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A STUDY OF SKIN TEMPERATURES OF CONICAL BODIES IN SUPERSONIC FLIGHT

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SUMMARY

A comparison is made between the time history of skin temperature measured on the nose of a V-2 missile and the time history of the temperature computed by the use of Eber's experimental relation for heat-transfer coefficients for conical bodies under supersonic conditions. The agreement obtained is felt to justify the use of Eber's relation in the calculation of skin temperatures under flight conditions. A general method developed for making such skin-temperature calculations is used to compute the variation of skin temperature with time for a wide range of values of the pertinent parameters. The results show that by proper selection of the basic parameters the increase of skin temperature during a limited time of flight can be held to structurally permissible values. Methods are given for taking into account, when necessary, the effects of solar heating and the radiation exchanged between the skin and atmosphere. Time histories of skin temperature are computed for hypothetical supersonic flight plans.

INTRODUCTION

Because of the high stagnation and boundary-layer temperatures associated with supersonic speeds, the effects of aerodynamic heating have been of much concern to the designers of supersonic air-borne structures. Values of skin temperature under steady-state conditions are calculated in reference 1. This paper shows the necessity of allowing for the variable specific heat of air in stagnation-temperature calculations and by allowing for radiation shows that skin temperatures can be lower than boundary-layer temperatures. The skin-temperature values calculated are high, however, from a structural or from an operational standpoint, and the results are based on an extension of subsonic heat-transfer coefficients to a supersonic wedge-shaped airfoil. For this extension no experimental verification is yet available.

Experimental information on aerodynamic-heating effects under supersonic conditions has been fragmentary. During the course of a series of V-2 missile flights at White Sands, N. Mex., the Naval Research Laboratory has, however, made a number of measurements of the variation of skin temperature with time. (See reference 2.) The data obtained on March 7, 1947 on the conical warhead of missile 21 are of particular interest since these data can be compared with the results of calculations based on an experimental relation for supersonic heat-transfer coefficient for conical bodies given in reference 3.

Since the results of reference 3 are in such a form that they could be used in general calculations of the variation of skin temperature with time, such a comparison is of particular interest, especially if it indicates that the results of reference 3 can be extended to flight conditions.

The purpose of the present paper is to show the order of agreement between the time history of skin temperatures measured on the V-2 missile and the time history of temperatures calculated by use of the results of reference 3. A general method of making such calculations is given which makes possible the calculation of the time history of skin temperature for a wide variety of structural and flight-path parameters. The equations given are sufficiently general that the effects of solar radiation can be investigated as well as the radiation received from space and the ambient atmosphere.

SYMBOLS

A area, square feet

B factor (see equation (B2))

C solar constant (0.1192 Btu/(sq ft)(sec))

c specific heat of skin material, Btu/(lb)($^{\circ}$F)

c_p specific heat of air at constant pressure, Btu/(lb)($^{\circ}$F)

G skin factor ($c\tau w$), Btu/(sq ft)($^{\circ}$F)

g acceleration due to gravity (32.1740 ft/sec^2)

h heat-transfer coefficient, Btu/(sec)(sq ft)($^{\circ}$F)

h_T enthalpy per unit mass of air corresponding to total temperature, Btu per pound

h_A enthalpy per unit mass of air corresponding to ambient temperature, Btu per pound

H altitude, feet

J mechanical equivalent of heat (778.27 ft-lb/Btu)

K	temperature recovery factor
k	thermal conductivity of air, Btu/(sec)(sq ft)(°F/ft)
l	characteristic length, feet
M	Mach number
Nu	Nusselt number (hl/k)
Pr	Prandtl number $(c_p \mu g/k)$
p	pressure of ambient atmosphere, pounds per square foot
p_0	pressure of standard atmosphere at sea level (2116.229 lb/sq ft)
Q	quantity of heat, Btu
R	Reynolds number $(Vl\rho/\mu)$
T	temperature, °F absolute
T_A	ambient atmospheric temperature, °F absolute
T_B	boundary-layer temperature, °F absolute
T_e	equilibrium skin temperature, °F absolute
T_m	mean skin temperature over time interval Δt, °F absolute
T_0	temperature of standard atmosphere at sea level (518.4° F abs.)
T_p	potential temperature, a fictitious temperature defined in equation (B2), °F absolute
T_S	skin temperature, °F absolute
T_T	total or stagnation temperature, °F absolute
t	time, seconds
V	velocity, feet per second
W	weight, pounds
w	specific weight of skin material, pounds per cubic foot
β	total apex angle of cone, radians (or deg)
γ	adiabatic exponent
δ	sky radiation factor (See appendix A.)

ϵ emissivity

ρ density of atmosphere, slugs per cubic foot

ρ_0 density of the standard atmosphere at sea level (0.002378 slugs/ft^3)

σ Stefan-Boltzmann radiation constant
$$\left(0.47594 \times 10^{-12} \frac{\text{Btu}}{(\text{sq ft})(\text{sec})(^{\circ}\text{F abs.})^4}\right)$$

τ skin thickness, feet (or in.)

μ coefficient of viscosity of air, pound-second per square foot

μ_0 coefficient of viscosity of air at sea-level conditions (3.7250×10^{-7} lb-sec/sq ft)

θ thermal lag constant, seconds

Subscripts:

1 start of time interval

2 end of time interval

A bar denotes average values in the time interval Δt.

BASIS FOR SKIN-TEMPERATURE CALCULATIONS

Heat-balance equation.- If the initial conditions and the specified flight path are given, the skin temperature can be evaluated as a function of time by means of the following differential equation which is derived in appendix A:

$$\frac{dT_S}{dt} = \frac{h\left(T_B - T_S\right) + \epsilon\sigma\left(\delta T_A{}^4 - T_S{}^4\right) + \epsilon C}{c\tau w} \tag{1}$$

Equation (1) takes into account the heat gained or lost by thermal conductivity in the boundary layer, the exchange of radiant energy between the skin and the surrounding atmosphere, and the heat gained from the sun. The physical properties of the skin material are taken into account in the product $c\tau w$ which in the rest of this paper is called the skin factor G. The equation applies to a thin skin under the assumption that no conduction or radiation of heat to other parts of the structure occurs.

Equation (1) is a nonlinear differential equation for which the explicit integral is not useful but which can be integrated by numerical methods. By use of average values of the various quantities over successive time intervals Δt, the relation between skin temperature and time can be expressed in the following equation:

$$T_{S_2} = T_{S_1} + \left\{\bar{h}\left(\bar{T}_B - \bar{T}_S\right) + \epsilon\sigma\left[\left(\bar{\delta}\bar{T}_A{}^4 + \frac{C}{\sigma}\right) - \bar{T}_S{}^4\right]\right\}\frac{\Delta t}{G} \tag{2}$$

The subscript 1 is used to denote the start of the interval and subscript 2, the end of the interval. The bar is used to denote average values in the time interval Δt.

In order to make use of equations (1) and (2), the values of heat-transfer coefficient h and boundary-layer temperature T_B must be known. Methods for determining these factors, based on experiments at supersonic speeds, have been given in reference 3 and are utilized in the present paper.

<u>Heat-transfer coefficient</u>.- The value of h to be used in equations (1) and (2) may be determined from reference 3 in which heat-transfer data were correlated on the basis of nondimensional parameters and a shape factor by means of the equation

$$Nu = \left(0.0071 + 0.0154\beta^{0.5}\right)R^{0.8} \tag{3}$$

This equation correlates wind-tunnel experiments on a series of conical models with apex angle β of 10° to 120° covering a Mach number range of 1.2 to 3.1 and a Reynolds number range of 2×10^5 to 2×10^6. An examination of the unpublished experimental data on which equation (3) is based shows that in computing the value of Nu and R, Eber based the conductivity and coefficient of viscosity on the boundary-layer temperature, while the density used was the density in the free stream ahead of the model. The characteristic length was the total length of the conical model as measured along the surface (length of generatrix), and the value of heat-transfer coefficient was the average value for the entire surface. The Prandtl number was taken as constant and ignored in determining equation (3), and only experimental data for which the shock wave was attached to the model were included.

In the extension of equation (3) to the calculation of skin temperatures in flight, selection of a temperature which characterizes conditions within the boundary layer, and also a characteristic length, are necessary in order to evaluate h. The experimental data of reference 3 were obtained under transient conditions which differed from equilibrium by, at most, 36° F. The characteristic temperature used as appropriate to such conditions is the boundary-layer temperature or stagnation temperature suitably corrected by the recovery factor K. Under flight conditions with a finite thickness of structural material, the skin temperature can be as much as several thousand degrees less than the boundary-layer temperature T_B. The skin temperature modifies the temperature of the air in the immediate vicinity of the skin and, consequently, the heat-transfer capacity of the boundary layer. There is, therefore, a question as to whether the value of h should be based on T_B, on T_S, or perhaps on some intermediate temperature.

For skin-temperature calculations in which conduction of heat along the thin skin may be neglected, as in equation (1), the local value of heat-transfer coefficient at any particular point is needed. In the extension of equation (3) to flight conditions it has been somewhat arbitrarily assumed that equation (3) can be used to obtain a local value of h if the characteristic length is taken as the distance of the particular point from the vertex of the conical body, measured along the surface. The selection of the proper characteristic length is also a part of the question of the range of Reynolds number over which equation (3) may be considered valid. In the calculations of the present paper, equation (3) has been used regardless of the value of R. In the absence of other experimental supersonic heat-transfer data, the validity of the assumptions about a characteristic temperature, characteristic length, and Reynolds number may be tested by comparison with the results of flight tests.

For convenience in calculation of the value of the heat-transfer coefficient, equation (3) may be rewritten in the following form:

$$h = \frac{0.0071 + 0.0154\beta^{0.5}}{l^{0.2}}\rho_0^{\ 0.8}\, k\left(\frac{V\rho}{\mu\rho_0}\right)^{0.8} \tag{4}$$

In the evaluation of equation (4), the values of μ and k corresponding to values of T_B less than 2400° F absolute may be based on table 3 of reference 4. For values of T_B greater than 2400° F absolute, approximate values for μ and k may be determined by use of the assumptions that the Prandtl number $c_p\mu g/k$ is equal to 0.65 at high temperatures and that

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{0.69} \tag{5}$$

Equation (5) is a tentative approximation based on extrapolation of the data in reference 4 and should be used only in the absence of experimental data on the viscosity of air at high temperatures.

Values of ρ/ρ_0 for pressure altitudes from sea level to 65,000 feet may be taken from tables of the NACA standard atmosphere (reference 5); values of ρ/ρ_0 for altitudes above 65,000 feet corresponding to the tentative standard atmosphere may be found in table V of reference 6. For any particular configuration (β and l specified) when the relationship between pressure, temperature, and altitude is known (from measurements or from tables for the standard atmosphere), the value of the heat-transfer coefficient h can be computed when altitude and velocity (or Mach number) are specified.

Boundary-layer temperature.- The boundary-layer temperature T_B may be evaluated from the equation

$$T_B = T_A + K\left(T_T - T_A\right) \tag{6}$$

Values of the temperature recovery factor K for conical bodies in supersonic flow are given in figure 1.

The stagnation temperature T_T corresponding to any specified values of velocity and ambient temperature is given by the following relation for adiabatic compressible flow at constant total energy

$$\int_{T_A}^{T_T} c_p \, dT + \int_{V}^{0} \frac{V \, dV}{Jg} = 0 \tag{7}$$

If the range of temperature change is such that c_p may be considered constant, equation (7) can be integrated directly, and the result of the integration can be expressed as

$$T_T = T_A\left(1 + \frac{\gamma - 1}{2}M^2\right) \tag{8}$$

Although this expression is a convenient one, values of T_T given by equation (8) are too large at high values of M. Equation (7) may, however, be conveniently evaluated by use of table 1 of reference 4 which gives values of the function

$$\int_{400}^{T} c_p \, dT$$

for the enthalpy or total heat of air over a range of temperature T from 300° F absolute to 6500° F absolute. In using this table, the stagnation temperature may be obtained from the relation

$$h_T = h_A + \frac{V^2}{2gJ} \tag{9}$$

Values of T_T from equation (9) are compared with those given by equation (8) with $\gamma = 1.4$ in table I. At a Mach number of 8 and $T_A = 392.4°$ F absolute, the stagnation temperature is 732° F less than the value given by equation (8) with $\gamma = 1.4$.

Skin factor.- Since the thickness, specific heat, and specific weight of the skin material enter the differential equation for skin temperature as the product of the three quantities, the results of an integration of equation (2) for a particular value of G apply to any structural material of high thermal conductivity and of sufficient thickness to give the specified value of G. Values of c and w for several commonly used aircraft structural materials are given in table II. The values of c for wrought iron and stainless steel are average values for which the temperature range is not specifically given in reference 7. In view of the large temperature variation of the specific heat of iron, allowance may sometimes need to be made for the change of G with temperature when calculations are specifically concerned with iron or steel. (See reference 8.) Values of G corresponding to various thicknesses of aircraft structural materials (and, for iron and steel to various temperatures) are shown in figure 2.

CALCULATION OF SKIN TEMPERATURE

The application of equations (1) or (2) to the determination of skin temperature requires that certain basic parameters and initial conditions be specified. These basic parameters which characterize the flight plan and the structure are Mach number M, altitude H, length l, cone angle β, emissivity ϵ, and skin factor G. In addition, when skin temperature is to be calculated as a function of time, the initial skin temperature must be specified.

Time-history calculations.- The variation of skin temperature with time while an airplane or missile follows a particular flight path in which either speed or altitude or both vary is often of much interest. Calculations of such a time history are easily performed by use of equation (2) in which average values of h, T_B, δ, and T_A are used. For preliminary design purposes, to determine the range of skin temperature expected and to determine the relative importance of the various basic parameters in limiting the temperature to structurally possible values during a flight of limited duration, it is convenient to use a hypothetical flight plan corresponding to the step function frequently used in the analysis of transient phenomena. In this hypothetical flight plan it is assumed that the altitude is constant, that the value of M is reached instantaneously at $t = 0$, and that the initial skin temperature is equal to the temperature of the ambient atmosphere. Time-history calculations using this flight plan are particularly simple; an outline of the method and an example are given in appendix B.

Equilibrium skin temperature.- The equilibrium skin temperature T_e is the skin temperature which would be reached after a sufficient lapse of time under steady-flight conditions at constant altitude. The value of T_e is independent of G and may be approximately equal to T_B, or very much less. The value depends on the value of h and the importance of the radiation terms. Equation (1) defines the value of T_e, since in the limit as $t \rightarrow \infty$, $\frac{dT_S}{dt} \rightarrow 0$ and equation (1) reduces to

$$\left[T_B + \frac{\epsilon\sigma}{h}\left(\delta T_A{}^4 + \frac{C}{\sigma}\right)\right] - \frac{\epsilon\sigma}{h} T_e{}^4 - T_e = 0 \tag{10}$$

For any specified conditions the term in the brackets is a constant which, once determined, is one of the two constants in an equation of the type $ax^4 + x - b = 0$. The positive real root of equation (10) may generally be located quickly by trial and error or by Newton's method of approximation as given in a number of standard mathematical works such as reference 9.

RESULTS AND DISCUSSION

Validity of skin-temperature calculations.- Skin temperatures have been measured at two points on the conical warhead of V-2 missile 21 (reference 2). These flight measurements can be used as an index of the applicability of equation (3) to flight and of the kind of errors to be expected when equations (1) and (3) are used for skin-temperature calculations under flight conditions.

Based on data supplied by the Naval Research Laboratory and also given in reference 2, the values of skin temperature measured on both the forward aluminum-alloy section and the rear steel section of the warhead of missile 21 are given in table III along with the radar-tracking data, the pressure and temperature of the ambient atmosphere, and the pertinent structural details. The values of pressure were determined from instruments located in the missile; the ambient temperature up to 50,000 feet was determined from radiosonde data, and temperatures above 50,000 feet are as reported in reference 2.

The time histories of skin temperature for the aluminum-alloy and the steel sections have been calculated by the methods outlined in the present paper. For the aluminum section, a value of skin factor of 0.34 was used, corresponding to a thickness of 0.105 inch. This value was selected since the dimensions of the temperature gage attached to the inside of the skin were so large in comparison with the skin thickness that the temperatures measured were judged to be more nearly representative of the increased thickness at the gage station. Because of the much greater heat capacity of the steel section, no correction was made for the local increase at the gage station, but the variation with temperature of the skin factor for steel, as shown in figure 2, was allowed for in making the calculations. In order to allow for radiation, a value of $\epsilon = 0.9$ was used as representative of the emissivity of the lacquer finish of the nose section. Since the effects of solar heating were found to be negligible, no attempt was made to allow for them.

The measured skin temperatures for this V-2 missile are shown in figure 3(a). Some of the flight-path data are shown in figure 3(b) which, for purposes of comparison, also shows the skin temperatures measured on the aluminum section. Also shown in figure 3(a) are two calculated time histories of skin temperature for both the aluminum-alloy and the steel sections. For the solid line in each case the value of h was based on T_B; for the dashed line the value of h was based on T_S.

The precision of the measured skin temperatures is given as $\pm 18^{\circ}$ F. (See table III.) When both the aluminum-alloy and the steel sections are considered, the agreement between measured and calculated skin temperatures is better for the calculations in which h is based on T_B. The measured temperatures lie between the calculated temperatures in the case of the aluminum-alloy section but are greater than either of the sets of calculated temperatures in the case of the steel section.

In order to make the calculations shown in figure 3(a), equation (3) has been used whether or not the value of R based on T_B fell in the range of the experiments of reference 3, that is, 2×10^5 to 2×10^6. Although at 100 seconds the value of R for the aluminum section had fallen to 100, during that part of the flight in which significant aerodynamic-heating effects occurred, 10 to 70 seconds, the value of R for both the aluminum and steel sections fell in the range 10^7 to 2×10^4, values which do not differ from the range of the experiments of reference 3 by more than a factor of 10. In view of the agreement obtained the following remarks are thought to be justified.

1. The experimental relation for heat-transfer coefficients for conical bodies under supersonic conditions

$$\mathrm{Nu} = \left(0.0071 + 0.0154\beta^{0.5}\right)R^{0.8}$$

given by Eber in reference 3 may be extended to flight conditions.

2. Satisfactory agreement between measured skin temperatures and temperatures calculated on the basis of Eber's equation has been obtained in the case of one flight over a Mach number range up to 4.8 and altitudes up to 250,000 feet.

3. In the extension of Eber's equation to flight conditions, better agreement between measured and calculated skin temperatures in the case of V-2 missile 21 was obtained when the boundary-layer temperature was used as the characteristic temperature than was obtained when the skin temperature was used, but one experiment is probably not enough to make a final conclusion about the best choice of temperature.

4. In the absence of further experimental data the characteristic length to be used in Eber's relation may be taken as the distance of the point under consideration from the vertex of the conical body, measured along the surface of the cone.

Range of skin temperature.- In view of the agreement between measured skin temperatures on V-2 missile 21 and the temperatures calculated by use of equation (2) and equation (3) with h based on T_B, a series of general calculations has been made. These calculations are designed to illustrate the range of skin temperatures which can be encountered in the design or operation of supersonic air-borne conical structures and to illustrate the relative significance of the six basic parameters in determining skin temperature. The following values of the basic parameters were selected for investigation:

H (ft)	M	G $\left(\frac{\text{Btu}}{(\text{ft}^2)(^\circ\text{F})}\right)$	l (ft)	ϵ	β (deg)
0	1	0.05	0.1	0	15
35,000	2	$\underline{.2}$	.5	0.05	$\underline{30}$
$\underline{80,000}$	$\underline{3}$	.5	$\underline{1.0}$	$\underline{.2}$	45
150,000	5	1.0	2.0	.5	60
250,000	8	1.5	5.0	1.0	90

Time histories of skin temperature and equilibrium skin temperatures were computed. The time-history calculations are for the convenient assumed flight plan in which the altitude remains constant, the value of M is reached instantaneously at $t = 0$, and the initial skin temperature is equal to T_A. In order to simplify the presentation and to reduce the number of calculations, the intermediate underlined values of the basic parameters were selected as representative. Each parameter was investigated for the five values given, while the other five parameters were held at the underlined representative values.

The results of the time-history calculations are given in figure 4, which shows the change in the time history of skin temperature for the various values of the basic parameters. The solid lines, indicating day conditions, include the maximum effect of solar heating. For the dashed lines, representing night conditions, the value of C was taken as zero in evaluating equation (2).

Since the time of flight of most supersonic missiles and airplanes is likely to be brief, figure 4 shows that the skin-temperature rise during a limited time of flight, with the proper selection of basic parameters, may be held to structurally permissible values. In particular, the value of T_S which is reached in any specified time at any specified Mach number is smaller at high altitudes, smaller for large values of G and l, and smaller for low values of β. For night conditions, increasing the value of ϵ always results in lower values of T_S. For day conditions, however, solar radiation can result in more rapid heating so long as the value of T_S is less than about 700° F absolute. The effect is small, however, and after the skin temperature has risen high enough so that the net radiation exchange results in a cooling of the surface, an appreciable reduction of skin temperatures can be achieved by employing a surface emissivity that approaches a value of 1.0. (See fig. 4(d).)

Since the rate of increase of T_S may become very small as T_S approaches its final or equilibrium value T_e, the time histories of skin temperature shown in figure 4 have not been extended beyond the time at which the initial temperature difference has fallen to about 10 percent of its initial value. The value of T_e is shown, however, in figure 5 for the same combinations of the basic parameters as were used in figure 4. The values for both day and night conditions illustrate the maximum effect of solar heating, which is seen to be small in all cases except at very high altitudes. For comparison, the value of T_B is also shown in figure 5. Values of T_B were computed by use of equations (6) and (9) and reference 4; values of T_e were computed by use of equation (10).

Skin temperatures for representative supersonic flight plans.- In order to illustrate the magnitude of aerodynamic-heating effects for practicable flight conditions, the time history of skin temperature has been computed for three hypothetical flight plans which are representative of the type of flight paths which might be useful for flight research with supersonic air-borne structures. The results, shown in figure 6, are for a point 1 foot back from the apex on a conical nose of total included angle β equal to 30°. Representative values of $\epsilon = 0.2$ and $G = 0.2$ Btu/(sq ft)(°F) have been chosen. This value of G is applicable to 0.034-inch-thick stainless steel or 0.061-inch-thick 24S-T aluminum alloy.

Flight plan A (fig. 6(a)) is a high-altitude run, followed by a constant-speed glide to lower altitudes. The skin temperature does not exceed 546° F absolute (86° F). Flight plan B (fig. 6(b)) is a run of 2-minutes duration at high speed and constant altitude. Because of the long time spent at high speeds, the skin temperature rises to 847° F absolute (387° F). The peak in temperature closely follows the peak in speed. Flight plan C (fig. 6(c)) is a short burst of speed to maximum Mach number, with full power, and a longitudinal acceleration of approximately 2.8g. Because of the high speeds involved, the temperature rise is very rapid and continues for approximately 20 seconds after the power is cut off. At this time the value of T_B has fallen below the value of T_S. The maximum temperature reached is 830° F absolute (370° F). Although the skin temperatures reached in the high-altitude run, plan A, are not excessive from a structural standpoint, the skin temperatures reached in plans B and C are excessive. Calculations for plan C indicate that an increase in skin factor from 0.2 to 0.5 would result in a decrease in maximum skin temperature of approximately 140° F to a value of 690° F absolute (230° F).

CONCLUSIONS

1. The experimental relation for heat-transfer coefficients for conical bodies under supersonic conditions given by Eber may be extended to flight conditions.

2. Satisfactory agreement between measured skin temperatures and temperatures calculated on the basis of Eber's equation has been obtained in the case of one flight over a Mach number range up to 4.8 and altitudes up to 250,000 feet.

3. In the extension of Eber's equation to flight conditions, better agreement between measured and calculated skin temperatures in the case of V-2 missile 21 was obtained when the boundary-layer temperature was used as the characteristic temperature than was obtained when the skin temperature was used, but one experiment is probably not enough to make a final conclusion about the best choice of temperature.

4. In the absence of further experimental data the characteristic length to be used in Eber's relation may be taken as the distance of the point under consideration from the vertex of the conical body, measured along the surface of the cone.

5. The skin temperatures which will be reached on supersonic missiles or airplanes may be very much less than either the equilibrium skin temperature or the boundary-layer temperature, at high altitudes, and also at low altitudes if the flight duration is short.

6. By proper selection of the basic parameters, the skin-temperature rise during a limited time of flight may be held to structurally permissible values.

7. Low values of skin temperature and low rates of increase of skin temperature at any given flight velocity are associated with

(a) High altitude

(b) High values for the skin of the product, specific heat times thickness times specific weight

(c) High values of emissivity

(d) Small cone angle

(e) Larger distances back of the nose.

8. The effect of solar heating on the skin temperature of supersonic air-borne structures is small at altitudes below 150,000 feet.

9. The radiation received from space and the outer atmosphere other than solar radiation may usually be neglected in the calculation of skin temperatures for supersonic air-borne structures.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
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APPENDIX A

DERIVATION OF THE DIFFERENTIAL EQUATION

FOR SKIN TEMPERATURE

The quantity of heat transferred in unit time through a fluid film is given as

$$dQ = hA\left(T_B - T_S\right) dt \tag{A1}$$

The heat lost by radiation is given by

$$dQ = A\epsilon\sigma {T_S}^4 \, dt \tag{A2}$$

and the radiant heat absorbed from space and the outer atmosphere is given by

$$dQ = A\epsilon\sigma\delta {T_A}^4 \, dt \tag{A3}$$

Equation (A3) is based on measurements made on the ground at different angles to the zenith reported in reference 10 in which it is shown that the radiant energy received from space and the atmosphere can be correlated with the local ambient atmospheric temperature if the atmosphere is taken as a black body of temperature T_A and emissivity δ. The quantity δ varies linearly with the square root of the proportion of the total atmosphere included in the path of the measurements. Since the proportion of the total atmosphere above any point to the proportion above a point at sea level is given by the ratio p/p_O, δ has been taken as proportional to $\sqrt{p/p_O}$ in the present paper. Although this extension to high altitudes of measurements made on the ground is admittedly an approximation, the validity of which must await experimental confirmation, the approximation is considered a better one than would be the case if some nominal black-body temperature were adopted for the atmosphere. The variation of δ with $\sqrt{p/p_O}$ is shown in figure 7, as adapted from reference 10.

The equation

$$dQ = \epsilon AC \, dt \tag{A4}$$

can be used to represent the maximum amount of radiant heat absorbed from the sun when the surface of the skin is perpendicular to rays of the sun. The value of the solar constant C is given in reference 11 as 1.94 calories per square centimeter per minute (0.1192 Btu/(sq ft)(sec)). Since the solar constant is defined as the average total energy received from the sun per unit area per unit time at the top of the earth's atmosphere, corrected to the sun's mean distance from the center of the earth, equation (A4) neglects the effects of season and the variation of atmospheric absorption with altitude. The emissivity ϵ is also used in equation (A4) instead of the absorptivity for solar radiation. Although the absorptivity of some materials of aircraft construction is higher than their emissivity, equation (A4) is thought to be a valid approximation for the maximum effect of solar radiation particularly for the range $0.2 < \epsilon < 1.0$. A more complete consideration of the effects of solar radiation, particularly as to the influence of selective absorption and emission on surface temperature, is beyond the scope of the present paper. A brief discussion is found in reference 12.

The heat required to raise the temperature of a body is

$$dQ = cW\ dT \tag{A5}$$

It is assumed (a) that the heat transferred to the interior of the cone can be neglected since the skin is backed by air only, (b) that radiation losses to other parts of the structure are negligible, (c) that radiation from the inner surface is balanced by radiation from the opposite side of the cone, (d) that the thermal conduction of heat away from the point under consideration due to a temperature gradient along the surface is negligible, and (e) that the skin is sufficiently thin that no temperature gradient exists in the skin perpendicular to the surface, due to the flow of heat at the external surface. Under these assumptions, for the part of a cone perpendicular to the solar radiation, the following equation, combining equations (A5), (A1), (A3), (A4), and (A2), would hold

$$cW \frac{dT}{dt}\ dt = hA\left(T_B - T_S\right)\ dt + A\epsilon\sigma\delta T_A{}^4\ dt + A\epsilon C\ dt - A\epsilon\sigma T_S{}^4\ dt \tag{A6}$$

Since the weight of the section of skin of area A can be expressed as

$$W = A\tau w \tag{A7}$$

and since in equation (A6) $dT \equiv dT_S$, the following basic differential equation for skin temperature is obtained

$$\frac{dT_S}{dt} = \frac{h\left(T_B - T_S\right) + \epsilon\sigma\left(\delta T_A^{\,4} - T_S^{\,4}\right) + \epsilon C}{c\tau w}$$

This equation (equation (1)) is given in a convenient general form in order to permit a definite evaluation of the contribution to dT_S/dt of the convection, radiation, and solar terms. In cases of practical interest, a brief study shows which of the terms in the equation may be omitted without the introduction of significant errors.

APPENDIX B

NUMERICAL CALCULATIONS WITH THE HEAT-BALANCE EQUATION

Numerical integration.- In the step-by-step numerical integration of equation (1), rearranging the terms as follows has been found convenient:

$$\frac{dT_S}{dt} = \frac{T_p - BT_S{}^4 - T_S}{\theta} \tag{B1}$$

where

$$\left.\begin{aligned} B &= \frac{\epsilon\sigma}{h} \\ T_p &= T_B + B\left(\delta T_A{}^4 + \frac{C}{\sigma}\right) \\ \theta &= \frac{G}{h} \end{aligned}\right\} \tag{B2}$$

The magnitude of B $((^{\circ}F\ abs.)^{-3})$ is a measure of the relative significance of the radiation and conduction terms. The quantity T_p which may be termed the potential temperature is a fictitious temperature resulting from all of the heating terms in equation (1). The quantity θ is the time constant of the thermal system and is a measure of the time (in seconds) required for an initial temperature difference to fall to 1/e times its original value (when radiation may be ignored).

Numerical integration over successive time intervals $t_1, t_2, \ldots t_n, t_{n+1}$ when neither the speed nor altitude is changing is a simple iterative arithmetic process which can be quickly carried out. For flight paths in which altitude or speed vary with time, the process is essentially no more complicated once the average values of T_B, h, and $\delta T_A{}^4$ have been determined for each time interval. For such varying flight paths, equation (2) would be written

$$T_{S_2} = T_{S_1} + \left(\overline{T}_p - \overline{B}T_m{}^4 - T_m\right)\frac{\Delta t}{\overline{\theta}} \tag{B3}$$

where

$$\bar{B} = \frac{\epsilon\sigma}{\bar{h}}$$

$$\bar{T}_p = \bar{T}_B + \bar{B}\left(\delta\bar{T}_A^{\,4} + \frac{C}{\sigma}\right)$$

$$\theta = \frac{G}{\bar{h}}$$

$$T_m = \frac{T_{S_1} + T_{S_2}}{2}$$

Although the temperature T_{S_2} which is reached at the end of the time interval Δt is not known in a step-by-step integration, a reasonable value may be assumed. The value of T_m corresponding to this assumption is then used to calculate a value T_{S_2}, and this first calculated value is compared with the assumed value. If there is a difference between the two values, rapid convergence is generally obtained if a new assumed T_{S_2} is used which lies midway between the assumed value and the first calculated value. Two or three iterations of this type generally give assumed and computed values of T_{S_2} which agree within 0.2° F, a value which is judged to be sufficiently close to prevent serious accumulative errors.

Approximate integral solution.- Equation (B1) can be integrated approximately by analogy with the integral of the basic equation for heat flow, generally known as Newton's law. The application of this law to thermal problems and, in particular, to thermometers has been given by a number of authors. (See, for example, reference 13.) The temperature of a thin sheet of skin initially at temperature T_1 which is plunged into a bath of temperature T_B, from which it is insulated by a layer with heat-transfer coefficient h, is given as a function of time by the equation

$$\int_{T_1}^{T} \frac{dT}{T_B - T} = \int_0^t \frac{dt}{G/h} = \frac{1}{\theta}\int_0^t dt \tag{B4}$$

The solution of equation (B4) is

$$\frac{T_B - T}{T_B - T_1} = e^{-t/\theta} \tag{B5}$$

If the term $T_p - BT^4$ in equation (B1) can be considered a constant over a small time interval Δt, then an integral of equation (B1) can be written as

$$\frac{\left(T_p - BT_m{}^4\right) - T_2}{\left(T_p - BT_m{}^4\right) - T_1} = e^{-\Delta t/\theta} \tag{B6}$$

where, as before, T_m is the mean value of T over the time interval Δt. Equation (B6) may be rewritten in a form suitable for step-by-step integration as

$$T_2 = \left(T_p - BT_m{}^4\right)\left(1 - e^{-\Delta t/\theta}\right) + T_1 e^{-\Delta t/\theta} \tag{B7}$$

Since T_m over the time interval Δt is not known, T_1 can be used as a first assumption for T_m, and the resultant value of T_2 used to make a better assumption. In view of the assumptions used in the derivation of equation (B7), values of $\Delta t/\theta$ should not exceed 0.25. A sample calculation made by use of equation (B6) is given in table IV. For a rapid approximation, the iterative process may be eliminated and the value of T_2 obtained by assuming that $T_m = T_1$ may be used as the value of T_1 for the next time interval.

Charts for the rapid evaluation of heat-transfer coefficient and equations for the quick calculation of the time history of skin temperatures during short-time supersonic flight are given in reference 14, which also shows the effect of acceleration on skin temperature.

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TABLE I

COMPARISON OF STAGNATION TEMPERATURES COMPUTED

BY USING CONSTANT AND VARIABLE SPECIFIC HEAT OF AIR

[Temperatures in °F abs.]

M	$T_A = 392.4$		$T_A = 518.4$	
	T_T $\gamma = 1.4$	T_T equation (9)	T_T $\gamma = 1.4$	T_T equation (9)
1	470	470	622	622
2	706	705	933	929
3	1098	1087	1451	1418
4	1647	1600	2177	2060
5	2352	2210	3110	2844
6	3142	2860	4251	3770
7	4232	3760	5599	4830
8	5412	4680	7154	6040

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TABLE II

THERMAL PROPERTIES OF MATERIALS

Specific heat and specific weight[a]		
Material	c (Btu/(lb)(°F))	w (lb/ft^3)
Aluminum		
Pure	[c]0.226	168.5
24S-T	[c].226	172.2
Wrought iron	.1138	483.6
Stainless steel	.142	492.5
Magnesium	[c].249	108.6
Specific heat of iron[b]		
T (°C)	T (°F abs.)	c (Btu/(lb)(°F))
0	492	0.1055
200	852	.1282
400	1212	.1509
600	1572	.1737

[a]Data from reference 7.
[b]Data taken from table entitled "Specific Heat - Variation with Temperature" of reference 8.
[c]For 0° to 100° C.

TABLE III

SKIN-TEMPERATURE MEASUREMENTS FOR V-2 MISSILE 21

FIRED MARCH 7, 1947[a]

[β, 26°; size of temperature gage, 0.014 in. by 0.6 in. by 3.5 in., approx.; surface finish, orange lacquer]

Time after launching (sec)	Altitude (km) (b)	Velocity (m/sec)	p (mm Hg)	T_A (°K)	T_S (Aluminum) (°C) (c)(d)	T_S (Steel) (°C) (c)(e)
0	1.2	0	645	280	22	30
10	1.7	109	600	279	22	30
20	3.5	260	480	267	22	30
25	5.0	348	410	261	22	30
30	7.0	438	300	246	25	30
35	9.4	530	210	228	35	31
40	12.3	646	130	218	45	40
45	15.9	810	76	215	62	55
47.5	18.0	900	52	215	77	65
50	20.4	995	33	215	87	75
52.5	23.0	1090	23	215	97	84
55	25.9	1200	14.5	216	108	92
57.5	29.0	1305	9.6	216	118	98
60	32.4	1420	5.8	217	124	105
65	39.9	1540	2.3	270	135	116
70	47.5	1490	.95	315	142	122
75	54.8	1440	.40	320	147	122
80	61.8	1390	.18	299	156	125
100	87.5	1200	.003	200	160	. . .

[a]Data supplied by Naval Research Laboratory and given in reference 2.
[b]To within ±0.5km.
[c]To within ±10° C.
[d]Thickness of 3S aluminum-alloy section, 0.091 in.; temperature gage at l = 1.5 ft.
[e]Thickness of spec. 48S5, grade M steel section, 0.109 in.; temperature gage at l = 2.6 ft.

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TABLE IV

SAMPLE TIME–HISTORY CALCULATION

$M = 3$
$H = 150,000$ ft
$\beta = 30^{\circ}$
$l = 1$ ft
$\epsilon = 0.2$
$F_S = 0.2$ Btu/ft^2 $^{\circ}$F
$T_S = 573.5^{\circ}$ F abs. at $t = 0$

$T_B = 1441^{\circ}$ F abs.
$h = 3.8 \times 10^{-4}$ Btu/(ft^2)(sec)($^{\circ}$F abs.)
$B = 2.504 \times 10^{-10}$ $^{\circ}$F abs.$^{-3}$
$\delta T_A{}^4 = 10.82 \times 10^{10}$ $^{\circ}$F abs.^{4}
$T_p = 1531^{\circ}$ F abs.

$\theta = 526.2$ sec
$\Delta t = 60$ sec
$\frac{\Delta t}{\theta} = 0.114025$
$e^{-\frac{\Delta t}{\theta}} = 0.89224$
$1 - e^{-\frac{\Delta t}{\theta}} = 0.10776$

$$T_2 = \left(1 - e^{-\frac{\Delta t}{\theta}}\right)\left(T_p - BT_m{}^4\right) + \left(e^{-\frac{\Delta t}{\theta}}\right) T_1$$

$$= 0.10776 \left(T_p - BT_m{}^4\right) + 0.89224 T_1$$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
t	T_S	T_1	T_m	$T_m{}^4$	$BT_m{}^4$	$T_p - BT_m{}^4$			T_2
			$\frac{(3) + (10)}{2}$	$(4)^4$	(5) × B	1531 − (6)	0.89224 × (3)	0.10776 × (7)	(8) + (9)
0	573.5	573.5	573.5	0.10818×10^{12}	27.1	1503.9	511.7	162.1	673.8
		573.5	623.6	.15122	37.9	1493.1	511.7	160.9	672.6
		573.5	623.0	.15118	37.8	1493.2	511.7	160.9	672.6
60	672.6	672.6	672.6	.20466	51.2	1479.8	600.1	159.5	759.6
		672.6	716.1	.26296	65.8	1465.2	600.1	157.9	758.0
		672.6	720.3	.26866	67.3	1463.7	600.1	157.7	757.8
120	757.8	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
600	1075.6	1075.6	1075.6	1.3384	335.1	1195.9	959.7	128.9	1088.6
		1075.6	1082.1	1.3711	343.3	1187.7	959.7	128.0	1087.7
		1075.6	1081.6	1.3686	342.7	1188.3	959.7	128.0	1087.7
660	1087.7								

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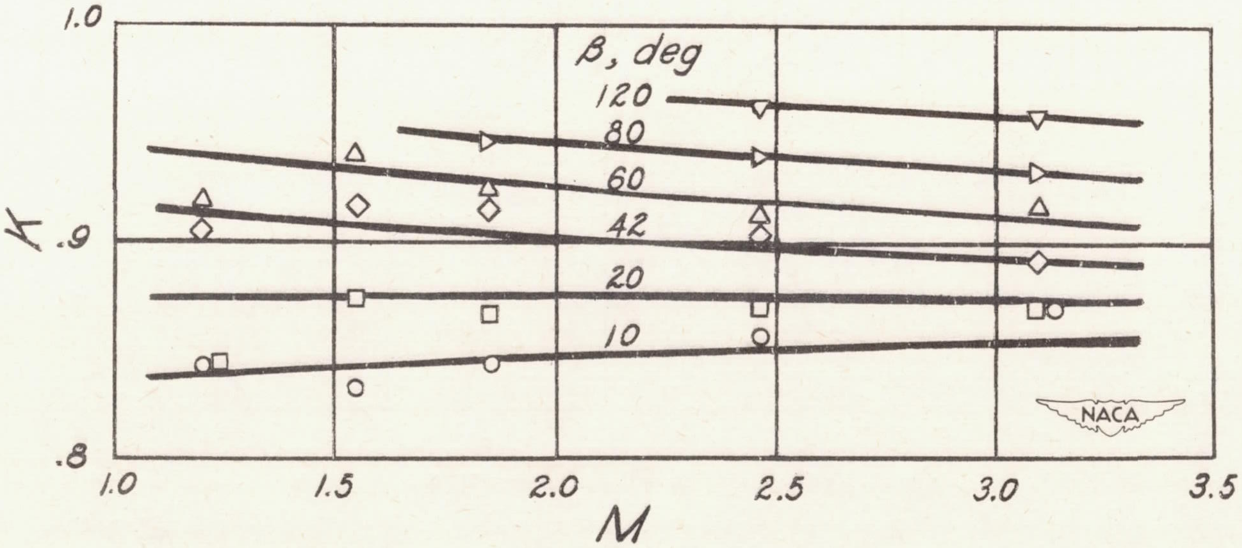


Figure 1.- Temperature recovery factor $K = \dfrac{T_B - T_A}{T_T - T_A}$ as a function of Mach number and cone angle. Data from reference 3.

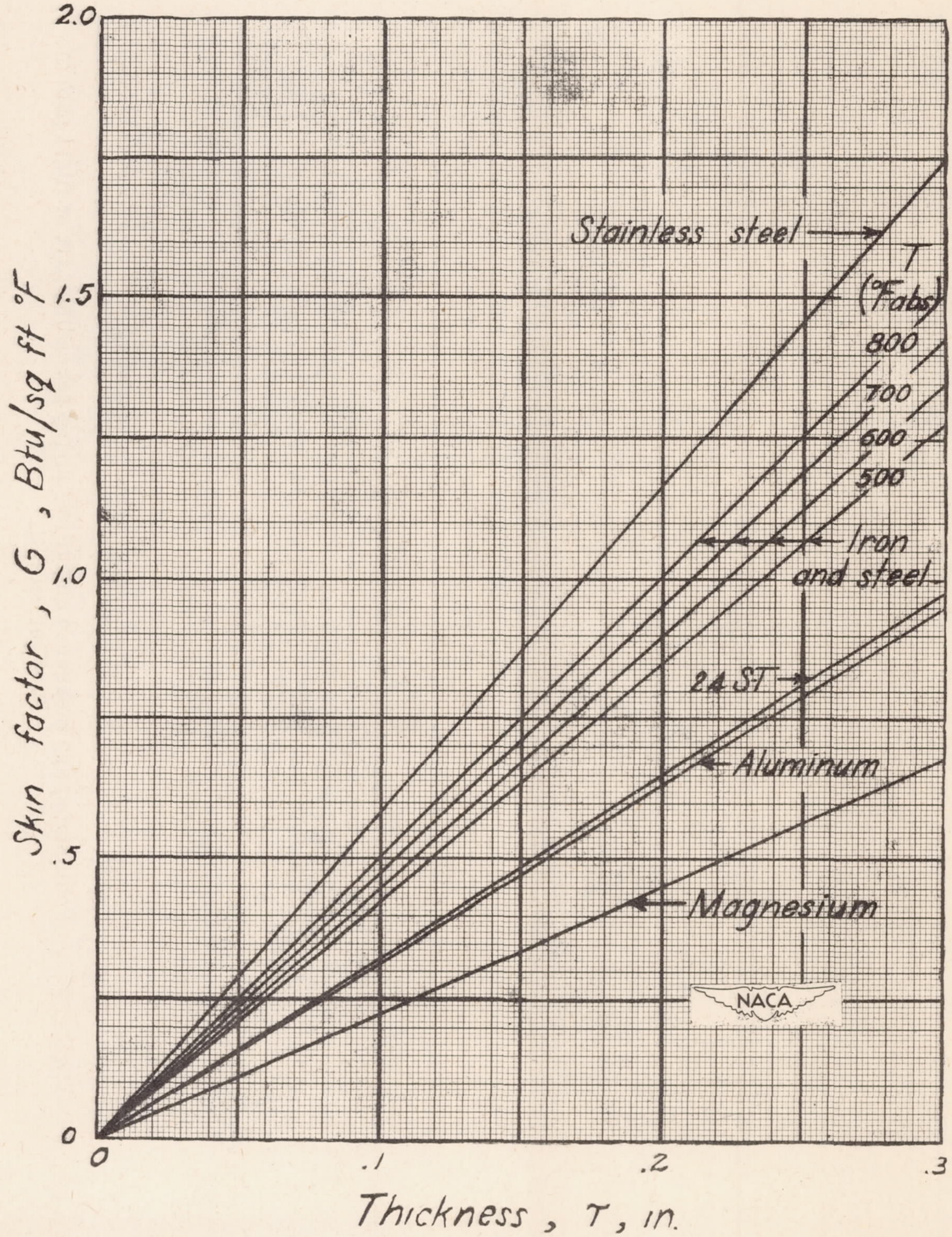
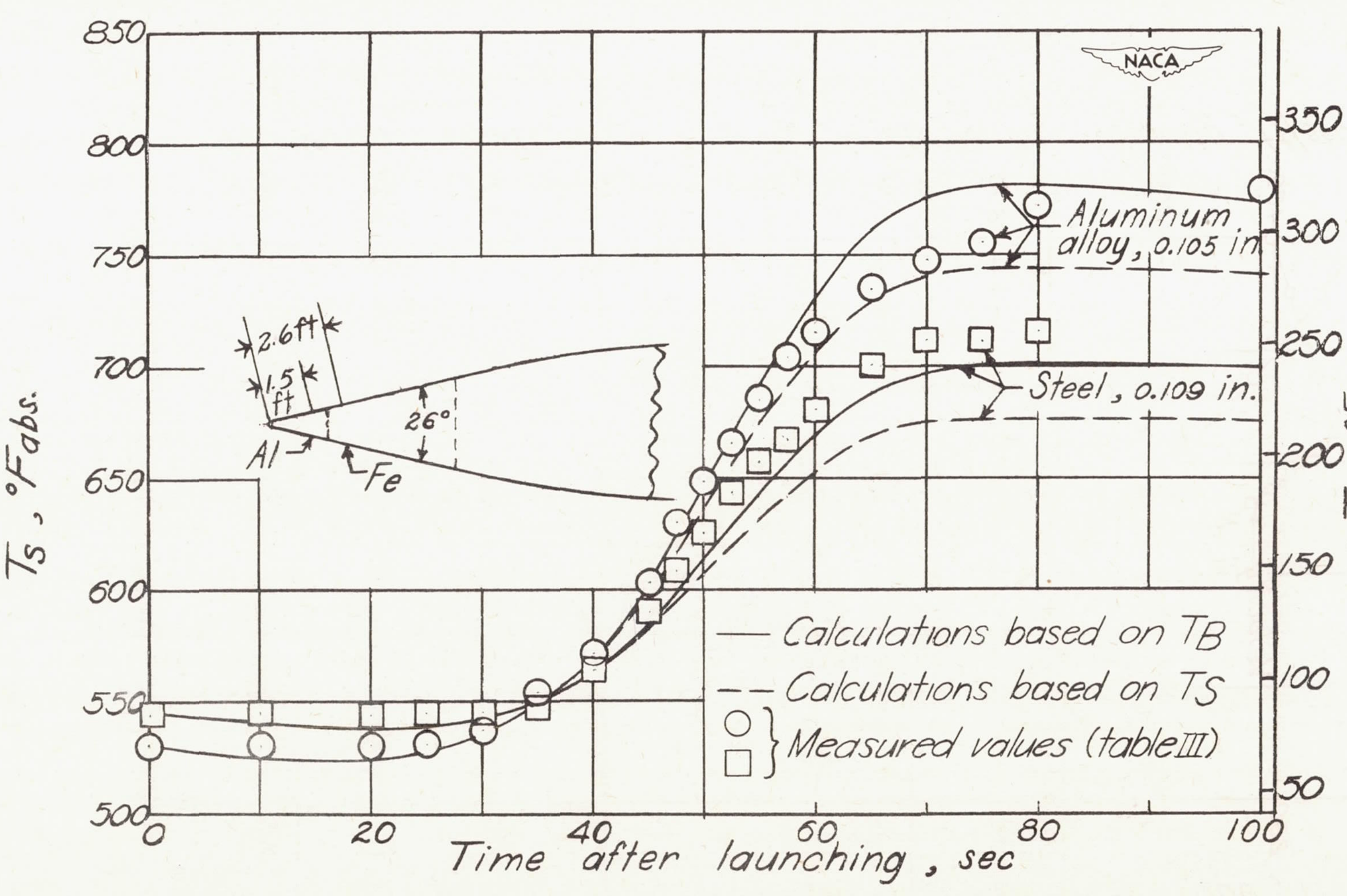
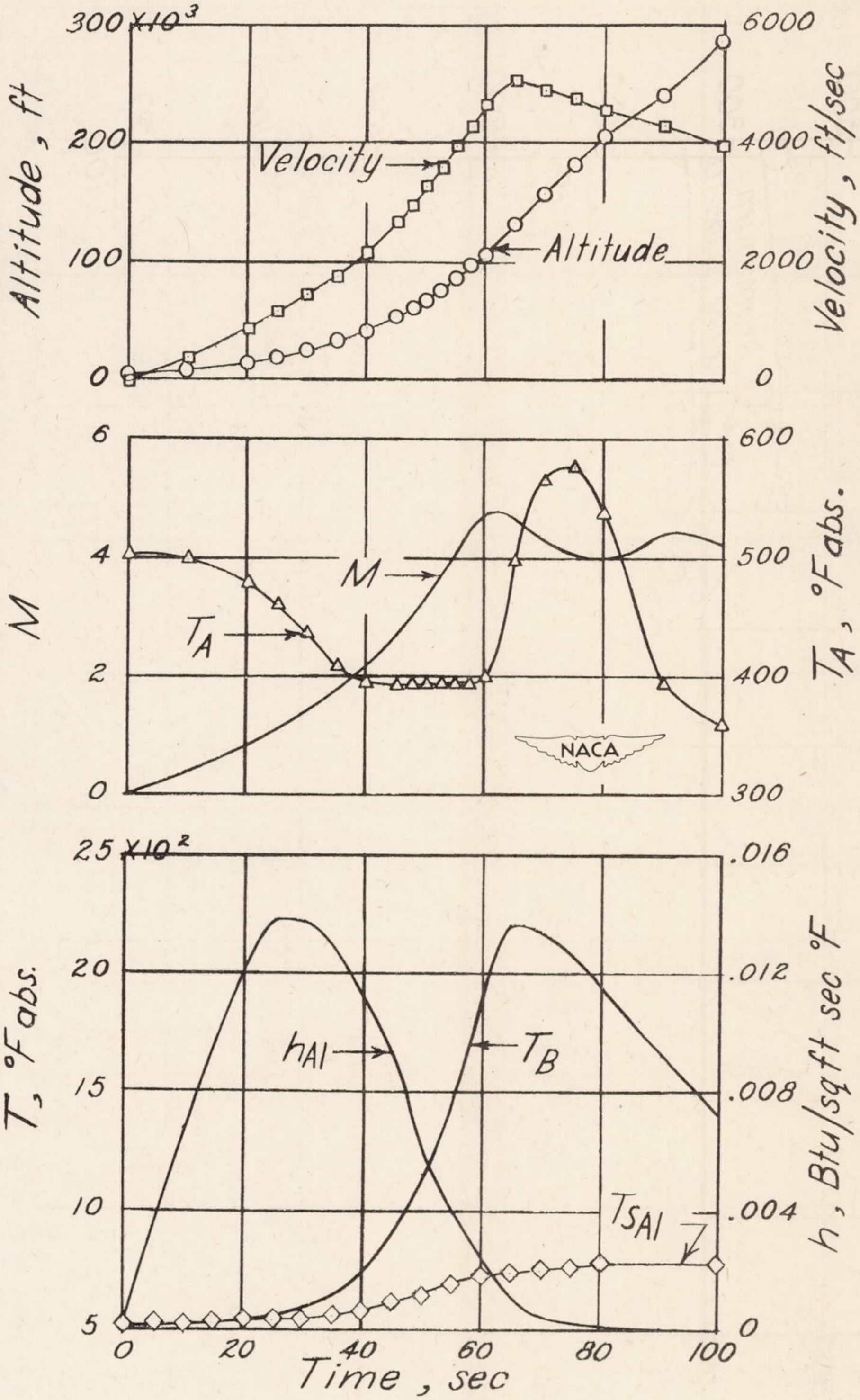


Figure 2.- Skin factor for various aircraft structural materials.



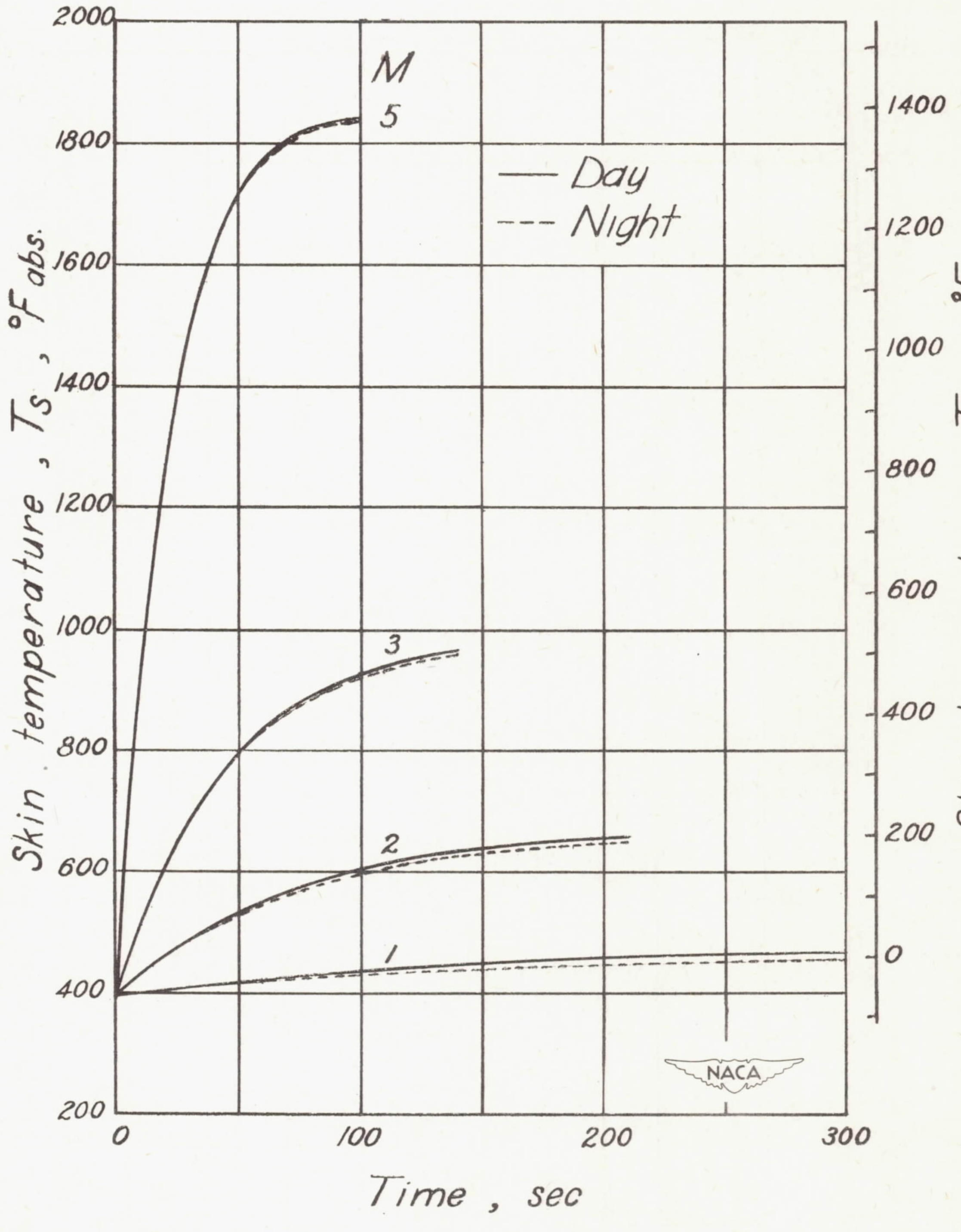
(a) Calculated and measured skin temperatures.

Figure 3.- Time history of calculated and measured quantities during ascending part of flight path of V-2 missile 21.



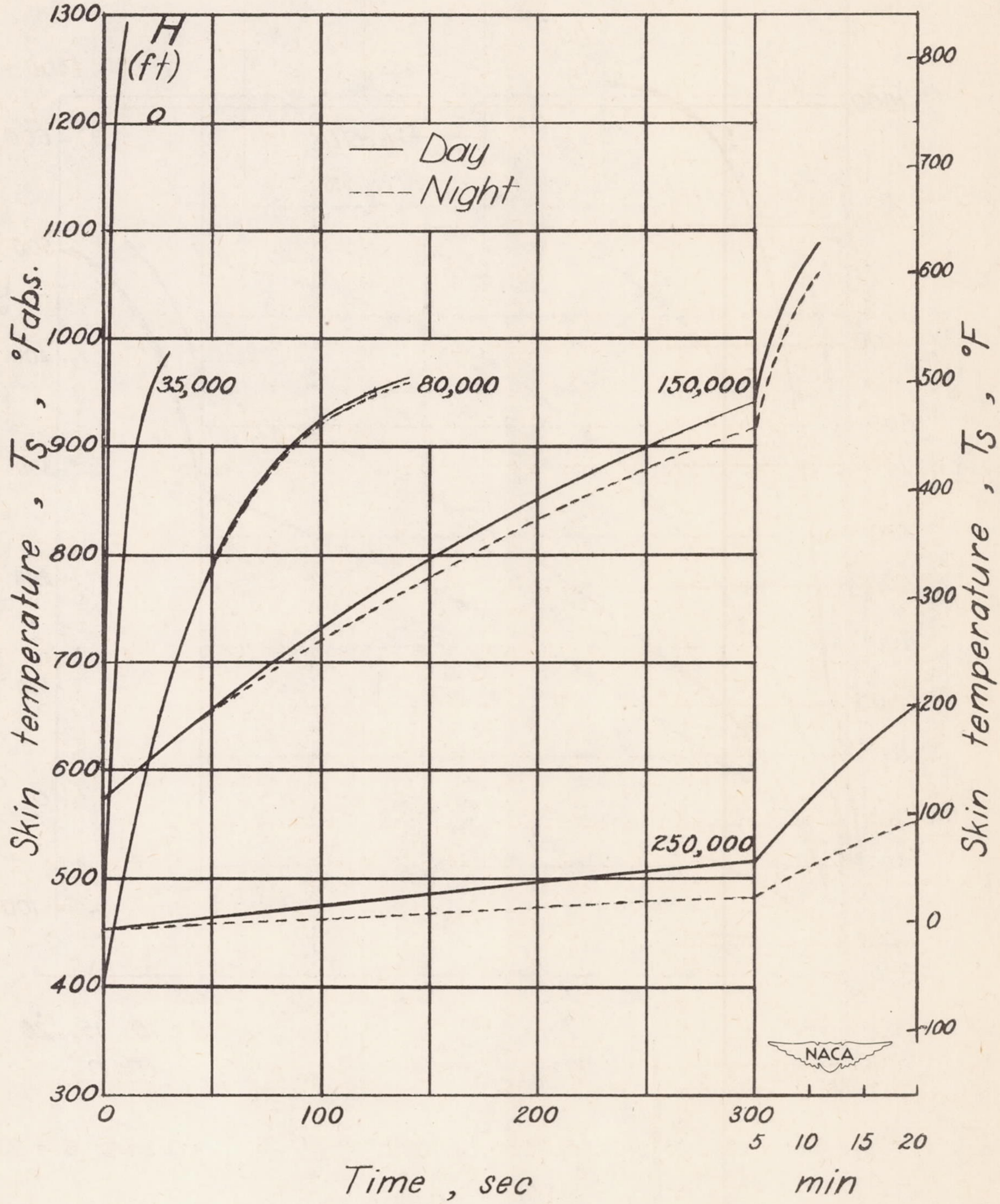
(b) Flight-path parameters.

Figure 3.- Concluded.



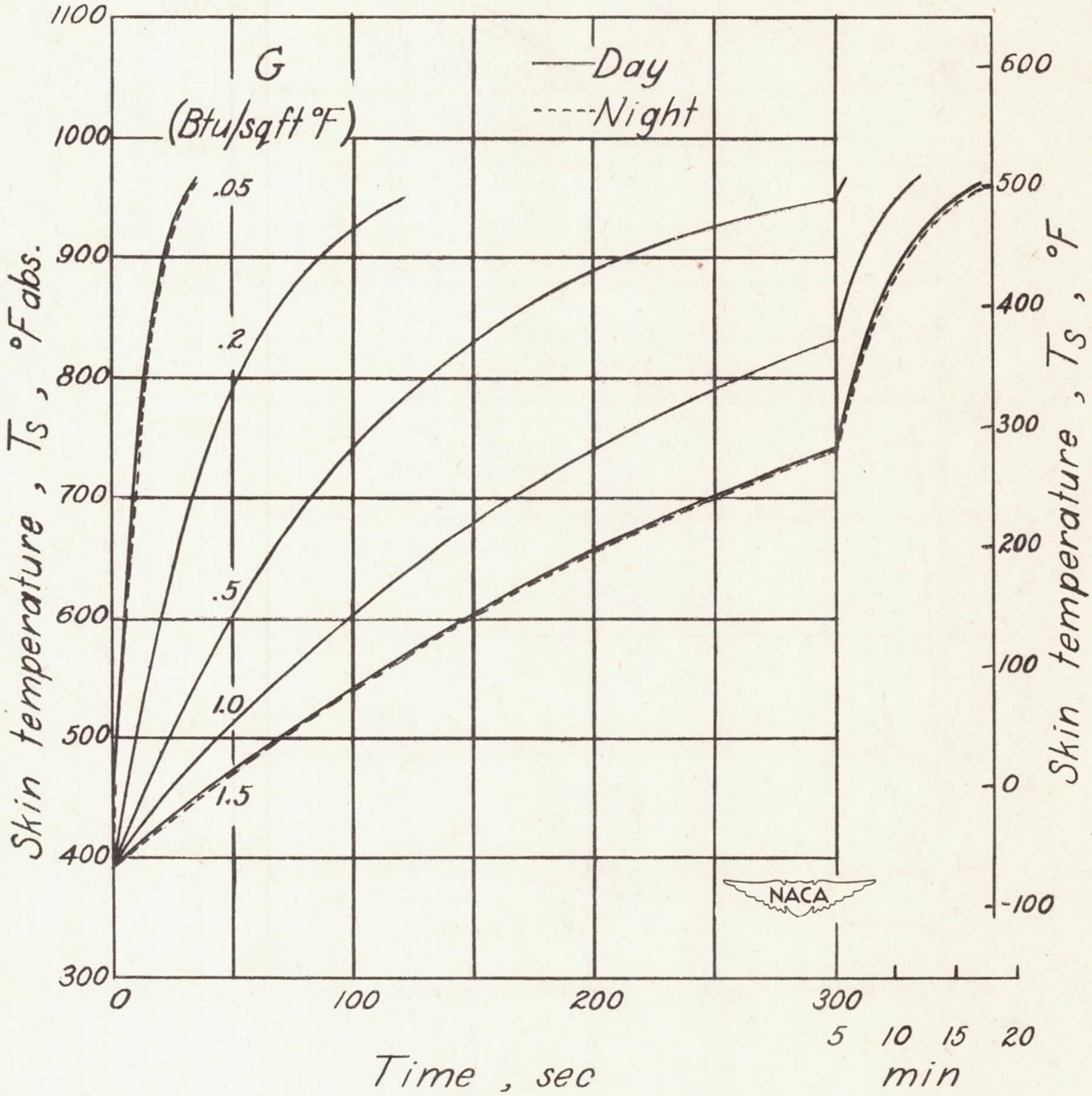
(a) Mach number M. $H = 80{,}000$ feet; $G = 0.2$ Btu/(sq ft)(°F); $\epsilon = 0.2$; $l = 1$ foot; $\beta = 30^{\circ}$.

Figure 4.- Change in skin-temperature time history for various values of design parameter.



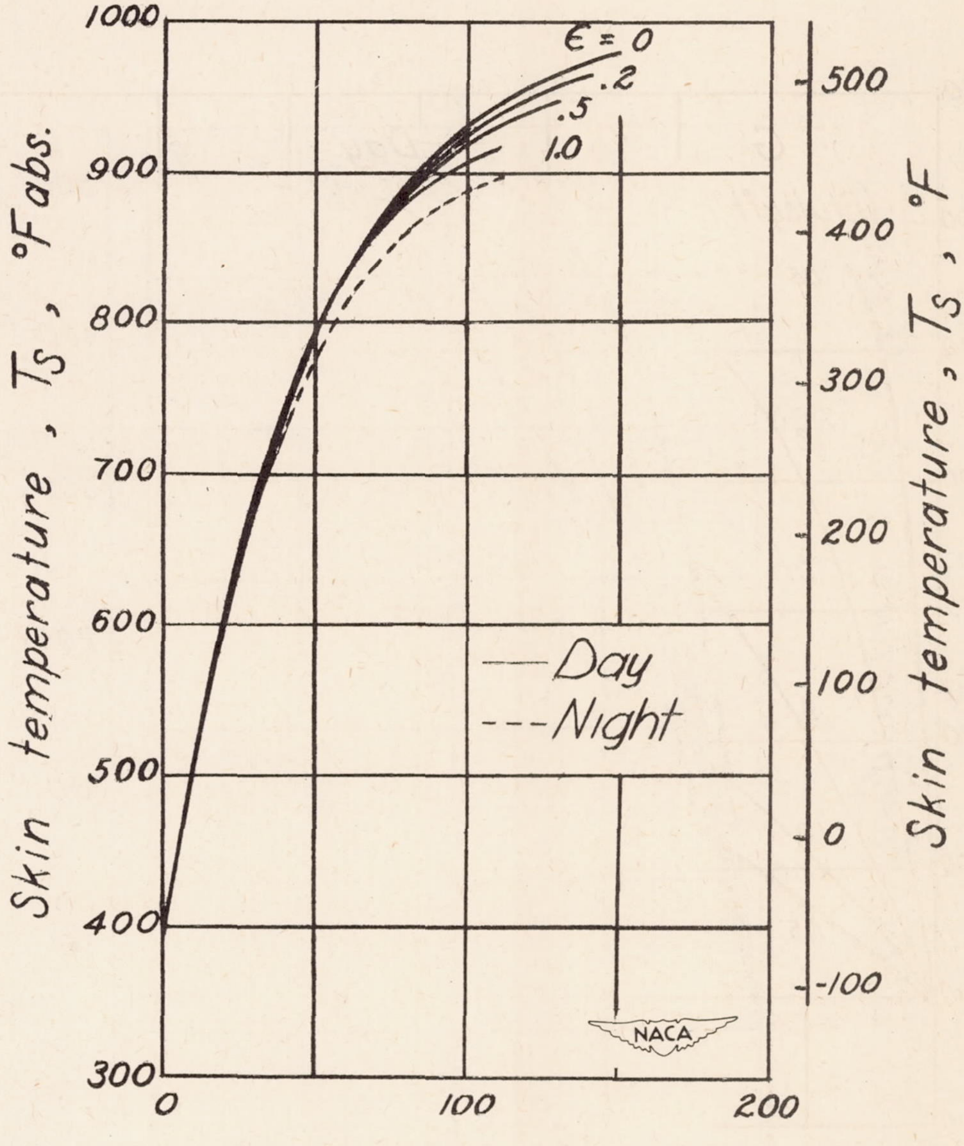
(b) Altitude H. M = 3; G = 0.2 Btu/(sq ft)(^{o}F); ϵ = 0.2; l = 1 foot; β = 30^{o}.

Figure 4.- Continued.



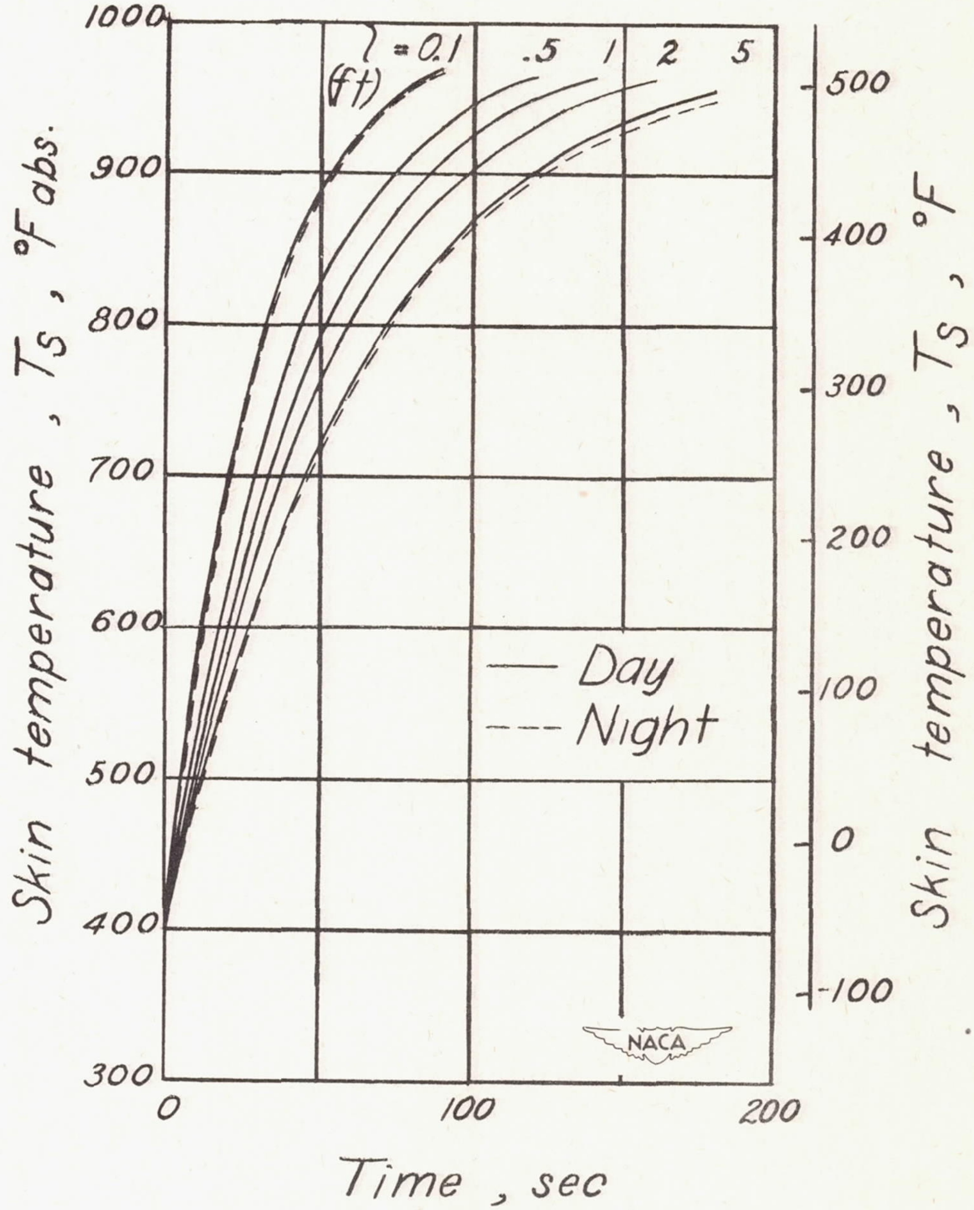
(c) Skin factor G. M = 3; H = 80,000 feet; ϵ = 0.2; l = 1 foot; β = 30°.

Figure 4.- Continued.



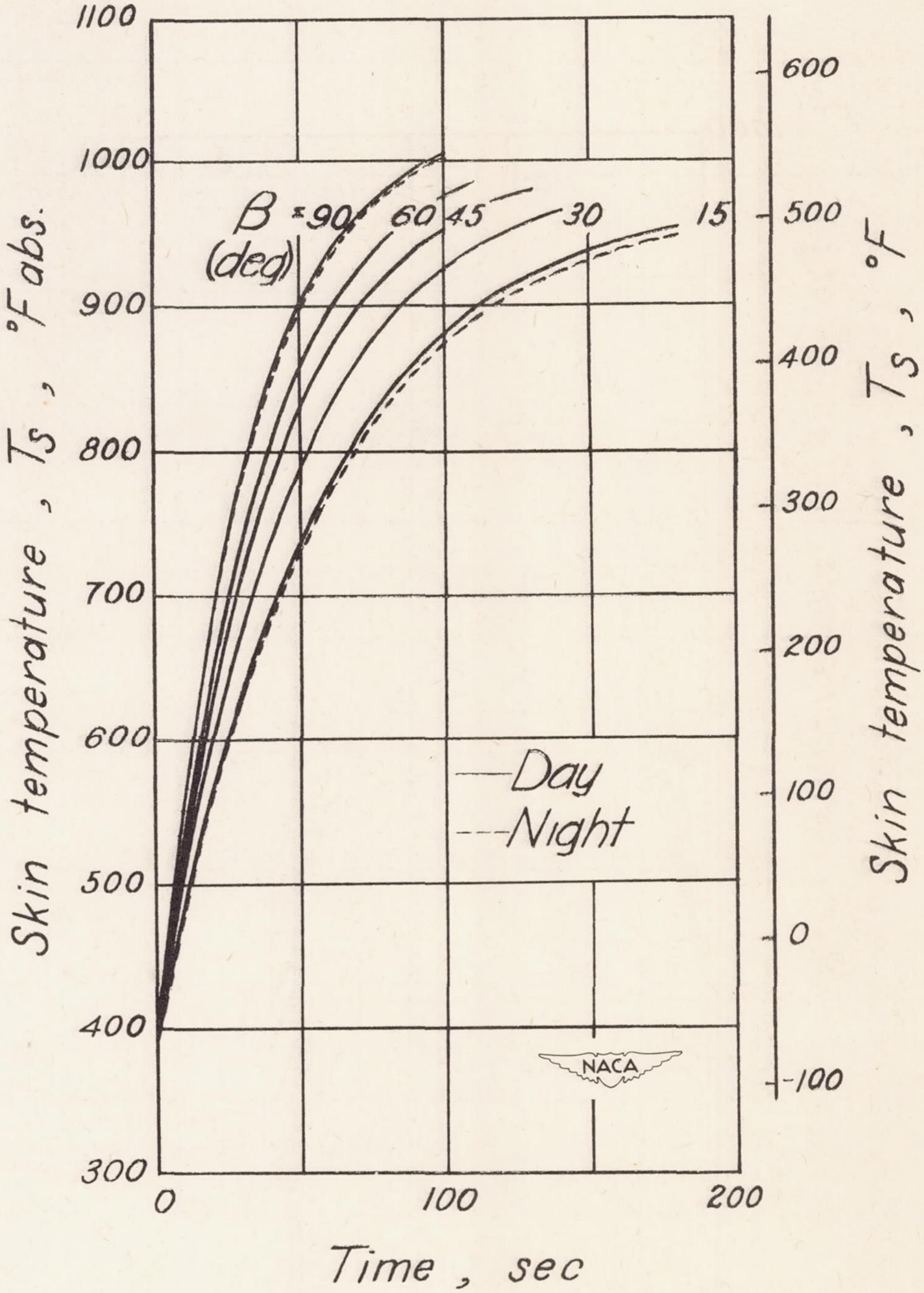
(d) Emissivity ϵ. $M = 3$; $H = 80{,}000$ feet; $G = 0.2$ Btu/(sq ft)($^{\circ}$F);
$l = 1$ foot; $\beta = 30^{\circ}$.

Figure 4.- Continued.



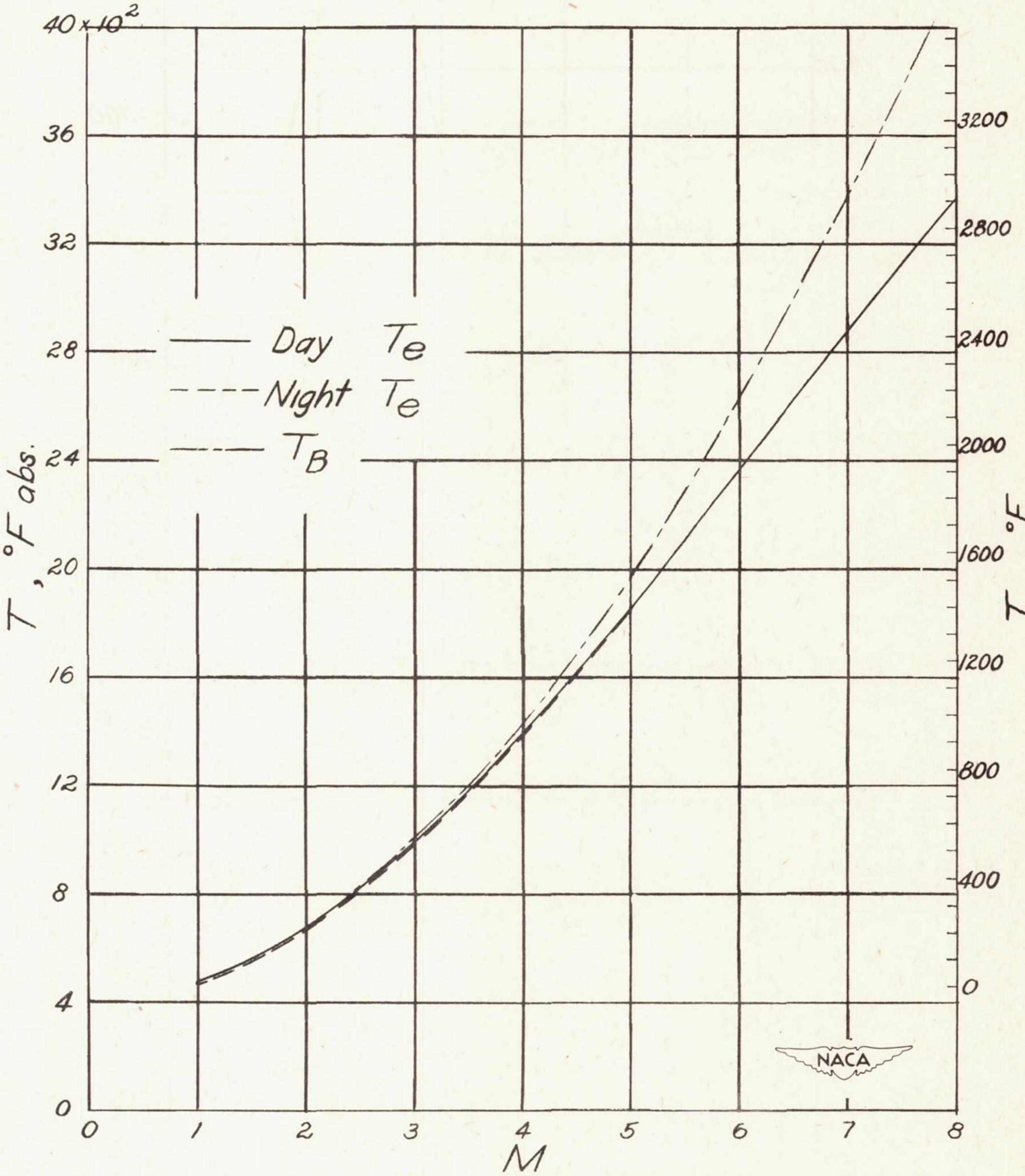
(e) Length l. M = 3; H = 80,000 feet; G = 0.2 Btu/(sq ft)(°F); ϵ = 0.2; β = 30°.

Figure 4.- Continued.



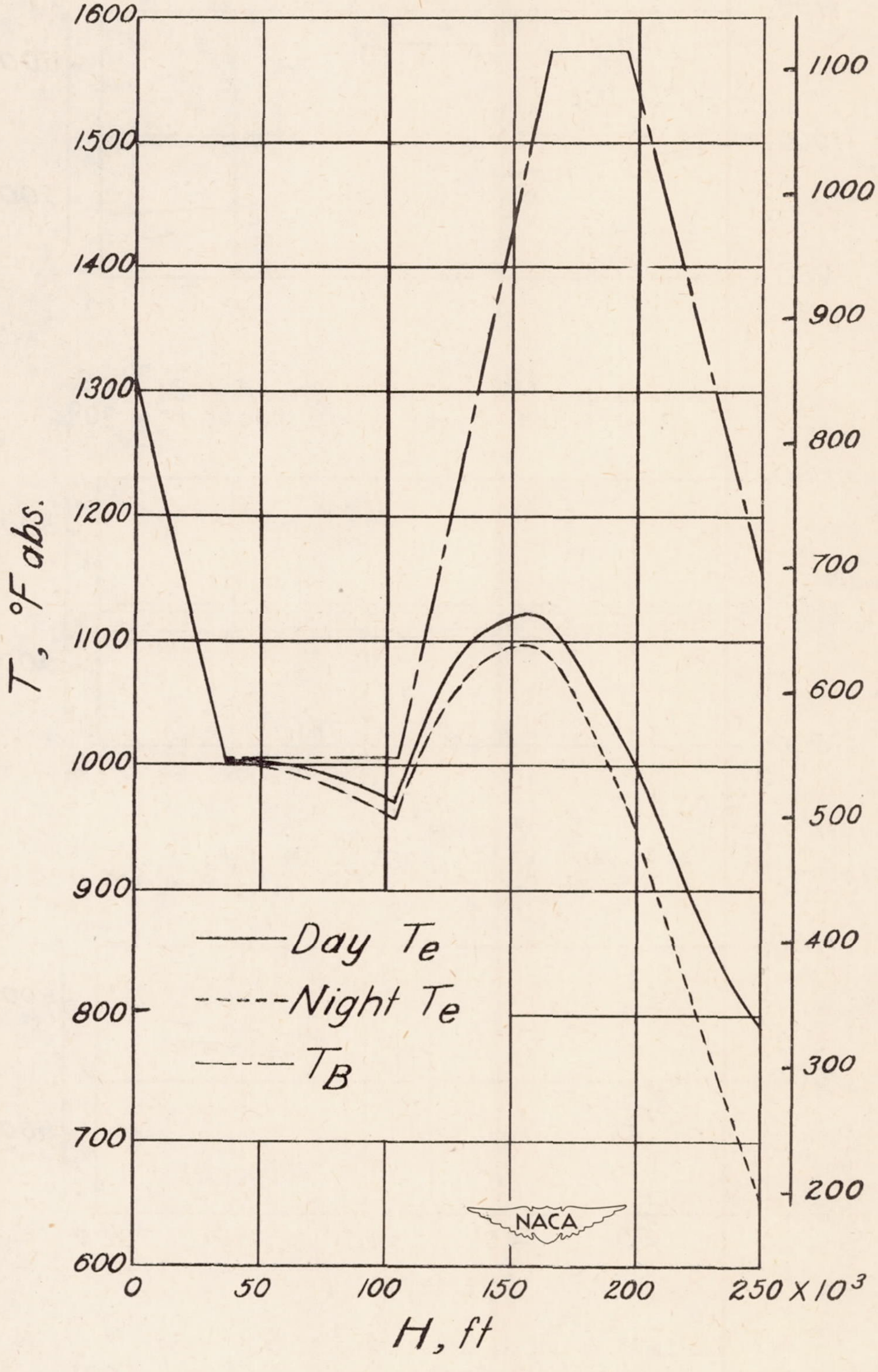
(f) Cone angle β. $M = 3$; $H = 80,000$ feet; $G = 0.2$ Btu/(sq ft)(°F); $\epsilon = 0.2$; $l = 1$ foot.

Figure 4.- Concluded.



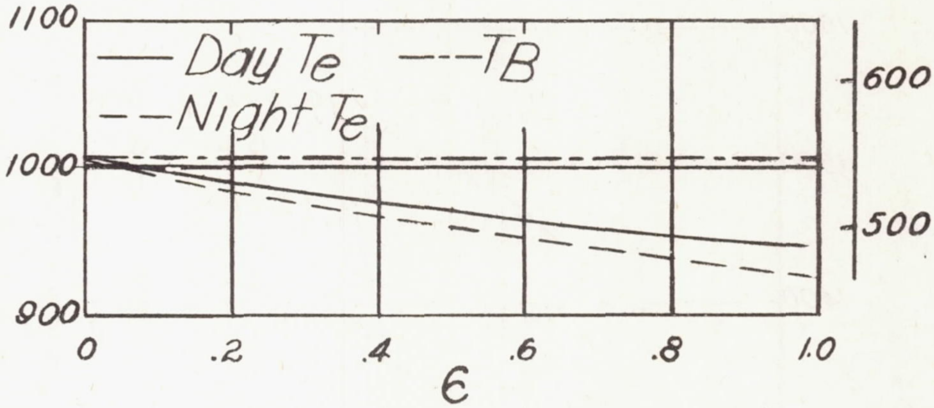
(a) Variation with Mach number. H = 80,000 feet; ϵ = 0.2; l = 1 foot; β = 30°.

Figure 5.- Variation of equilibrium skin temperature T_e; day and night conditions.

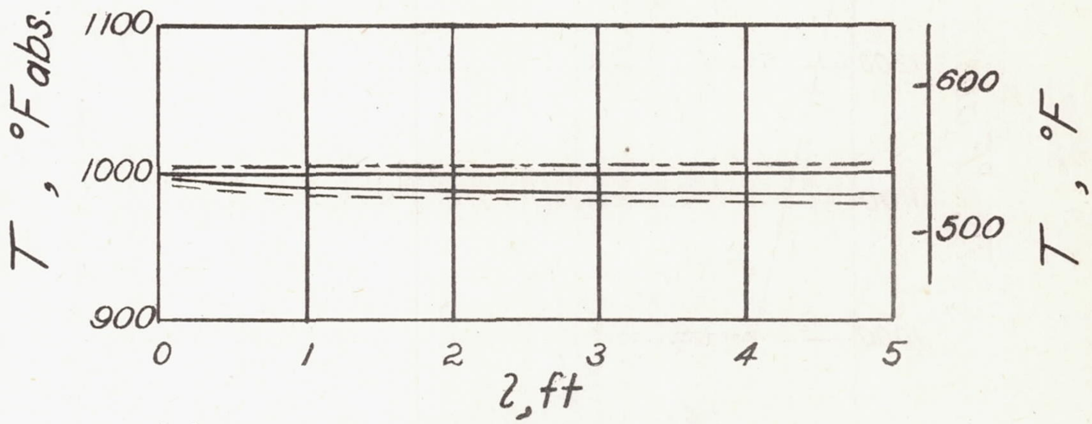


(b) Variation with altitude. M = 3; ϵ = 0.2; l = 1 foot; β = 30°.

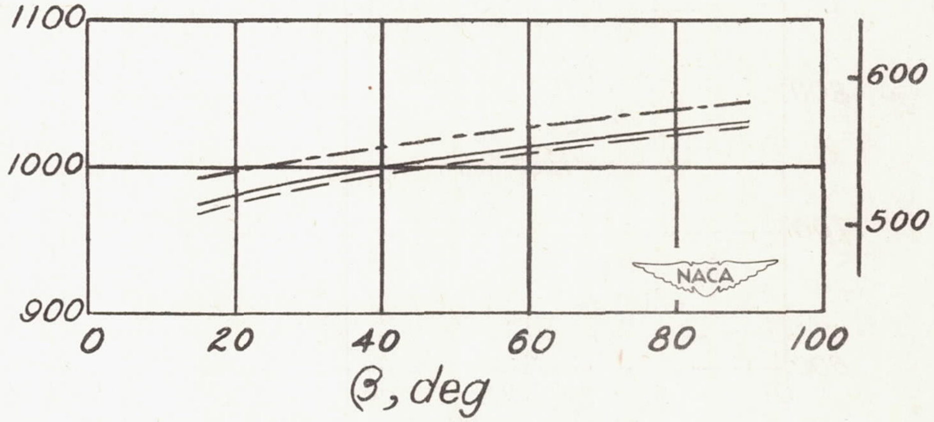
Figure 5.- Continued.



(c) Variation with emissivity. M = 3;
H = 80,000 feet; l = 1 foot; β = 30°.

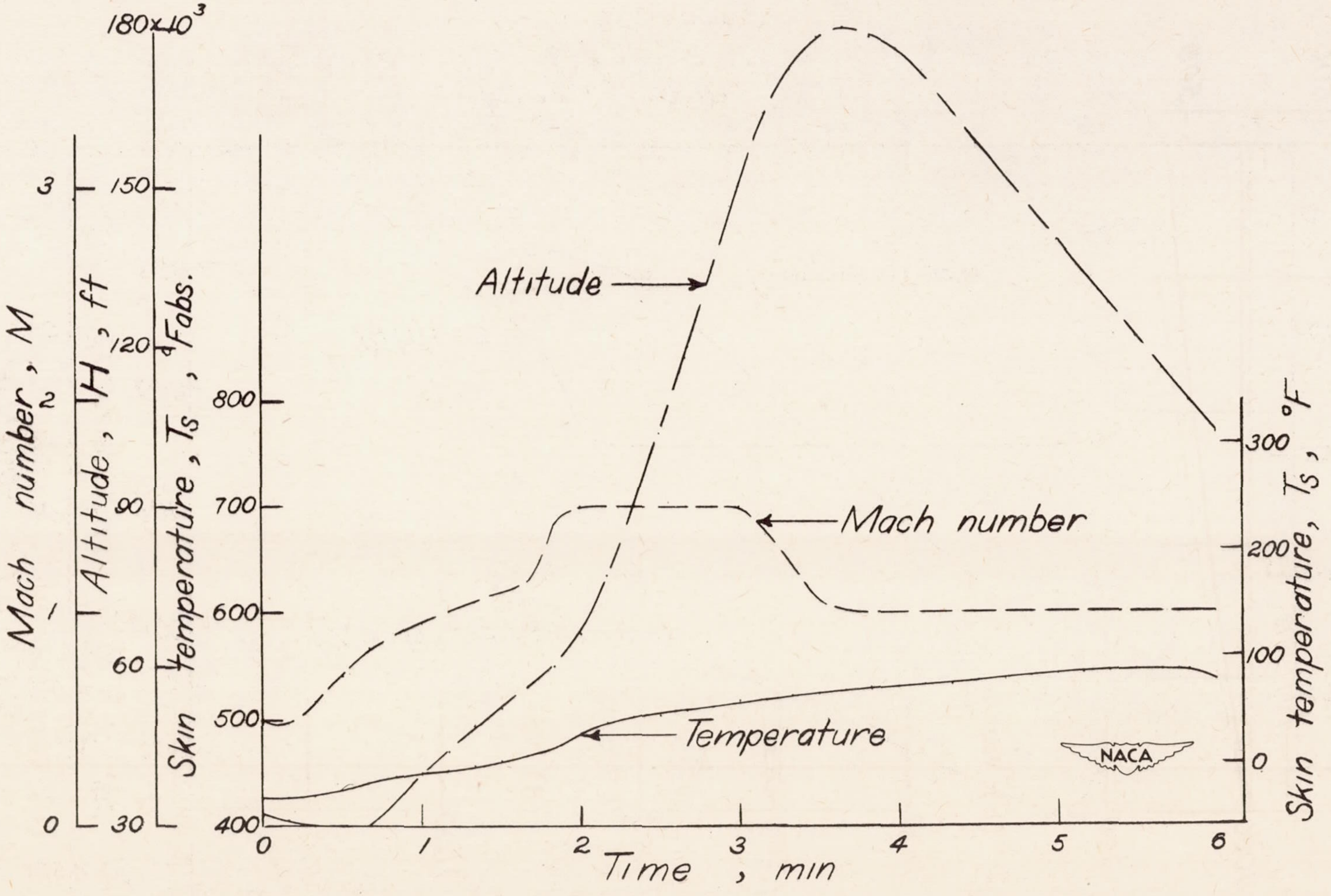


(d) Variation with length. M = 3;
H = 80,000 feet; ϵ = 0.2; β = 30°.



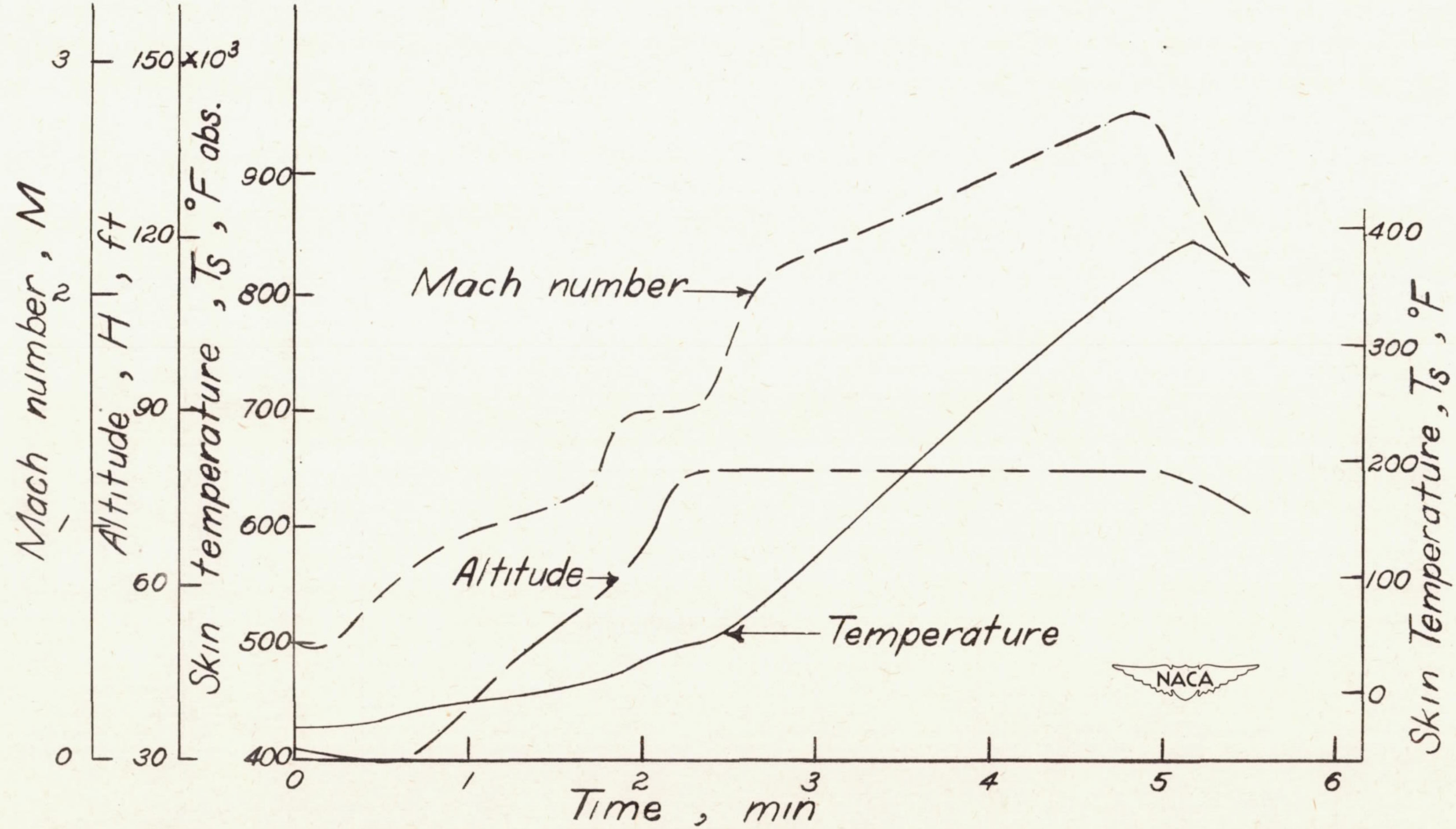
(e) Variation with cone angle. M = 3;
H = 80,000 feet; ϵ = 0.2; l = 1 foot.

Figure 5.- Concluded.



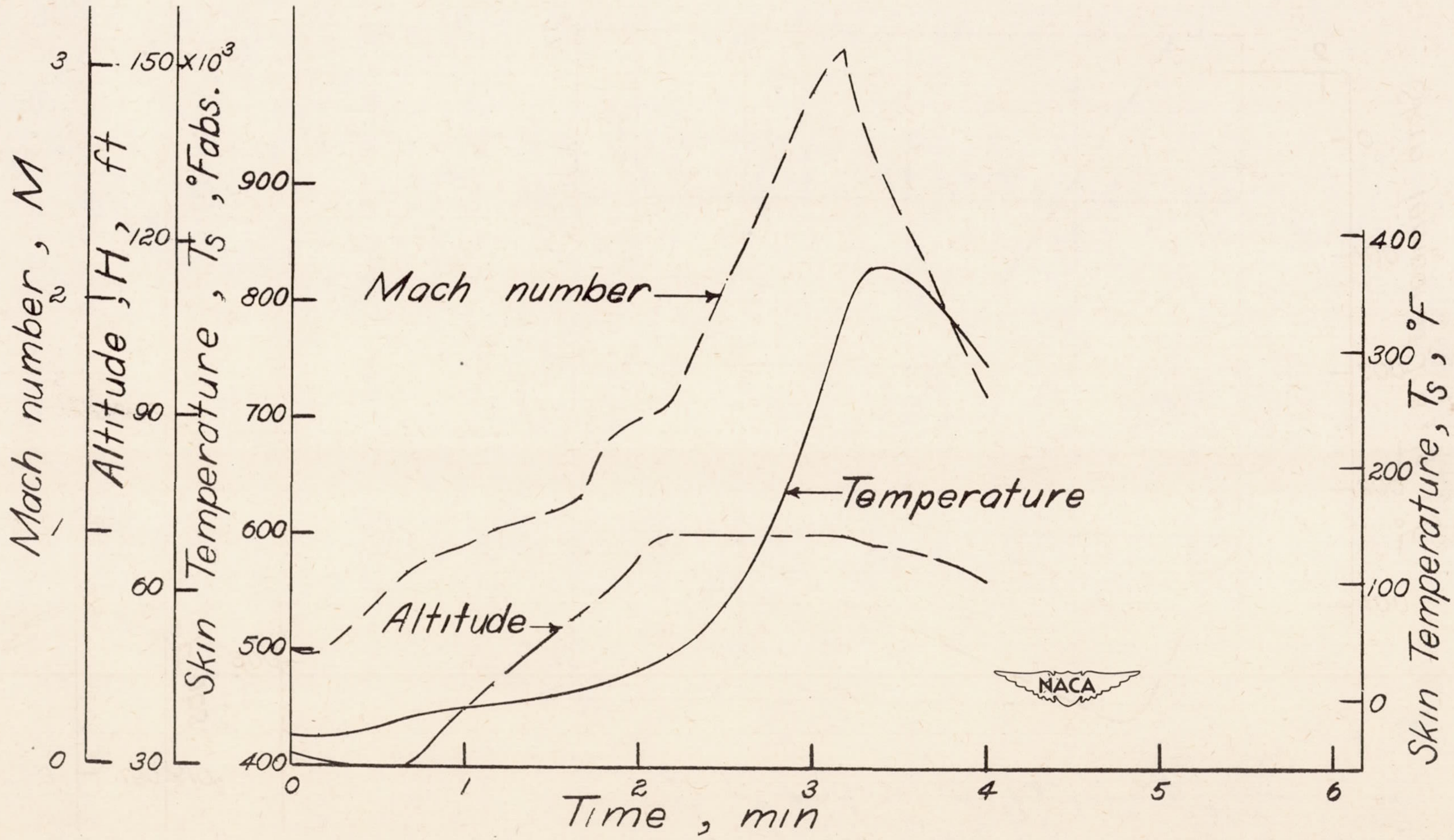
(a) Flight plan A.

Figure 6.- Time history of Mach number, altitude, and calculated skin temperature for hypothetical supersonic flight plans. l = 1 foot; G = 0.2 Btu/(sq ft)(°F).



(b) Flight plan B.

Figure 6.- Continued.



(c) Flight plan C.

Figure 6.- Concluded.

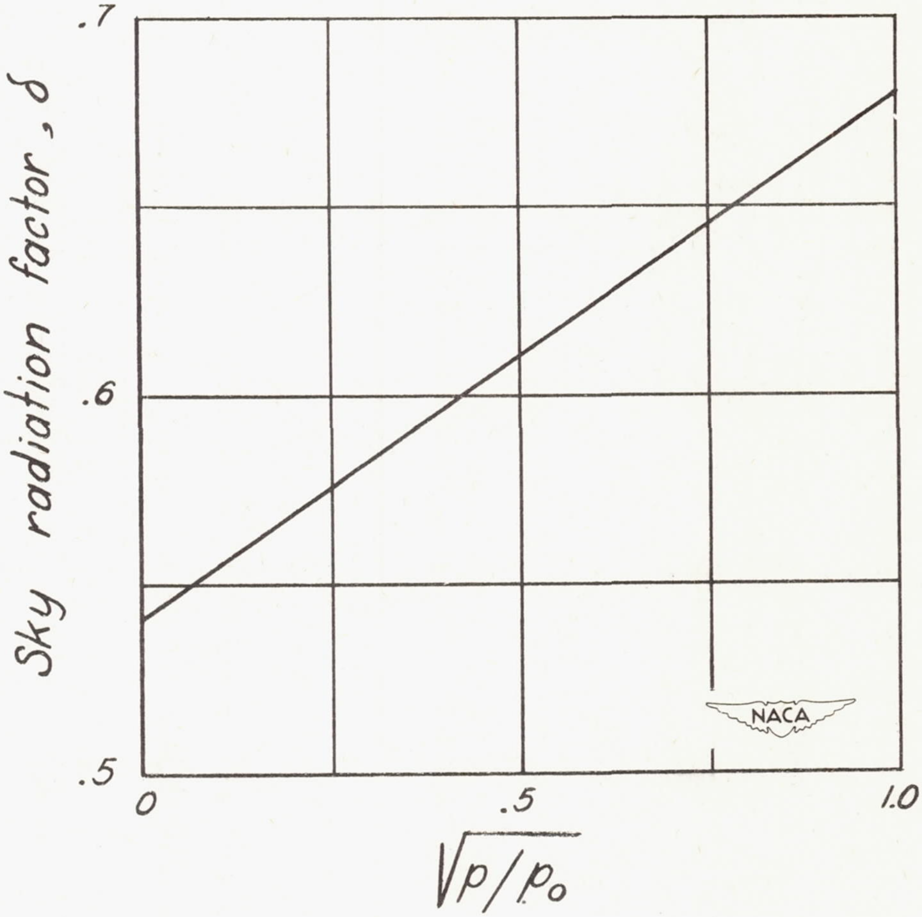


Figure 7.- Sky radiation factor. Data from reference 10.