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NOTE ON THE ACCURACY OF A METHOD FOR RAPIDLY CALCULATING THE INCREMENTS IN VELOCITY ABOUT AN AIRFOIL DUE TO ANGLE OF ATTACK

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SUMMARY

The accuracy of the method presented in NACA Rep. No. 824 for rapidly calculating the increment in the velocity distribution about a symmetrical airfoil due to angle of attack has been investigated by use of the exact potential-flow-theory solution of the velocity distribution about an arbitrary airfoil given in NACA Rep. No. 452. The analysis shows that the rapid method described in NACA Rep. No. 824 evaluates the effect of angle of attack on the net velocity distribution at any point on a symmetrical airfoil with a maximum error which can be expressed as an increment in velocity ratio by the relation $(1 - \cos\alpha)\sqrt{S_o}$ where α is the angle of attack and $\sqrt{S_o}$ is the velocity ratio on the airfoil at zero lift calculated according to the exact theory.

INTRODUCTION

A method is described in reference 1 which permits the rapid estimation of the pressure distribution at any lift coefficient about large numbers of airfoils from a rather limited amount of theoretical data. The method is based on the assumption that the velocity at any point on an airfoil can be resolved into components which are attributable to the induced velocity on the basic thickness form at zero lift, the load distribution due to angle of attack, and the load distribution due to camber. The accuracy of the method described in reference 1 for obtaining and applying the increment in velocity associated with the loading due to angle of attack has been open to some question. The purpose of the present paper is to give an exact account of the method by means of which the increment in velocity due to angle of attack is obtained and the inaccuracies to which the assumptions involved in the method may lead. The analysis is concerned only with symmetrical airfoils and does not consider any possible effects of camber on the velocity increments due to angle of attack.

COEFFICIENTS AND SYMBOLS

a	reference length
c	airfoil chord
x	distance along airfoil chord
c_l	section lift coefficient
c_n	section normal-force coefficient
α	section angle of attack
S	pressure coefficient $\left(\frac{H_o - p}{q_o}\right)$
H_o	free-stream total pressure
p	local static pressure
q_o	free-stream dynamic pressure
$\sqrt{S}$	velocity ratio for incompressible flow $\left(\frac{v}{V}\right)$
v	local velocity at some point on the airfoil
V	free-stream velocity
$\frac{\Delta v_a}{V}$	increment in velocity ratio caused by additional type loading due to angle of attack
P_R	resultant pressure coefficient; difference between local upper-surface and lower-surface pressure coefficients at the same chordwise position
Γ	circulation
$\left.\begin{matrix}\phi \\ K \\ \psi_o\end{matrix}\right\}$	parameters which are functions of airfoil geometry (reference 2)

Subscripts:

U	upper surface
L	lower surface
o	conditions calculated for airfoil at zero lift

DISCUSSION

The equation given in reference 1 for obtaining the net velocity ratio $\sqrt{S}$ at some point on a symmetrical airfoil at a lift coefficient c_l is

$$\sqrt{S} = \sqrt{S_o} \pm \frac{\Delta v_a}{V} c_l \tag{1}$$

Values of $\sqrt{S_o}$, the velocity ratio for the symmetrical airfoil at zero lift, and $\frac{\Delta v_a}{V}$, the incremental velocity ratio due to angle of attack for a lift coefficient of 1.0, are tabulated in reference 1 for a large number of airfoils. The sign convention followed is that all velocities which cause a flow in the direction of the trailing edge are positive. A negative sign is therefore employed with $\frac{\Delta v_a}{V}$ when the net velocity ratio $\sqrt{S}$ is calculated for the lower surface at positive lift coefficients.

The values of $\sqrt{S_o}$ presented in reference 1 for the various basic thickness forms at zero lift were calculated by the exact theory of Theodorsen and Garrick (reference 2). The values of $\frac{\Delta v_a}{V}$ were calculated by means of the following relation:

$$\frac{\Delta v_a}{V} = \frac{\sqrt{S_U} - \sqrt{S_L}}{2} \tag{2}$$

where $\sqrt{S_U}$ and $\sqrt{S_L}$ are the upper-surface and lower-surface velocity ratios, respectively, calculated by the exact theory for some lift coefficient c_l which is usually small. Values of $\frac{\Delta v_a}{V}$ for a lift coefficient of 1.0 were obtained by linearly scaling the values given by equation (2). Two basic assumptions are implicit in equations (1) and (2): first, that the mean of the upper- and lower-surface velocity ratios at any lift coefficient, given by

$$\sqrt{S_{mean}} = \frac{\sqrt{S_U} + \sqrt{S_L}}{2} \tag{3}$$

is the same as the velocity ratio calculated for the same point on the airfoil at zero lift $\sqrt{S_o}$; and second, that the incremental velocity ratio $\frac{\Delta v_a}{V}$ is a linear function of lift coefficient. The validity of these two assumptions will now be investigated.

From the exact potential-flow theory (reference 2), the following equations may be written for the velocity ratios on the upper and lower surfaces of a symmetrical airfoil:

$$\sqrt{S_U} = K\left[\sin(\alpha + \phi) + \sin \alpha\right] \tag{4}$$

$$\sqrt{S_L} = K\left[\sin(\alpha - \phi) + \sin \alpha\right] \tag{5}$$

where K and ϕ are functions of only the airfoil geometry and have the same value at corresponding points on the upper and lower surfaces of a symmetrical airfoil. The angle of attack is given by α. The velocity distribution about the airfoil at zero lift is, of course, given by the equation:

$$\sqrt{S_o} = K \sin \phi \tag{6}$$

Since ϕ is negative on the lower surface, equations (5) and (6) show that velocities which cause the flow to progress toward the trailing edge on the lower surface will have a negative sign. In order to be consistent with the previously defined sign convention, however, a negative sign must be placed in front of equation (5). Thus

$$\sqrt{S_L} = -K\left[\sin(\alpha - \phi) + \sin \alpha\right] \tag{7}$$

An indication of the accuracy of the assumption that

$$\sqrt{S_o} = \sqrt{S_{mean}}$$

can be obtained by substituting the expressions given by equations (4) and (7) in equation (3) and determining the manner in which the resultant expression differs from the exact equation (6) for $\sqrt{S_o}$. Performing the suggested substitution

$$\frac{\sqrt{S_U} + \sqrt{S_L}}{2} = K\left\{\frac{\left[\sin(\alpha + \phi) + \sin \alpha\right] - \left[\sin(\alpha - \phi) + \sin \alpha\right]}{2}\right\}$$

and expanding and simplifying give the following expression

$$\frac{\sqrt{S_U} + \sqrt{S_L}}{2} = \sqrt{S_{mean}} = K \sin \phi \cos \alpha \tag{8}$$

Equation (8) for the mean velocity ratio $\sqrt{S_{mean}}$ is seen to differ from the exact expression for the velocity ratio about the symmetrical airfoil at zero lift (given by equation (6)) only by the factor $\cos \alpha$. Since $\cos \alpha$ is nearly 1.0 for relatively small values of α, such as would be of interest, the assumption that $\sqrt{S_o} = \sqrt{S_{mean}}$ is seen to be justifiable. Furthermore, the approximation is seen to be in no way dependent upon the chordwise position along the airfoil.

In order to investigate the assumption that $\frac{\Delta v_a}{V}$ varies linearly with lift coefficient, the expressions given by equations (4) and (7) are substituted in equation (2) for the incremental velocity ratio

$$\frac{\Delta v_a}{V} = K\left\{\frac{\left[\sin(\alpha + \phi) + \sin \alpha\right] + \left[\sin(\alpha - \phi) + \sin \alpha\right]}{2}\right\} \tag{9}$$

where the upper-surface and lower-surface velocity ratios given by equations (4) and (7) are for some lift coefficient c_l. Simplifying equation (9) and dividing both sides by the lift coefficient c_l yield

$$\frac{\Delta v_a}{Vc_l} = \frac{K \sin \alpha}{c_l}\left(\cos \phi + 1\right) \tag{10}$$

An expression relating $\sin \alpha$ and c_l can be obtained from the following equation for the circulation Γ taken from reference 2

$$\Gamma = 4\pi V a e^{\psi_o} \sin \alpha \tag{11}$$

where ψ_o is a constant for any given airfoil, V is the free-stream velocity, and a is a constant which is approximately one-quarter of the airfoil chord c. The relation between a and c varies slightly for different airfoils, but this variation has no bearing on the present discussion. The lift coefficient and the circulation are related by the following expression:

$$\Gamma = \frac{c}{2} V c_l \tag{12}$$

Substituting from equation (12) in equation (11) gives the following relation between $\sin \alpha$ and c_l

$$\sin \alpha = \frac{c_l}{2\pi e^{\psi_0}} \tag{13}$$

The use of equation (13) with equation (10) for $\frac{\Delta v_a}{V}$ gives

$$\frac{\Delta v_a}{V} = \frac{K\left[\cos \phi + 1\right] c_l}{2\pi e^{\psi_0}} \tag{14}$$

Equation (14) shows that the incremental velocity ratio does, in fact, vary in an exactly linear manner with lift coefficient. Equation (14) also provides a very convenient means for calculating the values of $\frac{\Delta v_a}{V}$, since all the parameters involved in equation (14) must be evaluated for the calculation of the velocity distribution about the airfoil at zero lift.

The analysis presented has shown that for symmetrical airfoils the mean of the exactly calculated upper-surface and lower-surface velocity ratios for any lift coefficient is related to the exactly calculated velocity ratio at the same point on the airfoil at zero lift by the following expression:

$$\sqrt{S_{mean}} = \sqrt{S_o} \cos \alpha \tag{15}$$

and that the incremental velocity ratio $\frac{\Delta v_a}{V}$ varies in an exactly linear manner with lift coefficient. The only error which will be incurred in the net velocity ratio as a result of employing equation (1) together with the values of $\sqrt{S_o}$ calculated for the airfoil at zero lift and the values of $\frac{\Delta v_a}{V}$ calculated from equation (2) or (14) is therefore an increment in velocity ratio on the upper and lower surfaces which is given by $(1 - \cos \alpha)\sqrt{S_o}$.

With the aid of the results just obtained, the relation is now shown between the lift coefficient c_l employed in equation (1), for which the pressure or velocity distribution is desired, and the lift coefficient which is obtained from an integration of the pressure distribution calculated according to equation (1). The resultant pressure coefficient at some point along the airfoil chord P_R is defined as the difference between the upper-surface and lower-surface pressure coefficients at that point. An integration of P_R along the chord of an airfoil at some angle of attack gives an expression for the normal-force coefficient c_n

$$c_n = \int_0^{1.0} P_R \, d\left(\frac{x}{c}\right) \tag{16}$$

The lift coefficient is then given by the relation

$$c_l = \frac{1}{\cos \alpha} \int_0^{1.0} P_R \, d\left(\frac{x}{c}\right) \tag{17}$$

since the drag is zero for perfect fluid flow. In terms of the exactly calculated values of the velocity ratio for the basic thickness form at zero lift $\sqrt{S_o}$ and the values of $\frac{\Delta v_a}{V}$ for the lift coefficient under consideration, the resultant pressure coefficient is given approximately by equation (1) as

$$\begin{aligned} P_R &\approx \left[\sqrt{S_o} + \frac{\Delta v_a}{V}\right]^2 - \left[\sqrt{S_o} - \frac{\Delta v_a}{V}\right]^2 \\ &\approx 4 \sqrt{S_o} \frac{\Delta v_a}{V} \end{aligned} \tag{18}$$

The substitution of the approximate expression for P_R (equation (18)) in equation (16) gives

$$c_n \approx \int_0^{1.0} 4 \sqrt{S_o} \frac{\Delta v_a}{V} \, d\left(\frac{x}{c}\right) \tag{19}$$

whereas the accurate expression for c_n would be

$$c_n = \int_0^{1.0} 4 \sqrt{S_{mean}} \frac{\Delta v_a}{V} \, d\left(\frac{x}{c}\right) \tag{20}$$

Since $\sqrt{S_o} = \frac{\sqrt{S_{mean}}}{\cos \alpha}$, equation (19) can be written

$$c_n \approx \frac{1}{\cos \alpha} \int_0^{1.0} 4\sqrt{S_{mean}} \frac{\Delta v_a}{V} d\left(\frac{x}{c}\right)$$

Thus, the use of the approximation $\sqrt{S_o} \approx \sqrt{S_{mean}}$ is found to result in the evaluation of the lift coefficient c_l instead of the normal-force coefficient c_n, inasmuch as $c_l = \frac{c_n}{\cos \alpha}$.

CONCLUDING REMARKS

The accuracy of the method described in reference 1 for rapidly calculating the increments in the velocity distribution about a symmetrical airfoil due to angle of attack is in error only by a factor $\cos \alpha$ over the entire chord. In view of the discrepancies, resulting from the effects of viscosity, which are known to exist between airfoil velocity distributions calculated by any perfect fluid theory and those obtained experimentally, it is difficult to see how any situation might arise in which a theoretical, perfect-fluid velocity distribution about a symmetrical airfoil would be required to a degree of accuracy greater than that provided by the method described in NACA Rep. No. 824.

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