumed for the purpose of calculations in order to decrease the work required for the solution of the problem.

Figure 2 shows the three cylinders in their modified forms. In reality, Cylinder 35 had 16 stringers, 7 of which were omitted in the simplified setup. The cross-sectional areas of the stringers eliminated, however, were distributed evenly between the adjacent stringers that were left in the structure. Similarly, two rings were omitted from the complete portions of the cylinder and the cross section of one was added to the adjacent ring bordering the cutout and the other to the rigid end ring of the cylinder. At the same time the length of the field extending from the cutout to the end ring was assumed to be 9.6429 inches. This field length is $1\frac{1}{2}$ times the actual ring spacing and thus it is an intermediate value between the true distance from cutout to end ring and the actual ring spacing. It was not considered advisable to use a field length of three times the ring spacing in the calculations because long fields are weak in shear.

Cylinders 39 and 40 were built with only eight stringers. Consequently, changes in the structural arrangement were assumed only in connection with the rings. The changes were of the same nature as in the case of Cylinder 35.

As in previous work, the bending and torsional rigidities of the stringers were disregarded. The rings were considered resistant to bending in their own plane but weak in bending out of their plane as well as in torsion. The extensional and shearing rigidities of the rings were considered. The sheet panels were assumed to resist shear only, and the shear stresses were assumed to be distributed uniformly. The resistance of the sheet to extension and compression was taken approximately into account by adding an effective width of sheet to the stringers. In the present calculations the total width of the sheet was considered effective, since the stresses were calculated for small loads when the sheet is in a nonbuckled state. An effective width of sheet equal to the width of the ring was added to the ring when its cross-sectional properties were calculated. Because of these assumptions only the three displacement components as well as the rotation component about an axis parallel to the axis of the cylinder need be taken into consideration. Rotations about the tangential and radial axes are not resisted by either the stringer or the ring.

The vertical plane of a transverse section through the middle of the cylinder was regarded as a fixed reference plane relative to which the rigid end rings are tilted – and even twisted because of the asymmetric cutout – when the pure bending moments are applied to the end rings. The operations tables were set up for only one-quarter of the cylinder because the displacements in the four quarters are related by symmetry.

SETTING UP AND SOLVING THE OPERATIONS TABLES

A schematic arrangement showing the four quadrants of the operations tables for all three cylinders is given in table 1. As a rule, each entry in the operations tables (see, for instance, quadrant A, table 2) represents the magnitude and the sign of the generalized force, indicated at the left end of the row in which it appears, caused by the generalized unit displacement indicated at the top of the column. A generalized displacement is a displacement of the structure at a point in one of the directions of the axes, a rotation about one of these axes, or any combination of displacements and rotations of the structure at a group of points. A generalized force corresponding to a generalized displacement is the quantity – force, moment, or group of forces and moments – that gives the work done during the generalized displacement when multiplied by the generalized displacement. As was mentioned under STATEMENT OF PROBLEM

AND ASSUMPTIONS, the structure is considered to be rigidly clamped as regards every other generalized displacement when the effect of any one generalized unit displacement is sought.

The generalized forces in a reinforced monocoque cylinder caused by generalized unit displacements can be calculated when the solution of the so-called four-panel problem is known. The solution was given in reference 6. It is given in a slightly more convenient form in figures 3 to 6 of the present paper. These figures show the forces and moments at each of the nine corner points that are caused by generalized unit displacements of the middle point. The expressions are given in a form suitable for calculations even when each stringer and ring segment has a different but constant section and each panel a different but constant thickness. When a panel is in a buckled state, a reduced value should be used for its effective shear modulus G_{eff}. When a panel is absent, its shear modulus, or thickness, should be put equal to zero. The values of the shear flow-force coefficients α_t, α_r, and α_n as well as those of the influence coefficients $\widehat{tt}$, $\widehat{tr}$, $\widehat{tn}$, $\widehat{rr}$, . . ., must be obtained from reference 5.

Figures 7 to 10 give the solution of the four-panel problem for the case in which the curvature is opposite to that shown in the preceding four figures. The calculations with which this report deals indicated the desirability of two such sets of diagrams in order to reduce the likelihood of numerical errors and errors of sign in the operations tables.

Because of the symmetry of both the structure and the loading with respect to the plane of a transverse section through the middle of the cylinder, displacements of corresponding points must be the same on rings A and A', B and B', and C and C'. (See fig. 2.) Moreover, the loading is antisymmetric with respect to the horizontal plane containing the axis of the cylinder. Hence, displacements of corresponding points on stringers 1 and 1', 2 and 2', and so forth, must be antisymmetric. Their absolute magnitudes are equal and their signs can be determined from the following rules, which take care of the peculiarities of the sign conventions adopted: axial and radial displacements are of opposite sign, tangential displacements and rotations are of the same sign on the upper and lower halves of the cylinder. These symmetry considerations permit a reduction in the number of displacement quantities to be entered in the operations tables. Of the total of $4 \times 48 = 192$ possible generalized basic displacements in the case of Cylinder 39, a total of 108 could be omitted outright; 36 more displacements were considered only indirectly, as is shown by means of the following two typical examples.

When point B4 — the point of intersection of ring B with stringer 4 — is moved in the positive axial direction, point B4' must be moved the same distance in the negative axial direction because of the antisymmetry.

This combination of displacements causes twice as much shear strain in the panel bounded by rings A and B and stringers 4 and 4' as would be caused by the displacement of point B4 alone. Consequently, the forces and moments appearing because of the shear at points A4 and B4 will be doubled.

When point B4 is moved in the positive tangential direction, point B4' also must be moved the same distance in the tangential direction. Consequently, the shear strain in the panel bounded by rings A and B and stringers 4 and 4' is again subjected to the double amount of shear strain just as in the case discussed previously. Moreover, segment 4-4' of ring B is rotated but not shortened, whereas in the case of a tangential motion of point B4 alone a shortening also would take place. Consequently, 48 independent displacement quantities remain to be entered in the operations tables. This number is further reduced because of the end conditions. In the experiment the end rings were heavy and were rigidly attached to heavy end plates. For this reason, in the theory the end rings were assumed perfectly rigid and points on the end rings were permitted to participate only in rigid body displacements. Thus $4 \times 4 = 16$ further individual generalized displacements are eliminated; and three rigid-body displacements are introduced — namely, a rotation ω about the horizontal diameter, a rotation θ about the axis of the cylinder, and a vertical translation η of the end ring. Hence 35 unknown quantities remain.

When the pure bending moment is applied to the rigid end plate, the distribution of the forces to the stringers is not known. Obviously, it cannot be assumed according to the customary linear law because of the cutout in the structure. For this reason a rotation ω of the end ring was specified rather than a bending moment, and the corresponding bending moment was calculated only after the forces in the stringers were determined from the operations table. Hence, the forces and moments corresponding in the operations tables to the specified rigid body displacement ω were known quantities and had to be considered as the load terms in the equations. They are given in the last column of quadrants B and D of table 2.

It will be noted that the last two rows in the operations tables are denoted (1/2)Y (one-half the vertical shear force acting upon the end ring) and (1/2)(Q/r) (one-half the torque acting upon the end ring divided by the radius). This choice of the quantities to be entered in the last two rows results in a symmetric operations table.

The linear equations represented by the operations tables were then solved by a slightly modified version of Crout's method. In other words, the set of 34 displacement quantities causing forces and moments at all the points equal and opposite to those given in the last column of the operations tables (which are due to the specified rotation ω) was determined. These forces and moments listed in the last column are designated RHS to indicate right-hand-side members. It should be noted that two of the displacement quantities listed are the remaining two (unknown) rigid body displacements θ and η of the end ring. The generalized force corresponding to θ is a torque, that corresponding to η, a vertical shear force. Obviously, these two generalized displacements must be so chosen as to yield zero generalized forces when the external load applied to the cylinder is a pure bending moment. These two requirements are represented by the last two rows of the operations tables.

Similar considerations can be advanced in the case of the other two cylinders. The operations table of Cylinder 40 having the 135° cutout differs from table 2 only in quadrant D. This quadrant is given in table 3. In the case of Cylinder 35 all four quadrants are different. They are shown in table 4. In quadrant A of table 4 the columns of the tangential displacement and the rotation of point B1 correspond to two units each rather than to one. The doubling of these movements was undertaken in order to maintain the symmetry of the operations tables in spite of the assumptions regarding the simultaneous movements of points on the two sides of the horizontal plane of symmetry of the cylinder.

APPROXIMATE THEORY

Because of the great amount of work required for the solution of stress problems by the numerical method discussed, the possibility of using an approximate theory was investigated. The approximation amounted to neglecting all influences except that of the axial displacements. Physically the structure corresponding to the approximate theory would have rigid rings. Moreover, these rings would have to be supported in their own plane to provide reactions, since the shear forces and the torque acting upon the rings are not canceled in the approximate calculations.

The operations tables of the approximate theory are identical with those portions of the operations tables (tables 2 to 4) that involve only axial forces and displacements.

PRESENTATION AND DISCUSSION OF RESULTS

The displacements calculated for a rotation ω of the end ring amounting to 1×10^{-4} radian are presented in tables 5 to 7. The distortions of the rings corresponding to an applied bending moment of 20,000 inch-pounds are shown in figures 11 to 14.

The axial strains calculated from the displacements are plotted in figures 15 to 20, which also contain experimental results taken from reference 8 as well as calculated values corresponding to the approximate theory. The agreement between theory and experiment is satisfactory. The approximate theory is also in reasonable agreement with experiment in the complete portions of the cylinders. In the cutout portions the values calculated by the approximate theory are even slightly closer to the experimental points than the values obtained from the complete theory. The displacements calculated by the approximate theory are listed in table 8.

Figures 21 and 22 show the shear stresses in the sheet of the complete portions of the cylinders and the maximum bending stresses in the rings bordering the cutout. The absolute values of these stresses are very small. Moreover, they decrease in an oscillatory manner from the region of the neutral axis of the cylinder on the cutout side toward the neutral-axis location on the opposite side.

The bending moment required to cause a rotation of 1×10^{-4} radian of the rigid end ring with respect to the transverse plane of symmetry is 5075.45, 7845.90, and 4511.04 inch-pounds in the case of the cylinders having 45°, 90°, and 135° cutouts, respectively. It should be remembered that the construction of the cylinder with the 90° cutout was different from that of the other two.

CONCLUSIONS

During the course of the calculation of the stresses in three reinforced monocoque cylinders with side cutout, carried out by means of a numerical procedure developed in part IV in the present series of investigations, the following principal observations were made:

1. The problem can be stated mathematically by means of a set of simultaneous linear equations represented by the operations tables and the external loads. The operations tables can be set up without difficulty if use is made of the solutions of the four-panel problem contained in the present report, together with the coefficients presented in the tables and graphs given in NACA TN No. 999.

2. The equations can be solved by Crout's method at the rate of approximately 2 hours for each unknown quantity. This estimate does not allow for errors.

3. The calculated values of the normal strain in the stringers were in satisfactory agreement with the strains measured in the experiments of part IV of the present series of investigations.

4. The shear stress in the sheet and the bending stress in the rings were found to be very small.

5. An approximate method which considers only the axial displacements and thus does not satisfy all the equilibrium conditions gave results reasonably close to those obtained by the complete method.

Polytechnic Institute of Brooklyn
Brooklyn N. Y., July 3, 1946

REFERENCES

1. Southwell, R. V.: Relaxation Methods in Engineering Science, A Treatise on Approximate Computation. Clarendon Press (Oxford), 1940.

2. Hoff, N. J., Levy, Robert S., and Kempner, Joseph: Numerical Procedures for the Calculation of the Stresses in Monocoques. I – Diffusion of Tensile Stringer Loads in Reinforced Panels. NACA TN No. 934, 1944.

3. Hoff, N. J., and Kempner, Joseph: Numerical Procedures for the Calculation of the Stresses in Monocoques. II – Diffusion of Tensile Stringer Loads in Reinforced Flat Panel with Cutouts. NACA TN No. 950, 1944.

4. Hoff, N. J., Libby, Paul A., and Klein, Bertram: Numerical Procedures for the Calculation of the Stresses in Monocoques. III – Calculation of the Bending Moments in Fuselage Frames. NACA TN No. 998, 1946.

5. Hoff, N. J., Klein, Bertram, and Libby, Paul A.: Numerical Procedures for the Calculation of the Stresses in Monocoques. IV – Influence Coefficients of Curved Bars for Distortions in Their Own Plane. NACA TN No. 999, 1946.

6. Hoff, N. J., Boley, Bruno A., and Klein, Bertram: Stresses in and General Instability of Monocoque Cylinders with Cutouts. II – Calculation of the Stresses in a Cylinder with Symmetric Cutout. NACA TN No. 1014, 1946.

7. Hoff, N. J., and Boley, Bruno A.: Stresses in and General Instability of Monocoque Cylinders with Cutouts. I – Experimental Investigation of Cylinders with a Symmetric Cutout Subjected to Pure Bending. NACA TN No. 1013, 1946.

8. Hoff, N. J., Boley, Bruno A., and Viggiano, Louis R.: Stresses in and General Instability of Monocoque Cylinders with Cutouts. IV – Pure Bending Tests of Cylinders with Side Cutout. NACA TN No. 1264, 1948.

9. Crout, Prescott D.: A Short Method for Evaluating Determinants and Solving Systems of Linear Equations with Real or Complex Coefficients. Trans. A.I.E.E., vol. 60, 1941, pp. 1235–1240.

TABLE 1 - SCHEMATIC ARRANGEMENT SHOWING THE FOUR QUADRANTS OF THE OPERATIONS TABLES FOR ALL THREE CYLINDERS

A	B
C	D

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TABLE 2.- OPERATIONS TABLE FOR CYLINDER 39 WITH 45° CUTOUT.

	ξ_{C4}	u_{C4}	v_{C4}	w_{C4}	ξ_{B4}	u_{B4}	v_{B4}	w_{B4}	ξ_{C3}	u_{C3}	v_{C3}	w_{C3}	ξ_{B3}	u_{B3}	v_{B3}	w_{B3}
X_{C4}	−1179.6013728	11.577150	5.53227948	1.2284986	354.893966	−11.577150	−5.53227948	−1.2284986	9.5766229	−11.57715	−1.84409316	−1.2284986	9.5766229	11.57715	1.84409316	1.2284986
T_{C4}	11.577150	−43.7125289	−1.5108042	−5.30127106	11.577150	41.986749	2.2293185	4.455384	11.57715	12.2824313	1.5108042	−0.5133567	11.57715	13.995583	2.2293185	1.485128
R_{C4}	5.53227948	−1.5108042	−1.41480662	0.0769458	5.53227948	2.2293185	1.06530714	0.2365621	−1.84409316	1.5108042	0.07897288	−0.0769458	−1.84409316	−2.2293185	−0.35510238	−0.2365621
N_{C4}	1.2284986	−5.30127106	0.0769458	−2.0392002	1.2284986	4.455384	0.2365621	0.4727787	1.2284986	−0.5133567	0.0769458	0.1499924	1.2284986	1.485128	0.2365621	0.1575929
X_{B4}	354.893966	11.577150	5.53227948	1.2284936	−711.1977279	0	0	0	9.5766229	−11.57715	−1.84409316	−1.2284986	23.9415572	0	0	0
T_{B4}	−11.577150	41.986749	2.2293185	4.455384	0	−73.4294752	−2.27850223	−9.1174402	−11.57715	13.995583	2.2293185	1.485128	0	−19.8996684	−2.27850223	−0.5316707
R_{B4}	−5.53227948	2.2293185	1.06530714	0.2365621	0	−2.27850223	−2.47451086	0.2327457	1.84409316	−2.2293185	−0.35510238	−0.2365621	0	2.27850223	0.0395783	−0.2327457
N_{B4}	−1.2284986	4.4553840	0.2365621	0.4727787	0	−9.1174402	0.2327457	−3.92080749	−1.2284986	1.485128	0.2365621	0.1575929	0	−0.5316707	0.2327457	0.35251577
X_{C3}	9.5766229	11.57715	−1.84409316	1.2284986	9.5766229	11.57715	−1.84409316	−1.2284986	−1170.0247499	0	3.68818632	0	364.4705889	0	−3.68818632	0
T_{C3}	−11.57715	−12.2824313	1.5108042	−0.5133567	−11.577150	13.995583	2.2293185	1.485128	0	−31.4300976	0	−4.76791436	0	27.991166	0	2.970256
R_{C3}	−1.84409316	−1.5108042	0.07897288	0.0769458	−1.84409316	2.2293185	−0.35510238	0.2365621	3.68818632	0	−1.33583374	0	3.68818632	0	0.71020476	0
N_{C3}	−1.2284986	−0.5133567	0.0769458	0.1499924	−1.2284986	1.485128	−0.2365621	0.1575929	0	−4.76791436	0	−2.1001926	0	2.970256	0	0.3151858
X_{B3}	9.5766229	11.57715	−1.84409316	1.2284986	23.9415572	0	0	0	364.4705889	0	3.68818632	0	−687.2561707	0	0	0
T_{B3}	11.577150	13.995583	−2.2293185	1.485128	0	−19.8996684	2.27850223	−0.5316707	0	27.991166	0	2.970256	0	−53.5290068	0	−8.5857432
R_{B3}	1.84409316	2.2293185	−0.35510238	0.2365621	0	−2.27850223	0.0395783	0.2327457	−3.68818632	0	0.71020476	0	0	0	−2.43403256	0
N_{B3}	1.2284986	1.485128	−0.2365621	0.1575929	0	−0.5316707	−0.2327457	0.35251577	0	2.970256	0	0.3151858	0	−8.5857433	0	−4.27332326

QUADRANT A

— INDICATES NEGATIVE NUMBER

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	ξ_{C4}	u_{C4}	v_{C4}	w_{C4}	ξ_{B4}	u_{B4}	v_{B4}	w_{B4}	ξ_{C3}	u_{C3}	v_{C3}	w_{C3}	ξ_{B3}	u_{B3}	v_{B3}	w_{B3}
X_{C2}									9.5766229	11.57715	−1.84409316	1.2284986	9.5766229	−11.57715	1.84409316	−1.2284986
T_{C2}									−11.57715	−12.2824313	1.5108042	−0.5133567	−11.577150	13.995583	−2.2293185	1.485128
R_{C2}									−1.84409316	−1.5108042	0.07897288	0.0769458	−1.84409316	2.2293185	−0.35510238	0.2365621
N_{C2}									−1.2284986	−0.5133567	0.0769458	0.1499924	−1.2284986	1.485128	0.2365621	0.1575929
X_{B2}									9.5766229	11.57715	−1.84409316	1.2284986	23.9415572	0	0	0
T_{B2}									11.57715	13.995583	−2.2293185	1.485128	0	−19.8996684	2.27850223	−0.5316707
R_{B2}									1.84409316	2.2293185	0.35510238	0.2365621	0	−2.27850223	0.0395783	0.2327457
N_{B2}									1.2284986	1.485128	−0.2365621	0.1575929	0	−0.5316707	−0.2327457	0.35251577
X_{C1}																
T_{C1}																
R_{C1}																
N_{C1}																
X_{B1}																
T_{B1}																
R_{B1}																
N_{B1}																
$Y_A/2$																
$(Q_A/2)/T$																

QUADRANT C

— INDICATES NEGATIVE NUMBER

TABLE 2.- OPERATIONS TABLE FOR CYLINDER 39 WITH 45° CUTOUT - Concluded

	ξ_{C2}	u_{C2}	v_{C2}	w_{C2}	ξ_{B2}	u_{B2}	v_{B2}	w_{B2}	ξ_{C1}	u_{C1}	v_{C1}	w_{C1}	ξ_{B1}	u_{B1}	v_{B1}	w_{B1}	η_A	$\theta_A r$	−RHS
X_{C4}																			
T_{C4}																			
R_{C4}																			
N_{C4}																			
X_{B4}																	6.678900204	0	94.65079167
T_{B4}																	31.37288093	37.71758932	15.12627543
R_{B4}																	0.857401366	0	0.413391775
N_{B4}																	3.329103305	4.002365972	1.605110773
X_{C3}	9.5766229	11.577150	1.84409316	1.2284986	9.5766229	11.577150	1.84409316	1.2284986											
T_{C3}	11.577150	12.2824313	1.5108042	0.5133567	11.57715	13.995583	2.2293185	1.485128											
R_{C3}	1.84409316	1.5108042	0.07897288	0.0769458	1.84409316	2.2293185	0.35510238	0.2365621											
N_{C3}	1.2284986	0.5133567	0.0769458	0.1499924	1.2284986	1.485128	0.2365621	0.1575929											
X_{B3}	9.5766229	11.577150	1.84409316	1.2284986	23.9415572	0	0	0									16.124291456	0	228.507224866
T_{B3}	11.57715	13.995583	2.2293185	1.485128	0	19.8996684	2.27850223	0.5316707									12.995072772	37.71758932	6.26550843
R_{B3}	1.84409316	2.2293185	0.35510238	0.2365621	0	2.27850223	0.0395783	0.2327457									2.069950004	0	0.998016026
N_{B3}	1.2284986	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35251577									1.378959739	4.002365972	0.664858651

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	ξ_{C2}	u_{C2}	v_{C2}	w_{C2}	ξ_{B2}	u_{B2}	v_{B2}	w_{B2}	ξ_{C1}	u_{C1}	v_{C1}	w_{C1}	ξ_{B1}	u_{B1}	v_{B1}	w_{B1}	η_A	$\theta_A r$	−RHS
X_{C2}	1170.0247499	0	3.68818632	0	364.4705889	0	3.68818632	0	9.5766229	11.577150	1.84409316	1.2284986	9.5766229	11.57715	1.84409316	1.2284986			
T_{C2}	0	31.4300976	0	4.78791436	0	27.991166	0	2.970256	11.577150	12.2824313	1.5108042	0.5133567	11.57715	13.995583	2.2293185	1.485128			
R_{C2}	3.68818632	0	1.33583374	0	3.68818632	0	0.71020470	0	1.84409316	1.5108042	0.07897288	0.0769458	1.84409316	2.2293185	0.35510238	0.2365621			
N_{C2}	0	4.78791436	0	2.1891926	0	2.970256	0	0.3151858	1.2284986	0.5133567	0.0769458	0.1499924	1.2284986	1.485128	0.2365621	0.1575729			
X_{B2}	364.4705889	0	3.68818632	0	687.2561707	0	0	0	9.5766229	11.57715	1.84409316	1.2284986	23.9415572	0	0	0	16.124291456	0	228.507224868
T_{B2}	0	27.991166	0	2.970256	0	53.5298068	0	8.58574332	11.57715	13.995583	2.2293185	1.485128	0	19.8996684	2.27850223	0.5316707	12.995072772	37.71758932	6.26550843
R_{B2}	3.68818632	0	0.71020470	0	0	0	2.4349325	0	1.84409316	2.2293185	0.35510238	0.2365621	0	2.27850223	0.0395783	0.2327457	2.069950004	0	0.998016026
N_{B2}	0	2.970256	0	0.3151858	0	8.58574332	0	4.27332328	1.2284986	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35251577	1.378959739	4.002365972	0.664858651
X_{C1}	9.5766229	11.57715	1.84409316	1.2284986	9.5766229	11.577150	1.84409316	1.2284986	940.5670438	11.57715	1.84409316	1.2284986	900.7535174	11.57715	1.84409316	1.2284986			
T_{C1}	11.57715	12.2824313	1.5108042	0.5133567	11.577150	13.995583	2.2293185	1.485128	11.57715	15.7150488	2.93263735	2.39395718	11.577150	13.995583	2.2293185	1.485128			
R_{C1}	1.84409316	1.5108042	0.07897288	0.0769458	1.84409316	2.2293185	0.35510238	0.2365621	1.84409316	2.93263735	0.66791687	0.7020249	1.84409316	2.2293185	0.35510238	0.2365621			
N_{C1}	1.2284986	0.5133567	0.0769458	0.1499924	1.2284986	1.485128	0.2365621	0.1575929	1.2284986	2.39395718	0.7020249	1.0945963	1.2284986	1.485128	0.2365621	0.1575929			
X_{B1}	9.5766229	11.57715	1.84409316	1.2284986	23.9415572	0	0	0	900.7535174	11.57715	1.84409316	1.2284986	818.7507877	23.15430	3.68818632	2.4569972	6.678900204	0	94.65079167
T_{B1}	11.577150	13.995583	2.2293185	1.485128	0	19.8996684	2.27850223	0.5316707	11.577150	13.995583	2.2293185	1.485128	23.15430	45.4383092	1.4370286	6.1471581	31.37288093	37.71758932	15.12627543
R_{B1}	1.84409316	2.2293185	0.35510238	0.2365621	0	2.27850223	0.0395783	0.2327457	1.84409316	2.2293185	0.35510238	0.2365621	3.6881864	1.4370286	1.7643061	0.6270158	0.857401360	0	0.413391775
N_{B1}	1.2284986	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35251577	1.2284986	1.485128	0.2365621	0.1575929	2.4569972	6.1471581	0.6270158	3.6056216	3.329103305	4.002365972	1.605110773
$Y_A/2$																	72.3905188	0	34.90061462
$(Q_A/2)/r$																		152.4713037	0

QUADRANT D

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TABLE 2.- OPERATIONS TABLE FOR CYLINDER 39 WITH 45° CUTOUT.

	ξ_{C4}	u_{C4}	v_{C4}	w_{C4}	ξ_{B4}	u_{B4}	v_{B4}	w_{B4}	ξ_{C3}	u_{C3}	v_{C3}	w_{C3}	ξ_{B3}	u_{B3}	v_{B3}	w_{B3}
X_{C4}	$\overline{1779.6013728}$	11.577150	5.53227948	1.2284986	354.893966	$\overline{11.577150}$	$\overline{5.53227948}$	$\overline{1.2284986}$	9.5766229	$\overline{11.57715}$	$\overline{1.84409316}$	$\overline{1.2284986}$	9.5766229	11.57715	1.84409316	1.2284986
T_{C4}	11.577150	$\overline{43.7125289}$	$\overline{1.5108042}$	$\overline{5.30127106}$	11.577150	41.986749	2.2293185	4.455384	11.57715	$\overline{12.2824313}$	1.5108042	0.5133567	11.57715	13.995583	2.2293185	1.485128
R_{C4}	5.53227948	$\overline{1.5108042}$	$\overline{1.41480662}$	0.0769458	5.53227948	2.2293185	1.06530714	0.2365621	$\overline{1.84409316}$	1.5108042	0.07897288	0.0769458	$\overline{1.84409316}$	$\overline{2.2293185}$	$\overline{0.35510238}$	0.2365621
N_{C4}	1.2284986	$\overline{5.30127106}$	0.0769458	2.0392002	1.2284986	4.455384	0.2365621	0.4727787	1.2284986	$\overline{0.5133567}$	0.0769458	0.1499924	1.2284986	1.485128	0.2365621	0.1575929
X_{B4}	354.893966	11.577150	5.53227948	1.2284936	711.1977279	0	0	0	9.5766229	$\overline{11.57715}$	1.84409316	1.2284986	23.9415572	0	0	0
T_{B4}	$\overline{11.577150}$	41.986749	2.2293185	4.455384	0	$\overline{73.4294752}$	$\overline{2.27850223}$	9.11741402	$\overline{11.57715}$	13.995583	2.2293185	1.485128	0	$\overline{19.8896684}$	$\overline{2.27850223}$	$\overline{0.5316707}$
R_{B4}	$\overline{5.53227948}$	2.2293185	1.06530714	0.2365621	0	$\overline{2.27850223}$	$\overline{2.47451080}$	0.2327457	1.84409316	$\overline{2.2293185}$	$\overline{0.35510238}$	$\overline{0.2365621}$	0	2.27850223	0.0395783	$\overline{0.2327457}$
N_{B4}	$\overline{1.2284986}$	4.4553840	0.2365621	0.4727787	0	$\overline{9.11741402}$	0.2327457	$\overline{3.92080749}$	$\overline{1.2284986}$	1.485128	0.2365621	0.1575929	0	$\overline{0.5316707}$	0.2327457	0.35251577
X_{C3}	9.5766229	11.57715	$\overline{1.84409316}$	1.2284986	9.5766229	$\overline{11.57715}$	1.84409316	$\overline{1.2284986}$	$\overline{1170.0247499}$	0	3.68818632	0	364.4705889	0	$\overline{3.68818632}$	0
T_{C3}	$\overline{11.57715}$	$\overline{12.2824313}$	1.5108042	$\overline{0.5133567}$	$\overline{11.577150}$	13.995583	$\overline{2.2293185}$	1.485128	0	$\overline{31.4300976}$	0	$\overline{4.78791436}$	0	27.991166	0	2.970256
R_{C3}	$\overline{1.84409316}$	$\overline{1.5108042}$	$\overline{0.07897288}$	0.0769458	$\overline{1.84409316}$	2.2293185	$\overline{0.35510238}$	0.2365621	3.68818632	0	$\overline{1.33583374}$	0	3.68818632	0	0.71020476	0
N_{C3}	$\overline{1.2284986}$	$\overline{0.5133567}$	0.0769458	0.1499924	$\overline{1.2284986}$	1.485128	$\overline{0.2365621}$	0.1575929	0	$\overline{4.78791436}$	0	$\overline{2.1891926}$	0	2.970256	0	0.3151858
X_{B3}	9.5766229	11.57715	$\overline{1.84409316}$	1.2284986	23.9415572	0	0	0	364.4705889	0	3.68818632	0	$\overline{667.2561707}$	0	0	0
T_{B3}	11.577150	13.995583	2.2293185	1.485128	0	$\overline{19.8896684}$	2.27850223	$\overline{0.5316707}$	0	27.991166	0	2.970256	0	$\overline{53.5298068}$	0	$\overline{8.56574332}$
R_{B3}	1.84409316	2.2293185	$\overline{0.35510238}$	0.2365621	0	$\overline{2.27850223}$	0.0395783	0.2327457	$\overline{3.68818632}$	0	0.71020476	0	0	0	$\overline{2.45403256}$	0
N_{B3}	1.2284986	1.485128	0.2365621	0.1575929	0	$\overline{0.5316707}$	$\overline{0.2327457}$	0.35251577	0	2.970256	0	0.3151858	0	$\overline{8.5857433}$	0	$\overline{4.27332326}$

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X_{C2}									9.5766229	11.57715	$\overline{1.84409316}$	1.2284986	9.5766229	$\overline{11.57715}$	1.84409316	$\overline{1.2284986}$
T_{C2}									$\overline{11.57715}$	$\overline{12.2824313}$	1.5108042	$\overline{0.5133567}$	$\overline{11.577150}$	13.995583	$\overline{2.2293185}$	1.485128
R_{C2}									$\overline{1.84409316}$	$\overline{1.5108042}$	0.07897288	0.0769458	$\overline{1.84409316}$	2.2293185	$\overline{0.35510238}$	0.2365621
N_{C2}									$\overline{1.2284986}$	$\overline{0.5133567}$	0.0769458	0.1499924	$\overline{1.2284986}$	1.485128	$\overline{0.2365621}$	0.1575929
X_{B2}									9.5766229	11.57715	$\overline{1.84409316}$	1.2284986	23.9415572	0	0	0
T_{B2}									11.57715	13.995583	$\overline{2.2293185}$	1.485128	0	$\overline{19.8896684}$	2.27850223	$\overline{0.5316707}$
R_{B2}									1.84409316	2.2293185	$\overline{0.35510238}$	0.2365621	0	$\overline{2.27850223}$	0.0395783	0.2327457
N_{B2}									1.2284986	1.485128	$\overline{0.2365621}$	0.1575929	0	$\overline{0.5316707}$	$\overline{0.2327457}$	0.35251577
X_{C1}																
T_{C1}																
R_{C1}																
N_{C1}																
X_{B1}																
T_{B1}																
R_{B1}																
N_{B1}																
$Y_A/2$																
$(Q_A/2)/r$																

QUADRANT C

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TABLE 2.- OPERATIONS TABLE FOR CYLINDER 39 WITH 45° CUTOUT. - Concluded

	ξ_{C2}	u_{C2}	v_{C2}	w_{C2}	ξ_{B2}	u_{B2}	v_{B2}	w_{B2}	ξ_{C1}	u_{C1}	v_{C1}	w_{C1}	ξ_{B1}	u_{B1}	v_{B1}	w_{B1}	η_A	$\theta_A \Gamma$	−RHS
X_{C4}																			
T_{C4}																			
R_{C4}																			
N_{C4}																			
X_{B4}																	6.678900204	0	94.05070107
T_{B4}																	31.37286093	37.71756932	15.12627643
R_{B4}																	0.887401366	0	0.41339775
N_{B4}																	3.329103305	4.002356972	1.60511073
X_{C3}	9.5766229	11.577150	1.84409316	1.2284986	9.5766229	11.577150	1.84409316	1.2284986											
T_{C3}	11.577150	12.2824313	1.5108042	0.5133567	11.57715	13.995583	2.2293185	1.485128											
R_{C3}	1.84409316	1.5108042	0.07897288	0.0769458	1.84409316	2.2293185	0.35510238	0.2365621											
N_{C3}	1.2284986	0.5133567	0.0769458	0.1409024	1.2284986	1.485128	0.2365621	0.1575929											
X_{B3}	9.5766229	11.577150	1.84409316	1.2284986	23.9415572	0	0	0									16.124291456	0	228.50722466
T_{B3}	11.57715	13.995583	2.2293185	1.485128	0	19.6996884	2.27850223	0.5316707									12.995072772	37.71756932	0.28550843
R_{B3}	1.84409316	2.2293185	0.35510238	0.2365621	0	2.27850223	0.0395783	0.2327457									2.069950084	0	0.99601002
N_{B3}	1.2284986	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35251577									1.378959739	4.002356972	0.06485805

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	ξ_{C2}	u_{C2}	v_{C2}	w_{C2}	ξ_{B2}	u_{B2}	v_{B2}	w_{B2}	ξ_{C1}	u_{C1}	v_{C1}	w_{C1}	ξ_{B1}	u_{B1}	v_{B1}	w_{B1}	η_A	$\theta_A \Gamma$	−RHS
X_{C2}	170.0247489	0	3.66818632	0	364.4705889	0	3.66818632	0	9.5766229	11.577150	1.84409316	1.2284986	9.5766229	11.57715	1.84409316	1.2284986			
T_{C2}	0	31.4300976	0	4.78791436	0	27.991186	0	2.970256	11.577150	12.2824313	1.5108042	0.5133567	11.57715	13.995583	2.2293185	1.485128			
R_{C2}	3.66818632	0	1.33563374	0	3.66818632	0	0.71020476	0	1.84409316	1.5108042	0.07897288	0.0769458	1.84409316	2.2293185	0.35510238	0.2365621			
N_{C2}	0	4.78791436	0	2.1891926	0	2.970256	0	0.3151858	1.2284986	0.5133567	0.0769458	0.1409024	1.2284986	1.485128	0.2365621	0.1575929			
X_{B2}	364.4705889	0	3.66818632	0	687.2961707	0	0	0	9.5766229	11.57715	1.84409316	1.2284986	23.9415572	0	0	0	16.124291456	0	228.50722466
T_{B2}	0	27.991186	0	2.970256	0	53.5290086	0	4.58574332	11.57715	13.995583	2.2293185	1.485128	0	19.6996884	2.27850223	0.5316707	12.995072772	37.71756932	0.28550843
R_{B2}	3.66818632	0	0.71020476	0	0	0	2.4349325	0	1.84409316	2.2293185	0.35510238	0.2365621	0	2.27850223	0.0395783	0.2327457	2.069950084	0	0.99601002
N_{B2}	0	2.970256	0	0.3151858	0	6.58674932	0	4.27332326	1.2284986	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35251577	1.378959739	4.002356972	0.06485805
X_{C1}	9.5766229	11.57715	1.84409316	1.2284986	9.5766229	11.577150	1.84409316	1.2284986	940.5670438	11.57715	1.84409316	1.2284986	300.7536174	11.57716	1.84409316	1.2284986			
T_{C1}	11.57715	12.2824313	1.5108042	0.5133567	11.577150	13.995583	2.2293185	1.485128	11.57715	15.7150466	2.93263735	2.39395714	11.577150	13.995583	2.2293185	1.485128			
R_{C1}	1.84409316	1.5108042	0.07897288	0.0769456	1.84409316	2.2293185	0.35510238	0.2365621	1.84409316	2.03263735	0.66791687	0.7020249	1.84409316	2.2293185	0.35510238	0.2365621			
N_{C1}	1.2284986	0.5133567	0.0769458	0.1409924	1.2284986	1.485128	0.2365621	0.1575929	1.2284986	2.39395714	0.7020249	1.0945983	1.2284986	1.485128	0.2365621	0.1575929			
X_{B1}	9.5766229	11.57715	1.84409316	1.2284986	23.9415572	0	0	0	300.7536174	11.57715	1.84409316	1.2284986	518.7587877	23.15430	3.06818632	2.4569972	6.678900204	0	94.05070107
T_{B1}	11.577150	13.995583	2.2293185	1.485128	0	19.6996884	2.27850223	0.5316707	11.577150	13.995583	2.2293185	1.485128	23.15430	45.4383092	1.4370286	0.1471581	31.37286093	37.71756932	15.12627643
R_{B1}	1.84409316	2.2293185	0.35510238	0.2365621	0	2.27850223	0.0395783	0.2327457	1.84409316	2.2293185	0.35510238	0.2365621	3.6681864	1.4370286	1.7643061	0.6270158	0.887401366	0	0.41339775
N_{B1}	1.2284986	1.485128	0.2365621	0.1575929	0	0.5316707	0.2327457	0.35251577	1.2284986	1.485128	0.2365621	0.1575929	2.4569972	6.1471581	0.6270158	3.6056216	3.329103305	4.002356972	1.60511073
$Y_A/2$																	22.3905188	0	34.90081462
$(Q_A/2)/r$																		152.4713037	0

QUADRANT D

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TABLE 3.- OPERATIONS TABLE FOR CYLINDER 40 WITH 135° CUTOUT

	ξ_{C2}	u_{C2}	v_{C2}	w_{C2}	ξ_{B2}	u_{B2}	v_{B2}	w_{B2}	ξ_{B1}	u_{B1}	v_{B1}	w_{B1}	η_A	$\theta_A r$	−RHS
X_{C4}															
T_{C4}															
R_{C4}															
N_{C4}															
X_{B4}													6.678800284	0	94.65079167
T_{B4}													51.37288893	57.71758932	15.12627543
R_{B4}													0.857481366	0	0.413391775
N_{B4}													3.329103385	4.002365972	1.605110773
X_{C3}	9.5766229	11.577160	1.844093160	1.2284986	9.5766229	11.577150	1.844093160	1.2284986							
T_{C3}	11.577150	12.7824313	1.5108042	0.5133567	11.57715	13.905583	2.2293185	1.485128							
R_{C3}	1.84409316	1.5108042	0.078072880	0.0789458	1.84409316	2.2293186	0.355107380	0.2365621							
N_{C3}	1.2284986	0.5133567	0.0789458	0.1409024	1.2284986	1.485128	0.2365621	0.1575929							
X_{B3}	1.5766229	11.577150	1.84409316	1.2284986	23.9415572	0	0	0					16.12429456	0	228.5072248660
T_{B3}	11.57715	13.905583	2.2293185	1.485128	0	19.8098684	2.27850223	0.5316707					12.99507277	57.71758932	6.26550643
R_{B3}	1.84409316	2.2293185	0.355102380	0.2365621	0	2.27850223	0.0395783	0.7327457					2.06895600	0	0.99601602
N_{B3}	1.2284986	1.485128	0.2365621	0.1575929	0	0.5316707	0.7327457	0.3525577					1.378959730	4.002365972	0.06485865

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X_{C2}	940.5670438	11.57715	1.84409316	1.2284986	300.7535174	11.57715	1.84409316	1.2284986							
T_{C2}	11.57715	15.7150488	2.932637352	2.39395718	11.57715	13.905583	2.2293185	1.485128							
R_{C2}	1.84409316	2.93263735	0.66701867	0.7020249	1.84409316	2.2293185	0.355107238	0.2365621							
N_{C2}	1.2284986	2.39395718	0.7020249	1.0945963	1.2284986	1.485128	0.2365621	0.1575929							
X_{B2}	300.7535174	11.57715	1.84409316	1.2284986	604.3858534	11.57715	1.84409316	1.2284986	14.3640343	11.57715	1.84409316	1.2284986			
T_{B2}	11.57716	13.905583	2.2293185	1.485128	11.57715	39.5342238	2.2293185	7.1006532	11.57715	5.9040854	0.049183738	0.9534573			
R_{B2}	1.84409316	2.2293185	0.355102380	0.2365621	1.84409316	2.2293185	2.07068016	0.2365621	1.84409316	0.0491637	0.31552406	0.4693078			
N_{B2}	1.2284986	1.485128	0.2365621	0.1575929	1.2284986	7.1006532	0.2365621	4.1573030	1.2284986	0.9534573	0.4693078	0.51010867			
X_{C1}					14.3640343	11.577150	1.84409316	1.2284986	298.8440245	11.57715	5.532779460	1.2284986			
T_{C1}					11.577150	5.9040854	0.0491637	0.9534573	11.57715	51.4427262	0.04918373	4.66203002			
R_{C1}					1.84409316	0.04918373	0.31552406	0.4693078	5.53227048	0.04918373	1.40920372	0.4693078			
N_{C1}					1.2284986	0.9534573	0.4693078	0.51010867	1.2284986	4.66203002	0.4693078	3.44802879			
X_{B1}															
T_{B1}															
R_{B1}															
N_{B1}															
$Y_A/2$															
$(Q_A/2)/r$															

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TABLE 4.- OPERATIONS TABLE FOR CYLINDER 35 WITH 90° CUTOUT

	$2u_{B1}$	$2w_{B2}$	ξ_{B2}	u_{B2}	v_{B2}	w_{B2}	ξ_{C2}	u_{C2}	v_{C2}	w_{C2}	ξ_{B3}	u_{B3}	v_{B3}	w_{B3}	ξ_{C3}	u_{C3}	v_{C3}	w_{C3}
T_{B1}	51.0772816	11.23097464	23.1543	11.8081706	0.09836746	1.9088146												
N_{B1}	11.23097464	7.91627498	2.4569972	1.9088146	0.93861554	1.0202734												
X_{B2}	23.1543	2.4569972	840.748891	11.57715	1.84409316	1.2284988	252.6925472	11.6701416	0.91784628	0.2985833	47.4831144	0	0	0	19.1532456	11.6701416	0.91784628	0.2985833
T_{B2}	11.8081706	1.9088146	11.57715	189.14045787	14.78452633	14.01338488	11.6701416	7.11085847	0.5592470	0.18192727	0	72.64336253	15.94168964	10.65475158	11.6701416	7.11085847	0.5592470	0.18192727
R_{B2}	0.09836746	0.93861554	1.84409316	14.78452633	4.80220514	1.77156579	0.91784628	0.5592470	0.04398434	0.01430846	0	15.94168964	2.94740590	1.52788418	0.91784628	0.5592470	0.04398434	0.01430846
N_{B2}	1.9088146	1.0202734	1.2284988	14.01338488	1.77156579	5.132718547	0.2985833	0.18192727	0.01430846	0.0046546905	0	10.65475158	1.52788418	0.582867183	0.2985833	0.18192727	0.01430846	0.0046546905
X_{C2}			252.6925472	11.6701416	0.91784628	0.2985833	634.6906248	11.6701416	0.91784628	0.2985833	19.1532456	11.6701416	0.91784628	0.2985833	19.1532456	11.6701416	0.91784628	0.2985833
T_{C2}			11.6701416	7.11085847	0.5592470	0.18192727	11.6701416	49.37057847	6.931898	5.63314177	11.6701416	7.11085847	0.5592470	0.18192727	11.6701416	35.13657193	7.877637	5.39705463
R_{C2}			0.91784628	0.5592470	0.04398434	0.01430846	0.91784628	6.931898	1.87725134	1.43248446	0.91784628	0.5592470	0.04398434	0.01430846	0.91784628	7.877637	1.46637226	0.76155734
N_{C2}			0.2985833	0.18192727	0.01430846	0.0046546905	0.2985833	5.63314177	1.43248446	1.57760089	0.2985833	0.18192727	0.01430846	0.0046546905	0.2985833	5.39705463	0.76155734	0.29085781
X_{B3}			47.4831144	0	0	0	19.1532456	11.6701416	0.91784628	0.2985833	636.4825477	0	0	0	431.2528581	0.0929916	2.76193944	0.9299153
T_{B3}			0	72.64336253	15.94168964	10.65475158	11.6701416	7.11085847	0.5592470	0.18192727	0	123.13004067	17.55520783	15.40851288	0.0929916	21.10624147	1.6700715	1.66705527
R_{B3}			0	15.94168964	2.94740590	1.52788418	0.91784628	0.5592470	0.04398434	0.01430846	0	12.55620783	4.95730752	1.53500371	2.76193944	1.6700715	0.39906672	0.22225362
N_{B3}			0	10.65475158	1.52788418	0.582867183	0.2985833	0.18192727	0.01430846	0.0046546905	0	15.40851288	1.53500371	5.29031447	0.9299153	1.66705527	0.22225362	0.1622475905
X_{C3}			19.1532456	11.6701416	0.91784628	0.2985833	19.1532456	11.6701416	0.91784628	0.2985833	431.2528581	0.0929916	2.76193944	0.9299153	1406.6780491	0.0929916	2.76193944	0.9299153
T_{C3}			11.6701416	7.11085847	0.5592470	0.18192727	11.6701416	35.13657193	7.877637	5.39705463	0.0929916	21.10624147	1.6700715	1.66705527	0.0929916	65.08572727	5.99258658	5.02709806
R_{C3}			0.91784628	0.5592470	0.04398434	0.01430846	0.91784628	7.877637	1.46637226	0.76155734	2.76193944	1.6700715	0.39906672	0.22225362	2.76193944	5.99258658	2.9431602	0.73045058
N_{C3}			0.2985833	0.18192727	0.01430846	0.0046546905	0.2985833	5.39705463	0.76155734	0.29085781	0.9299153	1.66705527	0.22225362	0.1622475905	0.9299153	5.02709806	0.73045058	2.672196906

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X_{B4}											23.0415572	0	0	0	9.5760229	11.57715	1.84409316	1.2284986
T_{B4}											0	19.8996604	2.27650223	0.5316707	11.57715	13.985563	2.2293165	1.485128
R_{B4}											0	2.27650223	0.0595783	0.23274569	1.84409316	2.2293165	0.35510238	0.23656208
N_{B4}											0	0.5316707	0.23274569	0.33234577	1.2284986	1.485128	0.23656208	0.15759290
X_{C4}											9.5760229	11.57715	1.84409316	1.2284986	9.5760229	11.57715	1.84409316	1.2284986
T_{C4}											11.57715	13.985563	2.2293165	1.485128	11.57715	12.2824313	1.5108042	0.5133567
R_{C4}											1.84409316	2.2293165	0.35510238	0.23656208	1.84409316	1.5108042	0.07197288	0.07694582
N_{C4}											1.2284986	1.485128	0.23656208	0.15759290	1.2284986	0.5133567	0.07694582	0.1409924
X_{B5}																		
T_{B5}																		
R_{B5}																		
N_{B5}																		
X_{C5}																		
T_{C5}																		
R_{C5}																		
N_{C5}																		
$Y_A/2$																		
$(Q_A/2)/\pi$																		

QUADRANT C

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TABLE 4.- OPERATIONS TABLE FOR CYLINDER 35 WITH 90° CUTOUT - Concluded

	ξ_{B4}	u_{B4}	v_{B4}	w_{B4}	ξ_{C4}	u_{C4}	v_{C4}	w_{C4}	ξ_{B5}	u_{B5}	v_{B5}	w_{B5}	ξ_{C5}	u_{C5}	v_{C5}	w_{C5}	η_A	$\theta_A\Gamma$	-RHS
T_{B1}																	33.95778161	37.71758934	16.37236254
N_{B1}																	3.60338545	4.00236597	1.73735936
X_{B2}																	6.15043168	0	212.0082457
T_{B2}																	22.2257845	28.36302063	10.71604998
R_{B2}																	2.29169564	2.25630575	1.10500672
N_{B2}																	1.93894066	2.24437357	0.9334044
X_{C2}																			
T_{C2}																			
R_{C2}																			
N_{C2}																			
X_{B3}	23.9415572	0	0	0	9.5766229	11.57715	1.84409316	1.2284986									12.91897017	0	277.0845633
T_{B3}	0	19.8996604	2.27850223	0.5316707	11.57715	13.995583	2.2293185	1.485128									5.24690371	28.36302063	2.52976871
R_{B3}	0	2.27850223	0.0395783	0.23274569	1.84409316	2.2293185	0.35510238	0.23656208									0.41266436	2.25630575	0.19806406
N_{B3}	0	0.5316707	0.23274569	0.35251577	1.2284986	1.485128	0.23656208	0.15759290									0.13424930	2.24437357	0.06472472
X_{C3}	9.5766229	11.57715	1.84409316	1.2284986	9.5766229	11.57715	1.84409316	1.2284986											
T_{C3}	11.57715	13.995583	2.2293185	1.485128	11.57715	12.2824313	1.5106042	0.5133567											
R_{C3}	1.84409316	2.2293185	0.35510238	0.23656208	1.84409316	1.5106042	0.07897286	0.07694562											
N_{C3}	1.2284986	1.485128	0.23656208	0.15759290	1.2284986	0.5133567	0.07694580	0.1499924											

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	ξ_{B4}	u_{B4}	v_{B4}	w_{B4}	ξ_{C4}	u_{C4}	v_{C4}	w_{C4}	ξ_{B5}	u_{B5}	v_{B5}	w_{B5}	ξ_{C5}	u_{C5}	v_{C5}	w_{C5}	η_A	$\theta_A\Gamma$	-RHS
X_{B4}	1070.0009674	0	0	0	594.1570675	0	3.6881632	0	23.9415572	0	0	0	9.5766229	11.57715	1.84409316	1.2284986	16.12429145	0	360.97564828
T_{B4}	0	53.5298066	0	6.58574332	0	27.901166	0	2.970256	0	19.8996604	2.27850223	0.5316707	11.57715	13.995583	2.2293185	1.485128	12.99507275	37.71758932	0.2655064
R_{B4}	0	0	2.43493256	0	3.6881632	0	0.71020476	0	0	2.27850223	0.0395783	0.23274569	1.84409316	2.2293185	0.35510238	0.23656208	2.06950004	0	0.99696020
N_{B4}	0	6.58574332	0	4.27332326	0	2.970256	0	0.3151656	0	0.5316707	0.23274569	0.35251577	1.2284986	1.485128	0.23656208	0.15759290	1.37959873	4.00236597	0.65485865
X_{C4}	594.1570675	0	3.6881632	0	1659.0841657	0	3.6881632	0	9.5766229	11.57715	1.84409316	1.2284986	9.5766229	11.57715	1.84409316	1.2284986			
T_{C4}	0	27.901166	0	2.970256	0	31.4300076	0	4.7679436	11.57715	13.995583	2.2293185	1.485128	11.57715	12.2824313	1.5106042	0.5133567			
R_{C4}	3.6881632	0	0.71020476	0	3.6881632	0	1.3356574	0	1.84409316	2.2293185	0.35510238	0.23656208	1.84409316	1.5106042	0.07897286	0.07694562			
N_{C4}	0	2.970256	0	0.3151656	0	4.7679436	0	2.1691925	1.2284986	1.485128	0.23656208	0.1575929	1.2284986	0.5133567	0.07694580	0.1499924			
X_{B5}	23.9415572	0	0	0	9.5766229	11.57715	1.84409316	1.2284986	1094.0085246	0	0	0	584.5804446	11.57715	5.53227946	1.2284986	6.67690020	0	153.2480332
T_{B5}	0	19.8996604	2.27850223	0.5316707	11.57715	13.995583	2.2293185	1.485128	0	73.4294752	2.27850223	9.3691625	11.57715	41.906749	2.2293185	4.455384	31.37286093	37.71758032	15.1262754
R_{B5}	0	2.27850223	0.0395783	0.23274568	1.84409316	2.2293185	0.35510238	0.23656208	0	2.27850223	2.47450086	0.23274569	5.53227946	2.2293185	1.06530714	0.23656208	0.85740306	0	0.41339773
N_{B5}	0	0.5316707	0.23274569	0.35251577	1.2284986	1.485128	0.23656208	0.1575929	0	9.3691625	0.23274569	3.9206074	1.2284986	4.455384	0.23656208	0.4727767	3.32910336	4.00236597	1.60511072
X_{C5}	9.5766229	11.57715	1.84409316	1.2284986	9.5766229	11.57715	1.84409316	1.2284986	584.5804446	11.57715	5.53227946	1.2284986	1658.0606066	11.57715	5.53227946	1.2284986			
T_{C5}	11.57715	13.995583	2.2293185	1.485128	11.57715	12.2824313	1.5106042	0.5133567	11.57715	41.906749	2.2293185	4.455384	11.57715	43.7125240	1.5106042	5.30127106			
R_{C5}	1.84409316	2.2293185	0.35510238	0.23656208	1.84409316	1.5106042	0.07897286	0.07694562	5.53227946	2.2293185	1.00830714	0.23656208	5.53227946	1.5106042	1.41460002	0.07694562			
N_{C5}	1.2284986	1.485128	0.23656208	0.15759290	1.2284986	0.5133567	0.07694580	0.1499924	1.2284986	4.455384	0.23656208	0.4727767	1.2284986	5.30127106	0.07694582	2.0392002			
$Y_A/2$	16.12429145	12.99507275	2.06950004	1.37959873					6.67690020	31.37286093	0.85740306	3.32910336					145.6007706	0	70.2905464
$-(Q_A/2)/r$	0	37.71758032	0	4.00236597					0	37.71758032	0	4.00236597					0	584.94259760	0

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TABLE 5.— DEFLECTIONS AND ROTATIONS FOR CYLINDER WITH 45° CUTOUT

[M = 5075.45 in.-lb; η = 0.934679; θr = −0.182954; unit for values is 1×10^{-3} in. or radian]

		Ring A	Ring B	Ring C
Stringer 1'	ξ	−0.382683	−0.211436	45° cutout
	u	.680577	.000999	
	v	.357683	.000110	
	w	−.018295	−.013137	
Stringer 1	ξ	.382683	.211436	0.070479
	u	.680577	.000999	−.145643
	v	−.357683	−.000110	−.050187
	w	−.018295	−.013137	−.012965
Stringer 2	ξ	.923880	.462027	.154009
	u	.174729	−.086238	−.176519
	v	−.863531	−.232744	−.024713
	w	−.018295	−.028647	−.013301
Stringer 3	ξ	.923880	.461874	.153958
	u	−.540637	−.269938	−.183646
	v	−.863531	−.185258	.002972
	w	−.018295	−.010892	−.016637
Stringer 4	ξ	.382683	.191394	.063798
	u	−1.046485	−.376640	−.179948
	v	−.357683	−.086306	.003054
	w	−.018295	−.020508	−.018817
Stringer 4'	ξ	−.382683	−.191394	−.063798
	u	−1.046485	−.376640	−.179948
	v	.357683	.086306	−.003054
	w	−.018295	−.020508	−.018817

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TABLE 6.– DEFLECTIONS AND ROTATIONS FOR

CYLINDER WITH 90° CUTOUT

[M = 7845.90 in.-lb; η = 0.877023; θr = 0.128525; unit for values is 1 × 10−3 in. or radian]

		Ring A	Ring B	Ring C
Stringer 2'	ξ u v w	−0.707107 .748674 .620149 .012853	−0.443778 .316729 −.139502 −.081652	90° cutout
Stringer 1	ξ u v w	0 1.005548 0 .012853	0 .133452 0 .123108	
Stringer 2	ξ u v w	.707107 .748674 −.620149 .012853	.443778 .316729 .139502 −.081652	0.147921 .396281 −.614729 .095219
Stringer 3	ξ u v w	.923880 .464147 −.810264 .012853	.462609 .275678 −.327575 −.055475	.154210 .203256 −.356365 .093722
Stringer 4	ξ u v w	.923880 −.207097 −.810264 .012853	.461645 .034594 −.088412 .057352	.153875 .128796 .092635 .032331
Stringer 5	ξ u v w	.382683 −.681739 −.335622 .012853	.191577 −.003785 −.088206 .000093	.063867 .189788 .020932 .006447
Stringer 5'	ξ u v w	−.382683 −.681439 .335622 .012853	−.191577 −.003785 .088206 .000093	−.063867 .189788 −.020932 .006447

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TABLE 7.– DEFLECTIONS AND ROTATIONS FOR

CYLINDER WITH 135° CUTOUT

[M = 4511.04 in.-lb; η = 0.780683;
θr = -0.02370; unit for values
is 1 × 10^{-3} in. or radian]

		Ring A	Ring B	Ring C
Stringer 2'	ξ	-0.923880	-0.511401	135° cutout
	u	.275052	.142873	
	v	.721257	.276732	
	w	-.002370	-.093973	
Stringer 1'	ξ	-.382683	-.381188	
	u	.697557	-.095065	
	v	.298752	-.652765	
	w	-.002370	.045670	
Stringer 1	ξ	.382683	.381188	
	u	.697557	-.095065	
	v	-.298752	.652765	
	w	-.002370	.045670	
Stringer 2	ξ	.923880	.511401	0.170466
	u	.275052	.142873	.057603
	v	-.721257	-.276732	-.099144
	w	-.002370	-.093973	.025017
Stringer 3	ξ	.923880	.461519	.153841
	u	-.322452	-.075011	.043337
	v	-.721257	-.035912	.061874
	w	-.002370	.054762	.022328
Stringer 4	ξ	.382683	.191637	.063876
	u	-.744957	-.067597	.120387
	v	-.298752	-.049547	.088380
	w	-.002370	-.015939	-.003614
Stringer 4'	ξ	-.382683	-.191637	-.063876
	u	-.744957	-.067597	.120387
	v	.298752	.049547	-.088380
	w	-.002370	-.015939	-.003614
Stringer 3'				

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TABLE 8.– AXIAL DEFLECTIONS FOR APPROXIMATE SOLUTIONS

[Unit for values is 1 × 10^{-3} in.]

Angle of cutout	Stringer	Ring A	Ring B	Ring C	Average moment (in.–lb)
45°	1	0.382683	0.206436	0.071479	
	2	.923880	.432027	.0142009	
	3	.923880	.431874	.140958	
	4	.382683	.179394	.058798	
M(in.–lb)		5392	4788	4696	4959
90°	1	0	0	0	
	2	.707107	.426278	.143121	
	3	.923880	.450609	.149910	
	4	.923880	.443144	.146275	
	5	.382683	.183577	.060567	
M(in.–lb)		8155	7586	7533	7758
135°	1	.382683	.341188	----	
	2	.923880	.484401	.160466	
	3	.923880	.433019	.142341	
	4	.382683	.179637	.059176	
M(in.–lb)		4875	4272	4206	4451

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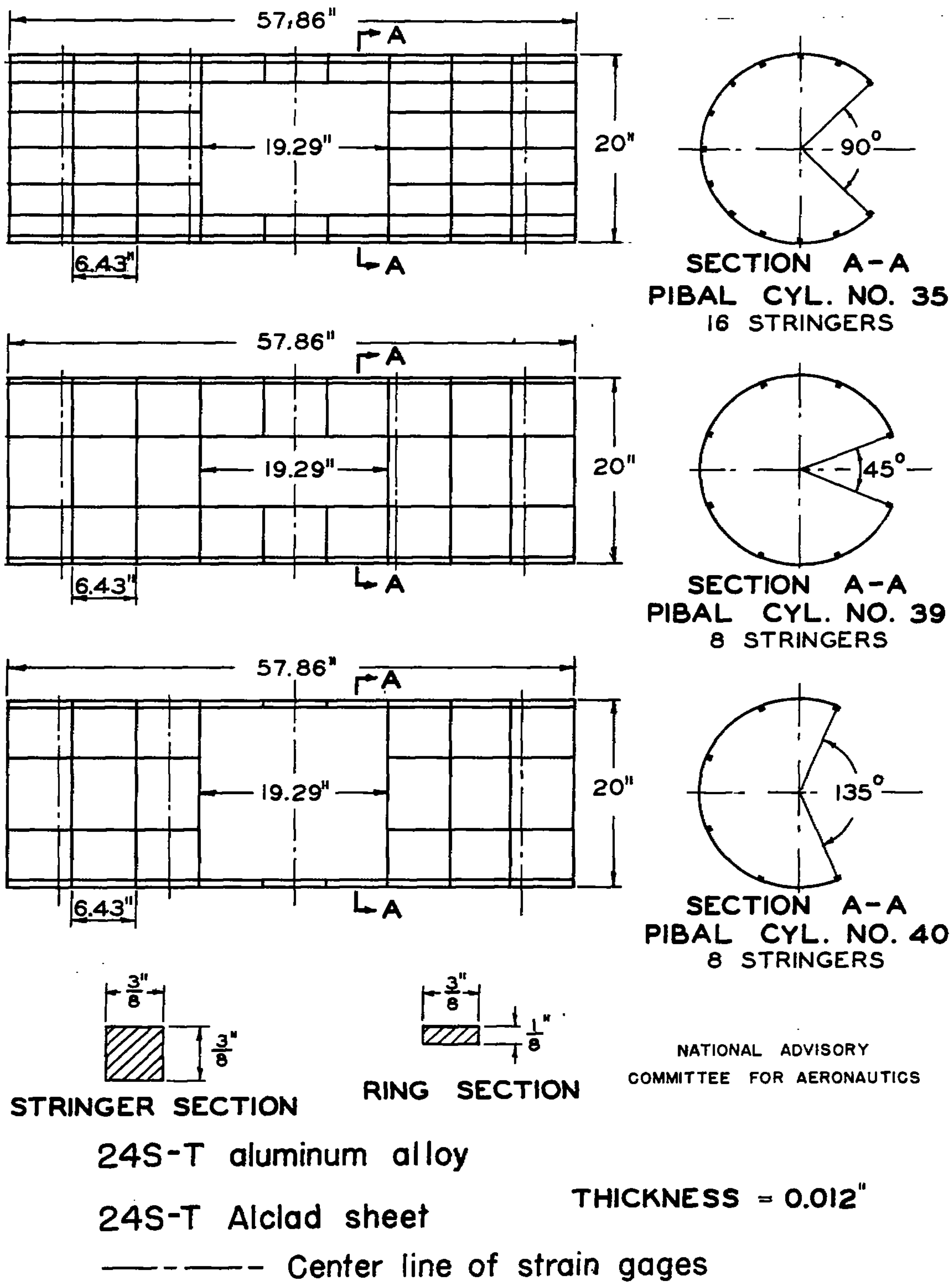

Figure 1.- Actual monocoque cylinders. (Described in reference 8.)

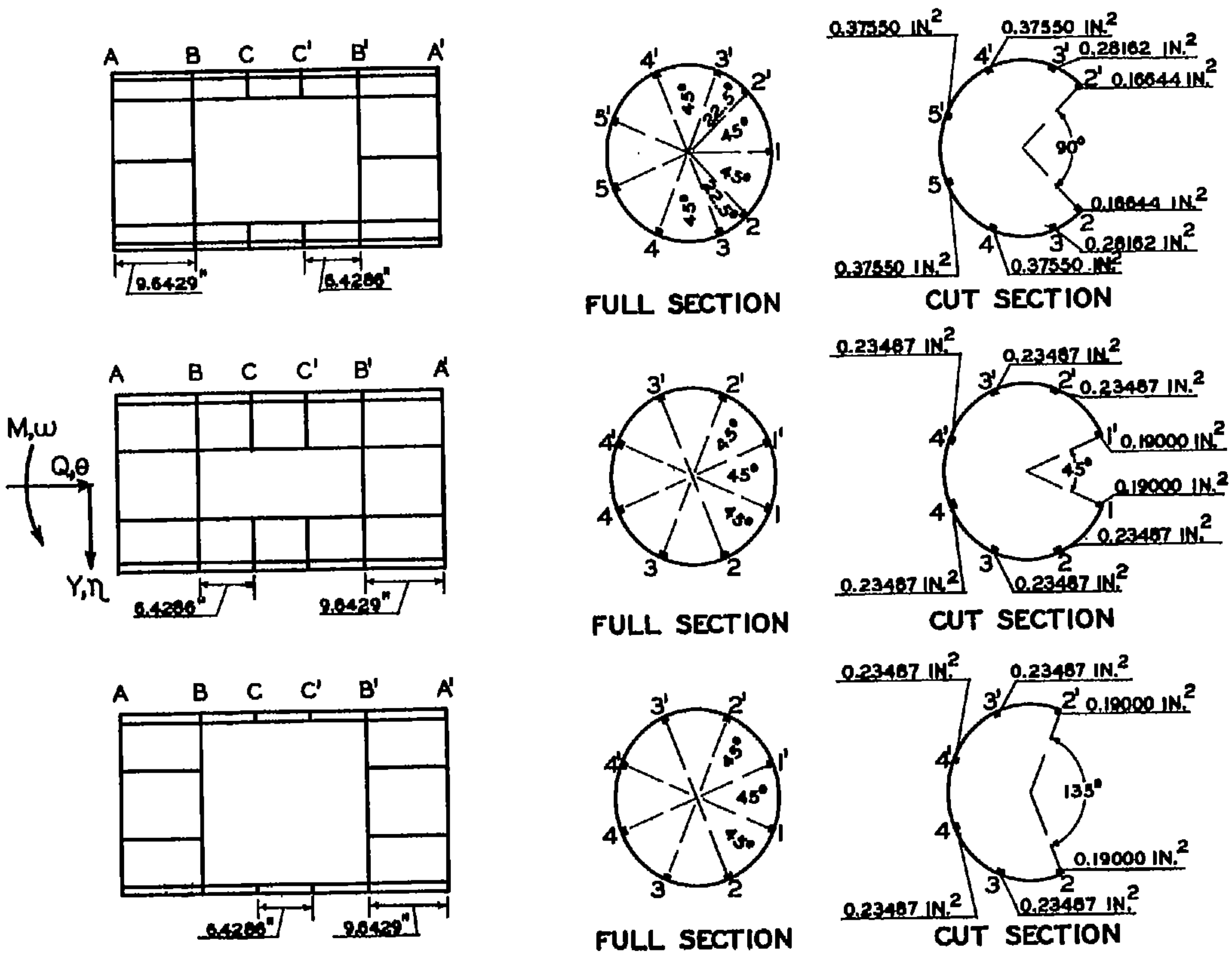

Figure 2.- Modified monocoque cylinders.

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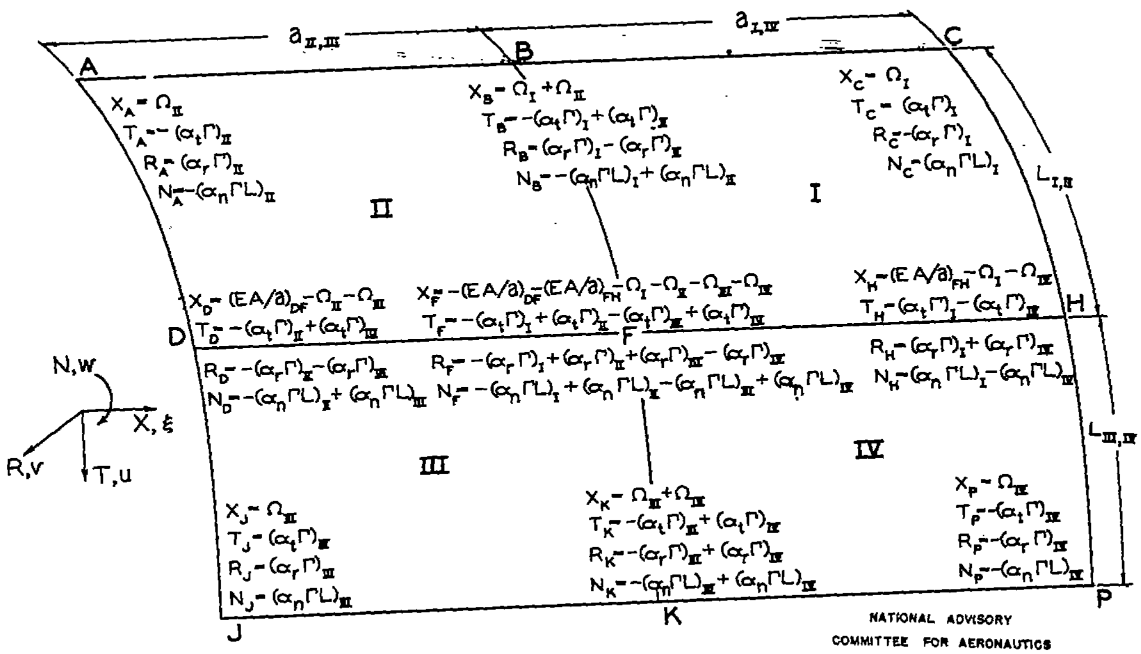

Figure 3.- Effect of unit axial displacement of F. Forces and moments acting on constraints.

$\Gamma = \frac{Gt}{2}$; $\Omega = \frac{Gta}{4L}$.

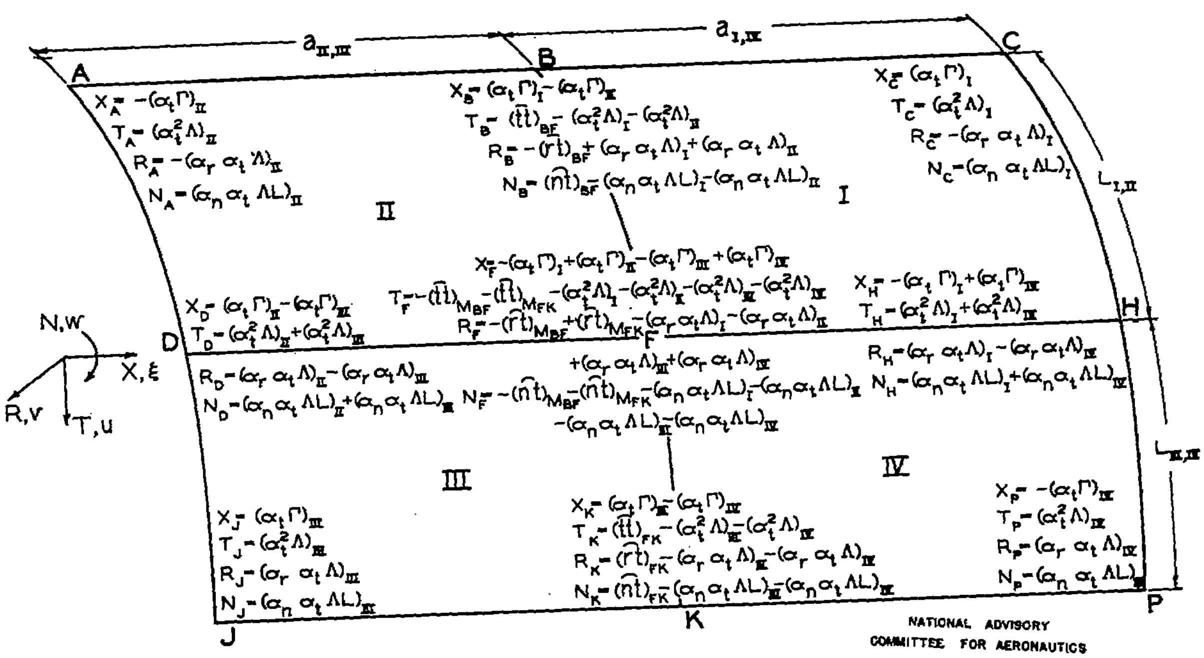

Figure 4.- Effect of unit tangential displacement of F. Forces and moments acting on constraints.

$\Gamma = \frac{Gt}{2}$; $\Lambda = \frac{GtL}{a}$.

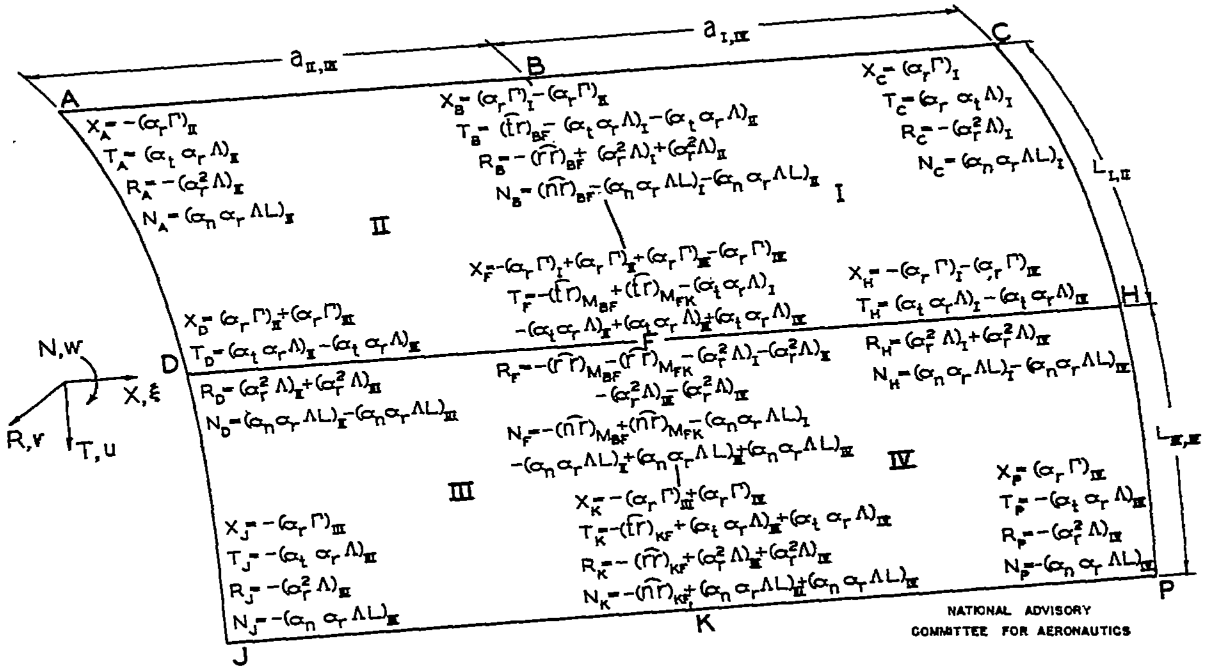

Figure 5.- Effect of unit radial displacement of F. Forces and moments acting on constraints. $\Gamma = \frac{Gt}{2}$; $\Lambda = \frac{GtL}{a}$

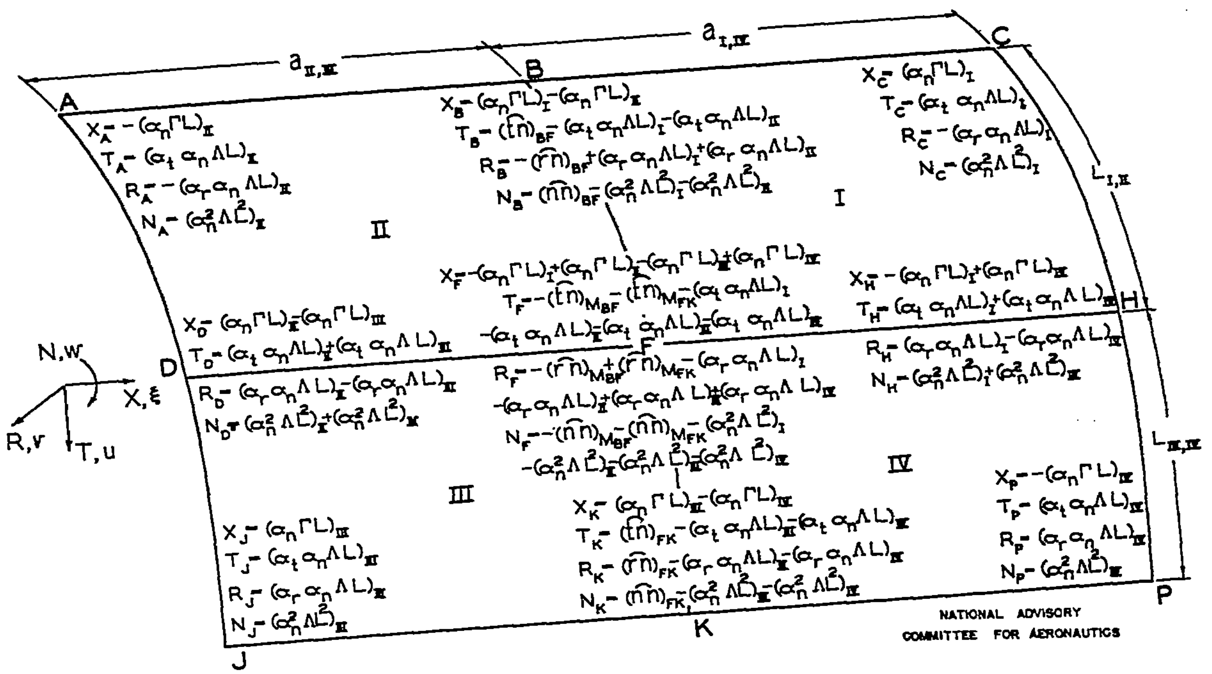

Figure 6.- Effect of unit rotation of F. Forces and moments acting on constraints. $\Gamma = \frac{Gt}{2}$; $\Lambda = \frac{GtL}{a}$.

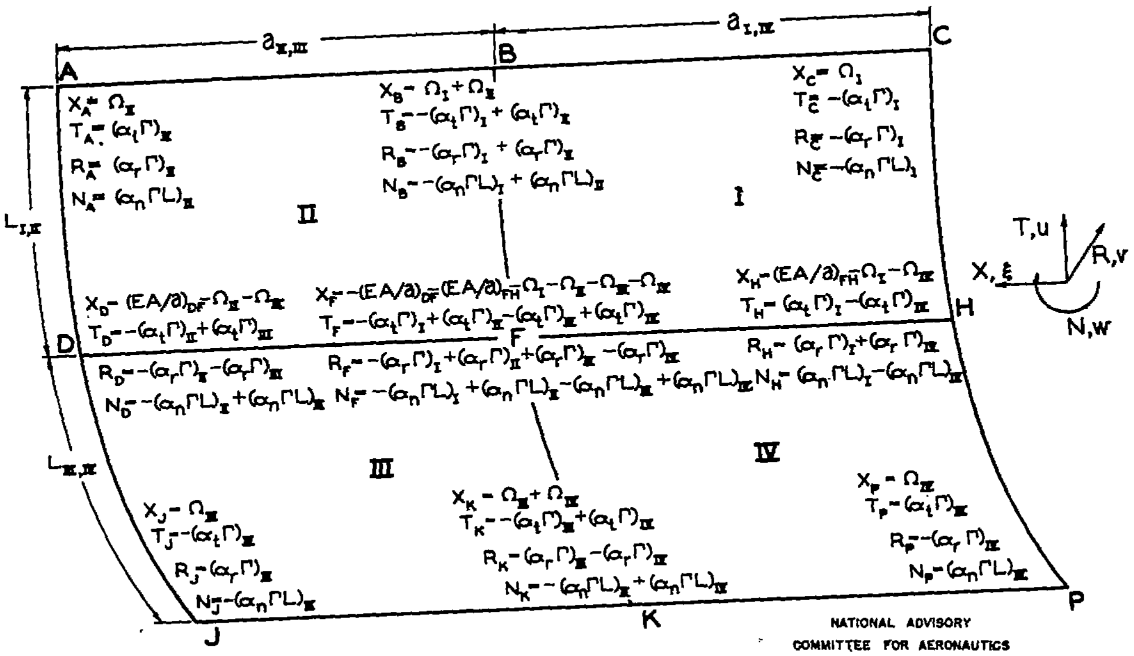

Figure 7.- Effect of unit axial displacement of F. Forces and moments acting on constraints. $\Gamma = \frac{Gt}{2}$; $\Omega = \frac{Gta}{4L}$. (Curvature opposite that in figs. 3 to 6.)

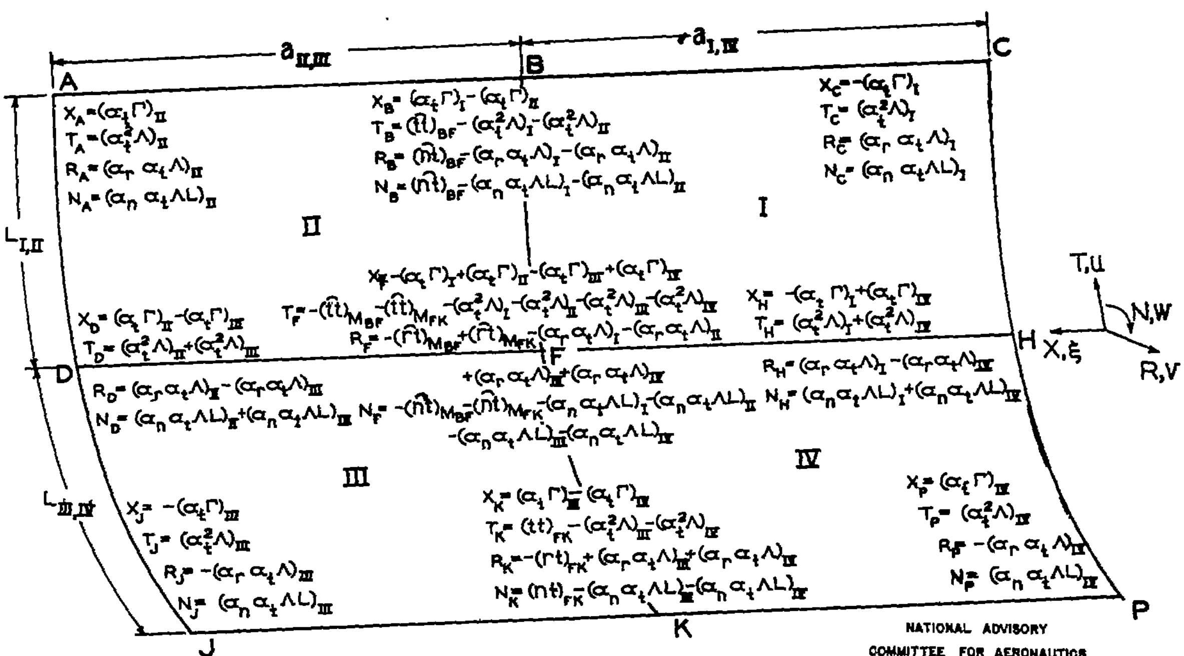

Figure 8.- Effect of unit tangential displacement of F. Forces and moments acting on constraints. $\Gamma = \frac{Gt}{2}$; $\Lambda = \frac{GtL}{a}$. (Curvature opposite that in figs. 3 to 6.)

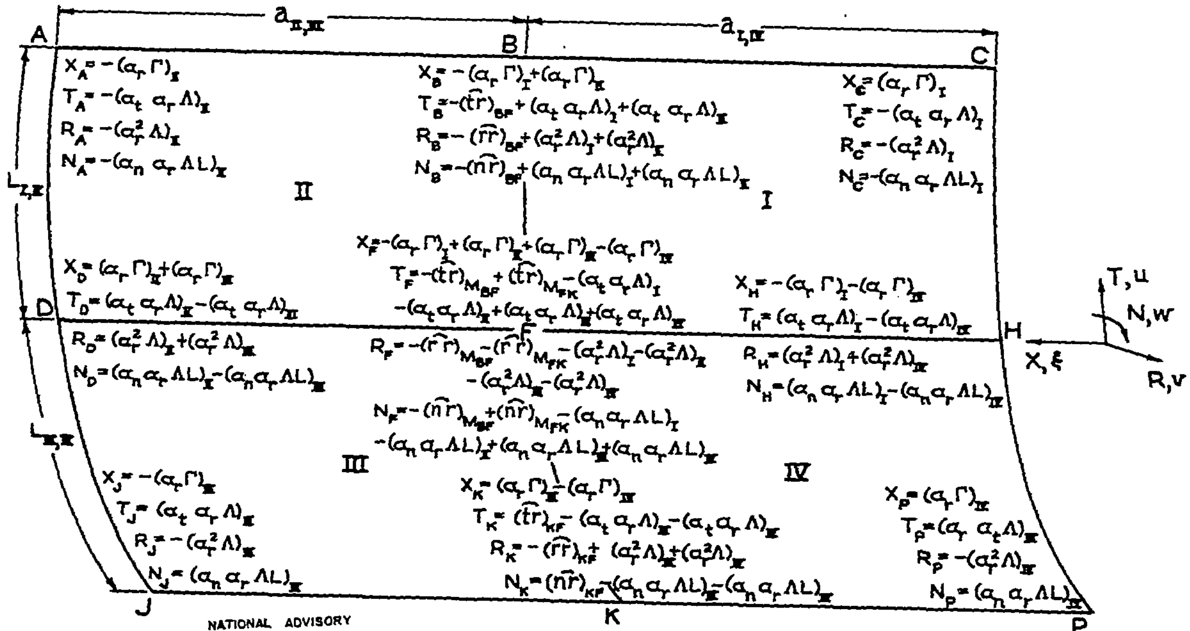

Figure 9.- Effect of unit radial displacement of F. Forces and moments acting on constraints. $\Gamma = \frac{Gt}{2}$; $\Lambda = \frac{GtL}{a}$. (Curvature opposite that in figs. 3 to 6.)

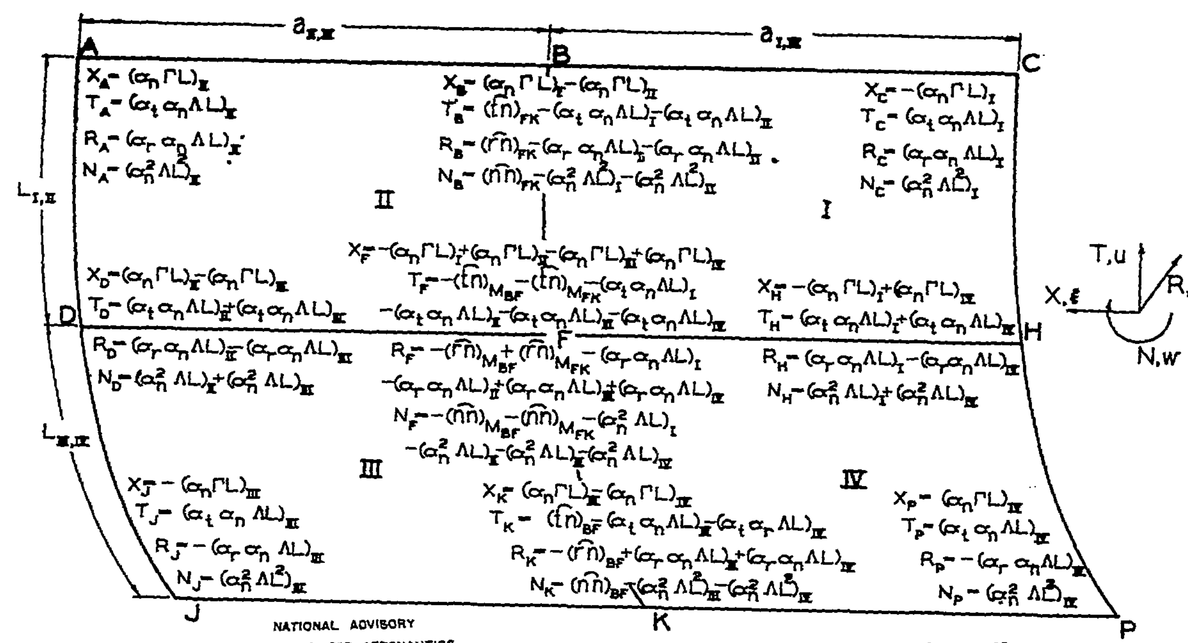

Figure 10.- Effect of unit rotation of F. Forces and moments acting on constraints. $\Gamma = \frac{Gt}{2}$; $\Lambda = \frac{GtL}{a}$. (Curvature opposite that in figs. 3 to 6.)

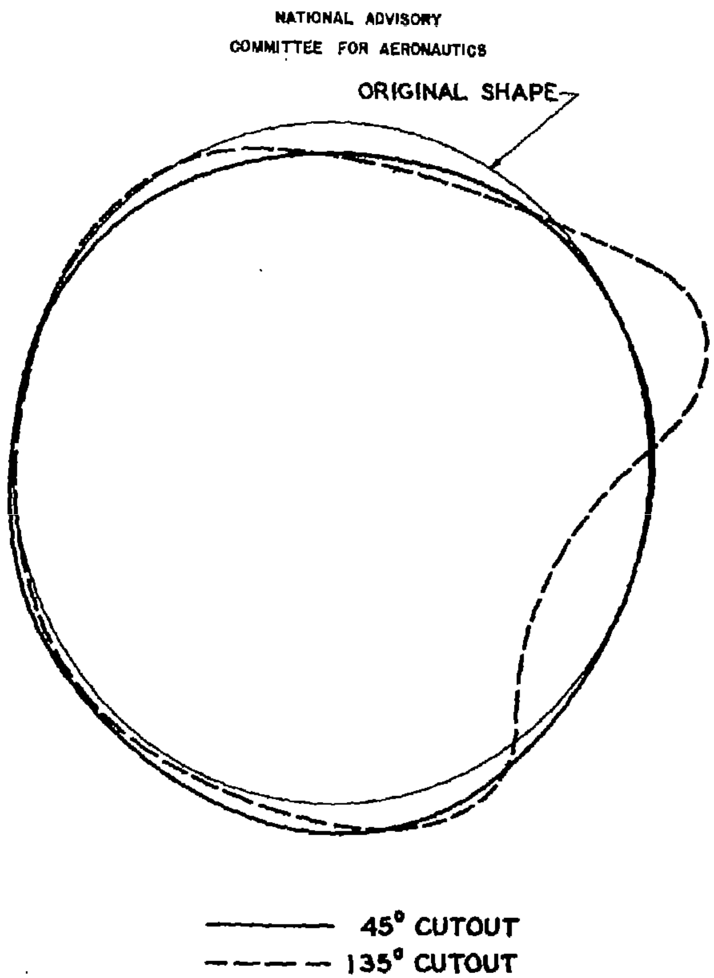

Figure 11.- Deflected shape of full rings in their own planes.

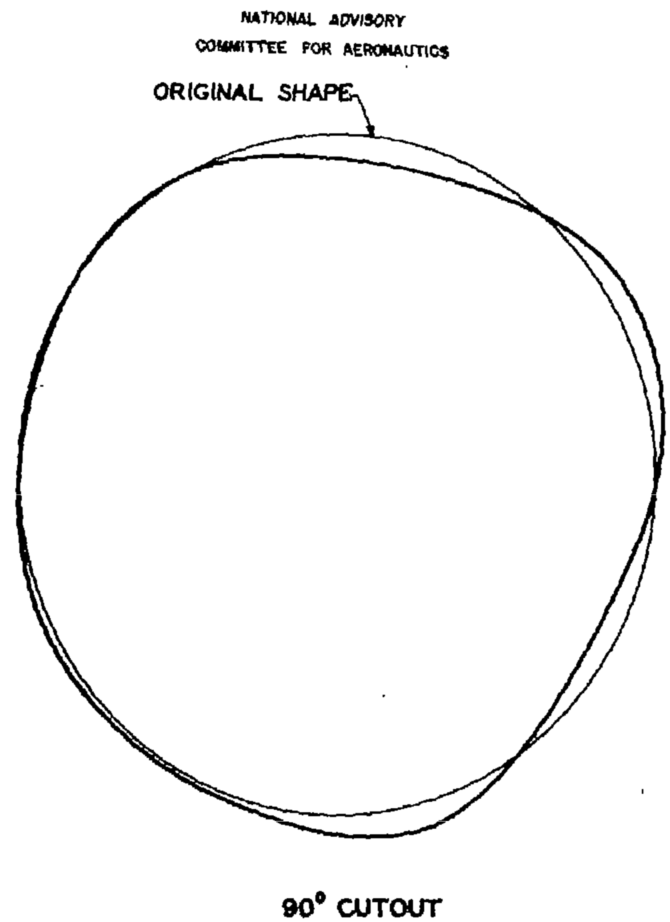

Figure 12.- Deflected shape of full ring in its own plane.

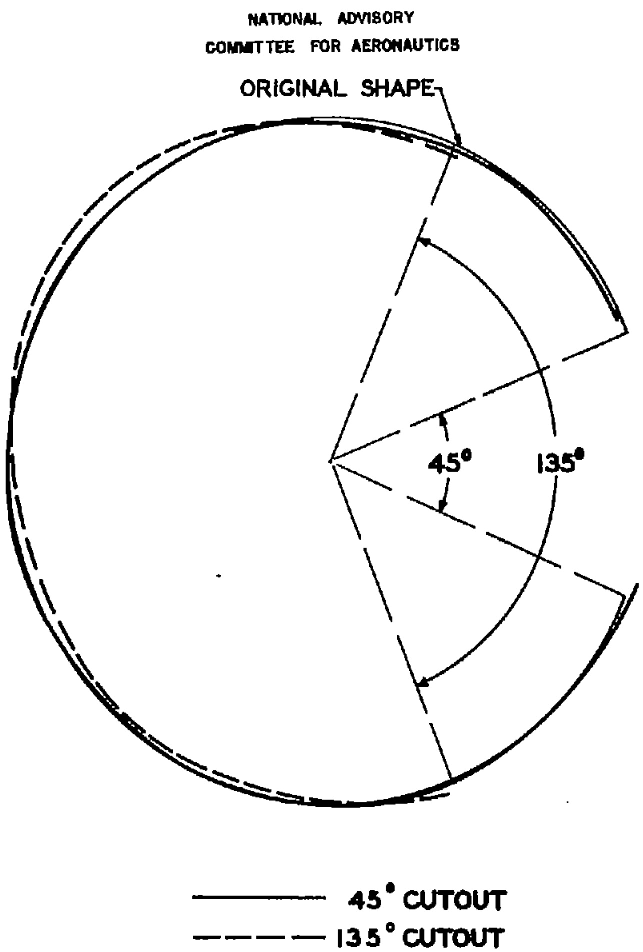

Figure 13.- Deflected shape of cut rings in their own planes.

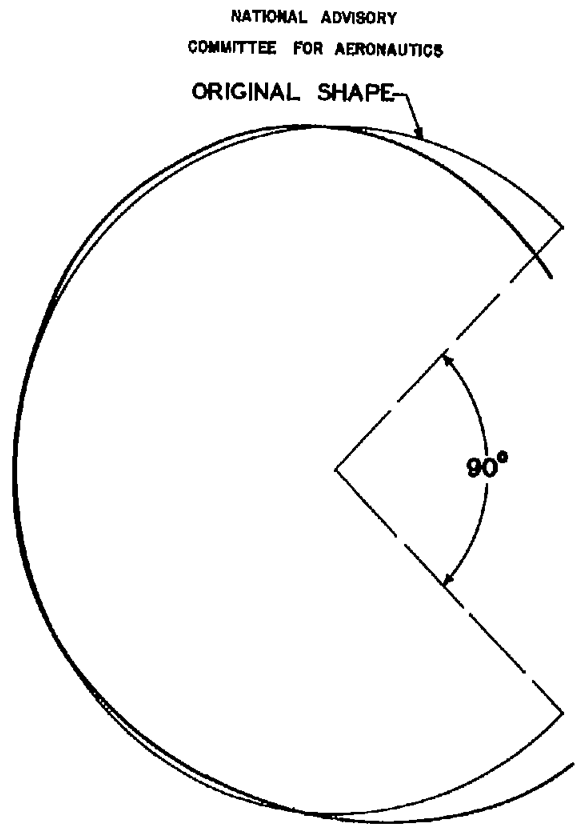

Figure 14.- Deflected shape of cut ring in its own plane.

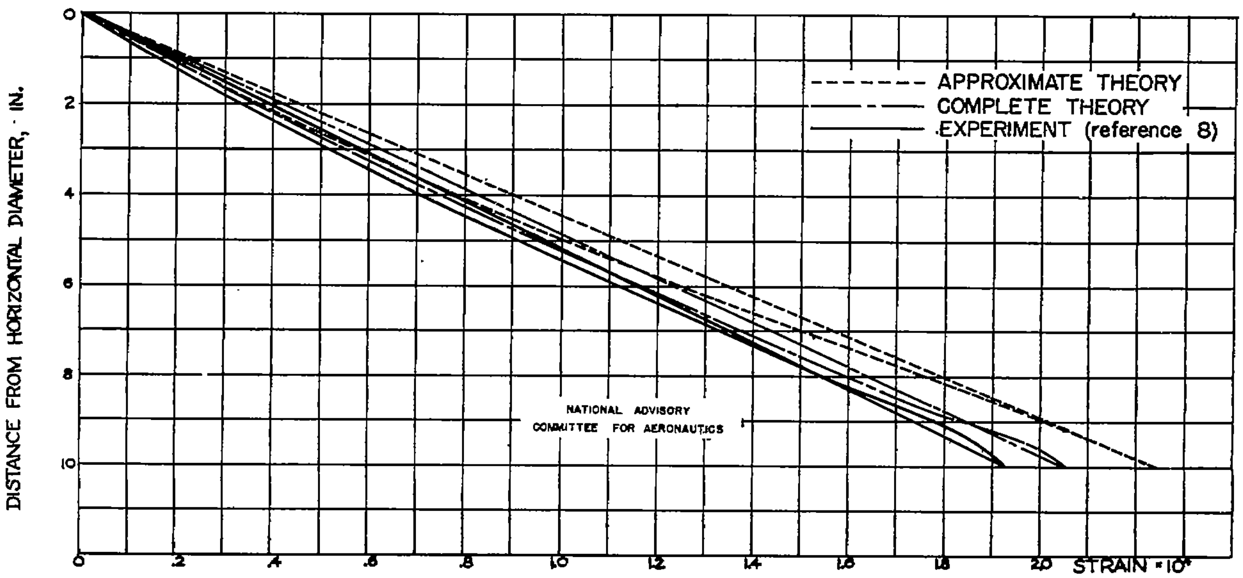

Figure 15.- Comparison of normal strain. Full section, 8 stringers, 45° cutout, M = 20,000 in-lb.

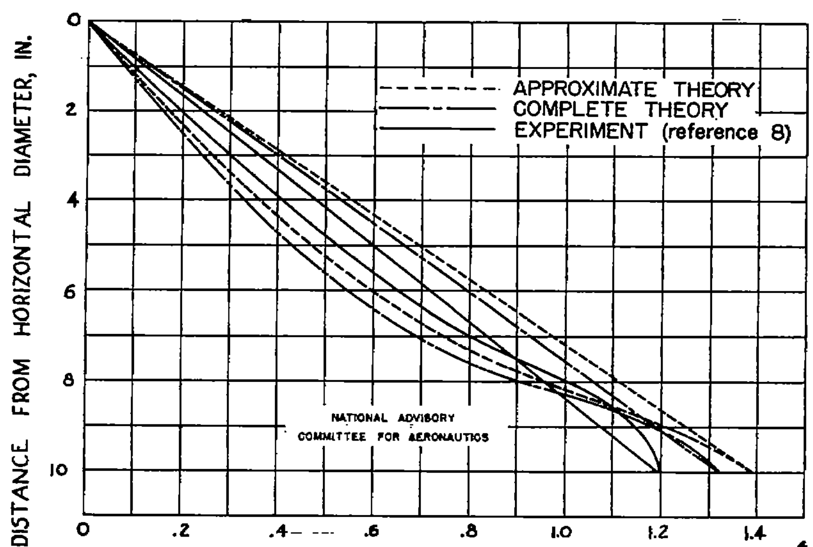

Figure 16.- Comparison of variation of normal strain. Full section, 16 stringers, 90° cutout, M = 20,000 in-lb.

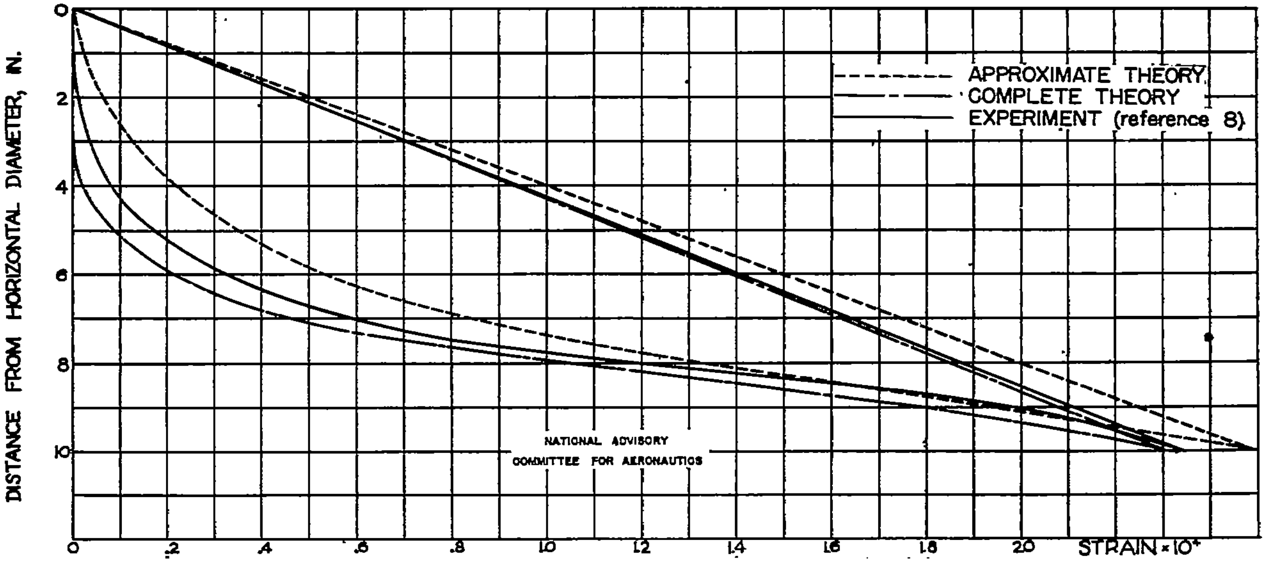

Figure 17.- Comparison of normal strain. Full section, 8 stringers, 135° cutout, M = 20,000 in-lb.

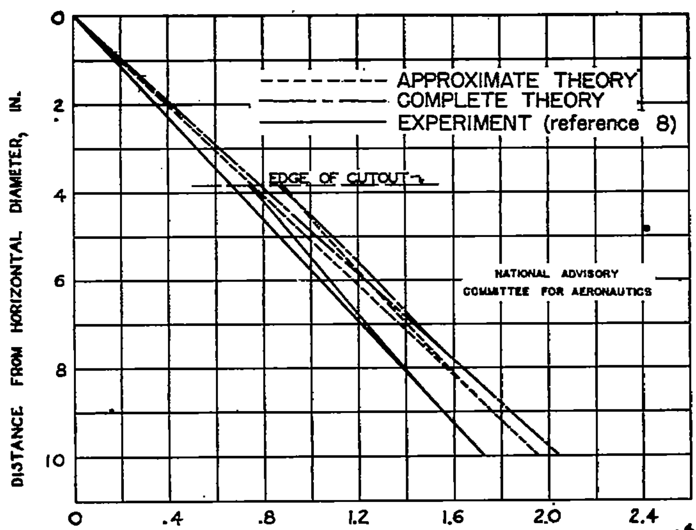

Figure 18.- Comparison of variation of normal strain. Cutout section, 8 stringers, 45° cutout, M = 20,000 in-lb.

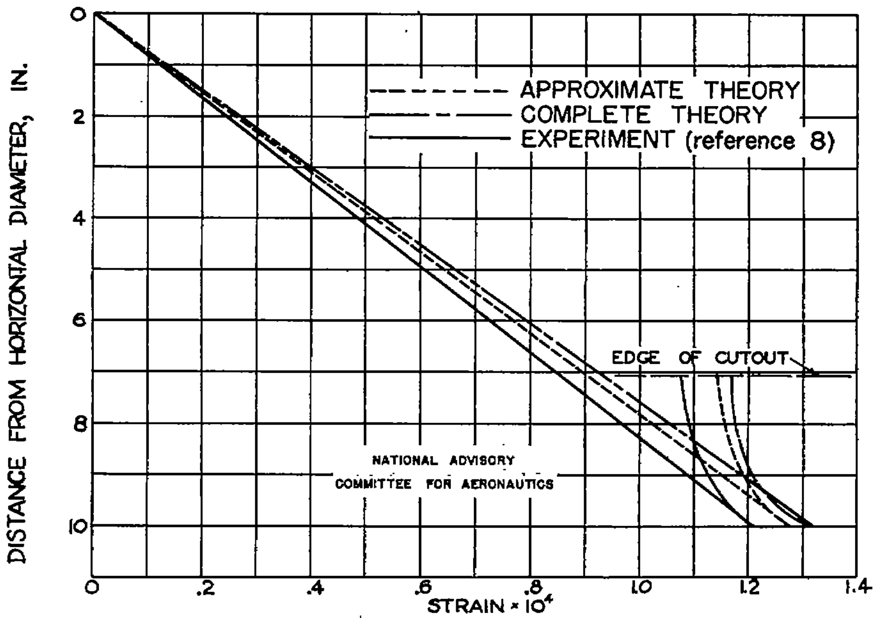

Figure 19.- Comparison of variation of normal strain. Cutout section, 16 stringers, 90° cutout, M = 20,000 in-lb.

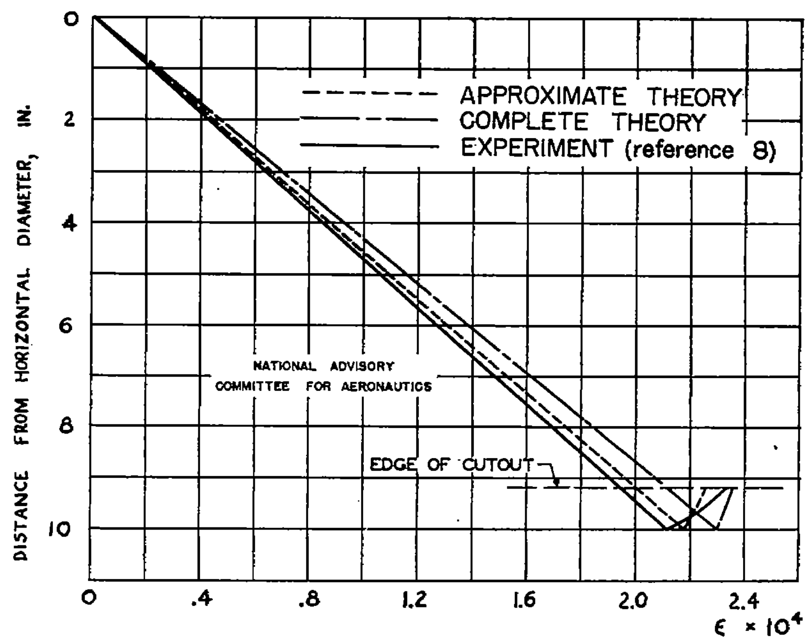

Figure 20.- Comparison of variation of normal strain. Cutout section, 8 stringers, 135° cutout, M = 20,000 in-lb.

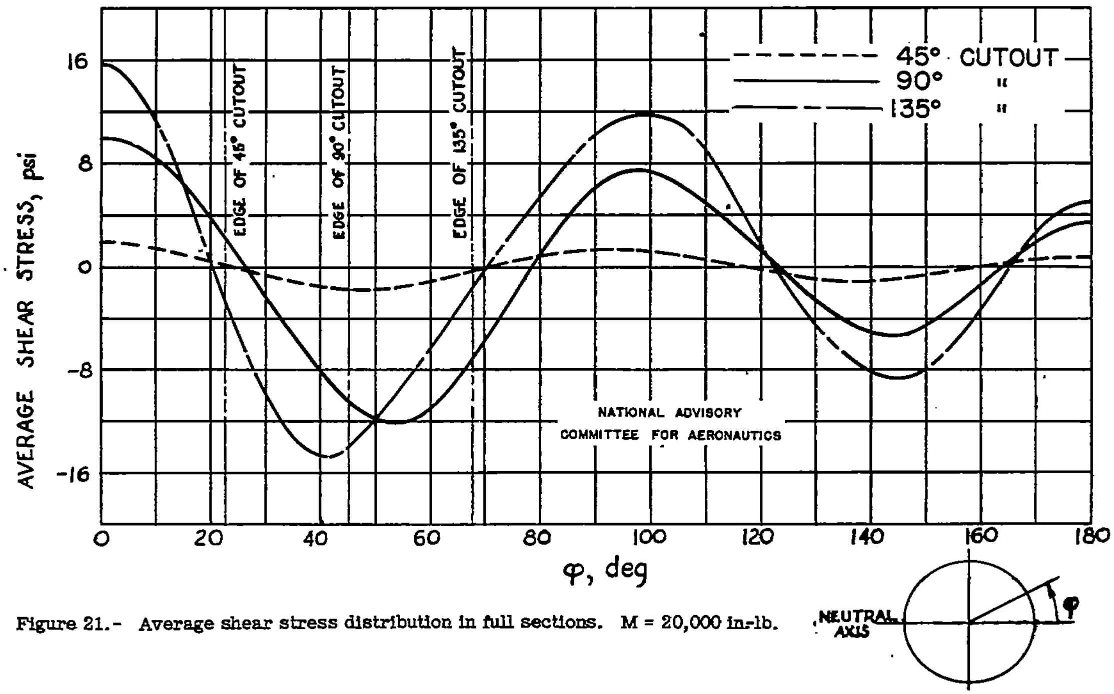

Figure 21.- Average shear stress distribution in full sections. M = 20,000 in-lb.

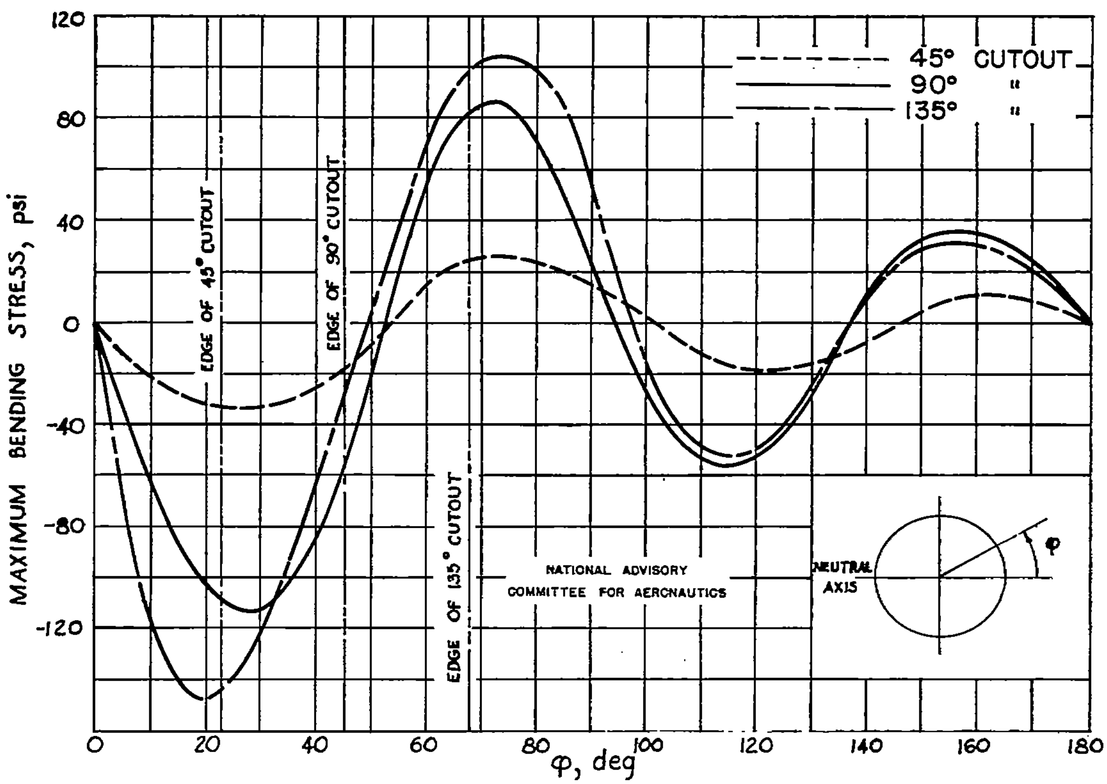

Figure 22.- Bending stress distribution in full rings. M = 20,000 in-lb.