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TECHNICAL NOTE

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BUCKLING STRESSES OF CLAMPED RECTANGULAR FLAT PLATES IN SHEAR

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SUMMARY

By consideration of antisymmetrical, as well as symmetrical, buckling configurations, the theoretical shear buckling stresses of clamped rectangular flat plates are evaluated more correctly than in previous work. The results given, which represent the average of upper- and lower-limit solutions obtained by the Lagrangian multiplier method, are within $1\frac{1}{4}$ percent of the true buckling stresses.

INTRODUCTION

The theoretical buckling stress in shear of a clamped rectangular plate has been found exactly only for the case of an infinitely long plate (reference 1); for plates of finite aspect ratio, approximate solutions of uncertain accuracy have been presented by several investigators. In reference 2, Cox gives buckling-stress coefficients computed by the Rayleigh-Ritz energy method for several aspect ratios, but no details of solution are included. Iguchi (reference 3) obtains approximate results by means of a series method that, as Smith points out in reference 4, does not provide definite information as to whether the results obtained are too high or too low. Smith then obtains upper limits to the true buckling stress by using Iguchi's deflection function in the Rayleigh-Ritz method. The numerical results published by these investigators differ by as much as 10 percent at some values of plate aspect ratio. Furthermore, these results were all apparently based on the assumption that, for each aspect ratio, the critical stress corresponds to a buckling pattern symmetrical (rather than antisymmetrical) about the plate midpoint.

The present paper gives theoretical buckling-stress coefficients computed from an analysis (reference 5) by the Lagrangian multiplier method. Both upper and lower limits to the true buckling stress were calculated, so that the maximum error in the final results is definitely known. Furthermore, in the present paper the assumption is not made that the symmetrical buckling pattern governs at all aspect ratios.

In reference 6 it is shown that, in shear buckling of a rectangular plate simply supported at the edges, an antisymmetrical buckling pattern is critical at certain aspect ratios. The antisymmetrical pattern was accordingly included in the present investigation, and the results indicate a range of aspect ratios in which the lowest buckling stress does occur with a configuration of antisymmetry.

SYMBOLS

a	length of plate
b	width of plate
β	plate aspect ratio (a/b)
t	thickness of plate
E	Young's modulus for material
μ	Poisson's ratio for material
D	flexural stiffness of plate $\left(\frac{Et^3}{12(1-\mu^2)}\right)$
τ	critical shear stress
k_s	critical shear stress coefficient in the formula $\tau = k_s \frac{\pi^2 D}{b^2 t}$

RESULTS AND DISCUSSION

The critical shear stress for a rectangular flat plate with clamped edges is given by the equation

$$\tau = k_s \frac{\pi^2 D}{b^2 t}$$

The solid curve in figure 1 gives the values of the shear stress coefficient k_s for values of aspect ratio β from 1 to 3. As shown in figure 1, the solid curve consists of parts of two distinct curves, one of which corresponds to symmetrical buckling, the other to antisymmetrical buckling, about the plate midpoint. The governing configuration at any aspect ratio, given by the solid line, is that which yields the lower buckling stress.

The values of k_s plotted in figure 1 represent the average of upper- and lower-limit solutions obtained from the analysis of reference 5. (See table 1.) In most cases, the lower-limit results were obtained from eleventh-order stability determinants, and the upper-limit results from ninth-order determinants. (See reference 5.) The evaluation of many terms in the determinants was considerably simplified by means of the computation aids discussed in the appendix to the present paper. As is seen from the data of table 1, the final results obtained must be within $1\frac{1}{4}$ percent of the true buckling stress coefficients.

It is to be expected that for values of β between 3 and ∞ the buckling stresses corresponding to symmetrical and antisymmetrical buckling will be very close to each other. Hence, the solid curve of k_s against $1/\beta$ shown in figure 2, faired through the data of table 1, can be used to estimate k_s for any value of β between 1 and ∞.

CONCLUDING REMARKS

From a consideration of both symmetrical and antisymmetrical buckling patterns, the shear buckling stresses of rectangular flat plates with clamped edges have been more correctly evaluated than in previous work wherein only symmetrical buckling patterns were considered. Through the use of the Lagrangian multiplier method, which permits the computation of both upper and lower limits to the true buckling stress, values known to be within $1\frac{1}{4}$ percent of the true buckling stress coefficients have been obtained.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., September 18, 1947

APPENDIX

COMPUTATION AIDS

In the determinants which form the stability criteria from which the buckling-stress coefficients are derived (reference 5), many terms are given in the form of infinite summations; that is, $\sum_{m=1}^{\infty} m^2 B_{mn}$ where

$$B_{mn} = \frac{A_{mn}}{A_{mn}{}^2 - k_s{}^2 \beta^6 m^2 n^2}$$

and

$$A_{mn} = 2\left(m^2 + \beta^2 n^2\right)^2$$

This summation, for a given value of n, converges very slowly, approximately as $\frac{1}{2m^2}$; however, the terms of the summation approach the value of $\frac{m^2}{A_{mn}}$ rapidly, as m becomes large. Thus, it may be assumed that above a certain value of the index m, say M, the terms of the desired series become equal to the corresponding terms of $\sum_{m=1}^{\infty} \frac{m^2}{A_{mn}}$. Thus, for example

$$\sum_{m=1}^{\infty} m^2 B_{m1} \approx \sum_{m=1}^{M} m^2 B_{m1} + \sum_{m=M+1}^{\infty} \frac{m^2}{A_{m1}}$$

$$\approx \sum_{m=1}^{M} m^2 B_{m1} - \sum_{m=1}^{M} \frac{m^2}{A_{m1}} + \sum_{m=1}^{\infty} \frac{m^2}{A_{m1}}$$

But

$$\sum_{m=1}^{\infty} \frac{m^2}{A_{m1}} = \sum_{m=1}^{\infty} \frac{m^2}{2\left(m^2 + \beta^2\right)^2}$$

$$= \frac{\pi}{8\beta}\left(\beta\pi + \coth \beta\pi - \beta\pi \coth^2\beta\pi\right)$$

Then

$$\sum_{m=1}^{\infty} m^2 B_{m1} \approx \sum_{m=1}^{M} m^2 B_{m1} - \sum_{m=1}^{M} \frac{m^2}{A_{m1}} + \frac{\pi}{8\beta}\left(\beta\pi + \coth \beta\pi - \beta\pi \coth^2\beta\pi\right)$$

In the case of $\beta = 2$, where $k_s \approx 10$, it was only necessary to take $M = 9$ to obtain sufficient accuracy.

All of the infinite summations found in the determinants for both the symmetrical and antisymmetrical lower-limit solutions can be evaluated in a similar manner. In the upper-limit determinants, there appear summations of a more complex type of term, such as

$$\sum_{m=1}^{\infty} \frac{m^2}{A_{m2}^{\;2} H_m}$$

where

$$H_m = \frac{1}{A_{m1}} + \frac{4}{A_{m2}}$$

Expanding the general term gives

$$\frac{m^2}{A_{m2}^{\;2} H_m} = \frac{m^2}{4\left(m^2 + 4\beta^2\right)^4 \left[\frac{1}{2\left(m^2 + \beta^2\right)^2} + \frac{4}{2\left(m^2 + 4\beta^2\right)^2}\right]}$$

$$= \frac{m^2\left(m^2 + \beta^2\right)^2}{2\left(m^2 + 4\beta^2\right)^2\left[\left(m^2 + 4\beta^2\right)^2 + 4\left(m^2 + \beta^2\right)^2\right]}$$

Dividing through the numerator by the denominator gives a sum of the form

$$\frac{A}{m^2} - \frac{B}{m^4} + \frac{C}{m^6} - \frac{D}{m^8} + \cdot \cdot \cdot$$

The first four terms of this sum are sufficient to provide a satisfactory approximation to the general term of the desired series at high values of m. Thus

$$\sum_{m=1}^{\infty} \frac{m^2}{A_{m2}{}^2 H_m} \approx \sum_{m=1}^{M} \frac{m^2}{A_{m2}{}^2 H_m} + \sum_{m=M+1}^{\infty} \left(\frac{A}{m^2} - \frac{B}{m^4} + \frac{C}{m^6} - \frac{D}{m^8}\right)$$

$$\approx \sum_{m=1}^{M} \frac{m^2}{A_{m2}{}^2 H_m} - \sum_{m=1}^{M} \left(\frac{A}{m^2} - \frac{B}{m^4} + \frac{C}{m^6} - \frac{D}{m^8}\right)$$

$$+ \sum_{m=1}^{\infty} \left(\frac{A}{m^2} - \frac{B}{m^4} + \frac{C}{m^6} - \frac{D}{m^8}\right)$$

where

$$A \equiv \frac{1}{10}, \quad B \equiv \frac{9.2\beta^2}{10}, \quad C \equiv \frac{58.44\beta^4}{10}$$

and

$$D \equiv \frac{318.208\beta^6}{10}$$

But since

$$\sum_{m=1}^{\infty} \frac{1}{m^2} = \frac{\pi^2}{6}$$

$$\sum_{m=1}^{\infty} \frac{1}{m^4} = \frac{\pi^4}{90}$$

$$\sum_{m=1}^{\infty} \frac{1}{m^6} = \frac{\pi^6}{945}$$

$$\sum_{m=1}^{\infty} \frac{1}{m^8} = \frac{\pi^8}{9450}$$

the desired summation becomes

$$\sum_{m=1}^{\infty} \frac{m^2}{A_{m2}{}^2 H_m} \approx \sum_{m=1}^{M} \frac{m^2}{A_{m2}{}^2 H_m} - \sum_{m=1}^{M} \left(\frac{A}{m^2} - \frac{B}{m^4} + \frac{C}{m^6} - \frac{D}{m^8}\right) + A\frac{\pi^2}{6} - B\frac{\pi^4}{90} + C\frac{\pi^6}{945} - D\frac{\pi^8}{9450}$$

Similar approximations are possible for the other infinite series of similar form in the upper-limit determinants.

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TABLE 1

SHEAR-STRESS COEFFICIENTS FOR VARIOUS ASPECT RATIOS OF PLATE

Symmetrical buckling					Antisymmetrical buckling				
β	Upper limit	Lower limit	Average	Maximum error (percent)	β	Upper limit	Lower limit	Average	Maximum error (percent)
1.00	[a]14.79	[a]14.64	14.71	0.54	----	------	------	----	----
1.25	[b]12.61	[c]12.35	12.48	1.05	----	------	------	----	----
1.50	[b]11.56	[c]11.45	11.50	.52	1.50	[d]12.08	[e]11.79	11.93	1.24
2.00	[b]10.60	[c]10.58	10.59	.10	2.00	[f]10.36	[e]10.32	10.34	.20
3.00	[b]9.67	[c]9.57	9.62	.52	3.00	[d]9.71	[e]9.64	9.67	.41
∞	------	------	[g]8.98	----	----	------	------	[g]8.98	----

[a]Sixth order determinant; p=q=3
[b]Ninth order determinant; p=3, q=2
[c]Eleventh order determinant; p=3, q=2
[d]Ninth order determinant; p=5, q=2
[e]Eleventh order determinant; p=5, q=2
[f]Eleventh order determinant; p=7, q=2
(See reference 5.)
[g]Obtained from exact solution in reference 1.

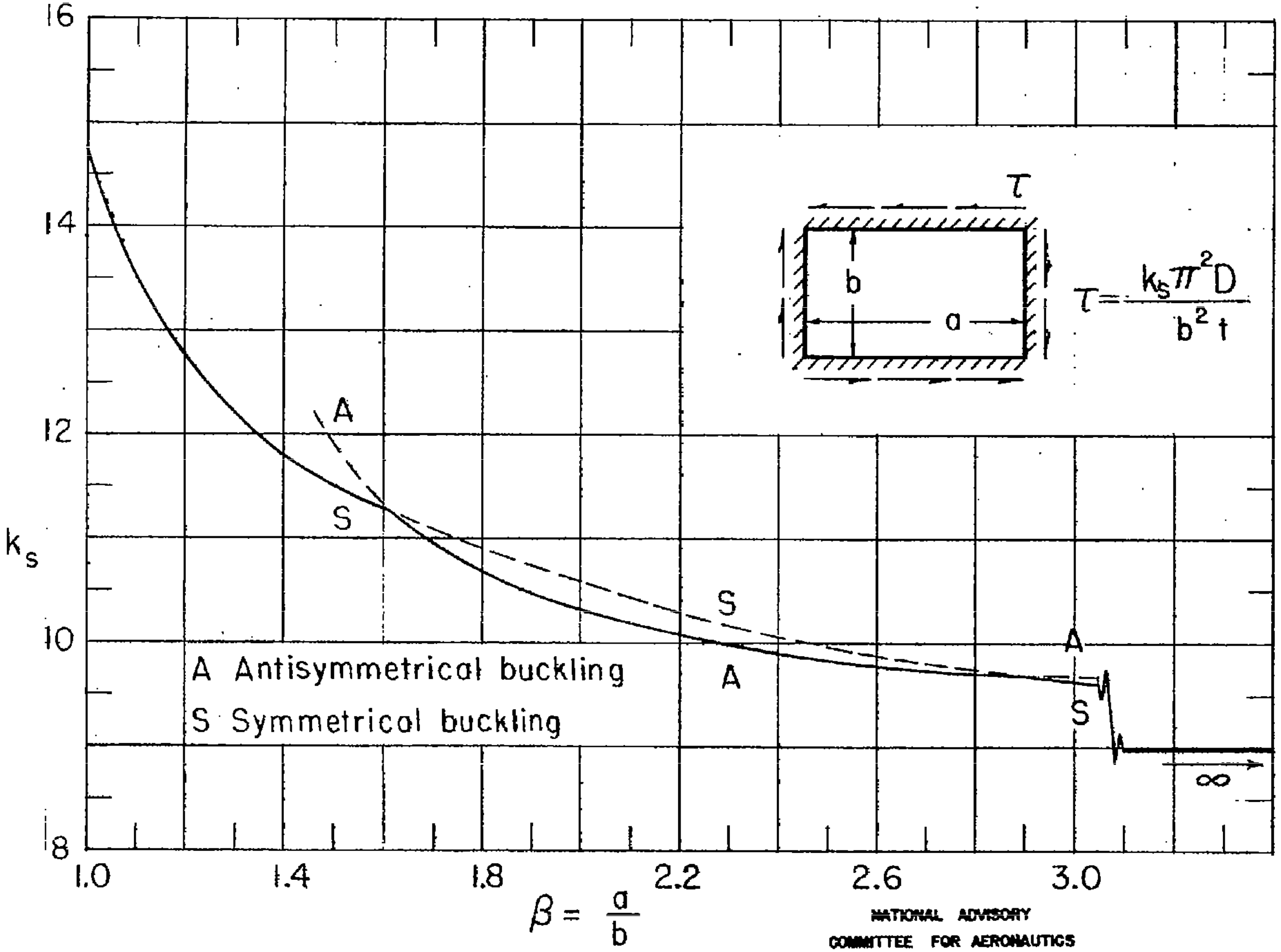

Figure 1.- Buckling stresses of clamped rectangular plates in shear.

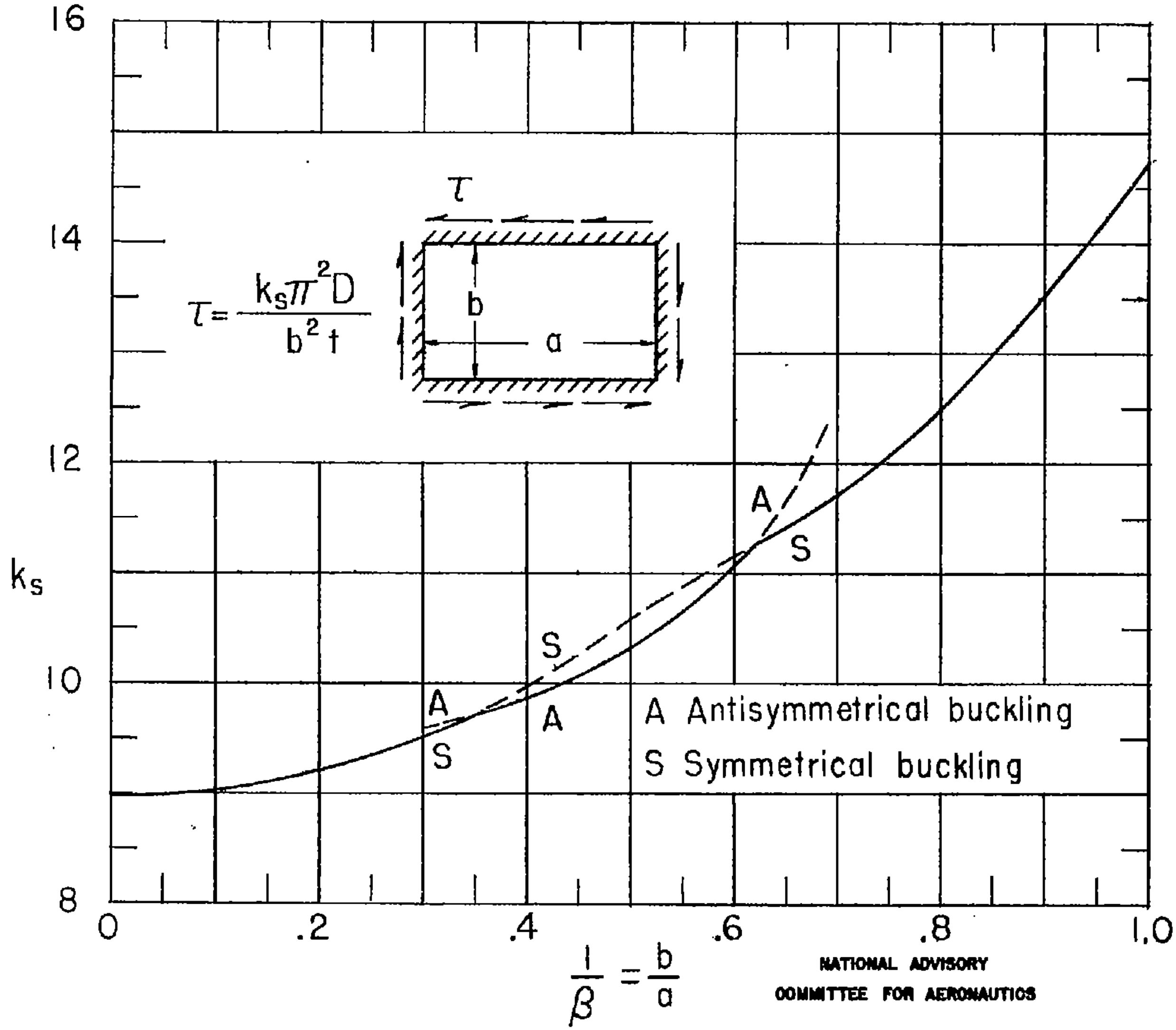

Figure 2.- Buckling stresses of clamped rectangular plates in shear.
($\frac{1}{\beta}$ as function of k_s.)