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TIME-DEPENDENT DOWNWASH AT THE TAIL AND THE PITCHING MOMENT DUE TO NORMAL ACCELERATION AT SUPERSONIC SPEEDS

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SUMMARY

The time-dependent downwash behind a wing in a supersonic stream is analyzed for the case when the angle of attack varies linearly with time. The result is applied to the calculation of the contribution of the horizontal tail to the pitching moment and lift due to normal acceleration of the airplane. The method employs an extension of an unpublished solution of the linearized potential equation for unsteady flow by Clifford S. Gardner. The pitching moment due to normal acceleration, together with the damping in pitch, determines the damping of the short-period mode of longitudinal oscillation for an airplane.

INTRODUCTION

The investigations of Garrick and Rubinow (reference 1) and others have shown that a two-dimensional wing may experience certain unstable torsional oscillations at low supersonic speeds. These oscillations are in pitch without coupled vertical oscillations of the center of gravity. The same behavior may also occur with three-dimensional wings. More generally, coupled oscillations in these two degrees of freedom, termed "short-period" oscillations, may be expected. The damping of these oscillations is governed by the sum of the damping-in-pitch derivative C_{m_q} and the pitching-moment coefficient due to the normal-acceleration derivative $C_{m_{\dot{\alpha}}}$, where $\dot{\alpha}$ is the time rate of change of angle of attack. The sum $C_{m_q} + C_{m_{\dot{\alpha}}}$ is therefore of great significance, inasmuch as its sign determines whether the motion is stable or unstable. An unstable sign usually arises from the component $C_{m_{\dot{\alpha}}}$.

The derivative $C_{m_{\dot{\alpha}}}$ for an airplane is compounded of a contribution from the wing and another from the horizontal tail. At low subsonic speeds the wing contribution is a negligible factor, but at supersonic speeds it may become significant. The relevant theory for the wing at supersonic speeds and some calculations for specific plan forms are given in references 1 to 7.

The contribution of the horizontal tail to $C_{m_{\dot{\alpha}}}$ depends to a large extent on the time-dependent downwash field of the wing when the angle of attack varies with time. Standard techniques based on the solution of Laplace's equation (that is, the assumption of incompressible flow) have been used for low subsonic speeds. These methods fail at higher speeds when the compressibility of the air must be taken into account. Then this time-dependent downwash, in the theory of small disturbances, must satisfy the time-dependent form of the Prandtl-Glauert equation. (See equation (1) herein.) In the present paper a solution of this problem is obtained for supersonic speeds, and some considerations are given for subsonic speeds.

The evaluation of the time-dependent downwash employs an extension of an unpublished paper by Clifford S. Gardner of New York University. (The relevant part of his work is set forth in an erratum sheet that was issued for reference 6 and is repeated in the body of the present paper.) It is shown that the problem of calculating the pressure distribution due to the time rate of change of angle of attack $\dot{\alpha}$ can be reduced to the well-known problems of calculating the pressure distributions due to steady angle of attack and to steady pitching. Gardner's velocity potential applies only ahead of regions affected by the trailing edge of the wing. The necessary modification for these regions, particularly the region behind the trailing edge, is considered herein.

The time-dependent downwash obtained in this manner is applied to the calculation of the contribution of the horizontal tail to the normal-acceleration derivatives $C_{m_{\dot{\alpha}}}$ and $C_{L_{\dot{\alpha}}}$. The derivation is limited to the case where the tail lies in the chord plane of the wing $(z = 0)$. In addition, formulas for the contribution of the wing to these derivatives, evaluated from Gardner's work, are given for convenience.

SYMBOLS

x, y, z	rectangular coordinate system fixed in horizontal wing: x-axis chordwise; y-axis spanwise; z-axis upward
t	time
V	stream velocity, directed parallel to x

a	speed of sound in free stream
M_0	stream Mach number (V/a)
$B = \sqrt{M_0^2 - 1}$	
ϕ	velocity potential
α	angle of attack
$\dot{\alpha} = \frac{\partial \alpha}{\partial t}$	(assumed constant herein)
q	angular velocity of pitching
ω	angular frequency (sinusoidal motion)
ψ	value of ϕ for $q = 1$, $\alpha = \dot{\alpha} = 0$
χ	value of ϕ for $\alpha = 1$, $q = \dot{\alpha} = 0$
χ_{tr}	part of χ associated with trailing vortex system of wing
u_1, v_1, w_1	x-, y-, and z-velocity components of specified potential flow (see text)
w	upwash due to angle of attack $(\alpha\chi_z)$ for $z = 0$
w_q	upwash due to pitching $(q\psi_z)$ for $z = 0$
ΔP	local pressure on lower surface of airfoil minus local pressure on upper surface (Lift/Unit area)
ρ	density of air
L	lift
M	pitching moment
C_L	lift coefficient $\left(\frac{L}{\frac{1}{2}\rho V^2 S}\right)$
C_m	pitching-moment coefficient $\left(\frac{M}{\frac{1}{2}\rho V^2 S\bar{c}}\right)$

$$C_{L_\alpha} = \left(\frac{\partial C_L}{\partial \alpha}\right)_{\alpha \longrightarrow 0}$$

$$C_{L_{\dot{\alpha}}} = \left[\frac{\partial C_L}{\partial\left(\frac{\dot{\alpha}\bar{c}}{2V}\right)}\right]_{\dot{\alpha} \longrightarrow 0}$$

$$C_{L_q} = \left[\frac{\partial C_L}{\partial\left(\frac{q\bar{c}}{2V}\right)}\right]_{q \longrightarrow 0}$$

$$C_{m_{\dot{\alpha}}} = \left[\frac{\partial C_m}{\partial\left(\frac{\dot{\alpha}\bar{c}}{2V}\right)}\right]_{\dot{\alpha} \longrightarrow 0}$$

S airfoil area

$\bar{c}$ airfoil mean aerodynamic chord

i airfoil incidence relative to x-axis

l x-coordinate of reference point in tail (tail arm)

Λ local sweep angle along trailing edge ($\Lambda(y)$)

$$F_1(x,y) = 1 + \frac{w}{\alpha V}$$

$$F_2(x,y) = \frac{M_0{}^2}{VB^2}\left[\bar{c}\left(\frac{w_q}{q\bar{c}}\right) - x\left(\frac{w}{\alpha V}\right) - \frac{x_{TE}\sqrt{M_0{}^2 \cos^2 \Lambda - 1}}{VM_0{}^2 \cos \Lambda} + \frac{1}{M_0{}^2}\int_{TE}^{x} \frac{w_{tr}}{\alpha V}\, dx\right]$$

$\bar{F}_1(l)$ average of $F_1(x,y)$ over tail span for $x = l$

$\bar{F}_2(l)$ average of $F_2(x,y)$ over tail span for $x = l$

$\bar{w}(l)$ average of $w(x,y)$ over tail span for $x = l$

$\bar{w}_q(l)$ average of $w_q(x,y)$ over tail span for $x = l$

Subscripts:

TE	wing trailing edge
T	tail (horizontal)

When x, y, z, or t are used as subscripts, the respective partial derivative is indicated. For example,

$$\phi_x = \frac{\partial \phi}{\partial x}$$

$$\phi_{xt} = \frac{\partial^2 \phi}{\partial x \, \partial t}$$

ANALYSIS

The linearized partial differential equation for unsteady supersonic flow is

$$B^2\phi_{xx} - \phi_{yy} - \phi_{zz} + \frac{2V}{a^2}\phi_{xt} + \frac{1}{a^2}\phi_{tt} = 0 \tag{1}$$

The boundary condition on a wing experiencing a constant time rate of change of angle of attack is

$$\phi_z = -\dot{\alpha}Vt \qquad (z \longrightarrow 0) \tag{2}$$

The disturbance potential for small disturbances must satisfy both equations (1) and (2).

In an unpublished paper Clifford S. Gardner has shown, in effect, that a suitable solution is

$$\frac{\phi}{\dot{\alpha}} = \frac{M_0{}^2}{B^2}\psi + \left(t - \frac{M_0{}^2 x}{VB^2}\right)\chi \tag{3}$$

where ψ is the steady-state potential corresponding to a unit pitching velocity about the y-axis, and χ is the steady-state potential corresponding to a unit angle of attack. (See also the section entitled "Discussion.") The fact that equation (3) is a solution can be verified by direct substitution into equations (1) and (2).

Thus, Gardner has shown that the time-dependent potential for an angle of attack $\dot{\alpha}t$ may be compounded of two time-free or steady-state potentials, one for a constant angle of attack, and the other for steady pitching. (Equation (3) is limited in direct application to flow regions uninfluenced by the trailing edge. This limitation will be elaborated later.) The first-order treatment of sinusoidal oscillations in reference 5 (equation (19) and the interpretation therein) leads to a relation corresponding to equation (3). The relation in reference 5 is derived, however, as an approximation for slow oscillations; whereas Gardner's relation is exact for a constant acceleration ($\dot{\alpha}$ = constant). Equation (3) is also contained, in effect, as a special case of a solution for a more general unsteady motion given in reference 8. The plan forms for which the derivation in reference 8 is applicable are not completely general in contradistinction to those considered herein.

The lift distribution at time $t = 0$ for the angle of attack $\dot{\alpha}t$ is obtained from the upper-surface potential by the linearized Bernoulli equation for unsteady motion,

$$\Delta P = 2\rho(V\phi_x + \phi_t)_{z=+0} \tag{4}$$

$$\Delta P = 2\rho V\dot{\alpha}\left(\frac{M_0{}^2}{B^2}\psi_x - \frac{M_0{}^2 x}{VB^2}\chi_x - \frac{\chi}{VB^2}\right)$$

$$\Delta P = \frac{\dot{\alpha}}{B^2}\left[M_0{}^2(\Delta P)_{q=1} - \frac{M_0{}^2 x}{V}(\Delta P)_{\alpha=1} - 2\rho\chi\right] \tag{5}$$

where $(\Delta P)_{q=1}$ is the lift distribution for unit steady pitching velocity about y-axis, $(\Delta P)_{\alpha=1}$ is the lift distribution for unit steady angle of attack, and equation (4) contains the implicit assumption that $(\phi)_{z=-0} = -(\phi)_{z=+0}$. The choice of time $t = 0$ eliminates the lift due to angle of attack and leaves only the increment due to time rate of change of angle of attack.

Integration to obtain the lift and moment and reduction to coefficient form yields

$$C_{L_{\dot{\alpha}}} = \frac{M_0^2}{B^2} C_{L_q} + \frac{2M_0^2}{B^2} C_{m_\alpha} - \frac{8}{B^2 S\bar{c}} \iint \frac{X}{V} \, dS \tag{6}$$

$$C_{m_{\dot{\alpha}}} = \frac{M_0^2}{B^2} C_{m_q} + \frac{2M_0^2}{B^2 S\bar{c}^2} \iint x^2 \left(\frac{\Delta P}{\frac{1}{2}\rho V^2}\right)_{\alpha=1} dS + \frac{8}{B^2 S\bar{c}^2} \iint x\left(\frac{X}{V}\right) dS \tag{7}$$

where the integrations are carried over the wing plan form.

Equations (6) and (7) provide an evaluation of the acceleration derivatives $C_{L_{\dot{\alpha}}}$ and $C_{m_{\dot{\alpha}}}$ for the wing in terms of quantities that are presumed to be known for the pitching wing and the wing at an angle of attack. The application of these equations is limited to wings with supersonic trailing edges. (The component stream velocity normal to the trailing edge is supersonic.) The reason for this limitation is shown later.

Time-Dependent Upwash at Tail

Wings with supersonic trailing edges.- For the evaluation of the contribution of a horizontal tail to $C_{L_{\dot{\alpha}}}$ and $C_{m_{\dot{\alpha}}}$, it is first necessary to evaluate the instantaneous upwash velocity ϕ_z at the tail location. Differentiation of equation (3) would afford this quantity if equation (3) were still valid behind the wing trailing edge. Equation (3) as applied behind the trailing edge still satisfies the differential equation (1) and the boundary-condition equation (2) (limited to the wing surface) if the values of ψ and X therein assume values appropriate to the region behind the wing, including the contribution from the trailing vortex sheet. Equation (5), however, shows that the lift fails to fall to zero there as it should: the terms in ψ_x and X_x, with ψ and X as previously defined, are known to vanish but the remaining term in X does not vanish.

Thus equation (3), in its present form, leads to a spurious lift behind the wing of amount $-2\rho X\dot{\alpha}/B^2$ per unit area. The situation may be corrected, however, by an application of Lagerstrom's concept of cancellation of lift. There is superposed behind the wing an additional flow u_1, v_1, w_1 that gives rise there to an equal lift of opposite sign. Note that this lift is independent of time. The value of u_1 along the upper surface $(z = +0)$ of the trailing vortex sheet is then

$$u_1(x,y,+0) = \frac{\chi\dot{\alpha}}{VB^2} \tag{8a}$$

according to the steady-state linearized Bernoulli equation, which is equation (4) with $\phi_t = 0$. The further evaluation of u_1 and w_1 (v_1 is not needed) is limited at first to the relatively simple case where the trailing edge of the wing is supersonic.

Equation (8a) and its counterpart for the lower surface constitute a boundary condition on u_1. The distribution of u_1 in space ($z \neq 0$) must be such that the corresponding potential satisfies the steady-state linearized potential equation, which is equation (1) without the time-dependency terms. (This potential, being time free, will also satisfy the complete equation (1).) By differentiation of equation (1) with respect to x, $u_1 = \frac{\partial\phi}{\partial x}$ is seen to satisfy the same partial differential equation as ϕ does. The part of χ,χ_{tr} that can be ascribed to the trailing vortex system of the wing satisfies the same boundary condition at $z = \pm 0$ (with the factor $\dot{\alpha}/VB^2$) and the same partial differential equation. The solution can be shown to be unique, and thus u_1 may be identified with $\chi_{tr}\dot{\alpha}/VB^2$ everywhere without restriction to $z = \pm 0$. Behind the wing $\chi = \chi_{tr}$ for $z = \pm 0$ but does not for other values of z. Equation (8a) is thus generalized to

$$u_1(x,y,z) = \frac{\chi_{tr}\dot{\alpha}}{VB^2} \tag{8b}$$

In what follows, however, the small difference between χ_{tr} and χ behind the wing is neglected to simplify the calculation.

Associated with this additional u-velocity u_1 is an upwash velocity w_1. This upwash may be determined from the irrotationality condition

$$\frac{\partial w_1}{\partial x} = \frac{\partial u_1}{\partial z}$$

By virtue of equation (8b) with χ in place of χ_{tr} this upwash is

$$\frac{\partial w_1}{\partial x} = \frac{\dot{\alpha}}{VB^2}\chi_z \tag{9}$$

For $z = 0$ there results upon integration

$$w_1 = w_{1,TE} + \frac{\dot{\alpha}}{VB^2}\int_{TE}^{x}\chi_z\,dx \tag{10}$$

The quantity $w_{1,TE}$ is the upwash induced just behind the trailing edge of the wing (leading edge of the superposed flow) by the superposed flow. This edge has been assumed supersonic; hence, in its vicinity the Ackeret two-dimensional flow equations may be applied to the flow components normal to the edge (that is, simple sweep theory may be applied). The following equation is obtained:

$$w_{1,TE} = -u_{1,TE} \frac{\text{R.P.}\sqrt{M_0^2 \cos^2 \Lambda - 1}}{\cos \Lambda} \qquad (z = +0)$$

where $\Lambda = \Lambda(y)$ is the local angle of sweep of the trailing edge. (The real part designation R.P. is superfluous here but has a later use.) By use of equation (8a) this relation may be incorporated in equation (10) in the form

$$w_1 = w_1(x,y,0) = \frac{\dot{\alpha}}{VB^2}\left(-\chi_{TE} \frac{\text{R.P.}\sqrt{M_0^2 \cos^2 \Lambda - 1}}{\cos \Lambda} + \int_{TE}^{x} \chi_z \, dx\right)_{z=+0} \qquad (11)$$

Equation (11) gives the upwash component w_1 of the flow u_1, v_1, w_1 that was superposed to cancel the spurious lift behind the wing. This equation applies to points behind the wing and in the same plane. The upwash w_1 is to be added to that computed as ϕ_z from equation (3).

Wings with subsonic trailing edges.- The foregoing development for w_1 has been limited to wings with supersonic trailing edges. A rigorous determination of the superposition flow u_1, v_1, w_1 for wings with subsonic trailing edges is much more difficult and the process is only indicated herein. For the wings with subsonic trailing edges, the steady potential flow u_1, v_1, w_1 to be superposed must satisfy the boundary condition, equation (8a), (and its counterpart for $z = -0$) behind the trailing edge, and must, in addition, satisfy the condition $w_1 = 0$ on the surface ($z = \pm 0$) ahead of the trailing edge within the region of the trailing-edge disturbance. A flow of this kind may itself be built up by superposition of simpler "mixed-wing" flows in a variant of the manner elaborated in reference 9.

A rigorous evaluation of w_1 may not be necessary for practical applications, however, if the region of the wing influenced by subsonic trailing edges is not large. A suitable approximate value of w_1 may be obtained by a modification of the procedure leading to equations (10) and (11). Equation (8a) relating the surface value of u_1 to the surface value of χ is still true for wings with subsonic trailing edges.

The generalization for values off the surface (equation (8b)), however, no longer holds rigorously. The assumption is now made that an equation of the form of equation (8b) holds approximately with χ in place of χ_{tr}. Then it follows that, to this approximation, equation (10) applies to wings with subsonic trailing edges.

For this application, the value of w_1 at the trailing edge, $w_{1,TE}$, is taken to be zero. This fact follows from the boundary condition $w_1 = 0$ ahead of the wing trailing edge, together with the observed fact that w_1 must be continuous across a boundary on one side of which w_1 is specified and on the other side of which u_1 is specified, both of these velocities being finite. Equation (11) may now be recognized to apply for both subsonic and supersonic trailing edges because of the restriction of the radical to its real part (R.P.). Thus, the term containing the radical, which is $w_{1,TE}$, automatically vanishes for subsonic trailing edges.

Remarks on wings in a completely subsonic stream.- The partial differential equation (1) applies equally well at subsonic and supersonic speeds. Thus, equation (3) as applied to subsonic speeds is still a solution of equation (1) that satisfies the boundary condition of equation (2) on the wing but yields a spurious lift in the wake. Again an additional flow u_1, v_1, w_1 is required to cancel this spurious lift behind the wing. The boundary conditions for this cancellation flow are specified as for wings in a supersonic stream with subsonic trailing edges, but in the present case the region of trailing-edge disturbance covers the entire wing. Accordingly, the incremental lift corresponding to u_1 affects the entire wing surface. Equations (5) to (7) do not include this important contribution.

The values of ψ and χ specified in equation (3) are potentials obtained from lifting-surface theory. Expressions obtained from lifting-line theory, however, may be applied in the region behind the wing. These expressions should serve for the calculation of the time-dependent upwash at the tail location.

Angle of attack at the tail.- The angle of the local flow relative to the tail chord is compounded of the tail incidence i_T, the airplane geometric angle of attack $\dot{\alpha}t$, and the upwash angle $\frac{\phi_z}{V} + \frac{w_1}{V}$:

$$\alpha_T = i_T + \dot{\alpha}t + \frac{\phi_z}{V} + \frac{w_1}{V}$$

The component ϕ_z is obtained by differentiating equation (3) (any z), and, for $z = 0$, w_1 is given by equation (11). In the following discussion, consideration is limited to the case where the tail lies in the chord plane of the wing ($z = 0$). Then, using equations (3) and (11),

$$\alpha_T = i_T + \dot{\alpha}t\left(1 + \frac{X_z}{V}\right)_{z=0} + \frac{\dot{\alpha}M_0^2}{VB^2}\left(\Psi_z - x\frac{X_z}{V} - \frac{X_{TE}R.P.\sqrt{M_0^2\cos^2\Lambda - 1}}{VM_0^2\cos\Lambda} + \frac{1}{M_0^2}\int_{TE}^{x}\frac{X_z}{V}\,dx\right)_{z=+0} \tag{12}$$

The term X_z in equation (12) may be identified as the upwash due to a unit wing angle of attack, and the term Ψ_z may be identified as the upwash due to a unit wing pitching velocity. It will be convenient to replace X_z and Ψ_z by means of the definitions:

$$\left.\begin{aligned} \frac{w}{\alpha V} &\equiv \frac{X_z}{V} \\ \frac{w_q}{q\bar{c}} &\equiv \frac{\Psi_z}{\bar{c}} \end{aligned}\right\} \tag{13}$$

Then

$$\alpha_T = i_T + \dot{\alpha}t\left(1 + \frac{w}{\alpha V}\right)_{z=0} + \frac{\dot{\alpha}M_0^2}{VB^2}\left[\bar{c}\left(\frac{w_q}{q\bar{c}}\right) - x\left(\frac{w}{\alpha V}\right) - \frac{X_{TE}R.P.\sqrt{M_0^2\cos^2\Lambda - 1}}{VM_0^2\cos\Lambda} + \frac{1}{M_0^2}\int_{TE}^{x}\frac{w}{\alpha V}\,dx\right]_{z=+0} \tag{14}$$

Abbreviate this to

$$\alpha_T = i_T + \dot{\alpha}t\,F_1(x,y) + \dot{\alpha}\,F_2(x,y) \tag{15}$$

Tail Contribution to $C_{m_{\dot{\alpha}}}$

Equation (14) or equation (15) gives the instantaneous angle of attack at any point (x,y) of a tail located in the $z = 0$ plane relative to a system of axes with origin in the wing. The effect of the variation of α_T in the y-direction may be presumed for the present purpose to be adequately taken care of by forming the arithmetic average over the span of the tail. Denote this average by means of a bar above F_1 and F_2 and above w and w_q in F_1 and F_2. The variation of α_T in the x-direction is more significant. For practical application, however, it will suffice to use, in effect, an average in the x-direction also. Thus, the angle of attack

$$\alpha_T = i_T + \dot{\alpha}t\,\overline{F}_1(l) + \dot{\alpha}\,\overline{F}_2(l) \qquad (z = 0) \tag{16}$$

where x is taken equal to the tail arm l, is presumed to apply uniformly over the entire tail area. The tail arm l is measured from the origin to some reference point on the tail, such as the tail center of gravity or the elevator hinge line.

Equation (16) forms the basis for obtaining the first approximation to the tail contribution to the derivatives $C_{L_{\dot{\alpha}}}$ and $C_{m_{\dot{\alpha}}}$. Thus, the tail lift at time $t = 0$ may be written

$$L_T = \frac{\partial L_T}{\partial \dot{\alpha}_T}\left(\dot{\alpha}_T\right)_{t=0} + \frac{\partial L_T}{\partial \alpha_T}\left(\alpha_T\right)_{t=0}$$

where $\dot{\alpha}_T = \frac{\partial \alpha_T}{\partial t}$. The choice of time $t = 0$ eliminates the lift due to the airplane angle of attack and leaves just the increment due to time rate of change of angle of attack. By virtue of equation (16) the tail lift may be written

$$L_T = \frac{\partial L_T}{\partial \dot{\alpha}_T}\left(\dot{\alpha}\overline{F}_1\right) + \frac{\partial L_T}{\partial \alpha_T}\left(i_T + \dot{\alpha}\overline{F}_2\right) \tag{17}$$

Then, upon differentiation with respect to $\dot{\alpha}$,

$$\frac{\partial L_T}{\partial \dot{\alpha}} = \frac{\partial L_T}{\partial \dot{\alpha}_T}\,\overline{F}_1 + \frac{\partial L_T}{\partial \alpha_T}\,\overline{F}_2$$

The pitching moment of the tail about its own reference axis is assumed in the present approximation to be small compared with the wing pitching moment. The only moment derivative with respect to $\dot{\alpha}$ contributed by the tail is therefore the component

$$\Delta\,\frac{\partial M}{\partial \dot{\alpha}} = -l\,\frac{\partial L_T}{\partial \dot{\alpha}}$$

Also, the $\dot{\alpha}$ lift derivative contributed is the component

$$\Delta \frac{\partial L}{\partial \dot{\alpha}} = \frac{\partial L_T}{\partial \dot{\alpha}}$$

In nondimensional coefficient form, these several equations become

$$C_{L_T} = \left(C_{L_{\dot{\alpha}}}\right)_T \frac{\dot{\alpha}\overline{F}_1\bar{c}}{2V} + \left(C_{L_\alpha}\right)_T \left(i_T + \alpha\overline{F}_2\right)$$

$$\frac{\partial C_{L_T}}{\partial \dot{\alpha}} = \left(C_{L_{\dot{\alpha}}}\right)_T \frac{\overline{F}_1\bar{c}}{2V} + \left(C_{L_\alpha}\right)_T \overline{F}_2 \tag{18}$$

$$\Delta C_{m_{\dot{\alpha}}} = -\frac{2VS_T l}{S\bar{c}^2} \frac{\partial C_{L_T}}{\partial \dot{\alpha}} \tag{19}$$

$$\Delta C_{L_{\dot{\alpha}}} = \frac{2VS_T}{S\bar{c}} \frac{\partial C_{L_T}}{\partial \dot{\alpha}} \tag{20}$$

Note that $\left(C_{L_\alpha}\right)_T$ and $\left(C_{L_{\dot{\alpha}}}\right)_T$ are merely the values of C_{L_α} and $C_{L_{\dot{\alpha}}}$ computed for the isolated tail considered as a wing in an undisturbed flow. The lift-curve slope C_{L_α} may be computed or estimated by well-known methods, and a formula for the evaluation of $C_{L_{\dot{\alpha}}}$ is given in equation (6). The functions $\overline{F}_1$ and $\overline{F}_2$ are defined in equations (13) to (16). The upwash parameters $w/\alpha V$ and $w_q/q\bar{c}$ therein may be plotted once and for all for a given wing for use in calculations of the present type.

DISCUSSION

Downwash Charts

The contribution of the horizontal tail to the derivative $C_{m_{\dot{\alpha}}}$ is seen to depend on a knowledge of the wing downwash due to angle of attack $-w/\alpha V$ and of the wing downwash due to pitching $-w_q/q\bar{c}$. (Furthermore, the tail contributions to the better-known derivatives C_{m_α} and C_{m_q} depend, respectively, on the same two quantities.)

Calculations and charts of $-w/\alpha V$ are already available for several types of wings at supersonic speeds (references 10 to 15), but data on $w_q/q\bar{c}$ or its average over the tail span $\bar{w}_q/q\bar{c}$ are lacking. This quantity is small compared with $\bar{w}/\alpha V$ and the following simple consideration is sufficient for its estimation.

It is easy to show that far behind the wing (compare references 10 and 11)

$$\frac{\bar{w}_q}{q\bar{c}} = \frac{C_{L_q}}{2C_{L_\alpha}} \frac{\bar{w}}{\alpha V} \qquad (x = \infty) \tag{21}$$

if the shape of the span loading curves for pitching and angle of attack are the same. For differing shapes, equation (21) will at least provide the order of magnitude of $\bar{w}_q/q\bar{c}$. The ratio $C_{L_q}/2C_{L_\alpha}$ is usually only a fraction of unity: for rectangular wings pitching about the midchord line the maximum value is 1/6; for triangular wings pitching about the $\frac{2}{3}$-chord line a representative value is 1/8. In the expression for $\bar{F}_2(l)$ (equations (14) and (15), with bars added) $\bar{w}_q/qc$ is multiplied by $\bar{c}$ and $w/\alpha V$ by $x = l$. Since the tail arm l is ordinarily two or three times the chord $\bar{c}$, the term in $\bar{w}_q/q\bar{c}$ is seen to be quite small in comparison with the term in $\bar{w}/\alpha V$. Accordingly, the term in $\bar{w}/q\bar{c}$ may frequently be neglected. In any event, equation (21) should provide a sufficiently good approximation.

Some Limitations on the Results

The final equations (14) to (20) are not rigorous. The approximation was introduced in the evaluation of the upwash w_1 associated with the cancellation flow for the spurious lift in the wake. The approximation lies in the integral that occurs in F_2 as defined in equation (14). The simulation of the exact integrand by $w/\alpha V$ was made there to render the calculations more tractable. For wings with supersonic trailing edges, the exact integrand is $\frac{\partial}{\partial z}\left(\frac{X_{tr}}{V}\right)$; this integrand is believed to differ only slightly from $w/\alpha V$. For wings with subsonic trailing edges, the exact integrand has not been obtained because of the difficulty. The exact integrand is believed not to differ greatly from $w/\alpha V$, provided the trailing-edge disturbance does not envelope a large proportion of the wing surface.

An error also occurs in equations (6) and (7) for $C_{L_{\dot{\alpha}}}$ and $C_{m_{\dot{\alpha}}}$ for the wing when the trailing edges are subsonic. The magnitude of the error likewise depends on the proportion of the wing surface enveloped by the trailing-edge disturbance.

The method of derivation used herein employs the assumption that the angle of attack varies linearly with time. The airplane, however, may be expected to experience sinusoidal variations of angle of attack with time, and a sinusoidal variation is assumed in solving the equations for dynamic stability. The values of $C_{m_{\dot{\alpha}}}$ and $C_{L_{\dot{\alpha}}}$ obtained herein are believed to be satisfactory approximations for sinusoidal motion so long as the square of the reduced frequency $\omega\bar{c}/2V$ is small compared with B^4/M^4. This result is inferred from the fact that equation (3) is still approximately true for slow sinusoidal motions. This result was proved in reference 5 (equation (19) and following discussions therein). An examination of the terms omitted therein in the power expansion in ω leads to the criterion for ω previously noted.

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