NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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CRITICAL AXIAL-COMPRESSIVE STRESS OF A CURVED RECTANGULAR PANEL WITH A CENTRAL LONGITUDINAL STIFFENER

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SUMMARY

A theoretical solution is presented for the critical axial-compressive stress of a simply supported curved rectangular panel having a central longitudinal stiffener offering no torsional restraint. The results are presented in the form of computed curves and tables.

Because a panel of moderate or large curvature buckles in compression at a stress lower than the theoretical value, a method is suggested to aid in determining the critical stress for use in design.

INTRODUCTION

As part of an investigation to determine whether the critical axial-compressive load of a curved rectangular panel can be increased by means of a central stiffener, panels having central chordwise stiffeners were treated in reference 1. The present paper gives an analysis for panels having a central longitudinal stiffener. By means of these two papers, the most effective way of reinforcing curved rectangular panels with a single central stiffener to resist axial compression can be determined.

A theoretical solution, based on small-deflection theory, is derived for the critical axial-compressive stress of a curved rectangular panel with a central longitudinal stiffener and theoretical curves present the axial-stress coefficient as a function of the dimensions of the panel and the flexural stiffness and cross-sectional area of the stiffener. Because unstiffened panels of moderate or large curvature buckle in axial compression at loads below the theoretical, a procedure is presented which modifies the theoretical solution for the critical compressive stress of stiffened curved panels and permits an approximation to be made for the actual compressive stress of stiffened panels.

SYMBOLS

a	axial dimension of panel
b	circumferential dimension of panel
k, m, n, p, q	integers
r	radius of curvature of panel
t	thickness of panel
w	displacement of point in median surface of panel in radial direction, positive outward
x	axial coordinate of panel
y	circumferential coordinate of panel
A	cross-sectional area of stiffener
D	flexural stiffness of panel per unit length $\left(\frac{Et^3}{12\left(1 - \mu^2\right)}\right)$
E	Young's modulus of elasticity
I	effective stiffener moment of inertia
L	length of cylinder
Z	curvature parameter $\left(\frac{b^2}{rt}\sqrt{1 - \mu^2}\right)$
k_x	axial-compressive-stress coefficient $\left(\frac{\sigma_x b^2 t}{\pi^2 D}\right)$
β	aspect ratio of panel $\left(\frac{a}{b}\right)$
γ	ratio of flexural stiffness of stiffener to flexural stiffness of plate $\left(\frac{EI}{Db}\right)$

δ ratio of cross-sectional area of stiffener to transverse cross-sectional area of plate $\left(\frac{A}{bt}\right)$

μ Poisson's ratio

σ_x critical axial-compressive stress

$\delta\left(y - \frac{b}{2}\right)$ Dirac δ function defined by $\int_{y_1}^{y_2} f(y)\ \delta\left(y - \frac{b}{2}\right)\ dy = f\left(\frac{b}{2}\right)$ where $y_1 < \frac{b}{2} < y_2$

$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2\ \partial y^2} + \frac{\partial^4}{\partial y^4}$

∇^{-4} inverse of ∇^4 defined by $\nabla^{-4}\left(\nabla^4 w\right) = w$

RESULTS AND DISCUSSION

Theoretical critical stresses.- The critical uniform axial-compressive stress of a simply supported curved rectangular panel with a centrally located longitudinal stiffener having zero torsional stiffness (see fig. 1) is determined from the equation

$$\sigma_x = \frac{k_x \pi^2 D}{b^2 t}$$

A theoretical solution giving k_x as a function of the dimensions of the panel and the flexural stiffness and cross-sectional area of the stiffener is derived in the appendix. The results of this solution are plotted in figure 2 to indicate the relationship between the stress coefficient k_x and $\frac{EI}{Db}$ (the ratio of the flexural stiffness of the stiffener to that of the plate) for specific values of the curvature parameter Z and δ (the ratio of area of stiffener to area of plate). The horizontal portions of the curves indicate the range in which buckling occurs with no deflection of the stiffener so that an increase in the flexural stiffness of the stiffener is not accompanied by a corresponding increase in the compressive-stress coefficient.

A sharp break is found to exist at the lower end of some of the curves. This break corresponds to a change in the buckle pattern from two half waves in the axial direction at low values of $\frac{EI}{Db}$ to one half wave in the axial direction as $\frac{EI}{Db}$ increases. This change in buckle pattern is due to the increasing effect of the column action of the stiffener as compared with the plate action of the sheet as the flexural stiffness of the stiffener increases. Because a simply supported column has its lowest critical load when it buckles into a single half wave, the panel will also buckle into one half wave in the axial direction when $\frac{EI}{Db}$ is sufficiently large. The break in the curves occurs at decreasing values of $\frac{EI}{Db}$, as the curvature parameter Z increases, because the plate strength increases with curvature, and, thus, less flexural stiffness of the stiffener is required to make column action predominate.

The decrease in the critical-stress coefficient k_x with increasing ratio of stiffener area to plate area is also explained by the column action of the stiffener under end load, because a column buckles at a decreasing stress if the area increases while the flexural stiffness remains constant.

Figure 2 also shows that the effect of the central longitudinal stiffener decreases with increasing values of the curvature parameter Z. As the curvature parameter Z increases, the number of buckles in the circumferential direction increases. When there is more than one buckle, additional flexural stiffness at the center of the panel has little effect because the stiffener remains almost straight.

A comparison of the theoretical values for the critical axial-compressive-stress coefficients of unstiffened rectangular curved panels and the coefficients corresponding to buckling of the stiffened plate with a node at the stiffener is presented in table 1. The percentage increase is a measure of the effectiveness of a central longitudinal stiffener in reinforcing curved panels. A comparison of the results of table 1 with the corresponding results of reference 1 indicates that a panel of moderate curvature ($Z < 30$) may be more effectively reinforced to resist axial compression by a longitudinal stiffener at the center than by a chordwise stiffener. Neither a central longitudinal stiffener nor a central chordwise stiffener can appreciably strengthen a panel of higher curvature ($Z > 30$). The comparison also shows that whereas a central chordwise stiffener does not strengthen a panel to any appreciable extent when the ratio of axial to circumferential dimension is greater than 1, a central longitudinal stiffener may considerably strengthen such a panel.

Estimate of design critical stresses.- Unstiffened panels of moderate and high curvature buckle in axial compression at a stress lower than the theoretical stress predicted on the basis of small-deflection theory. (See reference 2.) Curved panels with a central longitudinal stiffener, therefore, may also be expected to buckle at a stress lower than the theoretical stress. In the absence of direct experimental information, the following procedure, which makes allowances for the fact that an axially loaded stiffener will buckle at a stress close to the theoretical stress, is offered as a reasonable method for obtaining an estimate of the actual critical stress of the panel.

From the appropriate part of figure 2, determine the difference between the buckling stress of the stiffened panel and that of the unstiffened panel (reference 2). To this difference add either the design stress of an unstiffened panel having the same value of Z and r/t, as obtained from figure 3 or 4, or the flat-plate buckling stress, whichever is larger. The flat-plate values may be obtained from column (a) of table 1 for $Z = 0$. If the circumferential dimension of the panel is greater than the axial dimension, the design curves for simply supported cylinders from figure 3, taken from reference 3, should be used. If the axial length is equal to or greater than the circumferential length, the design curves for simply supported long curved plates from figure 4, taken from reference 2, should be used. These values are always conservative. The resulting stress should be a fair approximation of the actual critical stress of a stiffened panel.

CONCLUSIONS

The theoretical analysis shows that for certain curvatures a curved rectangular plate can be appreciably strengthened to resist axial compression without buckling by the use of a central longitudinal stiffener. The strengthening effect decreases as the curvature parameter increases. Semiempirical results are presented for the estimation of the axial-compressive buckling stress to be used in the design of a curved rectangular plate with a central longitudinal stiffener.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va., March 15, 1949

APPENDIX

THEORETICAL SOLUTION FOR CRITICAL AXIAL-COMPRESSIVE STRESS OF A CURVED RECTANGULAR PANEL WITH A CENTRAL LONGITUDINAL STIFFENER

Equation of equilibrium.- The critical uniform axial-compressive stress of a curved rectangular panel having a centrally located longitudinal stiffener of zero torsional stiffness (see fig. 1) may be obtained by solving the equation of equilibrium (reference 4)

$$D \nabla^4 w + \frac{Et}{r^2} \nabla^{-4} \frac{\partial^4 w}{\partial x^4} + EI\,\delta\left(y - \frac{b}{2}\right) \frac{\partial^4 w}{\partial x^4}$$

$$+ \sigma_x t \frac{\partial^2 w}{\partial x^2} + \sigma_x A\,\delta\left(y - \frac{b}{2}\right) \frac{\partial^2 w}{\partial x^2} = 0 \qquad (1)$$

The equation of equilibrium may be represented by

$$Q(w) = 0 \qquad (2)$$

Method of solution.- Equation (1) may be solved by the Galerkin method as outlined in references 4 and 5. As suggested in reference 4 for simply supported rectangular panels, the following series expansion is used for w

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \qquad (3)$$

The coefficients a_{mn} are determined by the condition that they satisfy the relationships

$$\int_0^a \int_0^b \sin \frac{p\pi x}{a} \sin \frac{q\pi y}{b} Q(w)\, dy\, dx = 0 \qquad \begin{array}{l}(p = 1, 2, 3, \ldots)\\(q = 1, 2, 3, \ldots)\end{array} \qquad (4)$$

Substituting for w from equation (3) in equation (4) and performing the indicated operations result in the following set of linear homogeneous algebraic equations for the coefficients a_{mn}:

$$a_{pq}\left[\left(p^2 + q^2\beta^2\right)^2 + \frac{12}{\pi^4}\frac{Z^2\beta^4p^4}{\left(p^2 + q^2\beta^2\right)^2} - k_xp^2\beta^2\right]$$

$$+ 2\left(\gamma p^4 - k_x\delta p^2\beta^2\right)\sin\frac{q\pi}{2}\sum_k a_{pk}\sin\frac{k\pi}{2} = 0 \tag{5}$$

$$\begin{pmatrix}p = 1, 2, \ldots\\ q = 1, 2, \ldots\end{pmatrix}$$

These equations may be written as two independent sets:

$$a_{pq}M_{pq} + 2\left(\gamma p^4 - k_x\delta p^2\beta^2\right)(-1)^{\frac{q-1}{2}}\sum_{k=1,3,\ldots}^{\infty} a_{pk}(-1)^{\frac{k-1}{2}} = 0 \tag{6}$$

$$\begin{pmatrix}p = 1, 2, 3, \ldots\\ q = 1, 3, 5, \ldots\end{pmatrix}$$

corresponding to buckling across the stiffener and

$$a_{pq}M_{pq} = 0 \qquad \begin{pmatrix}p = 1, 2, 3, \ldots\\ q = 2, 4, 6, \ldots\end{pmatrix} \tag{7}$$

corresponding to buckling with a node at the stiffener where

$$M_{pq} = \left(p^2 + q^2\beta^2\right)^2 + \frac{12}{\pi^4}\frac{Z^2\beta^4p^4}{\left(p^2 + q^2\beta^2\right)^2} - k_xp^2\beta^2$$

Equation (6) may be rearranged to yield

$$a_{pq} + \frac{2\left(\gamma p^4 - k_x\delta p^2\beta^2\right)}{M_{pq}}(-1)^{\frac{q-1}{2}}\sum_{k=1,3}^{\infty} a_{pk}(-1)^{\frac{k-1}{2}} = 0 \tag{8}$$

$$\begin{pmatrix}p = 1, 2, \ldots\\ q = 1, 3, \ldots\end{pmatrix}$$

Multiplication of each equation by $(-1)^{\frac{q-1}{2}}$ and summation with respect to q give

$$\sum_{q=1,3}^{\infty} a_{pq}(-1)^{\frac{q-1}{2}} + 2\left(\gamma p^4 - k_x \delta p^2 \beta^2\right) \sum_{q=1,3}^{\infty} \frac{1}{M_{pq}} \sum_{k=1,3}^{\infty} a_{pk}(-1)^{\frac{k-1}{2}} = 0 \qquad (9)$$

Dividing equation (9) by the quantity

$$\sum_{q=1,3}^{\infty} a_{pq}(-1)^{\frac{q-1}{2}} = \sum_{k=1,3}^{\infty} a_{pk}(-1)^{\frac{k-1}{2}}$$

and solving for γ yield

$$\gamma = \frac{-1}{2p^4 \sum\limits_{q=1,3}^{\infty} \frac{1}{M_{pq}}} + k_x \frac{\delta\beta^2}{p^2} \qquad (10a)$$

Equation (10a) in different form is

$$\frac{EI}{Db} = \frac{-1}{2p^4 \sum\limits_{q=1,3}^{\infty} \dfrac{1}{\left(p^2 + q^2\beta^2\right)^2 + \dfrac{12}{\pi^4} \dfrac{Z^2\beta^4 p^4}{\left(p^2 + q^2\beta^2\right)^2} - k_x p^2 \beta^2}} + k_x \frac{\delta\beta^2}{p^2} \qquad (10b)$$

The value of p to be used in equation (10) is that value which yields the maximum value of $\frac{EI}{Db}$ needed to maintain equilibrium of the panel-stiffener combination for given values of k_x, β, δ, and Z.

The solutions of equations (8) which correspond to buckling with a node at the stiffener are given by

$$M_{pq} = 0 \qquad \begin{pmatrix} p = 1, 2, \ldots \\ q = 2, 4, \ldots \end{pmatrix}$$

or

$$\left(p^2 + q^2\beta^2\right)^2 + \frac{12}{\pi^4} \frac{Z^2\beta^4 p^4}{\left(p^2 + q^2\beta^2\right)^2} - k_x p^2\beta^2 = 0 \qquad \begin{pmatrix} p = 1,\ 2,\ \ldots \\ q = 2,\ 4,\ \ldots \end{pmatrix}$$

REFERENCES

1. Batdorf, S. B., and Schildcrout, Murry: Critical Axial-Compressive Stress of a Curved Rectangular Panel with a Central Chordwise Stiffener. NACA TN No. 1661, 1948.

2. Batdorf, S. B., Schildcrout, Murry, and Stein, Manuel: Critical Combinations of Shear and Longitudinal Direct Stress for Long Plates with Transverse Curvature. NACA TN No. 1347, 1947.

3. Batdorf, S. B., Schildcrout, Murry, and Stein, Manuel: Critical Stress of Thin-Walled Cylinders in Axial Compression. NACA Rep. No. 887, 1947.

4. Batdorf, S. B.: A Simplified Method of Elastic-Stability Analysis for Thin Cylindrical Shells. II – Modified Equilibrium Equation. NACA TN No. 1342, 1947.

5. Duncan, W. J.: The Principles of the Galerkin Method. R. & M. No. 1848, British A.R.C., 1938.

TABLE 1.– CRITICAL AXIAL-COMPRESSIVE-STRESS COEFFICIENTS FOR BUCKLING WITHOUT STIFFENER AND FOR BUCKLING WITH NODE AT STIFFENER OFFERING ZERO TORSIONAL RESTRAINT

$\frac{a}{b}$	$Z = \frac{b^2}{rt}\sqrt{1-\mu^2}$	$k_x = \frac{\sigma_x t b^2}{\pi^2 D}$ (a) Buckling without stiffener	$k_x = \frac{\sigma_x t b^2}{\pi^2 D}$ (b) Buckling with node at stiffener	Percentage increase $\frac{(b)-(a)}{(a)} \times 100$
$\frac{2}{3}$	0	4.70	17.4	269
	5	5.35	17.5	228
	10	7.32	18.1	147
	30	21.1	23.7	13
	100	70.2	70.4	0
	1000	703	708	1
1	0	4.00	16.0	300
	5	4.77	16.2	240
	10	7.08	16.8	137
	30	21.1	22.9	9
	100	70.3	70.4	0
	1000	703	709	1
1.5	0	4.34	16.0	269
	5	5.05	16.2	221
	10	7.18	16.8	134
	30	21.2	22.9	8
	100	70.2	70.8	1
	1000	704	708	1
2	0	4.00	16.0	300
	10	7.08	16.8	137
	30	21.1	22.9	9
	100	70.3	70.3	0
	1000	704	708	1

NACA

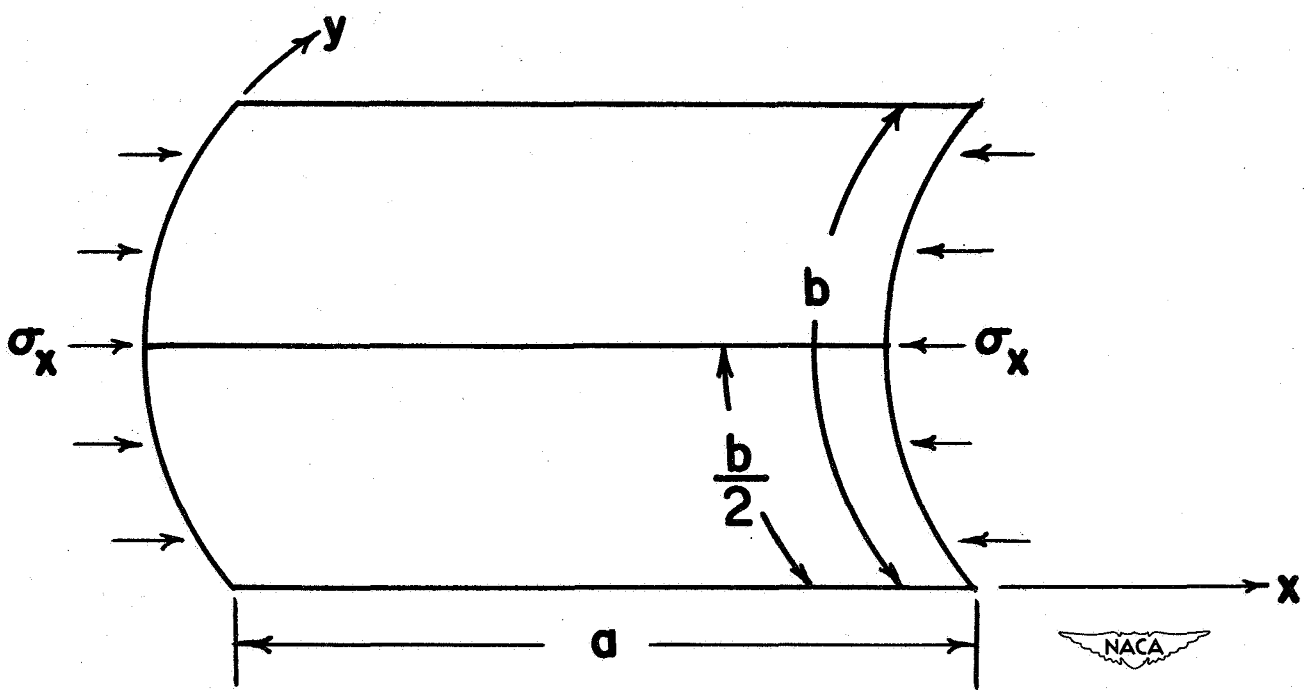
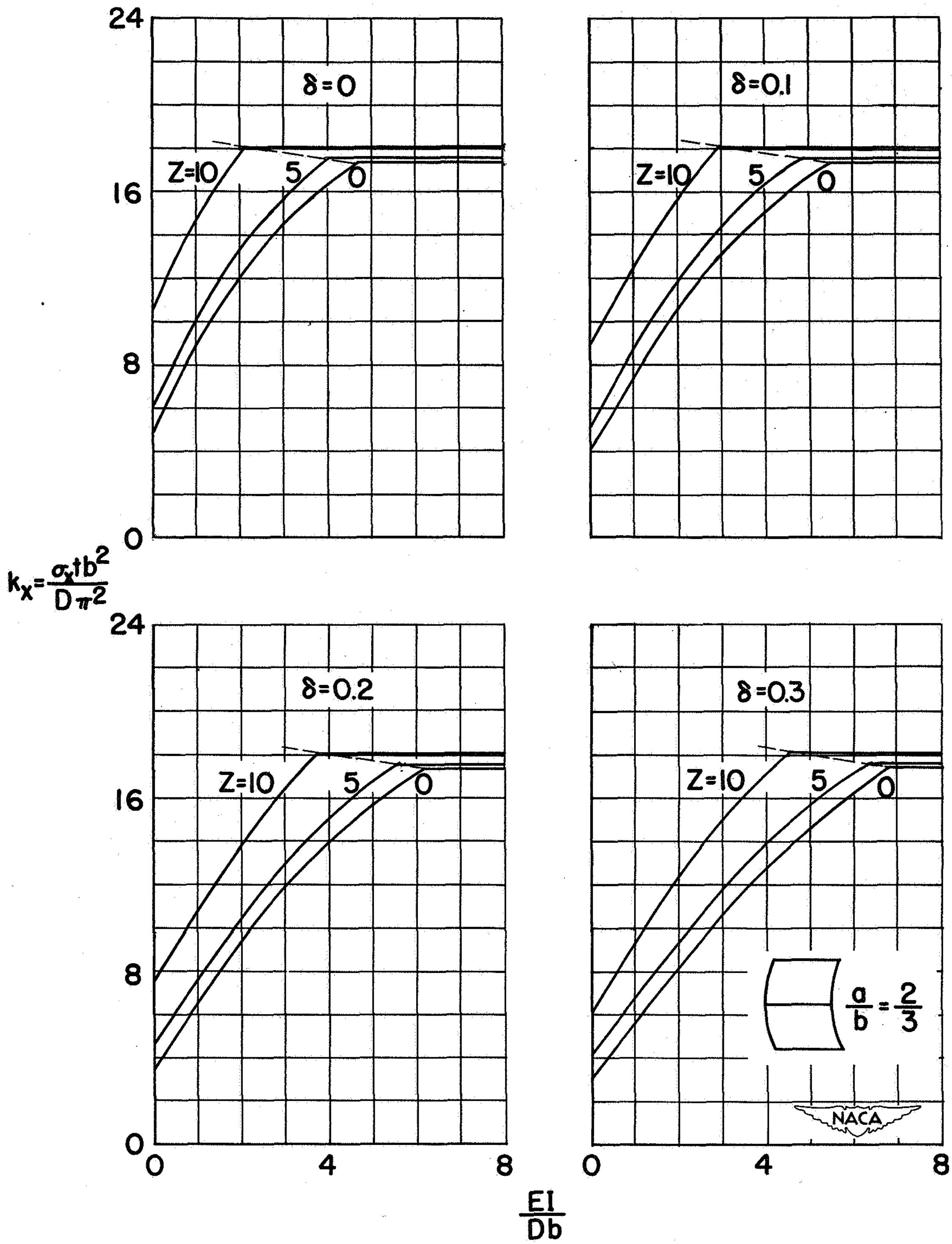
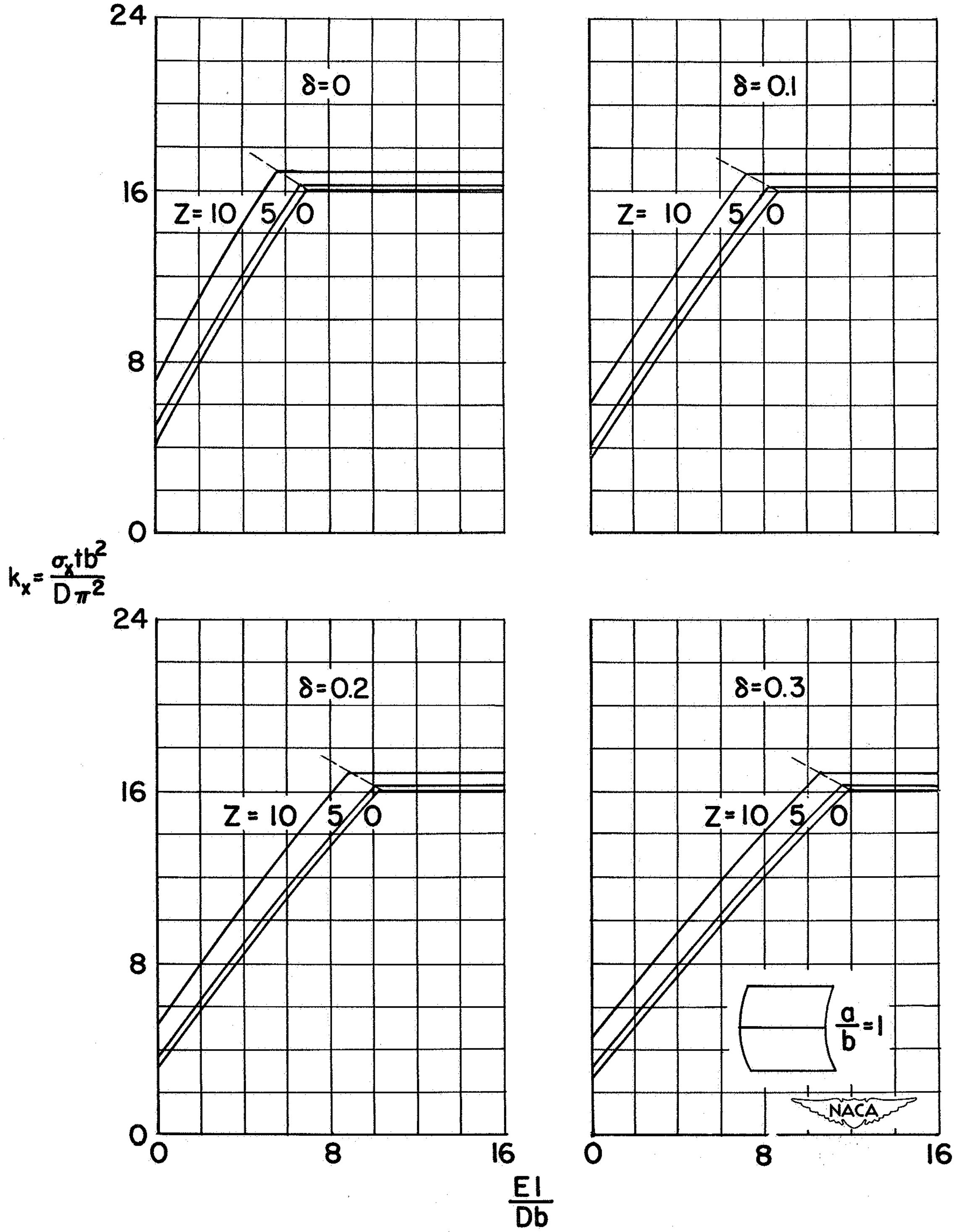


Figure 1.– Coordinate system used in the analysis.



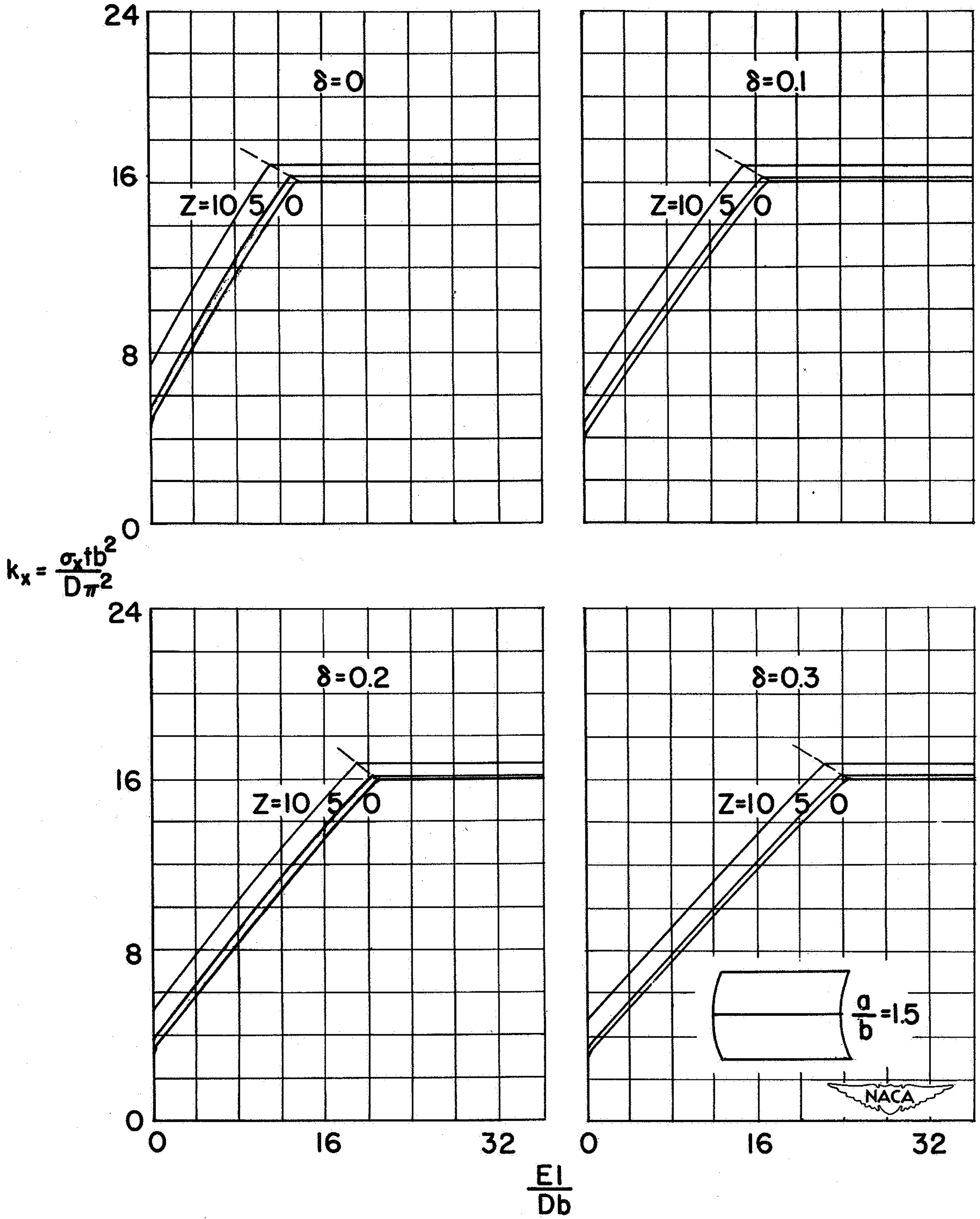
(a) $\frac{a}{b} = \frac{2}{3}$.

Figure 2.– Critical axial–compressive–stress coefficients for rectangular curved plates with a central longitudinal stiffener.



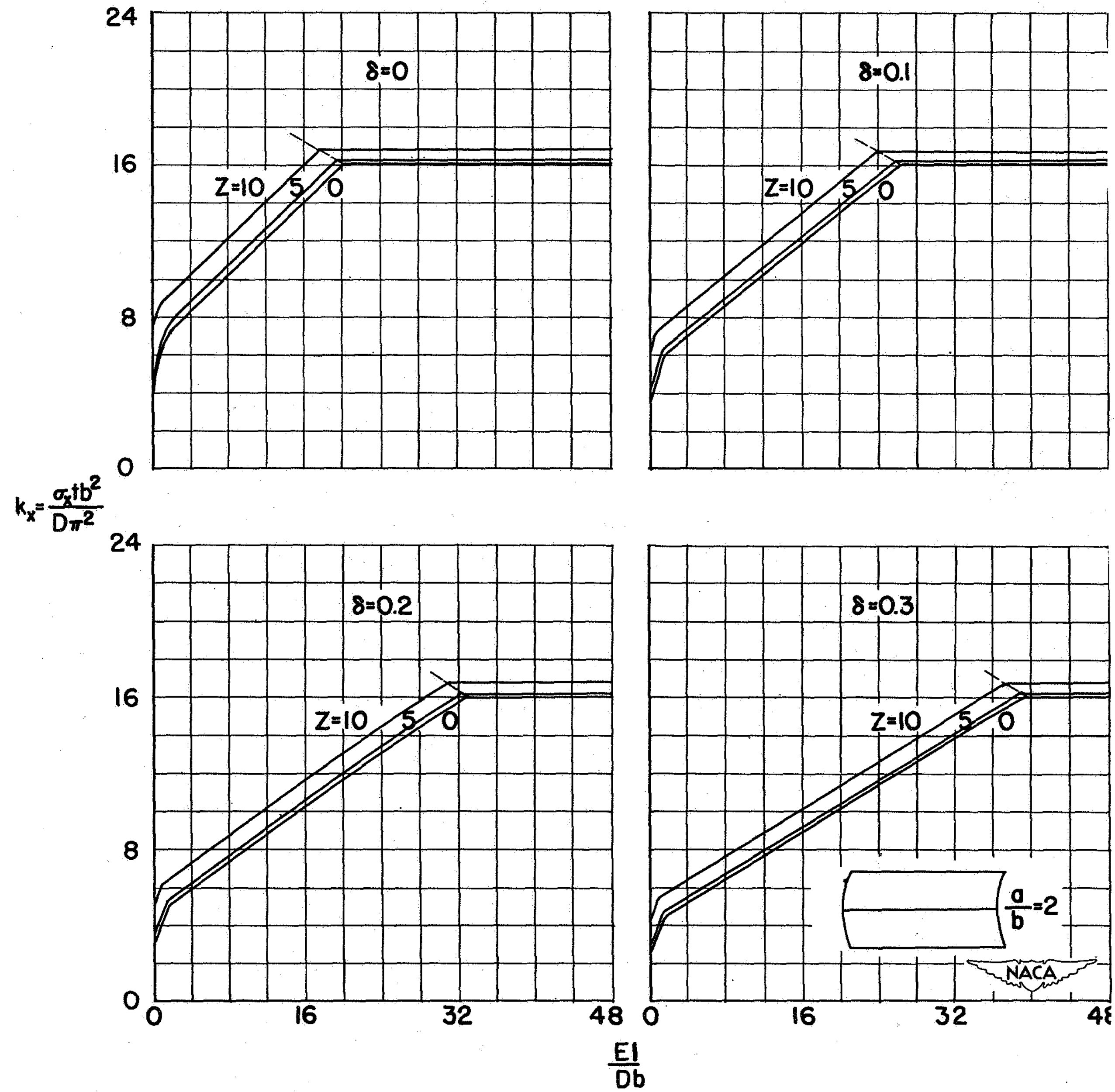
(b) $\frac{a}{b} = 1.$

Figure 2.– Continued.



(c) $\frac{a}{b} = 1.5$.

Figure 2.– Continued.



24
16
8
0
δ=0
Z=10
5
0
δ=0.1
Z=10
5
0
$k_x = \frac{\sigma_x t b^2}{D\pi^2}$
δ=0.2
Z=10
5
0
δ=0.3
Z=10
5
0
0
16
32
48
$\frac{a}{b}=2$
NACA
$\frac{EI}{Db}$

(d) $\frac{a}{b} = 2.$

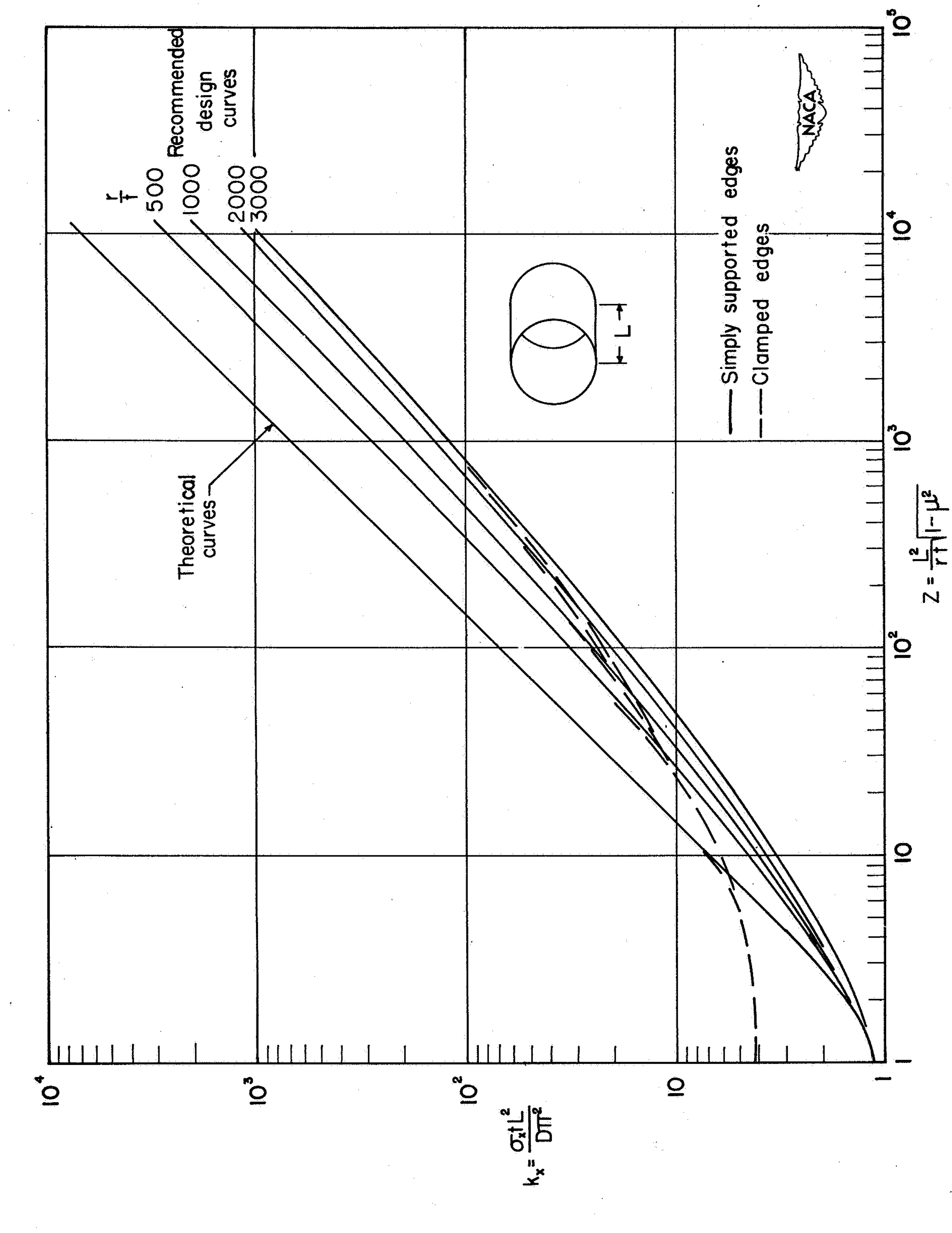


Figure 3.— Critical axial-compressive-stress coefficients for simply supported cylinders. (Figure adapted from reference 3.)

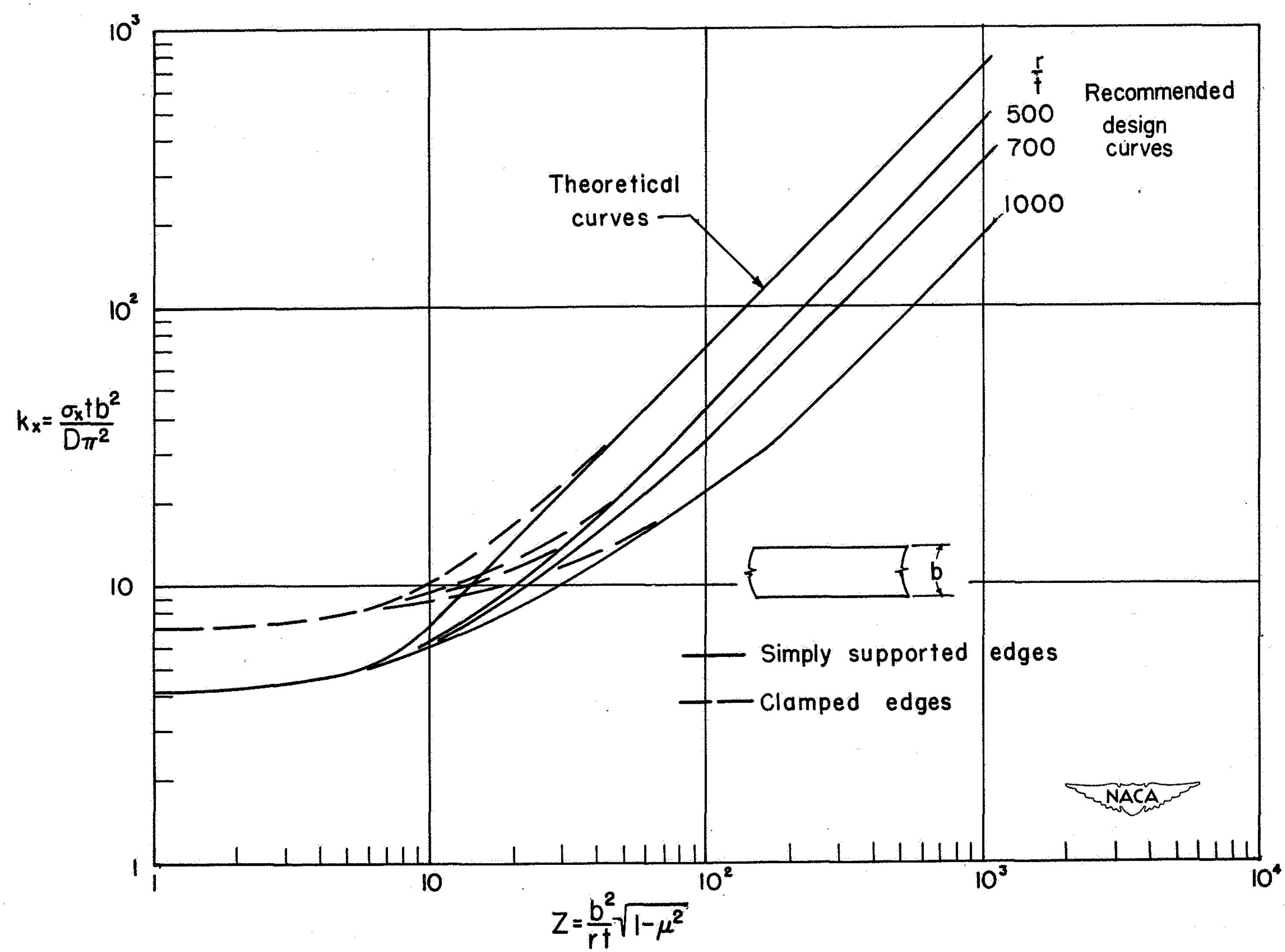


Figure 4.– Critical axial–compressive–stress coefficients for a simply supported, infinitely long strip with transverse curvature. (Figure adapted from reference 2.)