NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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APPRAISAL OF METHOD OF FLUTTER ANALYSIS BASED ON CHOSEN MODES BY COMPARISON WITH EXPERIMENT FOR CASES OF LARGE MASS COUPLING

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SUMMARY

The present paper reports the results of a series of flutter studies including comparisons of experimental results with calculations based on a Rayleigh type analysis, in which chosen modes are assumed. The model studied was a straight uniform cantilever wing of high aspect ratio and carried a single concentrated weight. An extensive set of experimental flutter data existed in which the mass coupling varied over a wide range. The theoretical results of a differential-equations approach, not requiring chosen modes, in which good agreement with experimental results had been obtained were also available for a number of these cases. An unusual opportunity was therefore afforded to appraise the validity of the assumptions involved in the more universally applicable Rayleigh type analysis. In general, a deterioration in agreement between the experimental and the approximate theoretical results is noted as mass coupling increases. Computed results are found to be high, that is unconservative, for weights ahead of the elastic axis but conservative for weights behind the elastic axis.

INTRODUCTION

The study of flutter embraces two main categories: the aerodynamics of unsteady flows and the mechanics of vibrating structures. In order to obtain practical solutions to the flutter problem, many simplifying assumptions are necessary in each category. In particular, the aerodynamic part of the problem has customarily been simplified by the use of theoretically derived two-dimensional air forces. In the structural part of the problem, it has been found convenient to assume that the motion of the wing during flutter may be represented by a finite number of terms of a series of chosen modal functions.

Various investigators have indicated that the validity of these assumptions has never been conclusively established. Jordan (reference 1) discusses the problem in relation to the German research effort both before and during the war and states that many theoretical results have been obtained, but that they lack adequate experimental backing. Many papers exist in British literature in which the problem has been considered. (See, for example, reference 2.) The problem has also been under study in the United States. Among the investigators was Loring (reference 3), who made a Rayleigh type analysis, in which certain modal shapes were assumed, and compared his results with experimental results in reference 4. A similar comparison with experimental results was made in reference 5. More recently, Goland and Luke (reference 6) have published a solution involving a differential-equation analysis not requiring the assumption of modal shapes. Their analysis was applied numerically to a few examples and compared with the results of the Rayleigh type analysis but included no comparison with experiment. In all of these cases, satisfactory agreement was found between theoretical and experimental results or between one theory and another. The experiments serving as bases for comparison consisted, however, of isolated cases in which little or no mass coupling was involved. The need existed, therefore, for a systematic study of a more general nature, which involved both experiment and theory for cases of larger mass coupling. Such a systematic study forms the basis of three closely related papers (references 7, 8, and the present paper). The experimental basis is given in reference 7, in which are reported the results of an extensive testing program intended to provide a sufficient number of flutter cases covering a range of mass coupling that might serve to appraise aspects of the various analytical methods. The wing used in this experimental work was a straight uniform cantilever of fairly high aspect ratio.

In the second paper of the series (reference 8), the differential-equation type analysis of reference 6 was used and extended to include a weight at any spanwise and chordwise station. Good agreement between theory and experiment was obtained for all cases studied. The results indicated that the differential-equation approach properly accounted for the structural part of the problem and that the theoretical two-dimensional air forces were sufficient for the conditions investigated.

The differential-equation procedure, not requiring the assumption of modes, yields most satisfactory results for a uniform wing but becomes unwieldy when applied to a nonuniform wing. Careful examination of the commonly used approximate methods, in which selected modes are employed and which are of more universal applicability, is therefore necessary and desirable. The purpose of the present paper is to give the results of such an investigation in which a Rayleigh type analysis is employed and which uses the experimental data of reference 7. Since the two-dimensional air forces were adequate in the differential-equation analysis of the problem for these data, presumably their use herein will permit a separation and examination of the mode approximations involved.

In the Rayleigh type analysis the assumption is made that the flutter mode may be approximated by use of a finite number of terms of a series of certain selected modal functions. The accuracy of the result depends, in general, on the choice of the modal shapes that make up the series and on the number of terms of the series used.

These shapes may be any arbitrary set of functions; the usual choice is either the coupled or the uncoupled modes of oscillation of the system in a vacuum since these functions satisfy structural boundary conditions. The choice between coupled and uncoupled modes at present remains a matter of preference of the individual investigator. The problem also arises as to the number of terms of the series required to obtain a practical answer sufficiently reliable for use in the prediction of flutter in aircraft. The amount of computation increases rapidly as additional degrees of freedom, or modes, are considered. Certain basic theoretical questions of the convergence of iterative methods remain an unsettled problem. Wielandt (reference 9) has initiated some theoretical work along these lines. Although none of these theoretical considerations are dealt with herein, some quantitative information on the problem is given and some of the parameters upon which the required number of modes depends are indicated.

SYMBOLS

A_{ch}, $A_{c\alpha}$, A_{ah}, $A_{a\alpha}$ air-force coefficients as given in reference 3

b wing half-chord

c dimensionless square matrix describing dynamic air forces acting on system

e_w nondimensional distance between center of gravity of concentrated weight and elastic axis based on half-chord, positive for center-of-gravity positions behind the elastic axis

EI bending rigidity of wing

f flutter frequency, cycles per second

GJ torsional rigidity of wing

I_r reference moment of inertia $\left(m_o l b^2\right)$

I' mass moment of inertia of wing section about elastic axis per unit length, including moment of inertia of concentrated weight at its proper spanwise station

I_{EA} mass moment of inertia of wing about elastic axis

I_w mass moment of inertia of concentrated weight about wing elastic axis

k — reduced-frequency parameter $\left(\frac{b\omega}{v}\right)$

l — semispan of wing (denoted by s in reference 4)

m_o — reference mass, taken as mass of wing per unit length

m' — mass per unit length, including mass of concentrated weight

r_α — nondimensional radius of gyration relative to elastic axis $\left(\sqrt{\frac{I'}{m_o b^2}}\right)$

v — flutter velocity

v_o — experimental flutter speed for wing without weight

W — weight of model wing

W_w — weight of concentrated weight

x — spanwise coordinate measured from wing root

$\bar{x}_\alpha$ — nondimensional distance from elastic axis to wing-section center of gravity based on half-chord, including chordwise displacement of concentrated-weight center of gravity at its proper spanwise station

Δ' — dimensionless matrix describing the elastic properties of system (denoted by f in reference 4)

Γ — mass ratio $\left(\frac{I_r}{\pi\rho b^4 l}\right)$

ρ — mass density of air

ω — angular flutter frequency, radians per second

ω_r — reference angular flutter frequency, radians per second

ξ — dimensionless square matrix describing inertia properties of system (denoted by a in reference 4)

τ dimensionless frequency ratio $\left(\left(\frac{\omega_r}{\omega}\right)^2\right)$

ν stiffness parameter $\left(\frac{GJ}{EI}\frac{l^2}{b^2}\right)$

ν_r reference stiffness (denoted by K_r in reference 4) $\left(EI\,\frac{b^2}{l^3}\right)$

$\phi_{h_1}, \phi_{h_2}, \phi_{\alpha_1}, \phi_{\alpha_2}$ modal functions in first bending, second bending, first torsion, and second torsion, respectively

ANALYSIS

A mechanical system such as a wing, considered as a continuous structure, possesses an infinite number of degrees of freedom and is therefore capable of vibrating in any of an infinite number of displacement forms. In order to describe the flutter of a wing in which the true displacement form is unknown, a correct analysis should, theoretically, therefore be carried out by considering an infinite series of harmonic modes.

An analysis of the Rayleigh type presumes that a good approximation to the flutter mode may be achieved by including the first few terms of the infinite series. The problem exists as to what constitutes a sufficient number of terms. The choice of modal functions to be employed is arbitrary; the usual preference is either the coupled or the uncoupled modes of vibration of the structure since these functions satisfy the structural boundary conditions. An interesting exchange of ideas concerning this preference was made between R. H. Scanlan and Goland and Luke in the Discussion cited with reference 6. (The term "uncoupled mode," as employed in the present paper, refers to an imagined constrained mode in which, for pure bending, the chordwise distribution of mass is considered to act at the elastic axis of the wing with no torsional deformation occurring. For pure torsion, the elastic axis is considered restrained against bending. The term "coupled mode" is employed herein in a limited sense and refers to a combination of bending and torsional deflections appropriate to the natural normal harmonic vibrations of the freely oscillating (undamped) system.) For the purpose of the present analysis, uncoupled modes have been selected; however, the use of coupled modes should be investigated further. Of course, for actual airplane structures the choice of the active modes for the complete structure, particularly for the empennage, may be extremely difficult.

The purpose of the present paper is essentially that of appraising the accuracy of the use of uncoupled modes in a Rayleigh type analysis with a study of the number of modes required to indicate a reasonable approach to the experimental result. The primary interest is in the results and significance of a number of numerical applications of such an analysis in which uncoupled modes are used. No attempt has been made in this paper, therefore, to present either derivation or details of the method. The form given in the analysis is that given by Loring in references 3 and 4. Although matrix notation is used for consistency with references, no knowledge of matrix methods other than of the solution of a determinant is required. Of course, other procedures, also based on Lagrangian equations, lead to the same results as the procedure of Loring; for example, that given by Smilg and Wasserman in reference 10.

The matrixes associated with the energies of system may be of order n, which makes the analysis adaptable to any desired number of degrees of freedom. The form given is for order 4, which is the maximum number of uncoupled modes employed in the present analysis.

The matrix associated with the kinetic energy of the system for the variable mass due to the addition of a concentrated weight is as follows:

$$I_r\xi = I_r \begin{vmatrix} \int_0^1 \phi_{h_1}{}^2 \frac{m'}{m_o} \frac{dx}{l} & \int_0^1 \phi_{h_2}\phi_{h_1} \frac{m'}{m_o} \frac{dx}{l} & \int_0^1 \phi_{h_1}\phi_{\alpha_1} \frac{m'\bar{x}_\alpha}{m_o} \frac{dx}{l} & \int_0^1 \phi_{h_1}\phi_{\alpha_2} \frac{m'\bar{x}_\alpha}{m_o} \frac{dx}{l} \\ \int_0^1 \phi_{h_1}\phi_{h_2} \frac{m'}{m_o} \frac{dx}{l} & \int_0^1 \phi_{h_2}{}^2 \frac{m'}{m_o} \frac{dx}{l} & \int_0^1 \phi_{h_2}\phi_{\alpha_1} \frac{m'\bar{x}_\alpha}{m_o} \frac{dx}{l} & \int_0^1 \phi_{h_2}\phi_{\alpha_2} \frac{m'\bar{x}_\alpha}{m_o} \frac{dx}{l} \\ \int_0^1 \phi_{h_1}\phi_{\alpha_1} \frac{m'\bar{x}_\alpha}{m_o} \frac{dx}{l} & \int_0^1 \phi_{h_2}\phi_{\alpha_1} \frac{m'\bar{x}_\alpha}{m_o} \frac{dx}{l} & \int_0^1 \phi_{\alpha_1}{}^2 \frac{I'}{m_o b^2} \frac{dx}{l} & \int_0^1 \phi_{\alpha_1}\phi_{\alpha_2} \frac{I'}{m_o b^2} \frac{dx}{l} \\ \int_0^1 \phi_{h_1}\phi_{\alpha_2} \frac{m'\bar{x}_\alpha}{m_o} \frac{dx}{l} & \int_0^1 \phi_{h_2}\phi_{\alpha_2} \frac{m'\bar{x}_\alpha}{m_o} \frac{dx}{l} & \int_0^1 \phi_{\alpha_1}\phi_{\alpha_2} \frac{I'}{m_o b^2} \frac{dx}{l} & \int_0^1 \phi_{\alpha_2}{}^2 \frac{I'}{m_o b^2} \frac{dx}{l} \end{vmatrix}$$

where $I_r = m_o l b^2$ and $\frac{I'}{m_o b^2} = r_\alpha{}^2$.

The matrix associated with the potential energy is

$$\nu_r\Delta' = \nu_r \begin{vmatrix} \int_0^1 \ddot{\phi}_{h_1}{}^2 \frac{dx}{l} & 0 & 0 & 0 \\ 0 & \int_0^1 \ddot{\phi}_{h_2}{}^2 \frac{dx}{l} & 0 & 0 \\ 0 & 0 & \nu\int_0^1 \dot{\phi}_{\alpha_1}{}^2 \frac{dx}{l} & 0 \\ 0 & 0 & 0 & \nu\int_0^1 \dot{\phi}_{\alpha_2}{}^2 \frac{dx}{l} \end{vmatrix}$$

where, as in reference 4,

$$\nu_r = \frac{EI}{l}\left(\frac{b}{l}\right)^2$$

and

$$\nu = \frac{GJ}{EI}\left(\frac{l}{b}\right)^2$$

The dots above ϕ indicate the derivatives with respect to time. Observe that this is a diagonal matrix, which implies that the potential energy is expressible as a quadratic form involving no cross-stiffness terms. Structural damping terms can be included in this matrix or in a separate matrix associated with dissipative forces. However, as in reference 8, for the purpose of the numerical work the structural damping has been kept zero.

The matrix associated with the air forces is as follows:

$$c = \begin{vmatrix} \int_0^1 \phi_{h_1}{}^2 A_{ch} \frac{dx}{l} & \int_0^1 \phi_{h_1}\phi_{h_2} A_{ch} \frac{dx}{l} & \int_0^1 \phi_{h_1}\phi_{\alpha_1} A_{c\alpha} \frac{dx}{l} & \int_0^1 \phi_{h_1}\phi_{\alpha_2} A_{c\alpha} \frac{dx}{l} \\ \int_0^1 \phi_{h_1}\phi_{h_2} A_{ch} \frac{dx}{l} & \int_0^1 \phi_{h_2}{}^2 A_{ch} \frac{dx}{l} & \int_0^1 \phi_{h_2}\phi_{\alpha_1} A_{c\alpha} \frac{dx}{l} & \int_0^1 \phi_{h_2}\phi_{\alpha_2} A_{c\alpha} \frac{dx}{l} \\ \int_0^1 \phi_{h_1}\phi_{\alpha_1} A_{ah} \frac{dx}{l} & \int_0^1 \phi_{h_2}\phi_{\alpha_1} A_{ah} \frac{dx}{l} & \int_0^1 \phi_{\alpha_1}{}^2 A_{a\alpha} \frac{dx}{l} & \int_0^1 \phi_{\alpha_1}\phi_{\alpha_2} A_{a\alpha} \frac{dx}{l} \\ \int_0^1 \phi_{h_1}\phi_{\alpha_2} A_{ah} \frac{dx}{l} & \int_0^1 \phi_{h_2}\phi_{\alpha_2} A_{ah} \frac{dx}{l} & \int_0^1 \phi_{\alpha_1}\phi_{\alpha_2} A_{a\alpha} \frac{dx}{l} & \int_0^1 \phi_{\alpha_2}{}^2 A_{a\alpha} \frac{dx}{l} \end{vmatrix}$$

where the air-force coefficients A_{ch}, $A_{c\alpha}$, A_{ah}, and $A_{a\alpha}$ are, as in reference 3, the two-dimensional coefficients developed by Theodorsen (see reference 11).

The determinantal flutter equation is formed by a linear combination of the three matrixes and is given as

$$\left| -\Gamma\xi + c + \tau\Delta' \right| = 0$$

where, as in reference 3,

$$\Gamma = \frac{I_r}{\pi\rho b^4 l}$$

$$\tau = \left(\frac{\omega_r}{\omega}\right)^2$$

$$\omega_r = \sqrt{\frac{\nu_r}{\pi\rho b^4 l}}$$

The solution of the determinant results in the flutter condition and yields the critical values of k and τ; therefore, flutter velocity and frequency may be found from the relations

$$V = \frac{\omega_r b}{k\sqrt{\tau}}$$

and

$$\omega = \frac{\omega_r}{\sqrt{\tau}}$$

APPLICATION AND DISCUSSION OF RESULTS

The actual computation of the uncoupled modes needed in the application was accomplished by the use of the differential-equation development given in reference 8. For a nonuniform structure, however, a more general method would have to be employed and an iteration process such as that of reference 12 could be used.

Many procedures exist for solving the flutter determinant. In the procedure employed herein, the structural parameters were assigned their values in the various matrixes. The expansion of the flutter determinant resulted in simultaneous real and imaginary equations which were solved for the pair of values k (the reduced frequency parameter) and τ (containing the flutter frequency) which also satisfied the flutter determinant.

Initial analyses were made by use of two uncoupled modes and, in certain selected cases, three or four uncoupled modes were used. The calculated results are compared with experimental results in table I and figures 1 to 4. Values are given for both velocity and frequency at flutter.

Attention is first directed to the calculated results obtained by employing two degrees of freedom, namely, first bending and first torsion. For the wing alone, reasonable agreement between calculated and experimental results was obtained as shown by case 1 of the table. Also, for cases in which the center of gravity of the weight was located close to the wing elastic axis ($e_w = 0.034$) and moved spanwise (cases 2 to 7), very good agreement was found; the calculated results were not more than 5 percent from those obtained experimentally. For the cases in which the center of gravity of the weight was slightly ahead of the elastic axis ($e_w = -0.360$) and for various spanwise positions (cases 8 to 10), a loss in accuracy was noted with a maximum discrepancy of 23 percent. For the cases in which the center of gravity of the weight was near the leading edge ($e_w = -0.818$), no solution to the flutter determinant could be obtained for any of the spanwise positions represented in the table by cases 12 to 15. For case 11 with the weight at the 11-inch spanwise station and the same leading-edge chordwise position ($e_w = -0.818$), a solution was obtained which was 14 percent high. For the cases in which the center of gravity of the weight was behind the wing elastic axis ($e_w = 0.500$), the spanwise variations are represented by cases 16 to 18. For the most inboard position (case 16) good agreement with experiment was obtained. The analyses of cases 17 and 18 yielded values which were below the experimental value and from flutter considerations, therefore, were conservative. In general, increased mass coupling reduces the agreement between experimental results and those obtained from calculations when two degrees of freedom were used. For cases with large mass coupling the structure should be allowed greater flexibility and, therefore, would require the use of more modes.

Of the previous cases considered a certain number have been selected for further investigation. For the continued analyses of these cases a third degree of freedom, the uncoupled mode in second bending, was added. With this addition the analyses of cases 1, 4, and 10 yielded values closer to the experimental values, although the reduction in the discrepancy for case 10 was only slight. The analyses using three degrees of freedom were also extended to three cases for which no solution to the flutter determinant could be obtained. Solutions resulted in each case, but the agreement varied with experimental results. The flutter speed calculated for case 12 was over 80 percent above the experimental value. The results obtained for cases 13 and 15 were within 14 percent and 11 percent, respectively, of the experimental values. Figure 1 shows, however, that the trend of the calculated curve for these two cases was similar to the unusual trend of the experimental curve.

A further extension of the analysis to include a fourth degree of freedom, namely, the uncoupled mode in second torsion, was applied to cases 12, 13, and 15. The analysis of case 12 employing the fourth degree of freedom reduced the discrepancy between theory and experiment from 80 percent to approximately 30 percent. The differential-equation approach of reference 8 yielded a value that was only 7 percent above the experimental value and was the maximum discrepancy obtained by this method. The addition of the fourth degree of freedom to the analyses of cases 13 and 15 had no appreciable effect on the value of the calculated flutter velocity. Although the percentage error has been discussed at a given spanwise station, the large value of the slope of the experimental curve for cases 12 to 15 may make such a discussion somewhat misleading, since a small displacement in the spanwise weight position may affect the result. Another basis for comparison could be used, for example some other distance between the theoretical and experimental curves.

For all cases in which the center of gravity of the weight was located forward of the elastic axis the results were unconservative, but for most cases with the center of gravity of the weight located behind the elastic axis the results were conservative.

CONCLUSIONS

Comparisons of experimental results of the cases studied with calculations based on a Rayleigh type analysis, in which chosen modes are assumed, give the following conclusions:

1. The use of two uncoupled modes in the theoretical analysis gave good agreement with experiment for the wing alone or for the wing carrying a concentrated weight near the elastic axis.

2. Increased mass coupling, obtained by placing the weight either ahead of or behind the elastic axis, required the consideration of more degrees of freedom to produce satisfactory results.

3. With the weight forward of the elastic axis the calculated results based on theory were high and, therefore, unconservative. With the weight behind the elastic axis the calculated results based on theory were conservative.

4. The analysis indicated that the degree of approximation to the experimental value generally improved with the addition of more degrees of freedom to the analysis.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va., March 28, 1949

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TABLE I.- EXPERIMENTAL AND CALCULATED RESULTS

Case	Weight (a)	Span position (percent span from root) (b)	$\frac{W_w}{W}$	e_w	$\frac{I_w}{I_{EA}}$	Experimental results			Calculated results for 2 modes $\phi_{h_1}, \phi_{\alpha_1}$			Calculated results for 3 modes $\phi_{h_1}, \phi_{h_2}, \phi_{\alpha_1}$			Calculated results for 4 modes $\phi_{h_1}, \phi_{h_2}, \phi_{\alpha_1}, \phi_{\alpha_2}$		
						f (cps)	$\frac{v}{b\omega}$	v (fps)	f (cps)	$\frac{v}{b\omega}$	v (fps)	f (cps)	$\frac{v}{b\omega}$	v (fps)	f (cps)	$\frac{v}{b\omega}$	v (fps)
1	None	0	0	0	0	22.1	7.22	334	25.2	6.10	321	23.90	6.80	340	----	----	---
2	7e	23.0	.954	.034	1.56	23.3	6.42	312	23.3	6.42	314	-----	-----	----	----	----	---
3	7e	43.7	.954	.034	1.56	17.6	7.76	286	19.4	7.39	300	-----	-----	----	----	----	---
4	7e	60.5	.954	.034	1.56	14.5	9.80	298	15.1	9.75	309	15.25	9.52	304	----	----	---
5	7e	72.9	.954	.034	1.56	13.0	10.85	296	13.8	10.50	304	-----	-----	----	----	----	---
6	7e	83.3	.954	.034	1.56	11.4	13.10	314	12.3	12.20	313	-----	-----	----	----	----	---
7	7e	100.0	.954	.034	1.56	8.9	18.20	338	9.7	15.80	326	-----	-----	----	----	----	---
8	7c	23.0	.940	-.360	2.04	19.6	7.93	326	23.2	7.50	365	-----	-----	----	----	----	---
9	7c	43.7	.940	-.360	2.04	16.5	10.00	346	18.6	10.00	394	-----	-----	----	----	----	---
10	7c	68.4	.940	-.360	2.04	14.0	12.40	364	16.3	13.15	448	15.70	13.42	443	----	----	---
11	7a	23.0	.917	-.818	4.26	17.4	8.88	324	21.3	8.28	370	-----	-----	----	----	----	---
12	7a	35.4	.917	-.818	4.26	[d] 16.3 26.8	[d] 11.04 7.02	382	(c)	(c)	(c)	14.21	23.52	699	32.8	7.27	498
13	7a	95.8	.917	-.818	4.26	21.8	8.09	368	(c)	(c)	(c)	18.00	7.64	421	18.2	7.37	410
14	7a	97.9	.917	-.818	4.26	21.6	7.48	339	(c)	(c)	(c)	-----	-----	----	----	----	---
15	7a	100.0	.917	-.818	4.26	21.4	7.14	320	(c)	(c)	(c)	25.50	6.65	354	24.8	6.06	355
16	7f	33.3	.917	.500	2.27	17.2	7.02	252	20.2	6.08	257	-----	-----	----	----	----	---
17	7f	50.0	.917	.500	2.27	14.3	7.65	229	13.1	6.53	179	-----	-----	----	----	----	---
18	7f	100.0	.917	.500	2.27	7.5	17.10	268	8.1	14.02	239	-----	-----	----	----	----	---

[a] From reference 7.
[b] Length of wing equals 48 inches.
[c] No solution.
[d] See footnote a of table on p. 15 of reference 8.

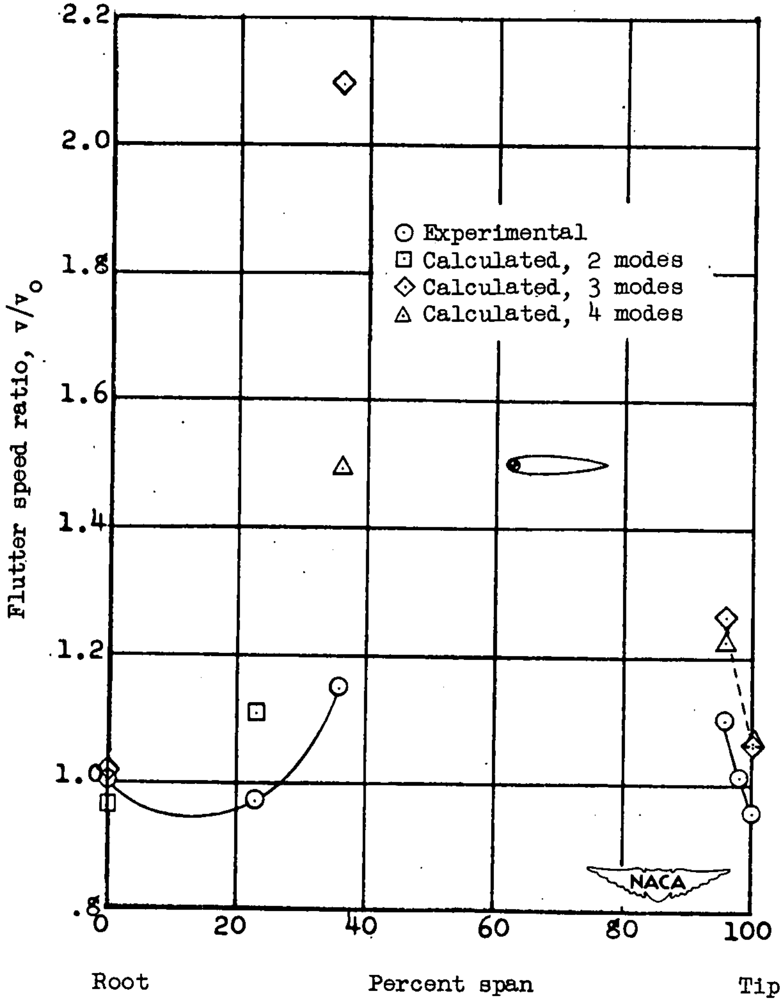

Figure 1.- Comparison of calculated and experimental flutter speeds for weight 7a (see reference 7).

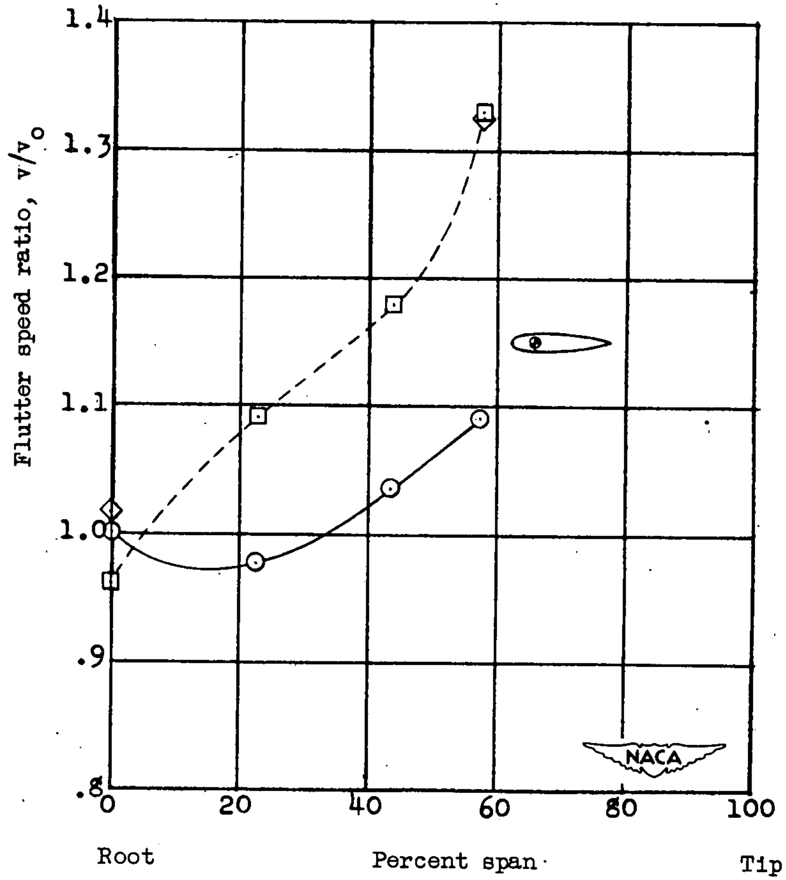

Figure 2.- Comparison of calculated and experimental flutter speeds for weight 7c (see reference 7).

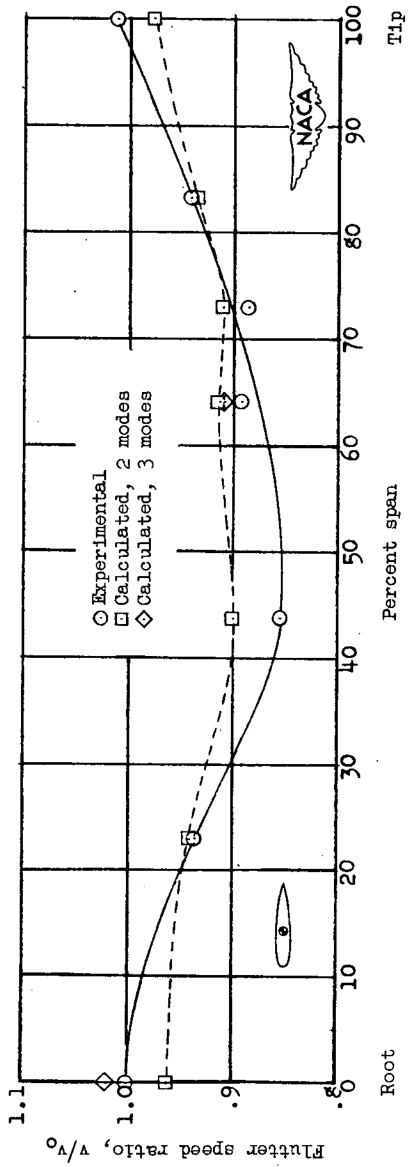

Figure 3.- Comparison of calculated and experimental flutter speeds for weight 7e (see reference 7).

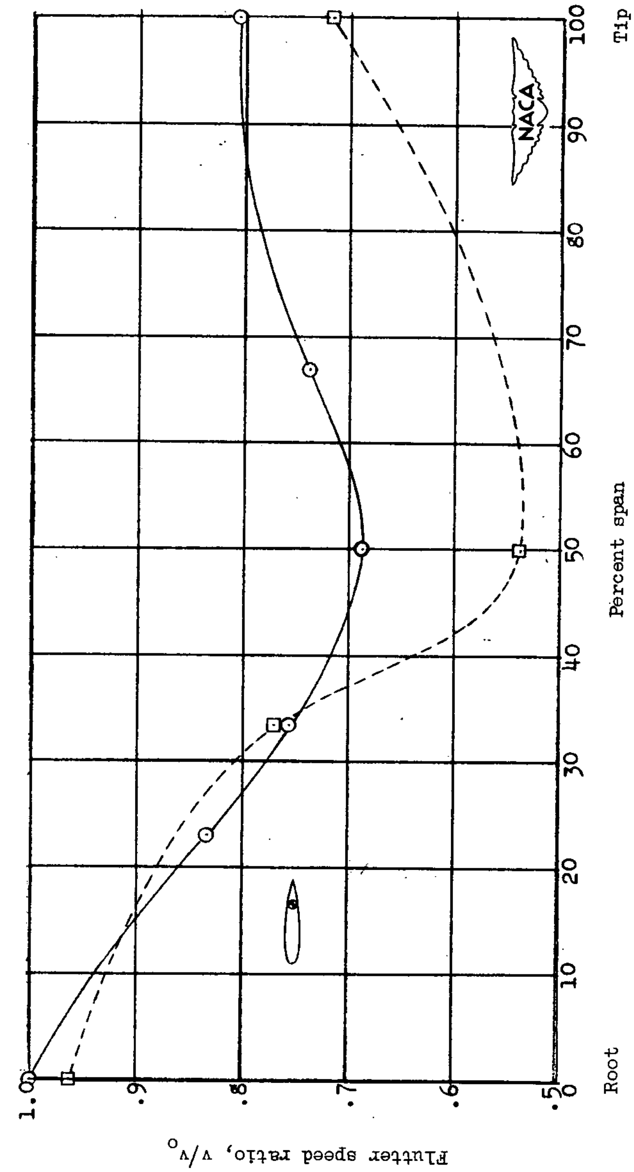

Figure 4.- Comparison of calculated and experimental flutter speeds for weight 7f (see reference 7).