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EXPERIMENTAL ANALYSIS OF A PRESSURE-SENSITIVE SYSTEM FOR SENSING GAS TEMPERATURE

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SUMMARY

Various theoretical and practical aspects of a pressure-sensitive system for the measurement of gas temperatures in gas-turbine-engine combustion chambers are analyzed. An experimental setup was operated under controlled conditions of temperature, weight flow, and wall temperature. The gas temperature after combustion was obtained by application of a relation equating thermodynamic conditions before and after combustion. This method of gas-temperature sensing appears practical for high-temperature applications. The accuracy with which gas temperature may be determined by this method is within ±2 percent. Measurements made by the thermodynamic method were used as a temperature standard for comparison of temperature data obtained from conventional thermocouple probes.

INTRODUCTION

With the advent of gas-turbine engines, the controls designer has been confronted with the serious problem of operating engines not only at high temperatures for maximum efficiency, but also at temperatures low enough to be within the safe operating range for the materials used.

It therefore becomes imperative to incorporated temperature-limiting controls to prevent engine failure by overheating. Temperature can be used both as a primary control parameter and as a temperature-sensing parameter. Any means of sensing gas temperature for application to controls must meet the following requirements: (1) have an output that is a simple function of true gas temperature under various conditions of weight flow, temperature of surrounding materials, density, composition of gas, and temperature level; (2) provide an output that is easily incorporated in a control; (3) have a rapid response to transient conditions; (4) have an extended temperature range; (5) have a long life; and (6) be independent of material characteristics subject to change with time and use.

The most commonly used methods for measuring temperature employ thermocouples or resistance-wire thermometers, both of which have the advantages of simplicity and durability. These devices, however, do not provide sufficiently ideal outputs when consideration is given to the inherent errors resulting from radiation, heat conduction, and velocity losses that occur at elevated gas temperatures. In addition, chemical action on instrument materials can result in a change of calibration as well as in a reduction in life, which is a particularly difficult factor to contend with because the mass of the unit must be large in order to lengthen life; whereas the requirement of rapid response to a transient demands that the mass be small. At temperatures below 1000° F these errors can be reasonably neglected in control work. At elevated temperatures, however, such errors cannot be neglected, particularly radiation errors, because these errors appear as the difference of the fourth-power functions of the two temperatures involved.

As a possible means of circumventing present difficulties, a theoretical analysis was conducted at the NACA Lewis laboratory using the thermodynamic properties of gases as basic parameters. An equation has been developed based on these properties (reference 1). This equation, based upon the expression for weight flow, correlates gas temperatures with measured pressures before and after combustion. Such pressure measurements are independent of radiation or heat-conduction effects, and therefore permit an evaluation of the true gas temperature (within the accuracy of pressure-measuring instrumentation) as long as the ideal gas laws apply.

Various theoretical and practical aspects of the pressure-sensitive system for gas-temperature measurement are discussed herein. The system was applied to an experimental burner setup operating under controlled conditions of temperature level, weight flow, and test-section-wall temperature. Several methods of measuring weight flow were analyzed to determine their applicability to the system. Particular emphasis is given to a discussion of the pitot-static method of determining weight flow by analyzing the velocity profile across a test section at elevated temperatures.

For comparison purposes, the pressure-sensitive method (reference 1) is considered as a reference system for sensing temperature. Data obtained with several conventional thermocouple probes were compared with data from this reference system over a range of temperature levels, weight flows, and duct-wall temperatures.

1222

ANALYSIS

Various types of fluid meter, including orifices, nozzles, venturis, and pitot tubes, or combinations of such units, may be employed to evaluate thermodynamic relations of reference 1 in terms of gas temperature. Experimental data reported herein were obtained using an orifice before combustion (station 1) and a pitot-static tube after combustion (station 2). The general weight-flow equation, which applies for all fluid meters, is

$$W = \rho AV \tag{1}$$

(Definitions of symbols used in this section are given in appendix A.)

Orifice Equation

The specific equation for flow through an orifice, developed from equation (1), is

$$W_1 = A_1 K_1 E_1 Y_1 \sqrt{\rho_1 \Delta P_1 2g} \tag{2}$$

The measurements required by equation (2) for the determination of weight flow at station 1 can be readily and accurately obtained using an orifice installation, as recommended in reference 2.

Pitot-Tube Equation

Discussion of pitot-tube method. - A pitot-static tube was used at station 2 because of the convenience of this method when dealing with high-temperature gases, inasmuch as it offers negligible resistance to gas flow, has a rapid response (reference 3), and is simple in construction. The specific equation for weight flow when applying the pitot-static tube for flow measurement at station 2, as developed from equation (1), is

$$W_2 = E_2 A_2 \varphi'_2 \sqrt{2g_2 \rho_2 \Delta P_2} \tag{3}$$

When using this method it is important to realize that the measured value of ΔP (the total pressure minus the static pressure) is that of the specific point at which the pressure-sensing probe is located. In general, the flow or velocity profile across a section is not flat, therefore ΔP is not constant across that section. In order to make the system applicable to the temperature equation, it is necessary that a single-point indication be proportional to the total weight flow through the section. It therefore becomes essential to determine the relation that exists between a measured value of ΔP, which is quite readily obtained, and the effective value of ΔP, which is a function of the average velocity of the gas. If it is assumed that the effects of changes in boundary layer on weight flow over the range of temperature operation considered remain essentially constant, the ratio of the effective ΔP to a measured ΔP can be evaluated.

In order to successfully apply equation (3) for a given apparatus, whether it be an aircraft power plant or a simple straight-through pipe, the relation between a measured ΔP and the effective ΔP must be consistent $(\Delta P_{eff}/\Delta P_{meas} = C$, a constant). If this consistency does not exist, the method presented herein for gas-temperature evaluation cannot be applied. It should be noted, however, that a consistent ratio of $\Delta P_{eff}/\Delta P_{meas}$ will be considered as such only in regard to the individual application and the accuracy of gas-temperature evaluation desired. For example, in one installation an evaluation of gas temperature to within ±10 percent may be suffient; the ratio $\Delta P_{eff}/\Delta P_{meas}$ therefore may vary ±10 percent and be considered consistent. In another installation, where a gas-temperature-evaluation accuracy of ±2 percent is desired, a ±3 percent variation in $\Delta P_{eff}/\Delta P_{meas}$ could not be considered consistent. For the data presented, the evaluation of $\Delta P_{eff}/\Delta P_{meas}$ was made over an extended temperature range for fully developed turbulent flow in a pipe; furthermore, ΔP was measured at the center of the gas stream so that $\Delta P_{meas} = \Delta P_{max}$.

Evaluation of $\Delta P_{eff}/\Delta P_{max}$. - If a fully developed laminar-flow velocity profile is realized in the section, it is possible to determine the ratio $\Delta P_{eff}/\Delta P_{max}$ from theoretical considerations. Because experimental data were obtained for turbulent-flow conditions only, experimental methods had to be employed in order to evaluate the ratio.

The velocity profile in a duct cannot be defined by a simple mathematical relation when turbulent-flow conditions are obtained. The profile is more nearly uniform under these conditions than for laminar flow. Figure 1 shows a typical curve of the velocity ratio

V/V_{max} as a function of Reynolds number. These data were recorded by Stanton and Pannell in 1914 and were obtained from reference 4. The curve shows that in the laminar region A the velocity ratio is of the order 0.5. A very rapid change from 0.5 to 0.72 is noted in the transition region B. In the fully developed turbulent region C, the velocity ratio is nearly constant at a value of approximately 0.78. It should be noted, however, that these numerical values probably would not apply to flow conditions that exist in turbojet engines. The important point is that a consistant velocity ratio must exist before a single pitot tube can be used to determine ΔP_{eff} for any application.

A direct method of evaluating $\Delta P_{eff}/\Delta P_{max}$ is to obtain actual ΔP profile data by making a traverse of the section using a pitot tube. Particular attention must be given to recording accurately data obtained near the pipe walls, because a steep gradient exists in that region.

Inasmuch as

$$W = \rho AV \tag{1}$$

where V, the average gas velocity, is a function of $\sqrt{\Delta P_{eff}}$ when ρ is a function of p/RT (appendix B), and A is a function of radius squared, a value of ΔP_{eff} can be obtained from considerations of ΔP (measured). In order to obtain this value, it is necessary to plot the square root of the measured pressures against the square of the radius. Figure 2 shows that $\sqrt{\Delta P_{eff}}$ is proportional to A', the area under the curve, and $\sqrt{\Delta P_{max}}$ is proportional to area A' plus A'', the hatched area. Therefore

$$\frac{\sqrt{\Delta P_{eff}}}{\sqrt{\Delta P_{max}}} = \frac{A'}{A' + A''}$$

or

$$\frac{\Delta P_{eff}}{\Delta P_{max}} = \left(\frac{A'}{A' + A''}\right)^2 = C \tag{4}$$

The ratio of $\Delta P_{eff}/\Delta P_{max}$ can be determined quite readily from an analysis of measured pressures.

The average ΔP, calculated from a traverse of the section with a pressure probe, does not have the same significance as the effective ΔP based on the considerations of an average velocity across that section.

Another method of evaluating the ratio $\Delta P_{eff}/\Delta P_{max}$ can be used if the gas temperature after combustion (station 2) is known for at least part of the temperature range. For this case, the weight flow at stations 1 and 2 can be equated and solved for ΔP_{eff}; the ratio can then be found by using a measured value of ΔP_{max}. In order to equate the weight flow at stations 1 and 2, it is necessary to account for the addition of fuel:

$$W_2 = W_1 + f = \left(1 + \frac{f}{W_1}\right) W_1 \tag{5}$$

The weight flow at station 2 (equation (3)) is equal to the weight flow at station 1 (equation (2)) times $\left(1 + \frac{f}{W_1}\right)$. When a thermocouple is used to determine gas temperature at station 2, as it was for this experimental analysis, errors can be expected at temperatures exceeding 1000° F, at which temperature radiation errors of thermocouples become prominent. The weight-flow evaluation method was used as a check on the pressure-survey method only in the range in which errors of thermocouples could be neglected. A theoretical discussion of thermocouples and their inherent errors is given in appendix C.

Temperature Equation

General equation. - The orifice and pitot-static-tube equations for weight flow may be equated:

$$E_2 A_2 \varphi'_2 \sqrt{2g\rho_2 \Delta P_2} = A_1 K_1 E_1 Y_1 \sqrt{\rho_1 \Delta P_1 2g} \left(1 + \frac{f}{W_1}\right) \tag{6}$$

When the density ρ is replaced by its equivalent $\frac{p}{RT}$ (appendix B), equation (6) can be solved for T_2.

$$T_2 = \left(\frac{E_2}{E_1} \frac{\varphi'_2}{Y_1} \frac{A_2}{K_1 A_1} \frac{1}{1 + \frac{f}{W_1}} \right)^2 \left(\frac{p_2}{p_1} \right) \left(\frac{\Delta P_2}{\Delta P_1} \right) T_1 \tag{7}$$

Equation (7) represents the complete temperature equation with which experimental data discussed herein will be concerned.

Simplified temperature equation. - In reference 1 it is shown that, based on analytical considerations, the term

$$\left(\frac{E_2}{E_1} \frac{\varphi'_2}{Y_1} \frac{A_2}{K_1 A_1} \frac{1}{1 + \frac{f}{W_1}} \right)^2$$

of equation (7) is very nearly constant. Considering the term as a constant, the temperature equation becomes

$$T_2 = NT_1 \left(\frac{p_2}{p_1} \right) \left(\frac{\Delta P_2}{\Delta P_1} \right) \tag{8}$$

For the results presented herein, each factor of the coefficient N has been taken into consideration and the analytical conclusions substantiated.

APPARATUS AND PROCEDURE

A schematic representation of the apparatus used to obtain the data presented is given in figure 3. Air was drawn from the atmosphere and compressed to a pressure of approximately 3.5 pounds per square inch gage by two commerical blowers connected in series, then ducted through the combustion chambers and test sections. The maximum air flow available was approximately 6.5 pounds per second at a pressure of 2.5 pounds per square inch gage in the test sections. An orifice designed and installed according to A.S.M.E. specifications (reference 2) was located at station 1, downstream of the blowers, for the determination of air flows. Located downstream of station 1 were four combustion chambers capable of heating the maximum air flow to a temperature of 2500° F. A flame shield and gas mixer, installed to shield the thermocouples in test sections from radiation from the flame front and also to effect uniform gas mixing, was located immediately downstream of the combustion chambers. The gas leaving the flame shield and gas mixer was ducted through a 10-inch-diameter Inconel pipe for approximately 15 feet, whereupon it entered test sections 1, 2, and 3, also 10 inches in diameter and arranged in series, and was finally discharged to the atmosphere.

The instrumentation at test section 1 (fig. 4), designated station 2, consisted of: a six-tube static-pressure survey rake; a six-thermocouple survey rake incorporating single-shielded thermocouples; three thermocouples embedded in the wall of the test section; and a total-pressure rake. A total-head tube of 0.030-inch outside diameter, which could be positioned along the pipe diameter during a run for the purpose of obtaining a complete total-pressure-profile survey was also used.

Test section 2, illustrated in figure 5, contained eight thermocouples having bead diameters of 3/8, 1/4, 3/16, 5/32, 1/8, 3/32, 1/16, and 1/32 inch, which were arranged in a ring concentric with the pipe. Three additional thermocouples were embedded in the wall of the test section.

Test section 3 (fig. 6) contained the following instrumentation: a thermocouple having an electrically heated shield; a commerical-type multiple-shield thermocouple; a gold-encased single-shield Bureau of Standards thermocouple (reference 5); three thermocouples embedded in the wall of the test section; and a pitot-static pressure tube in the gas stream. A cutaway drawing of the heated-shield thermocouple is presented in figure 7. During steady-state operation, the electric-heater coil supplied heat to the shield surrounding the thermocouple located in the gas stream until the indicated wall temperature of the shield equalled indicated gas temperature. When this equality existed, the gas temperature was recorded under the assumption that conduction and radiation losses were then minimized.

Operating conditions were established by maintaining the weight flow at a constant level and varying the fuel flow to the combustion chambers, whereby the time required for changes from one steady-state temperature level to another was held to a minimum. In order to present the effects of weight-flow variations on the operation of the various temperature-sensing probes, three weight flows were investigated over a temperature range from approximately 900° to 2200° F. The flows selected for these tests were approximately 4.5, 5, and 6.5 pounds per second. A plot of weight-flow variations at station 2 is presented in figure 8. For the temperature range considered, the gas velocities in the test sections varied between 300 and 800 feet per second with the various weight flows. The Mach number, however, varied only slightly for each setting, as can be seen from figure 9.

The effects of conduction and radiation losses on the thermocouple indications of gas temperature at constant reference-gas-temperature settings were investigated by controlling the test-section-wall temperature with cooling-water sprays acting on the outside walls of the test sections. For runs in which water cooling was not used, there was no variation in indicated wall temperature at any given reference gas temperature during variations of weight flow. This effect is illustrated in figure 10.

RESULTS AND DISCUSSION

Evaluation of Constant C

Profile survey. - The results of pressure-profile surveys taken at station 2 for reference gas temperatures of 550°, 1500°, and 2300° R are presented in figure 11. A plot of C for all operating gas temperatures is given in figure 12. Any differences in boundary layer caused by differences in gas temperature (reference 6) are small enough to cause a variation in C of less than 0.5 percent. On the basis of these results, a value for C of 0.87 was selected and used in the evaluation of the temperature equation (equation (7)) as applied to data presented herein.

Weight-flow equations. - The evaluation of C obtained by equating the weight flow at stations 1 and 2, using thermocouple data taken at these stations, is presented in figure 13. As mentioned in the section ANALYSIS and discussed in appendix C, inherent errors of the thermocouple should be considered at temperatures of approximately 1000° F (1460° R) and above, which is shown by the data presented in figure 13. A comparison between the weight-flow and the pressure-survey evaluations of C is shown in figure 14.

In view of the results obtained by the two methods of evaluation of C, and the simplicity of weight-flow method over the profile survey, the weight-flow method should be satisfactory for use in determining C in practical application of pressure-sensitive temperature-sensing systems, providing this determination is made at temperatures below 1000° F.

Pressure-Sensitive System as Temperature Standard

On the basis of the fundamental relations used in deriving the temperature equation (equation (7)) and the experimental evaluation of the constant C, the pressure-sensitive system is hereinafter considered a temperature standard. This standard, based upon the application of equation (7), introduces only errors of instrument reading; the temperature-sensing accuracy is therefore considered to be within ±2 percent.

Weighing is the fundamental method of fluid measurement and is the technique used by the A.S.M.E. to establish a flow-measurement standard for fluid meters, such as an orifice plate, pitot-static tube, nozzle, venturi, and so forth (reference 2). The A.S.M.E. has established flow-measurement accuracy of approximately ±1 percent for these various flow-measurement units. Installation and instrumentation requirements, as established by the A.S.M.E. in order to obtain this accuracy, were used in obtaining the data for the evaluation of the flow equations presented. Inasmuch as the maximum error in weight-flow measurement is approximately ±1 percent, in effect, the maximum deviation of any one parameter in equation (3) is limited to ±1 percent, and as weight-flow is evaluated at two separate stations, the maximum error deviation between the two stations is ±2 percent. Because density, and therefore temperature, is one of the parameters in equation (3), the greatest deviation of temperature is ±2 percent from a true absolute level. The state of development of pressure-measuring techniques is such as to insure a rapid response rate for the pressure-sensitive system (reference 3). The error involved in the use of the simplified form of the temperature equation (equation (8)) is shown to be quite small in figure 15, where values of the dimensionless coefficient N, plotted against gas temperature for all operating conditions, fall within a band of ±1.5 percent. Equation (8) was not used in connection with the temperature standard.

COMPARISON OF THERMOCOUPLES WITH TEMPERATURE STANDARD

No attempt is made to evaluate the various thermocouples in this discussion. A general explanation of the performance of the various thermocouples is given in the theoretical discussion of thermocouple operation in appendix C.

Radial Temperature Profile

A typical plot of gas-temperature profile, as measured by the single-shielded-thermocouple survey rake in test section 1, is compared to the temperature standard in figure 16 for reference gas temperatures of approximately 1450° and 2000° R. Because the gas temperature is uniform across the section, a comparison can be made between any thermocouple-temperature indication and the reference gas temperature given by the temperature standard or reference system.

Performance of Thermocouples

A performance plot is presented in figure 17, showing temperature deviations from the reference temperature of (1) a commercial multiple shield, (2) a single-shield survey rake across the pipe diameter (average of all temperatures), (3) a Bureau of Standards gold shield, (4) a NACA heated shield, and (5) a 3/8-inch-diameter junction thermocouple for weight flows of 4.6, 5.3, and 6.6 pounds per second. Data for the Bureau of Standards thermocouple were not obtainable for the mass flow of 6.6 pounds per second because the thermocouple burned out during the performance run.

A plot showing the percentage deviation from the calculated reference temperature for four of the five types of thermocouple is given in figure 18 for weight flows of 4.6, and 6.6 pounds per second. From the slope of the curves, it is apparent that increasing reference gas temperature considerably increases the percentage of error.

Influence of Thermocouple-Bead Diameter

Indications of gas temperature obtained from thermocouples of several bead diameters are compared with the reference temperature at 2525° R in figure 19. The data were extrapolated in order to obtain an intersection with the reference temperature at a thermocouple-bead diameter of zero. Without a reference temperature to indicate the true end condition of gas temperature at a thermocouple-bead diameter of zero, extrapolation of the data curves could be greatly in error. Also, the differences in temperature indication of the same thermocouple operating at a constant reference gas temperature but at various weight flows are quite apparent from the data. An equation representing the data curve would be impractical when added consideration is given to such factors as varying wall temperatures, Mach number, and reference levels.

Effect of Wall Temperature on Thermocouple Indications

The effect on thermocouple indication of changes in conduction and radiation losses at various reference gas temperatures were investigated by controlling test-section-wall temperatures with cooling-water sprays acting on the outside of the walls. A plot showing the deviation of several thermocouples from the temperature reference for a wall temperature of 710° R is given in figure 20. (The effects on thermocouple indications of changes in conduction and radiation losses may be seen when comparing figs. 20 and 17(c).) The percentage deviation is given in figures 21(a) and 21(b) for weight flows of 4.6 and 6.6 pounds per second, respectively. The slope of the curves of figure 19 shows that large errors can be encountered at the higher reference gas temperatures. A tabulation of results giving the performance of the temperature-sensing probes investigated at reference gas temperatures of 2000° and 2400° R and various weight-flow conditions, is given in table I.

CONCLUSIONS

Various theoretical and practical aspects of the pressure-sensitive systems for the measurement of gas temperatures have been discussed. The gas temperature after combustion was obtained by application of a relation equating thermodynamic gas conditions before and after combustion. The following conclusions may be drawn from this application:

1. Determination of gas temperatures after combustion from measurements of gas temperatures before combustion and gas pressures before and after combustion appears practical for high-temperature applications. The temperature-sensing accuracy of this method is within ±2 percent.

2. Temperature changes of the combustion gases are accompanied by pressure changes that are in effect instantaneous, are unaffected by errors associated with conventional thermocouples, and can be utilized in a control system.

3. By using the thermodynamic method as a temperature standard for comparison of thermocouples, it is determined that conventional thermocouples can be appreciably in error in their temperature indications at gas temperatures above approximately 1500° R.

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1222

APPENDIX A

SYMBOLS

The following symbols are used in the section ANALYSIS and in appendix B:

A area, sq ft

C constant, $\Delta P_{eff}/\Delta P_{max}$

E area multiplier for thermal expansion

f fuel flow, lb/sec

g acceleration due to gravity, ft/sec^2

K flow coefficient of orifice plate

N constant, $\left(\frac{E_2}{E_1} \frac{\varphi'_2}{Y_1} \frac{A_2}{K_1 A_1} \frac{1}{1 + \frac{f}{W_1}}\right)^2$

P total pressure, lb/sq ft absolute

p static pressure, lb/sq ft absolute

R gas constant

r ratio of static to total pressure, p/P

T total temperature, °R

t static temperature, °R

V average velocity, ft/sec

W weight flow, lb/sec

Y ratio of flow coefficient of gas to that for liquid at same Reynolds number (reference 2)

γ ratio of specific heats at constant pressure and constant volume

ΔP total pressure minus static pressure, P - p, lb/sq ft

ΔP_{eff} pressure of average velocity, lb/sq ft

ΔP_{max} pressure of maximum velocity, lb/sq ft

ΔP_{meas} measured value of total minus static pressure, lb/sq ft

ρ density, p/RT, lb/cu ft

φ conversion factor of hydraulic equation to compressible-flow equation

φ' particular value of φ when $\rho = p/RT$ (See equation B.)

Subscripts:

max maximum

1 station 1 (before combustion)

2 station 2 (after combustion)

1222

APPENDIX B

COMPRESSIBLE-FLOW EQUATIONS

The hydraulic equation for incompressible flow may be multiplied by an appropriate conversion factor φ to obtain the exact equation for compressible flow. The expression for the conversion factor φ may be derived from the compressible-flow equation by factoring out the hydraulic equation so that the remaining factor is the expression for the conversion factor φ.

Bernoulli's theorem for compressible flow may be written as

$$V = \left[\frac{2g\gamma}{\gamma - 1}\left(\frac{P}{\rho_t} - \frac{p}{\rho_0}\right)\right]^{\frac{1}{2}} \tag{B1}$$

The weight flow is

$$W = A\rho_0 V \tag{B2}$$

Substitution of equation (B1) in equation (B2) and replacement of stagnation density ρ_t with the equivalent adiabatic relation $\rho_0\left(\frac{P}{p}\right)^{\frac{1}{\gamma}}$ gives

$$W = A\left\{2g\left(\frac{\gamma}{\gamma - 1}\right)\rho_0\left[\frac{P}{\left(\frac{P}{p}\right)^{\frac{1}{\gamma}}} - p\right]\right\}^{\frac{1}{2}}$$

The free-stream density ρ_0 may be replaced by its equivalent $\frac{p}{Rt}$ and the equation simplified:

$$W = A\left\{2g\left(\frac{\gamma}{\gamma - 1}\right)\frac{p}{Rt}\left[\frac{P^{\left(1-\frac{1}{\gamma}\right)} - p^{\left(1-\frac{1}{\gamma}\right)}}{p^{-\frac{1}{\gamma}}}\right]\right\}^{\frac{1}{2}}$$

$$W = A\left\{2g\left(\frac{\gamma}{\gamma - 1}\right)\frac{p^2}{Rt}\left[\frac{P^{\left(1-\frac{1}{\gamma}\right)} - p^{\left(1-\frac{1}{\gamma}\right)}}{p^{\left(1-\frac{1}{\gamma}\right)}}\right]\right\}^{\frac{1}{2}}$$

$$W = A\left\{2g\left(\frac{\gamma}{\gamma - 1}\right)\frac{p^2}{Rt}\left[\frac{P}{p}^{\left(\frac{\gamma-1}{\gamma}\right)} -1\right]\right\}^{\frac{1}{2}} \qquad \text{(B3)}$$

This expression for compressible flow may be written as

$$W = A\varphi\sqrt{2g\rho\ (P - p)} \qquad \text{(B4)}$$

where

$$\varphi = \left\{\frac{1}{\rho\ (P - p)}\left(\frac{\gamma}{\gamma - 1}\right)\frac{p^2}{Rt}\left[\left(\frac{P}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\right\}^{\frac{1}{2}} \qquad \text{(B5)}$$

This expression for φ involves the density ρ, which also appears in equation (B4) and may be arbitrarily selected as a ratio involving total pressure P or static pressure p divided by total temperature T or static temperature t. For the case in which the density ρ is selected as p/RT, the conversion factor φ is designated φ' and equation (B5) is simplified as follows:

$$(\varphi')^2 = \frac{Tp\gamma\left[\left(\frac{P}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{t\ (P - p)(\gamma - 1)} \qquad \text{(B6)}$$

The adiabatic relation of the temperatures is

$$\frac{T}{t} = \left(\frac{p}{P}\right)^{\frac{1-\gamma}{\gamma}} \tag{B7}$$

This relation may be substituted into equation (B6) to obtain

$$(\varphi')^2 = \frac{\left(\frac{p}{P}\right)^{\frac{1-\gamma}{\gamma}} \gamma \left[\left(\frac{P}{p}\right)^{\frac{\gamma-1}{\gamma}} -1\right]}{\left(\frac{P - p}{p}\right) (\gamma - 1)}$$

If the pressure ratio p/P is set equal to r this equation becomes

$$(\varphi')^2 = \frac{\left(r^{\frac{1-\gamma}{\gamma}}\right)\left(r^{\frac{1-\gamma}{\gamma}} - 1\right) \gamma}{\left(\frac{1}{r} - 1\right) (\gamma - 1)}$$

or

$$(\varphi')^2 = \frac{r^{\frac{1}{\gamma}}\left(r^{\frac{1-\gamma}{\gamma}} - 1\right) \gamma}{(1 - r) (\gamma - 1)} \tag{B8}$$

Evaluation of the conversion factor φ' for various pressure ratios r show the error that may be expected from neglecting φ in the hydraulic equation (B4) where the density ρ is p/RT. The greatest deviation of the conversion factor φ' from 1 occurs at the critical pressure ratio, at which φ' is approximately 0.945 for the ratio of specific heats γ equal to 1.3 for air at a temperature of 3000° R.

APPENDIX C

CHARACTERISTICS OF THERMOCOUPLES

A brief summary of the significance of thermocouple indications is required in order to explain and to compare temperature data obtained from thermocouples with data obtained from the reference system.

When dealing with thermocouples it must be noted that the indicated temperature is that of the junction itself and not necessarily that of the medium in which the junction is immersed. A thermocouple probe inserted in a hot gas stream indicates the temperature attained by the thermal element when a condition of thermal equilibrium with its environment is reached. The indicated temperature is therefore a function of several modes of heat transfer, including convection, conduction, and radiation.

Velocity effect. - Actual total temperature is also a function of gas-stream velocity. This relation can be expressed as

$$T = T_0 + T_a$$

where

T_0 free-stream temperature

T_a temperature due to velocity, $\frac{V^2}{2gJC_p}$

J mechanical equivalent of heat

C_p specific heat at constant pressure

Standard probes are designed to partly stagnate the gas stream around the junction and thereby recover a part of the temperature due to velocity. Because probes do not recover all this temperature, they must be calibrated for this effect so that the corrected expression is

$$T = T_0 + \left(\frac{V^2}{2gJC_p}\right) r$$

where r is the recovery factor.

Inasmuch as the probe temperature is a function of "heat in" minus "heat out," these terms must be evaluated and analyzed to
1222 determine their effect on the final indicated temperature.

Heat in is basically a function of heat transfer by convection from the gas to the solid material of the thermocouple, which can be expressed as

$$q_c = Sh(T - T_p)$$

where

q_c "heat in"

S surface area in contact with gas

h heat-transfer coefficient

T_p temperature indicated by probe

The coefficient of heat transfer is an involved function of probe area and weight flow across the thermocouple as well as the viscosity and the composition of the gas. In order for the probe equilibrium temperature to approach total temperature, the difference $(T - T_p)$ must be small. It therefore becomes evident that in order to insure sufficient heat transfer by convection, the weight flow over the junction must be high.

Heat out (heat lost) is due to: (1) conduction of heat away from the junction through the leads of the thermocouple and, (2) radiation of heat to cooler surfaces. (It should be noted that if the walls of the duct containing the gas are at a higher temperature than the gas, then all heat flow is reversed.)

Conduction errors in indications are given by

$$T - T_p = \frac{T - T_L}{\cosh\sqrt{\frac{hcL^2}{kA}}} = \frac{T - T_L}{\cosh^{2} \frac{L}{D} \sqrt{\frac{hD}{k}}}$$

where

T_L temperature at distance L from junction

L some distance from junction along wires c circumference of wires

k cross-sectional thermal conductivity of thermocouple wires

A cross-sectional area of wires

D diameter of wires

From this relation it follows that in order to keep the conduction error small the L/D ratio and h must be large, whereas k must be small.

Heat loss due to radiation may be one of the most serious errors encountered in using thermocouples for sensing elevated gas temperatures. This fact is readily appreciated when an expression governing radiation error is analyzed.

The heat loss due to radiation may be expressed as

$$H_r = e\,\delta\,S\,(T_p^{\,4} - T_w^{\,4})$$

where

e emissivity of radiating body

δ Stefan-Boltzman constant

T_w wall temperature of radiating body

It should be noted that for a given installation of a thermocouple probe, the only variables concerned with the radiation error are the temperature of the junction and the temperature of the wall or radiating body. Slight changes in temperature appear as the fourth power, making the difference and the resulting radiation error quite appreciable.

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2. Anon.: Fluid Meters, Their Theory and Application. A.S.M.E. Res. Pub., pub. by Am. Soc. Mech. Eng. (New York), 4th ed., 1937, pp. 33-37.

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3. Delio, Gene J., Schwent, Glennon V., and Cesaro, Richard S.: Transient Behavior of Lumped-Constant Systems for Sensing Gas Pressure. NACA TN 1988, 1949.

4. McAdams, William H.: Heat Transmission. McGraw-Hill Book Co., Inc., 2d ed., 1942.

5. Fiock, Ernest F.: Thirteenth Monthly Report of Progress on the Development of Thermocouple Pyrometers for Gas Turbines. Nat. Bur. Standards, Jan. 21, 1947.

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TABLE I

PERFORMANCE OF TEMPERATURE - SENSING PROBES

Weight flow (lb/sec)	Mach Number	Velocity (ft/sec)	Calculated average gas temperature (°R)	Deviation of thermocouple from calculated gas temperature, percent							
				Nonshielded thermocouple		Single-shielded thermocouple		Gold-shielded thermocouple		Multiple-shielded thermocouple	
				Without cooling	With wall cooling	Without cooling	With wall cooling	Without cooling	With wall cooling	Without cooling	With wall cooling
4.60	0.196	425	2000	7.50	7.70	5.50	6.10	4.20	-----	3.40	3.10
6.55	.280	610	2000	6.65	8.50	4.60	7.30	----	-----	3.20	4.70
4.50	.214	510	2400	10.70	13.00	8.20	10.60	6.30	-----	5.00	6.60
6.35	.300	715	2400	8.75	13.00	5.70	11.00	----	-----	3.50	7.00

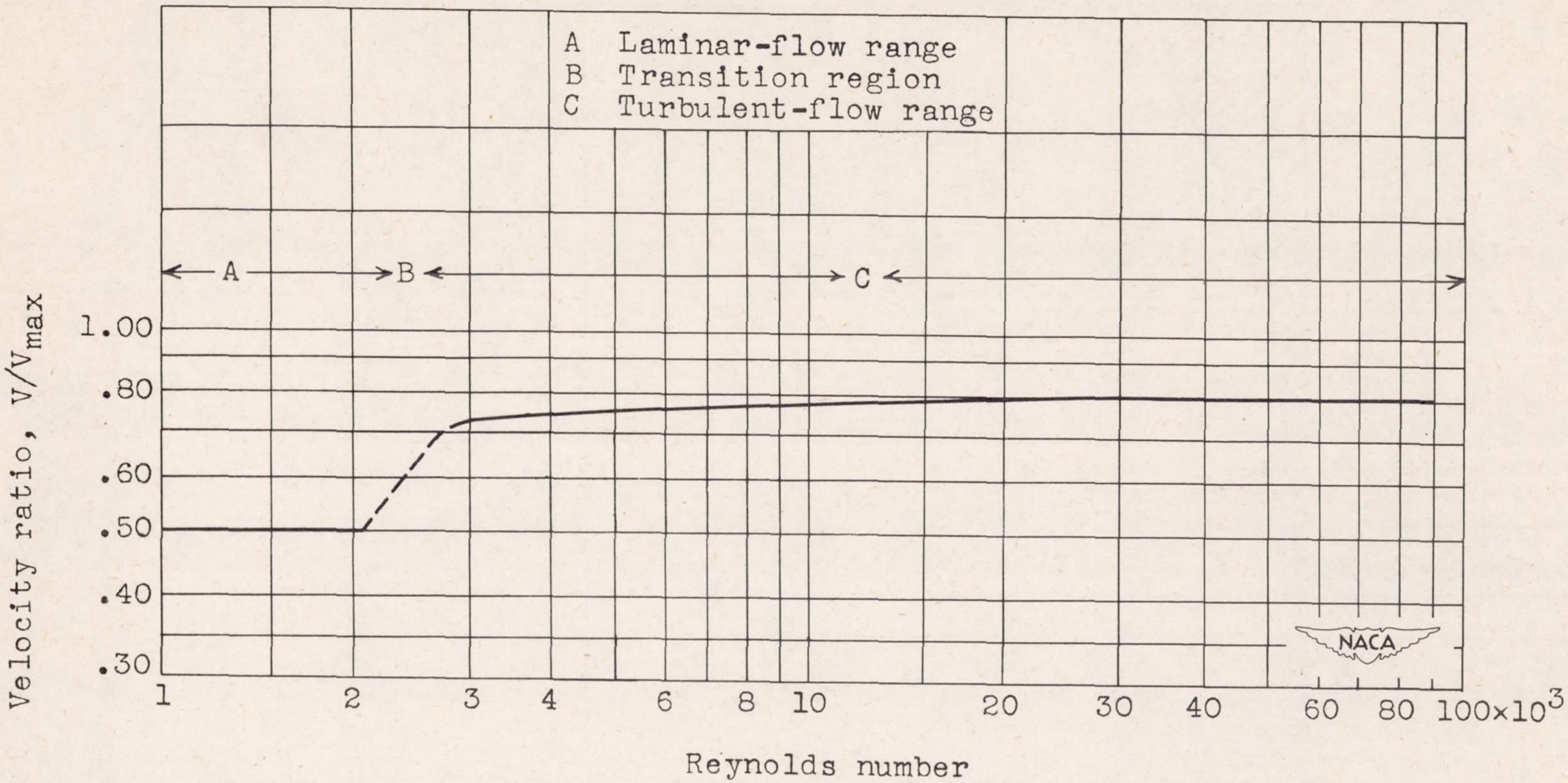

Figure 1. - Effect of Reynolds number on ratio of average to maximum velocity V/V_{max} for isothermal flow in smooth pipes. (Data obtained from reference 4.)

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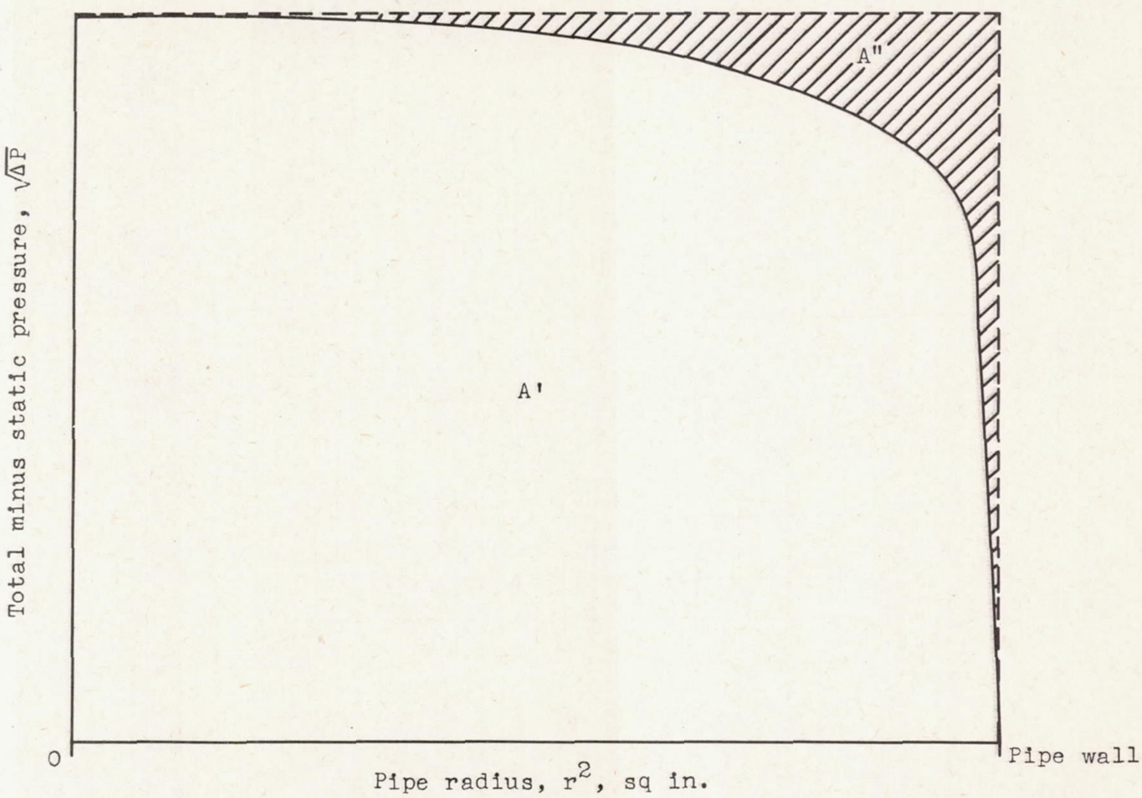

Figure 2. - Effect of Reynolds number on flow profile for turbulent flow.

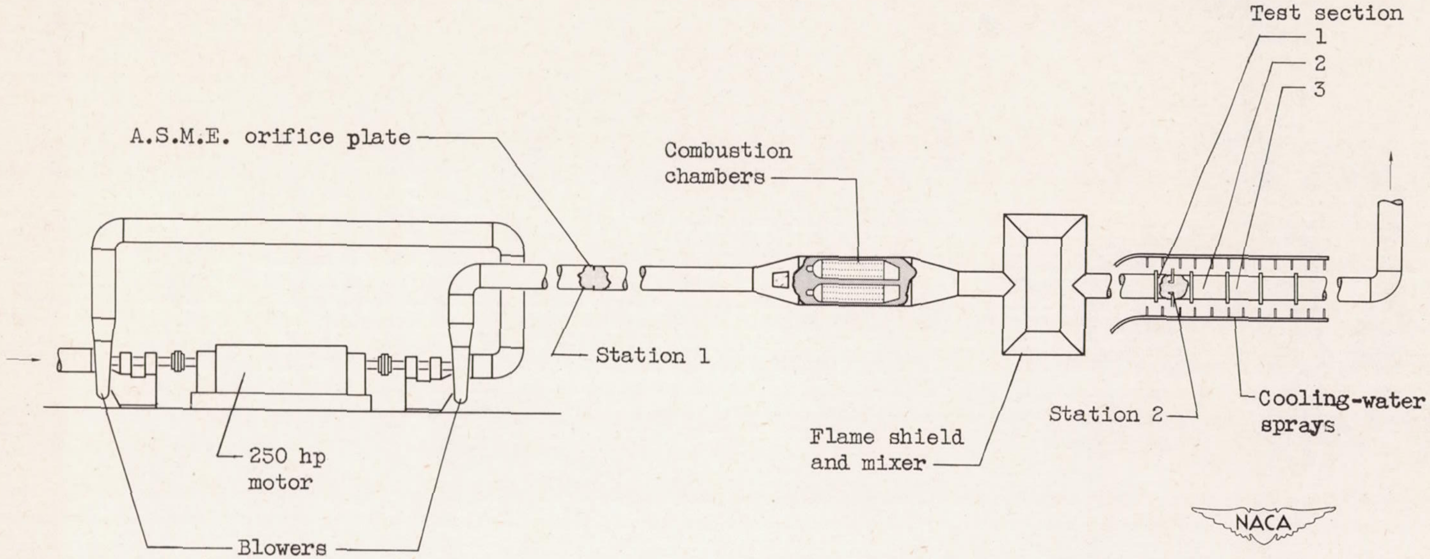

Figure 3. - Schematic representation of research apparatus.

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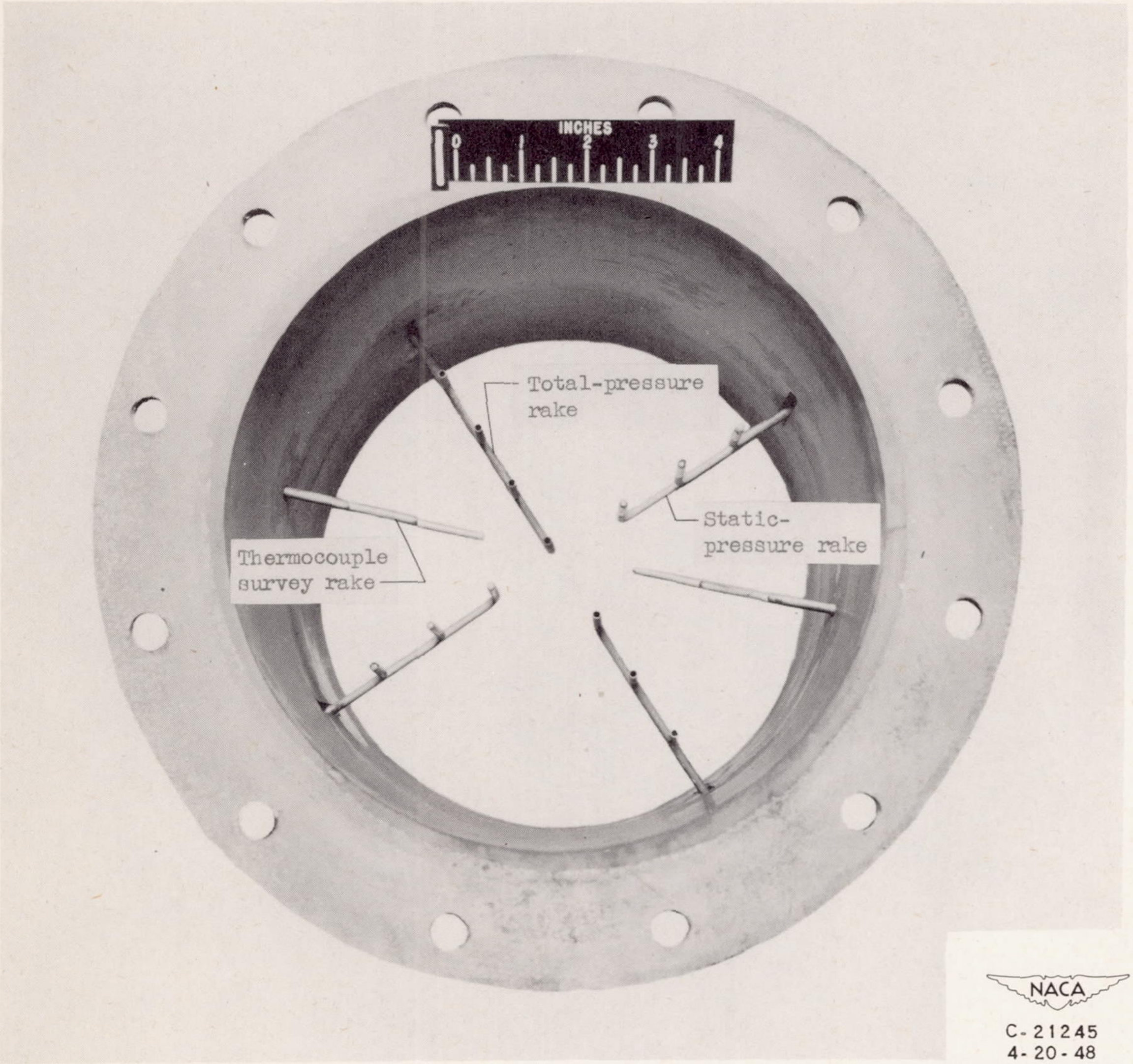

Figure 4. - Test section 1; pitot-static and thermocouple survey rake and instrumentation.

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Figure 5. - Test section 2; varying thermocouple-bead diameter.

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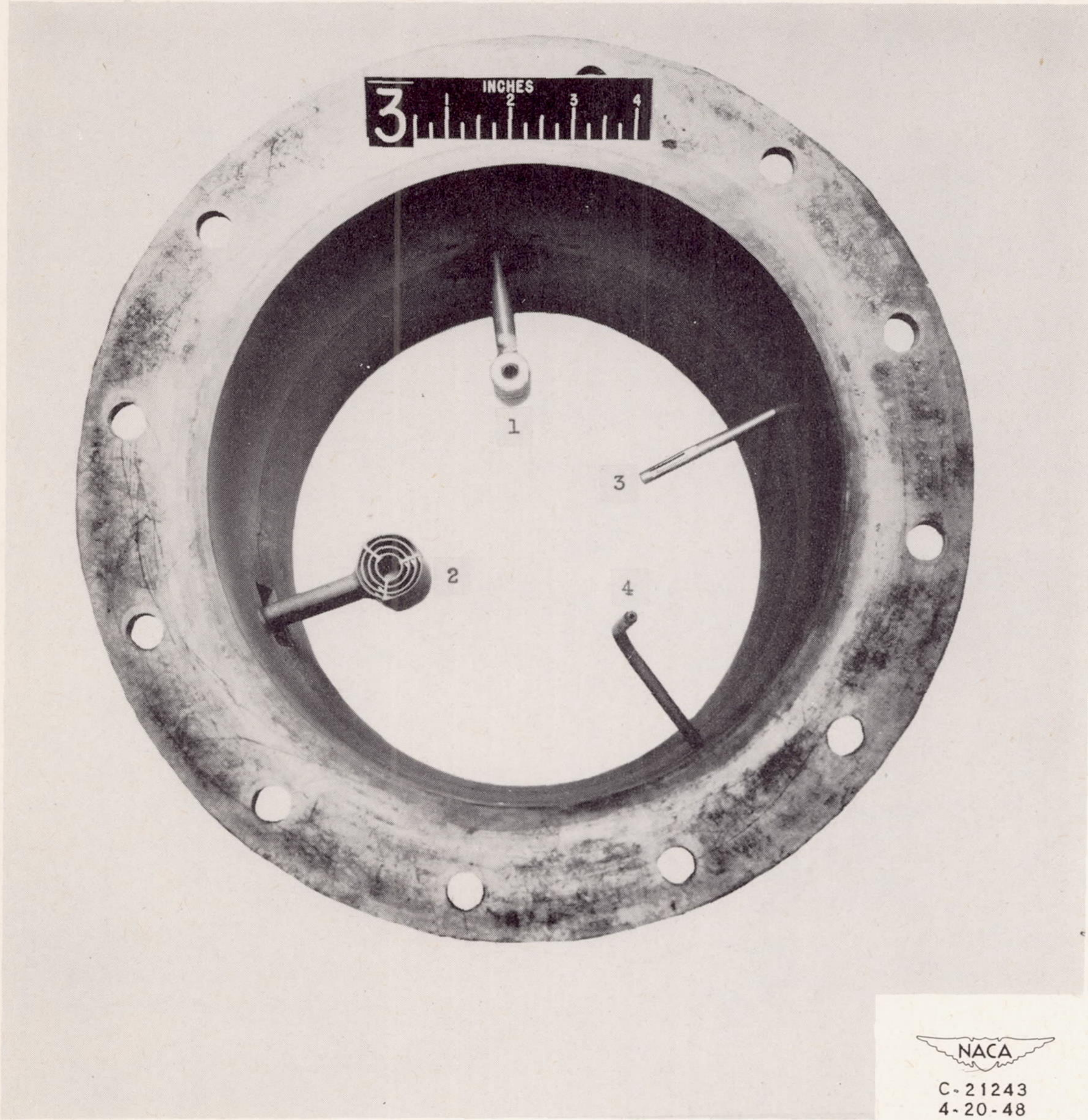


Figure 6. - Test section 3; (1) heated-shield thermocouple, (2) multiple-shield thermocouple, (3) Bureau of Standards gold-shield thermocouple, (4) pitot-static tube.

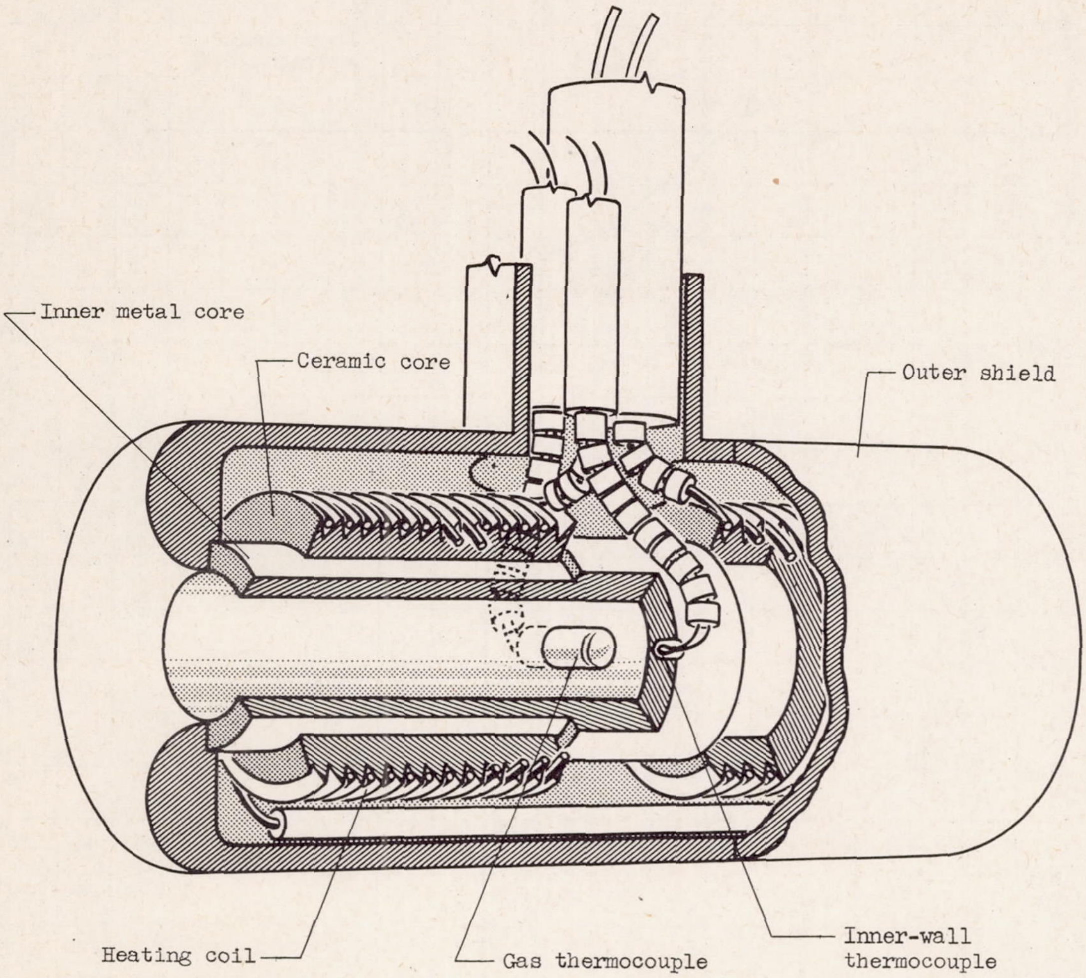


Figure 7. - Heated-shield component structure and assembly. (Drawn four times actual size.)

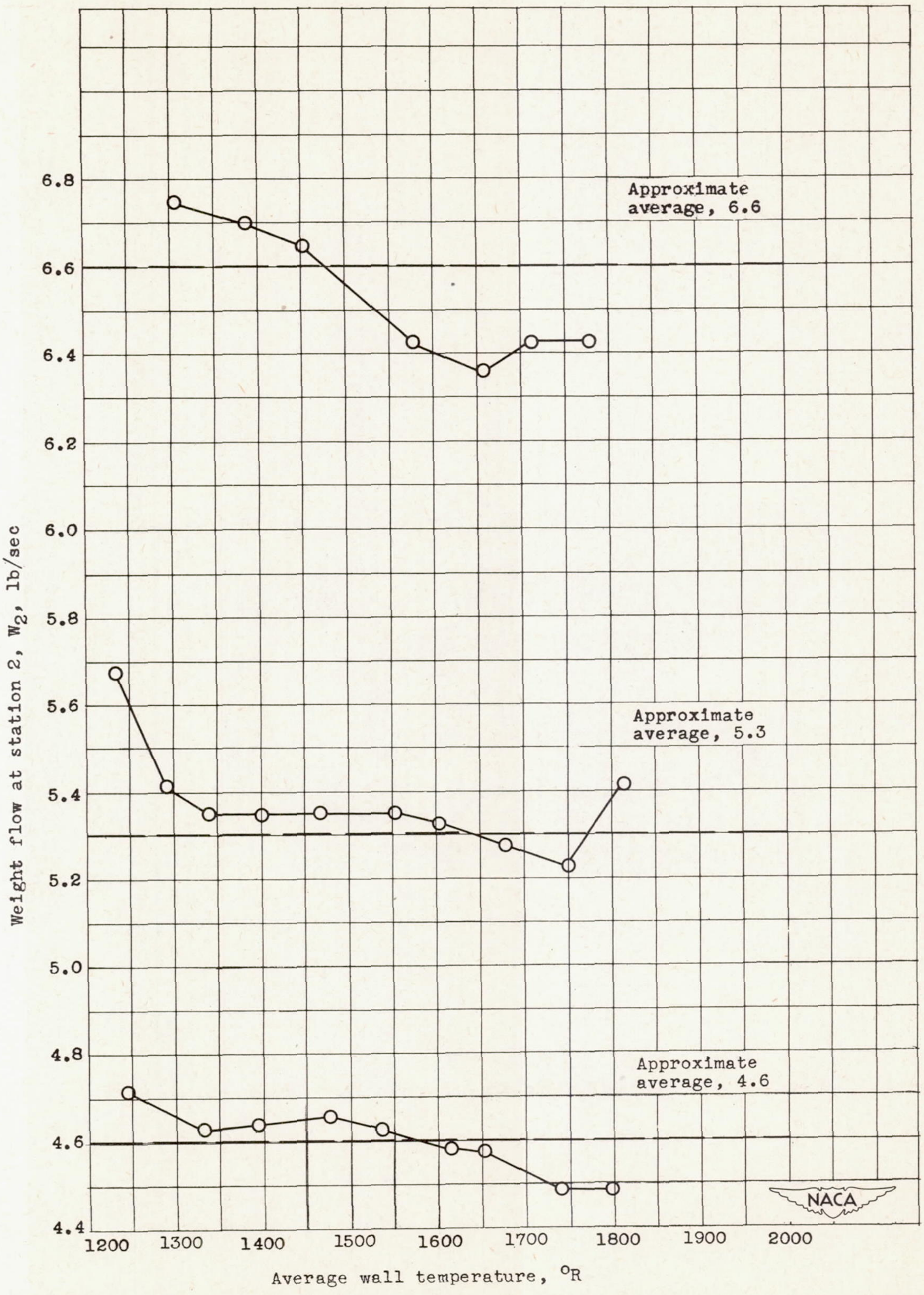


Figure 8. - Variation of weight flow with operating temperature.

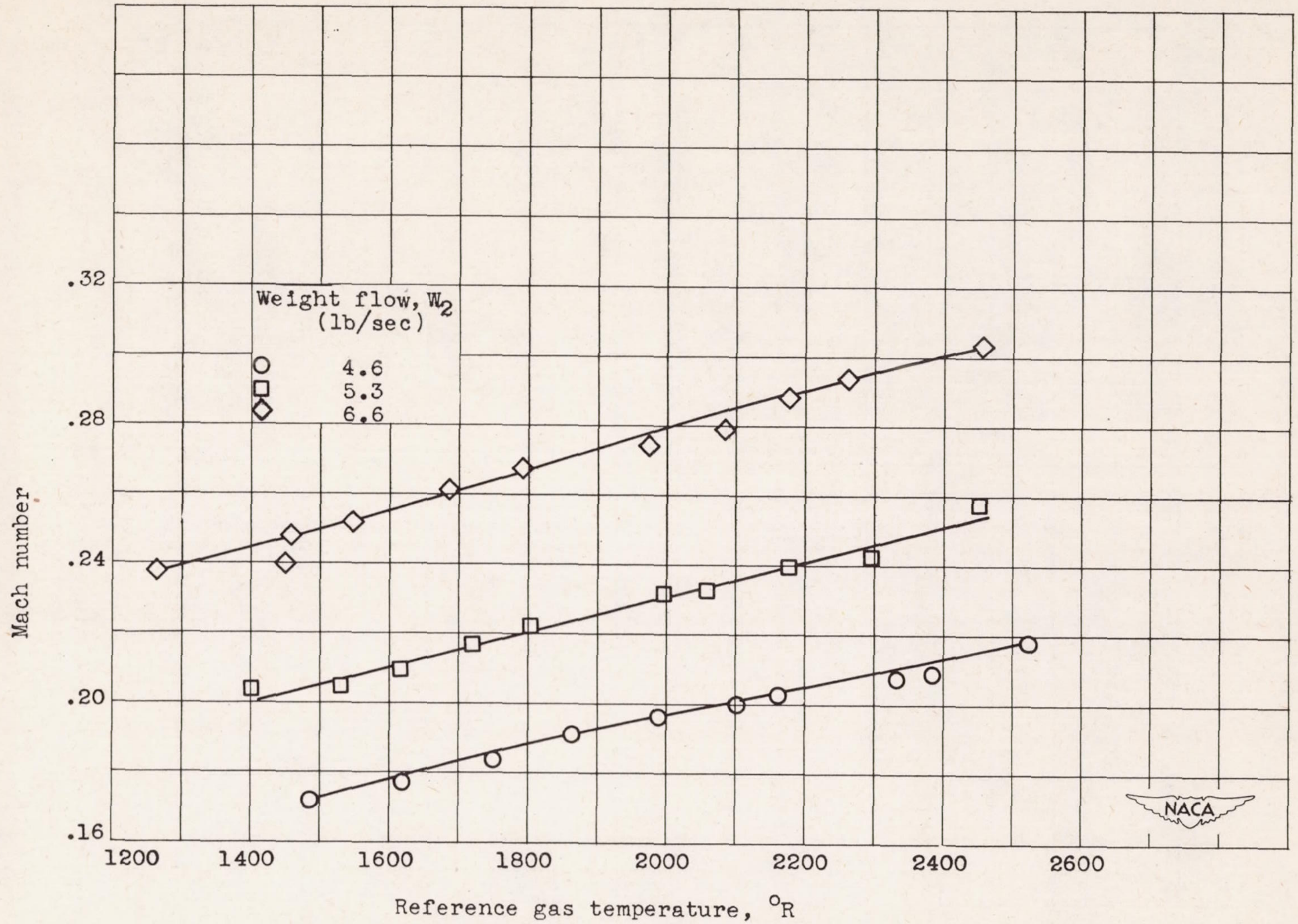


Figure 9. - Variation of Mach number with reference gas temperature under varying weight-flow conditions.

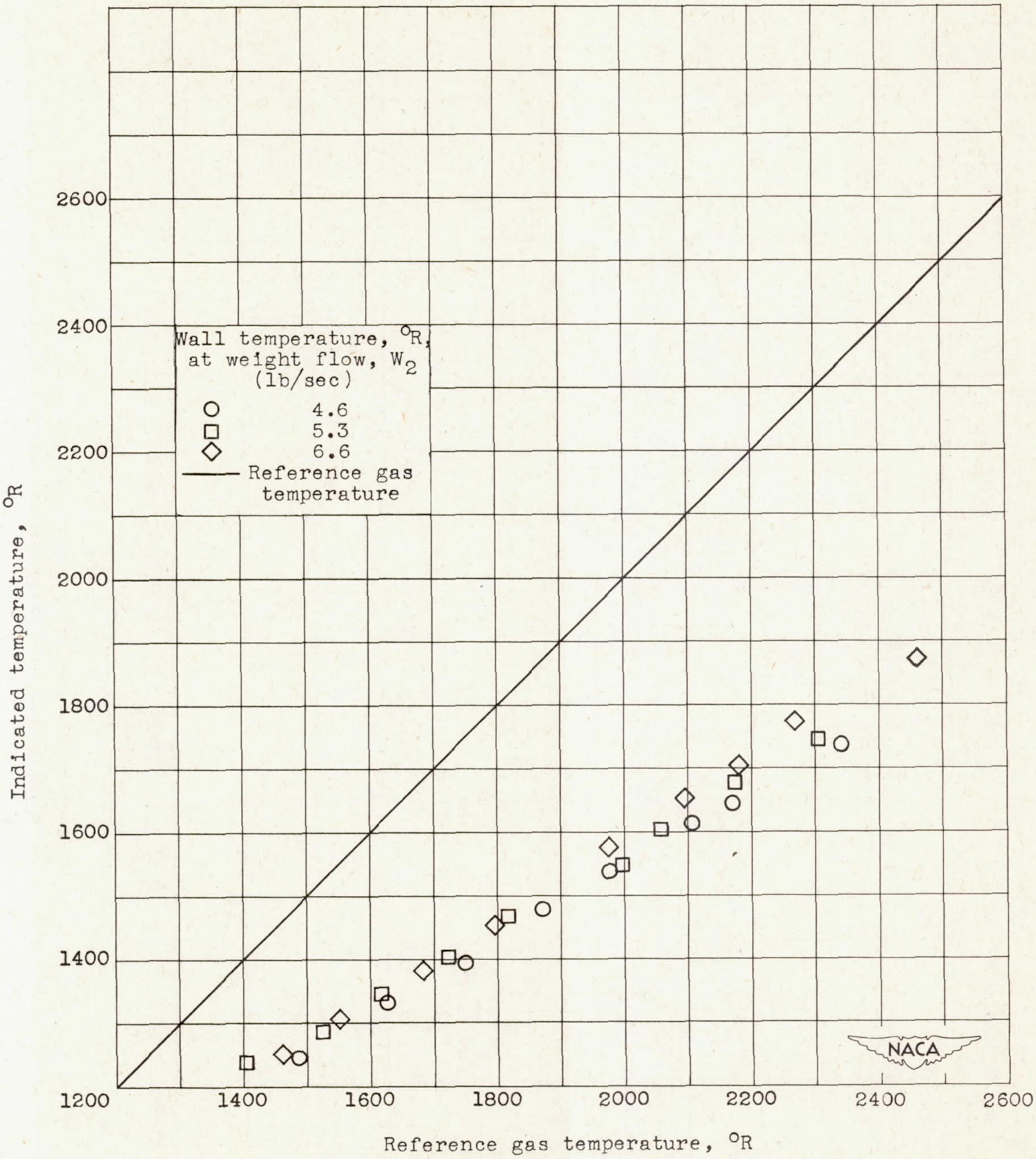
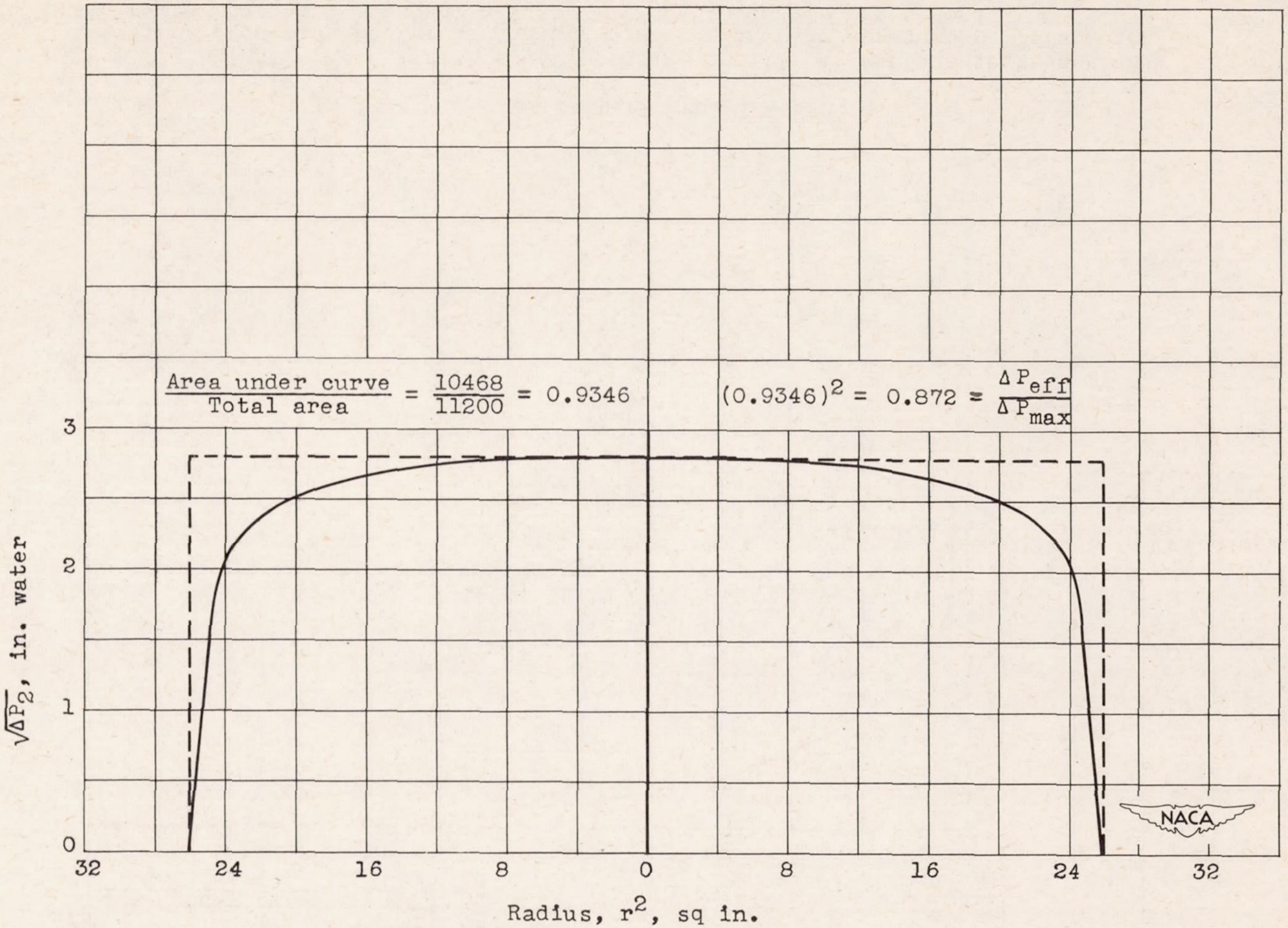
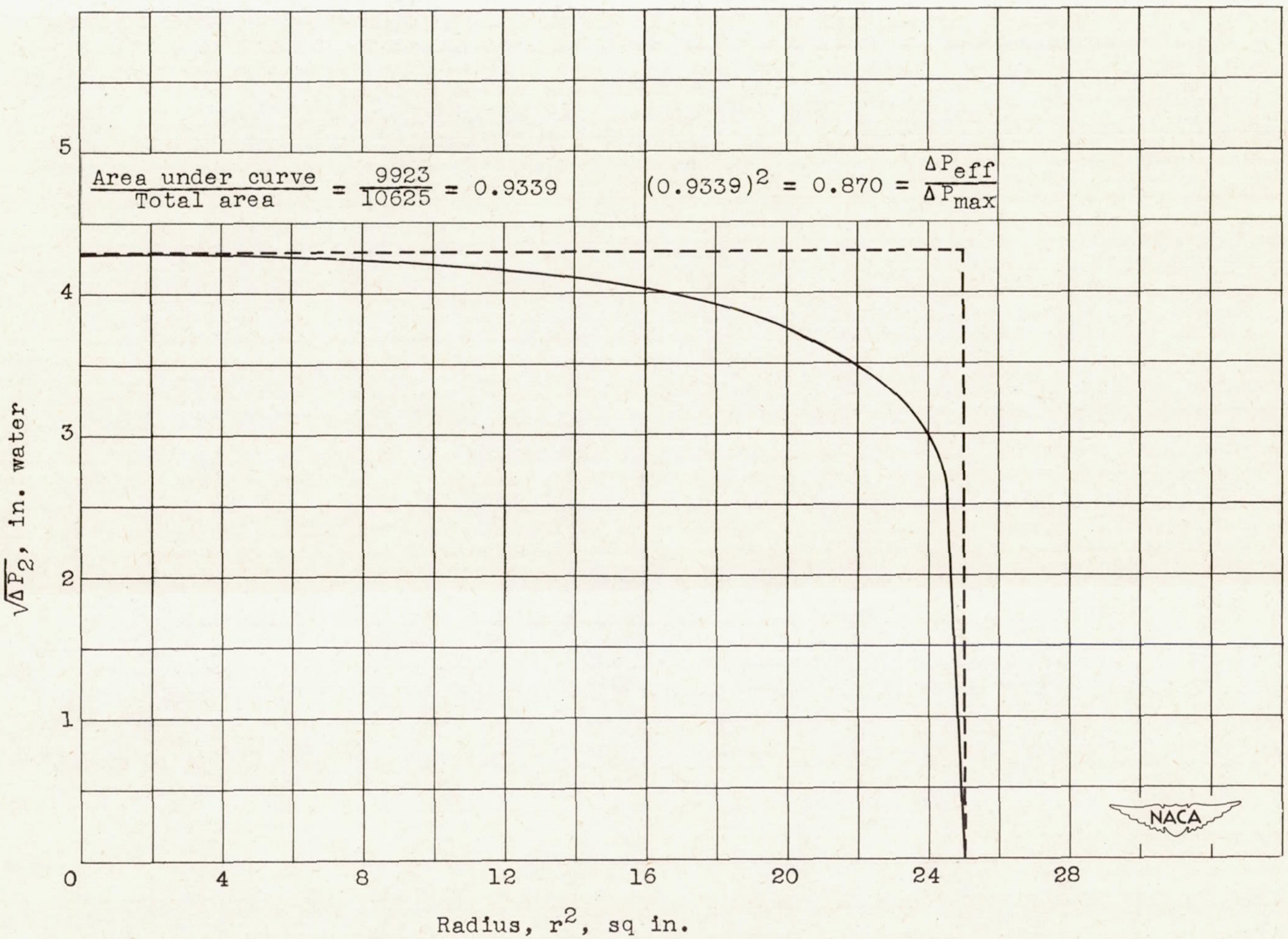


Figure 10. - Variation of test-section-wall temperature from reference gas temperature for all conditions.



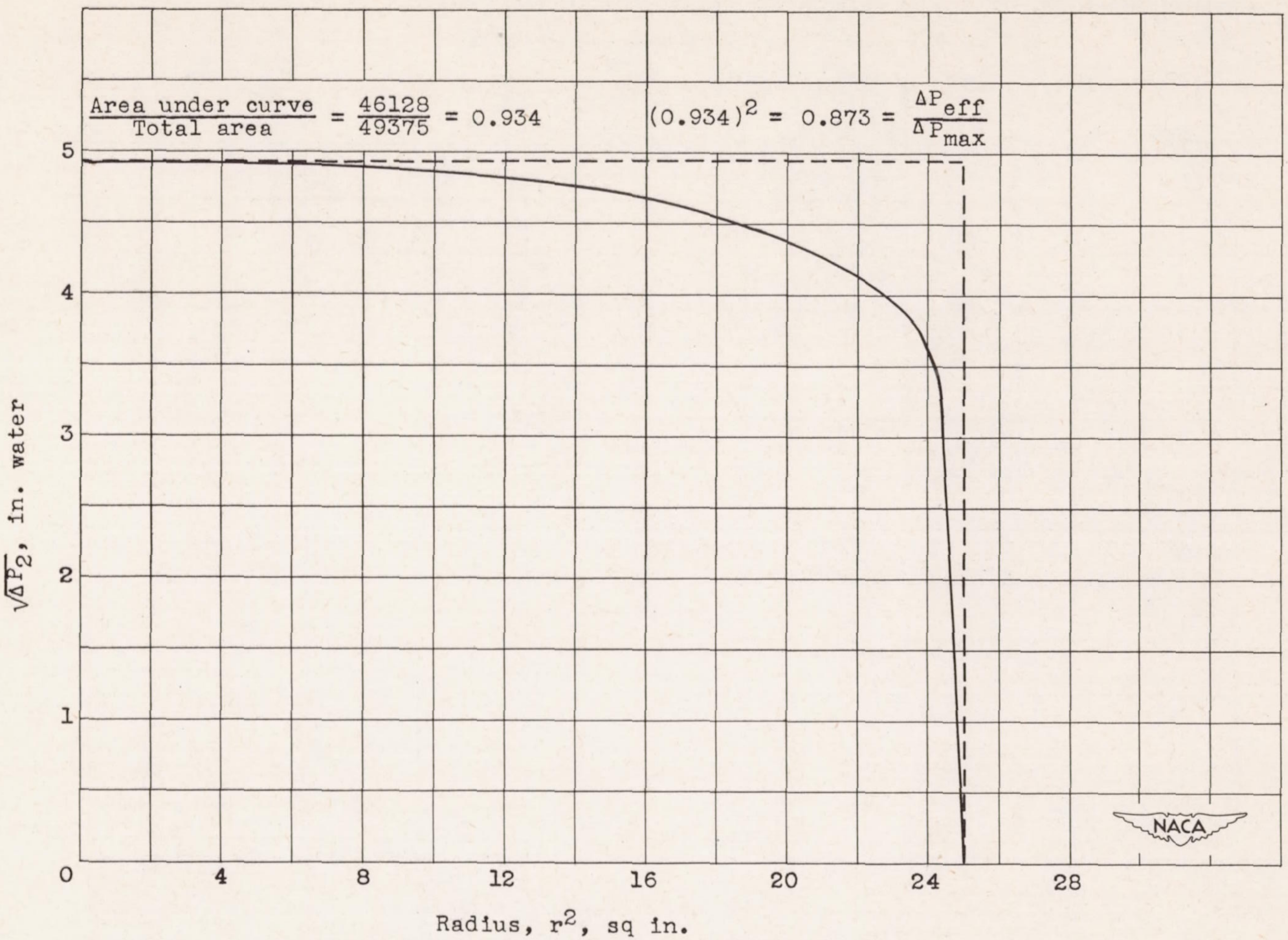
(a) Reference gas temperature, 550° R.

Figure 11. - Variation of pressure profile at various test-section locations from pipe center line along diameter of pipe at constant reference gas temperature and constant weight flow of 6.3 pounds per second.



(b) Reference gas temperature, 1500° R.

Figure 11. - Continued. Variation of pressure profile at various test-section locations from pipe center line along diameter of pipe at constant reference gas temperature and constant weight flow of 6.3 pounds per second.



(c) Reference gas temperature, 2300° R.

Figure 11. - Concluded. Variation of pressure profile at various test-section locations from pipe center line along diameter of pipe at constant reference gas temperature and constant weight flow of 6.3 pounds per second.

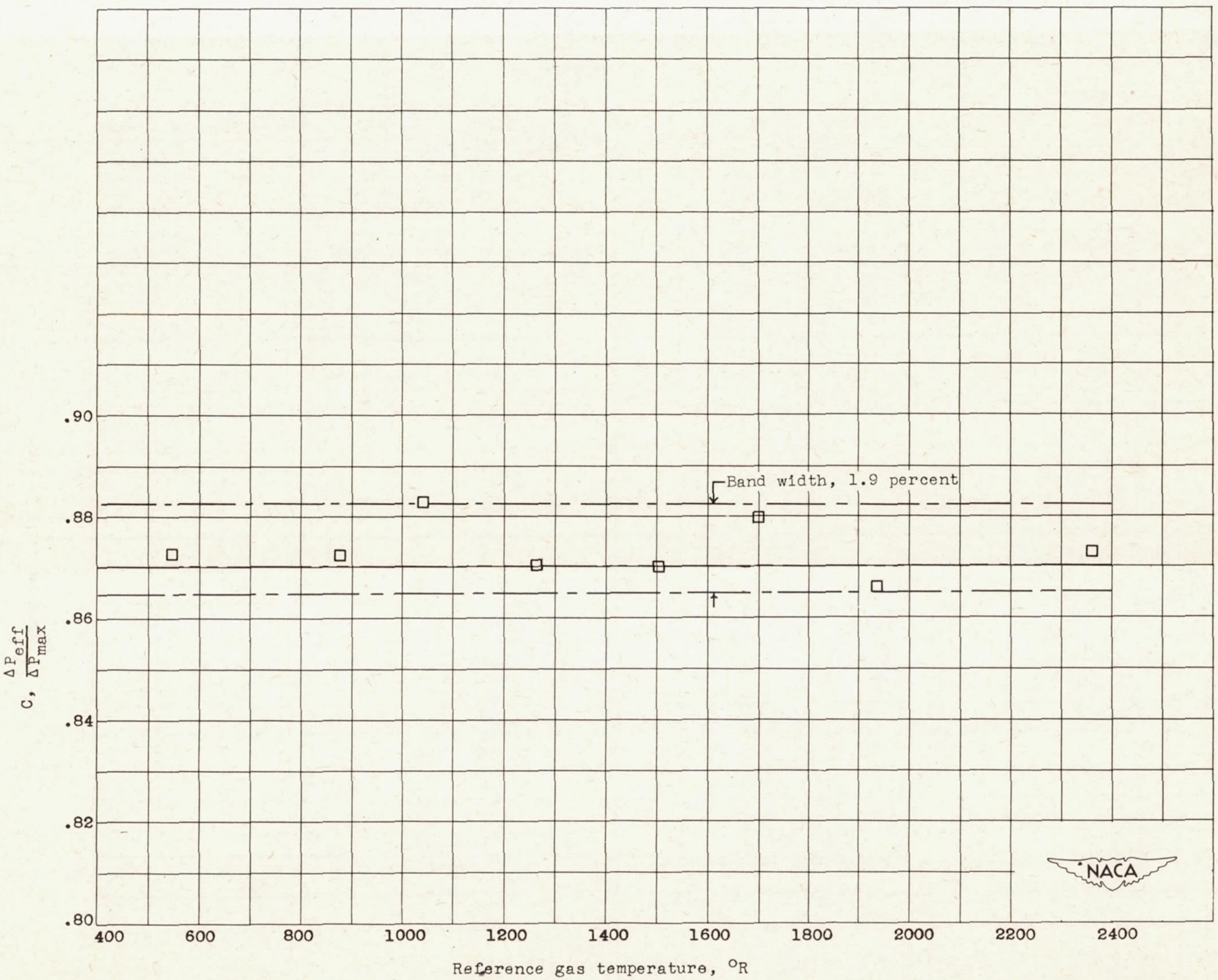


Figure 12. - Variation of C determined from pressure-profile surveys under varying gas-temperature operation.

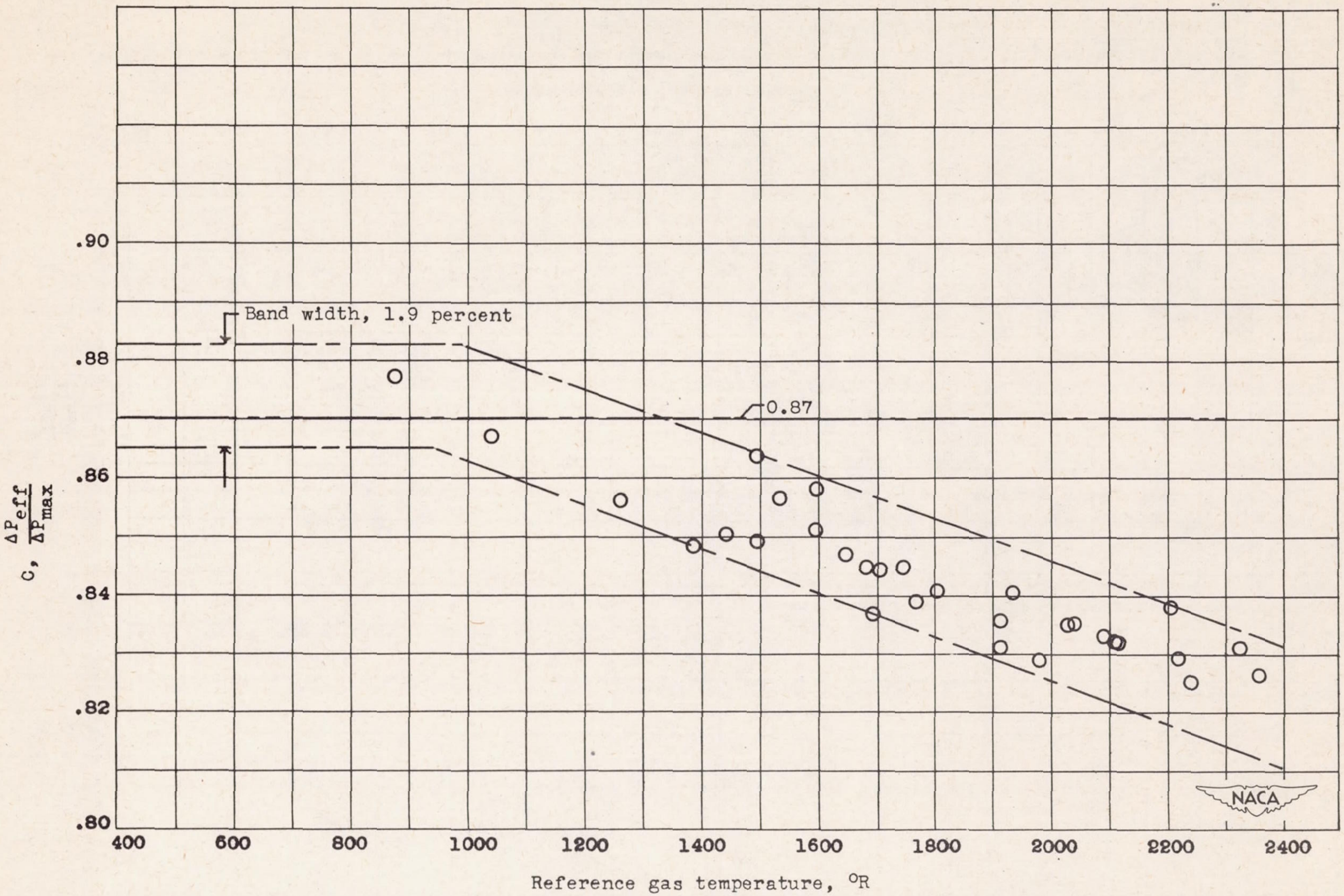


Figure 13. - Evaluation of C by weight-flow considerations under varying gas-temperature operation.

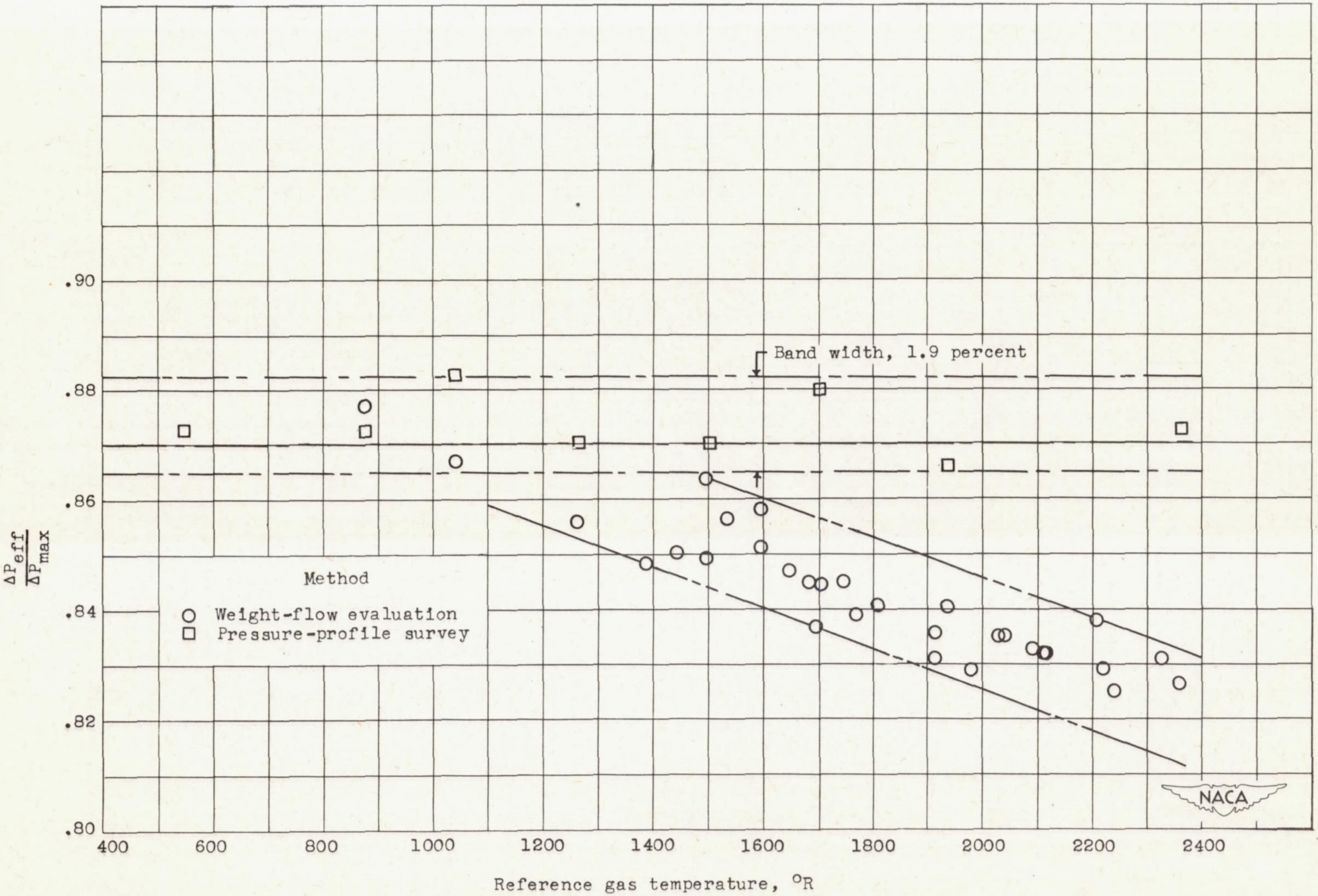


Figure 14. - Comparison between weight-flow evaluation method and pressure-profile survey method for determination of ratio $\frac{\Delta P_{eff}}{\Delta P_{max}}$ under varying gas-temperature operations.

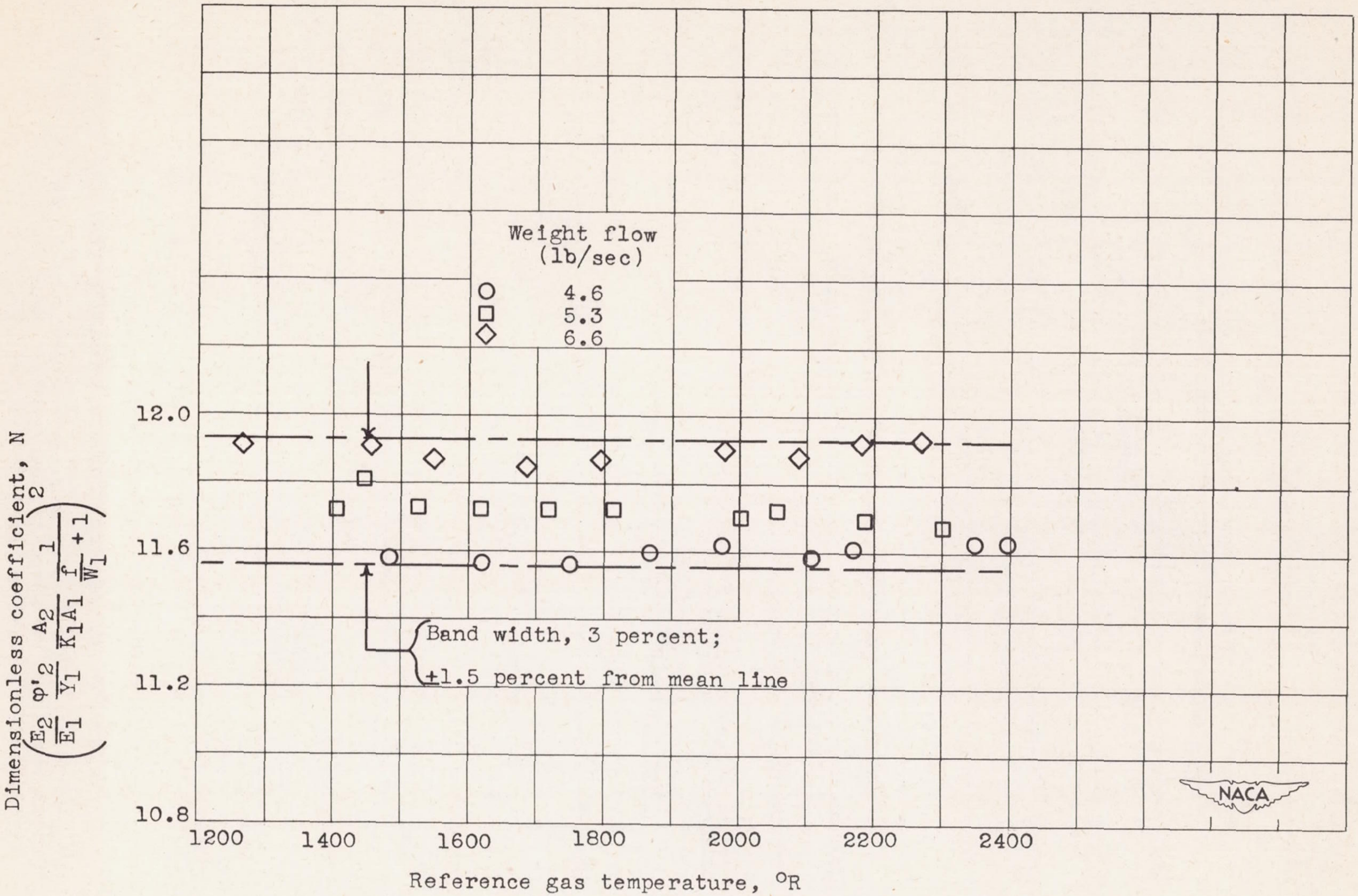
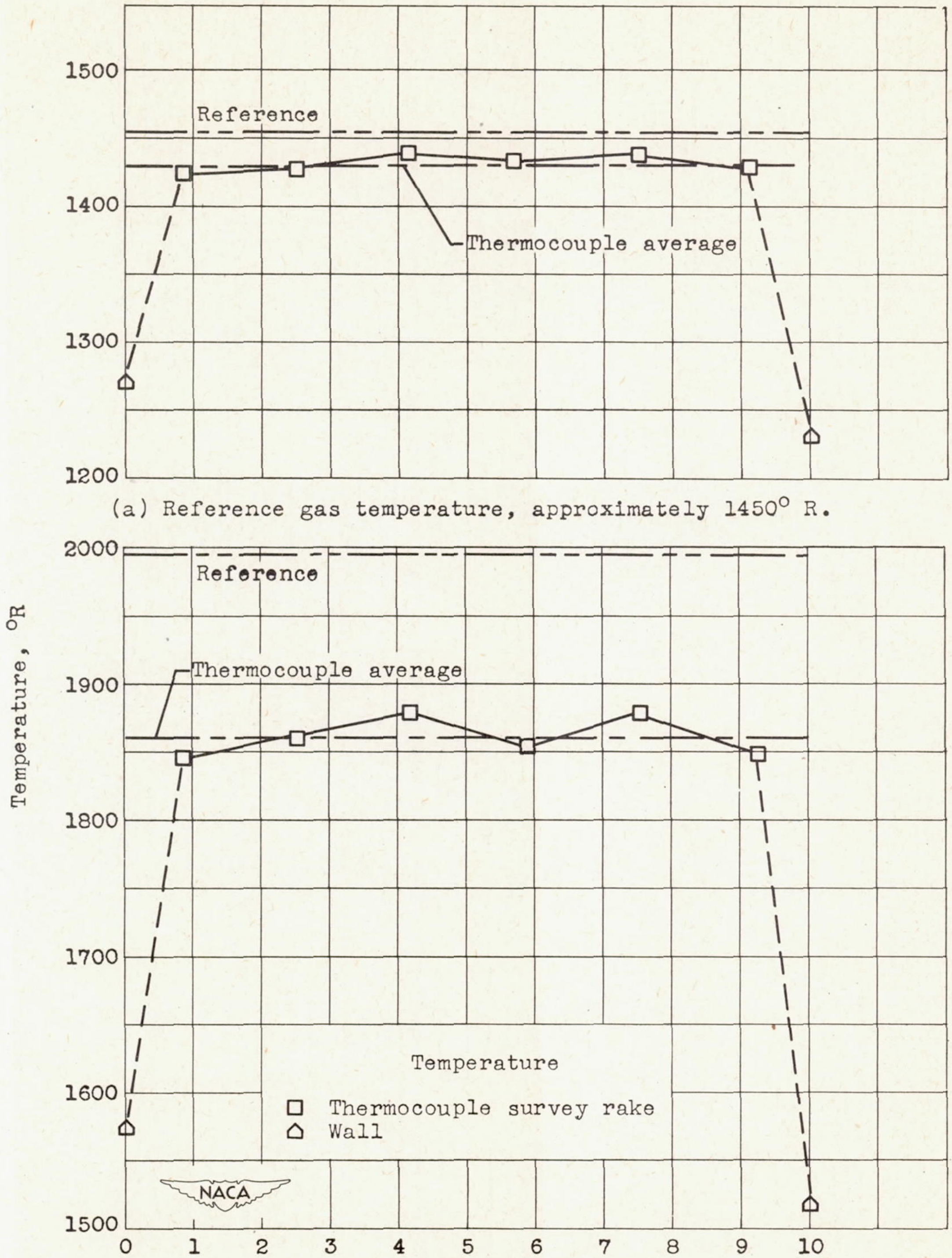


Figure 15. - Variations of dimensionless coefficient N for all conditions of gas flow and reference gas temperature.



(a) Reference gas temperature, approximately 1450° R.

(b) Reference gas temperature, approximately 2000° R.

Figure 16. - Comparison of gas-temperature profile in test section 1 measured by thermocouple rake to gas temperature indicated by reference system.

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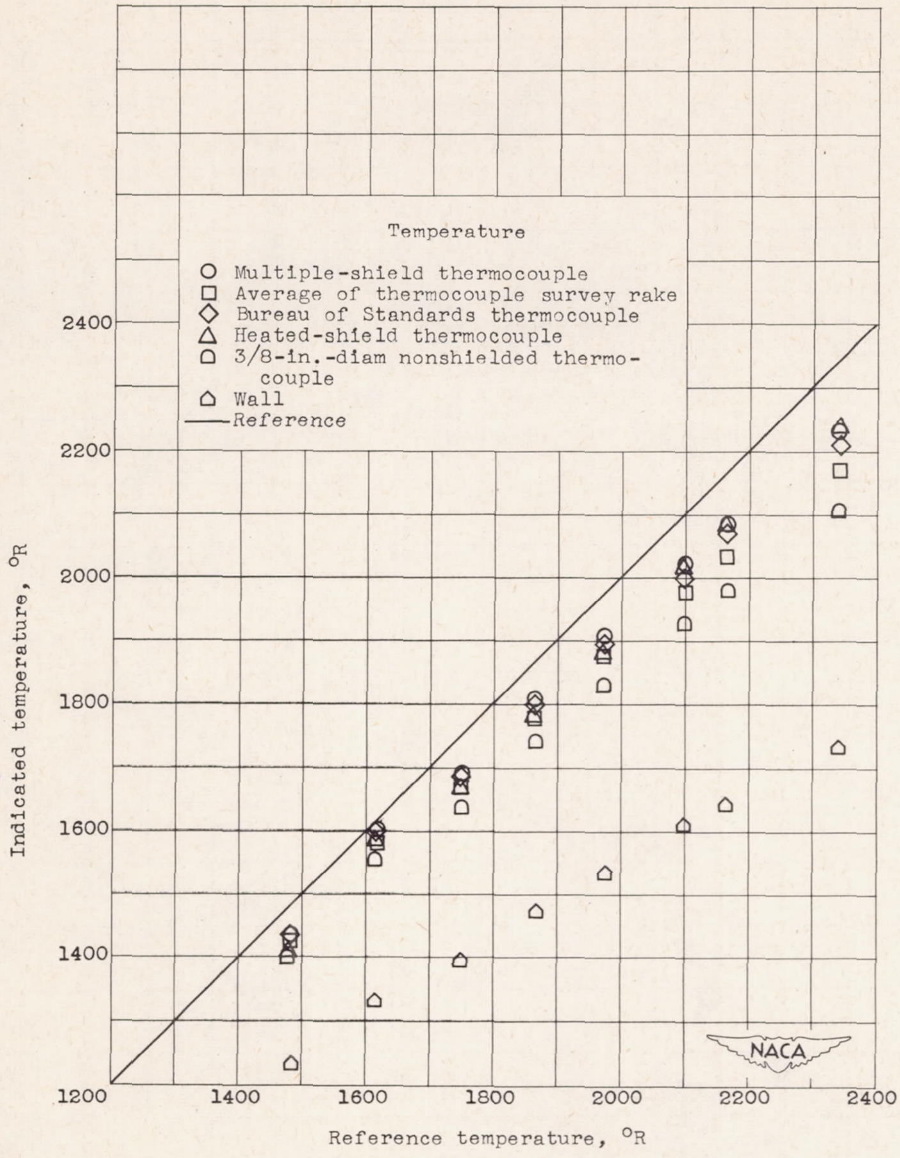
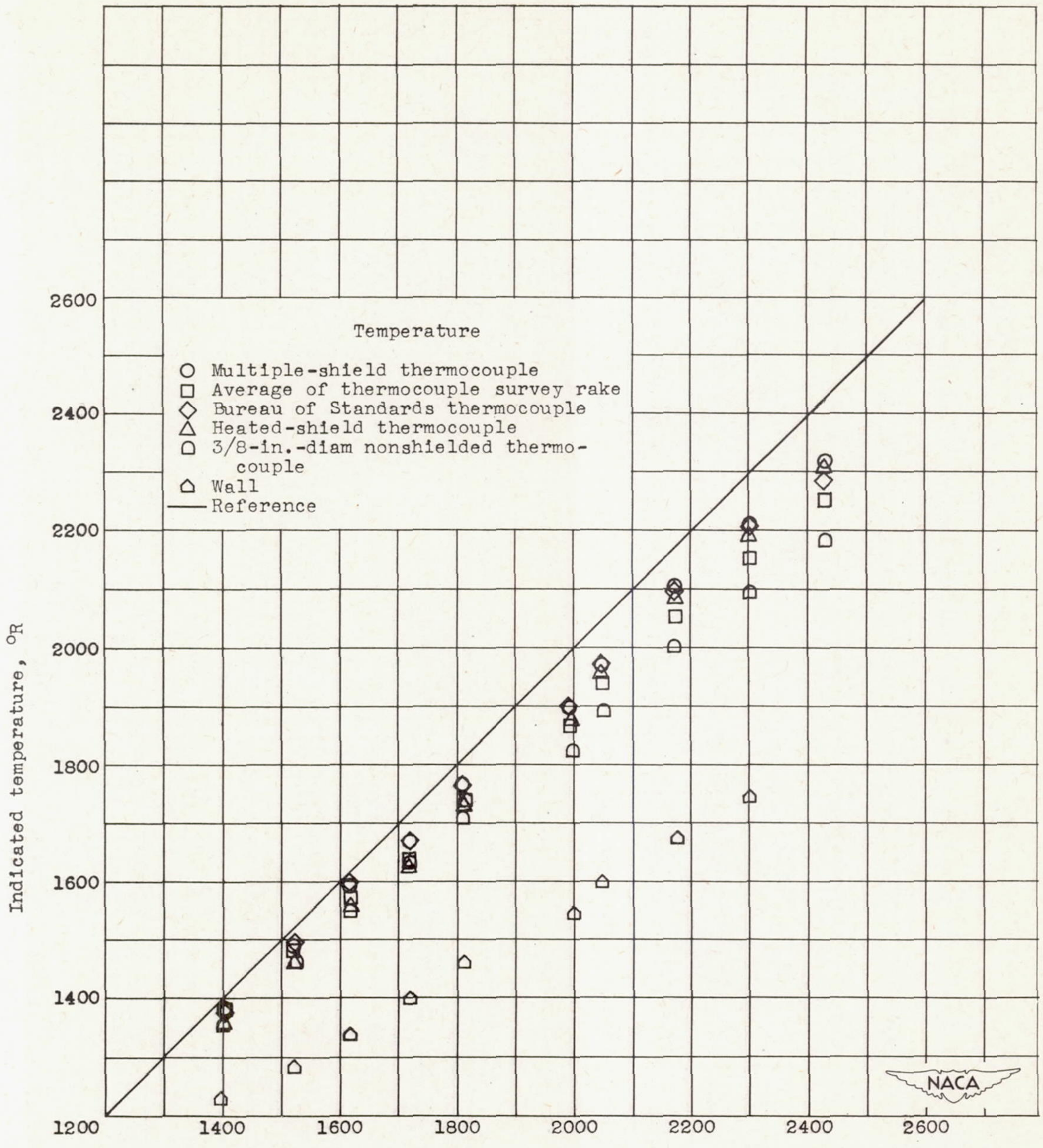


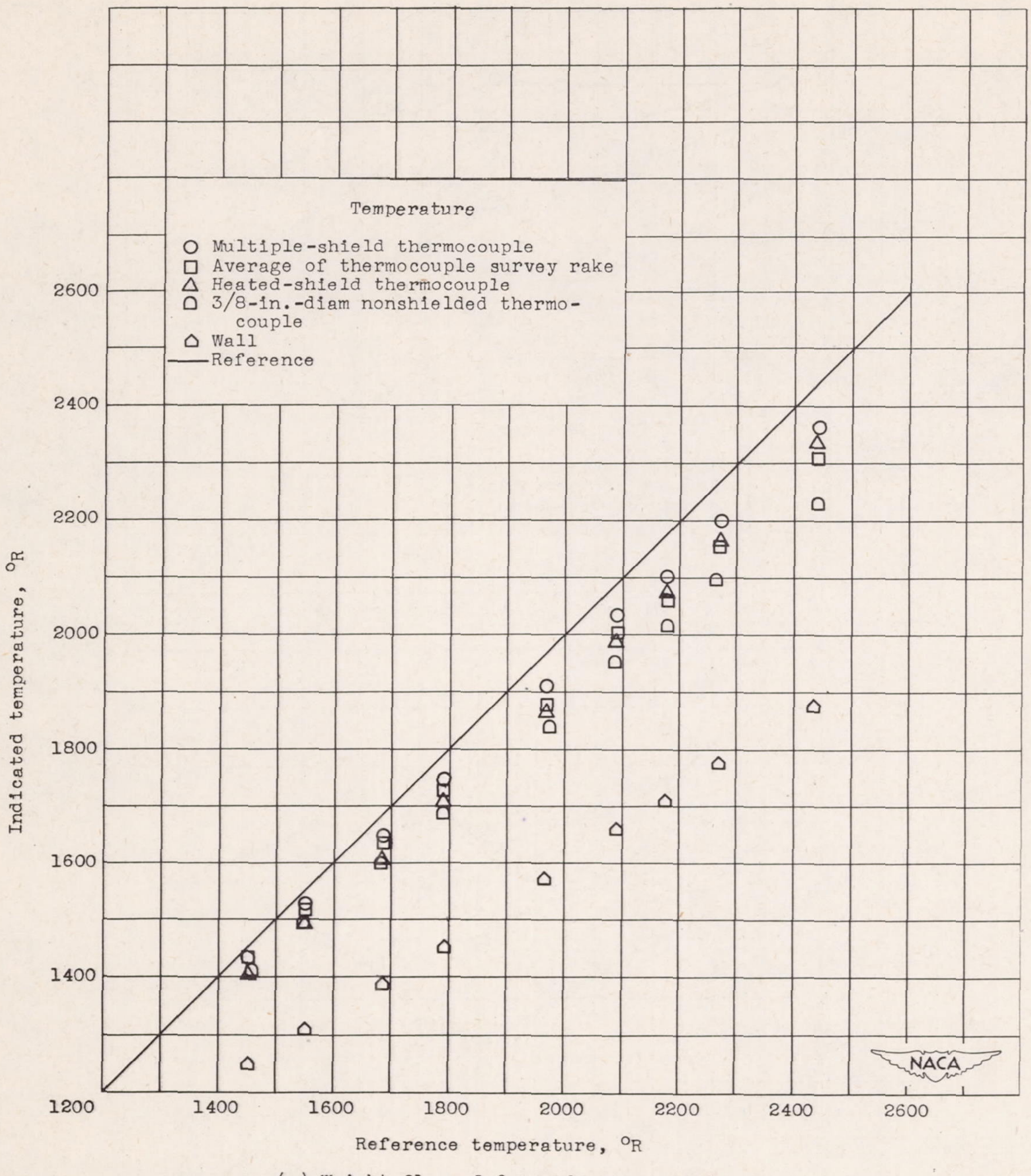
(a) Weight flow, 4.6 pounds per second.

Figure 17. - Variation of gas-temperature indications from conventional thermocouples with gas-temperature indications from reference system.



(b) Weight flow, 5.3 pounds per second.

Figure 17. - Continued. Variation of gas-temperature indications from conventional thermocouples with gas-temperature indications from reference system.



(c) Weight flow, 6.6 pounds per second.

Figure 17. - Concluded. Variation of gas-temperature indications from conventional thermocouples with gas-temperature indications from reference system.

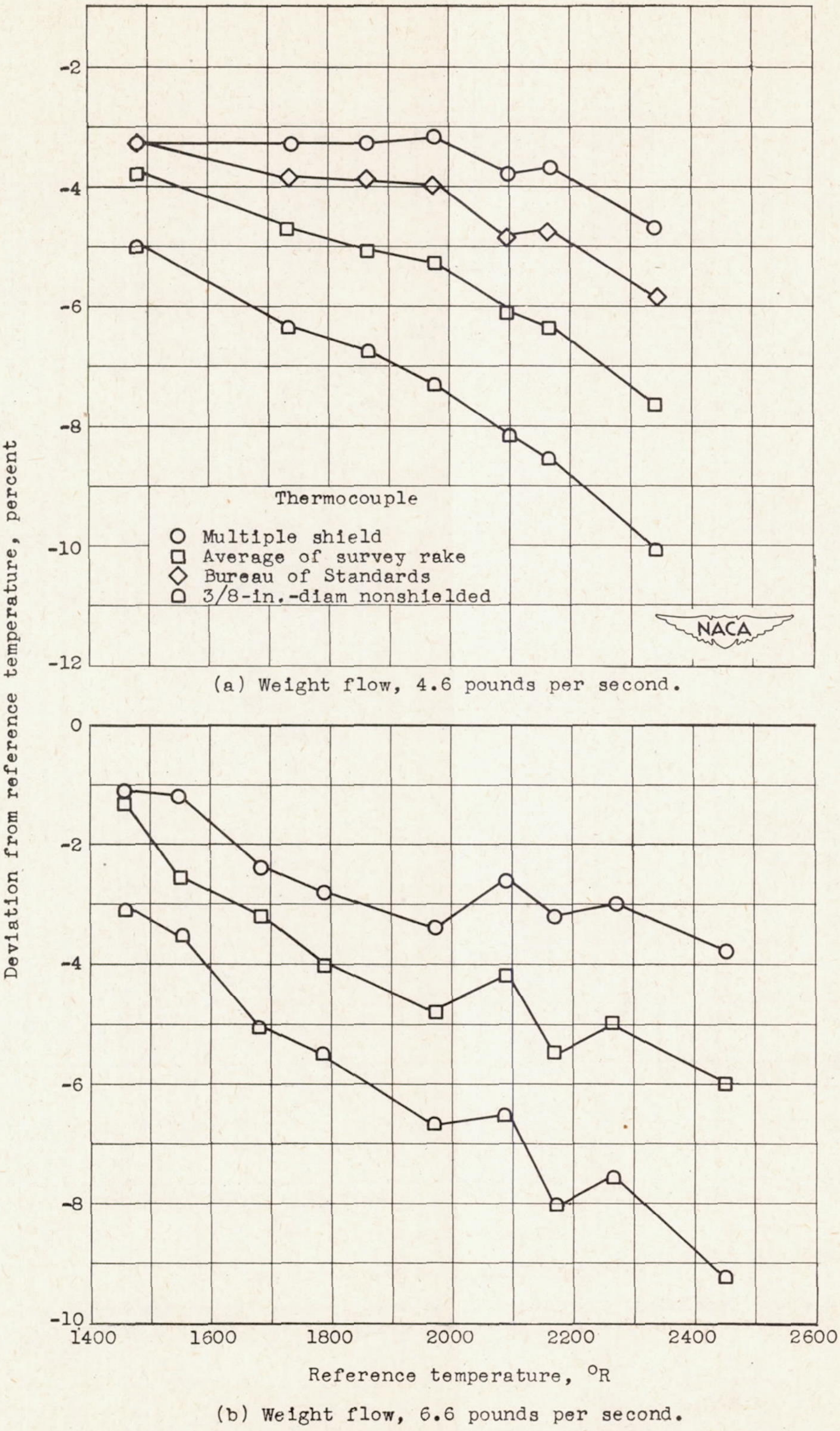


Figure 18. - Percentage deviations of conventional thermocouple temperature indications from pressure-sensitive system (reference temperature).

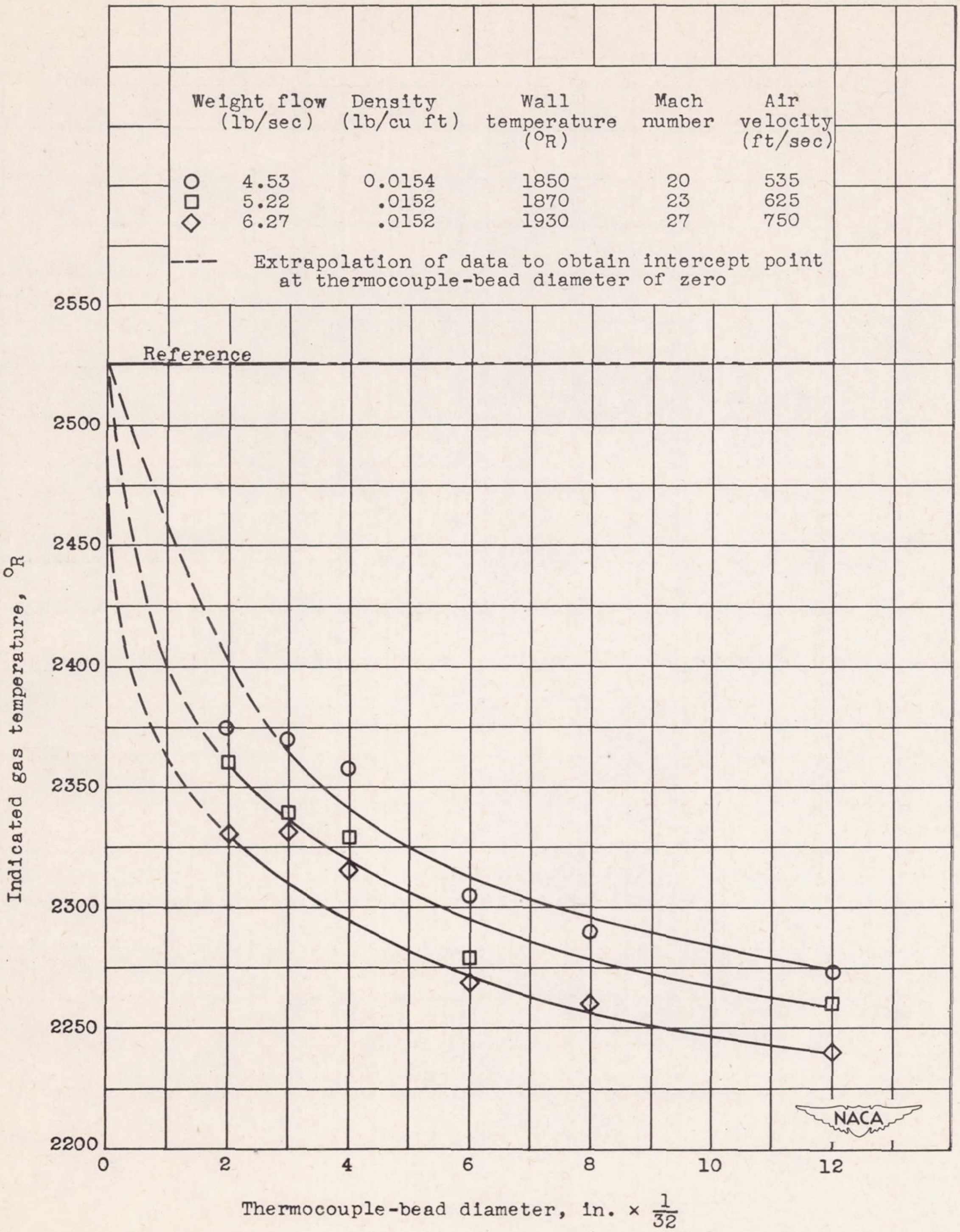


Figure 19. - Variation of gas-temperature indications for varying thermocouple-bead diameters at constant reference temperature.

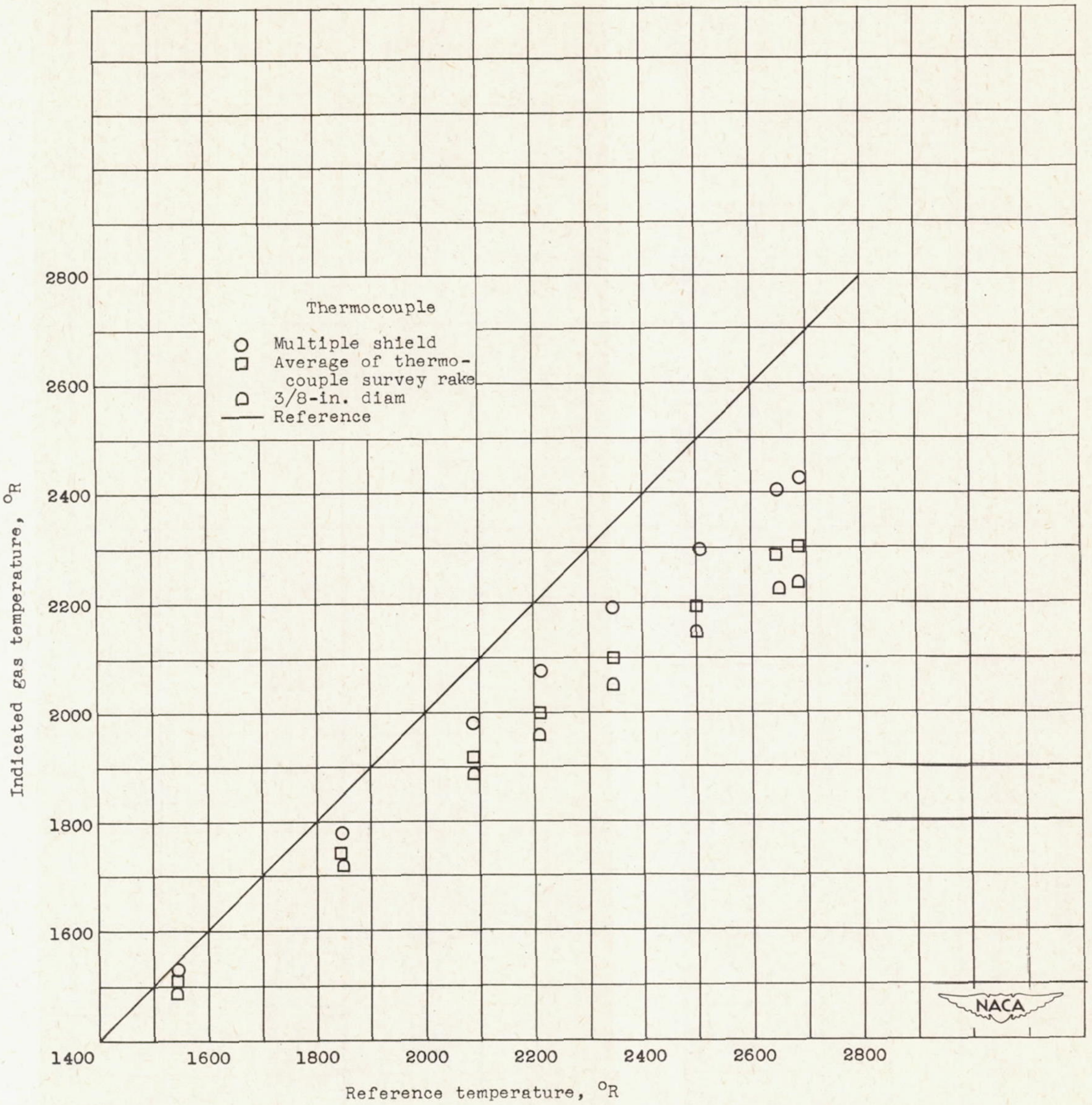
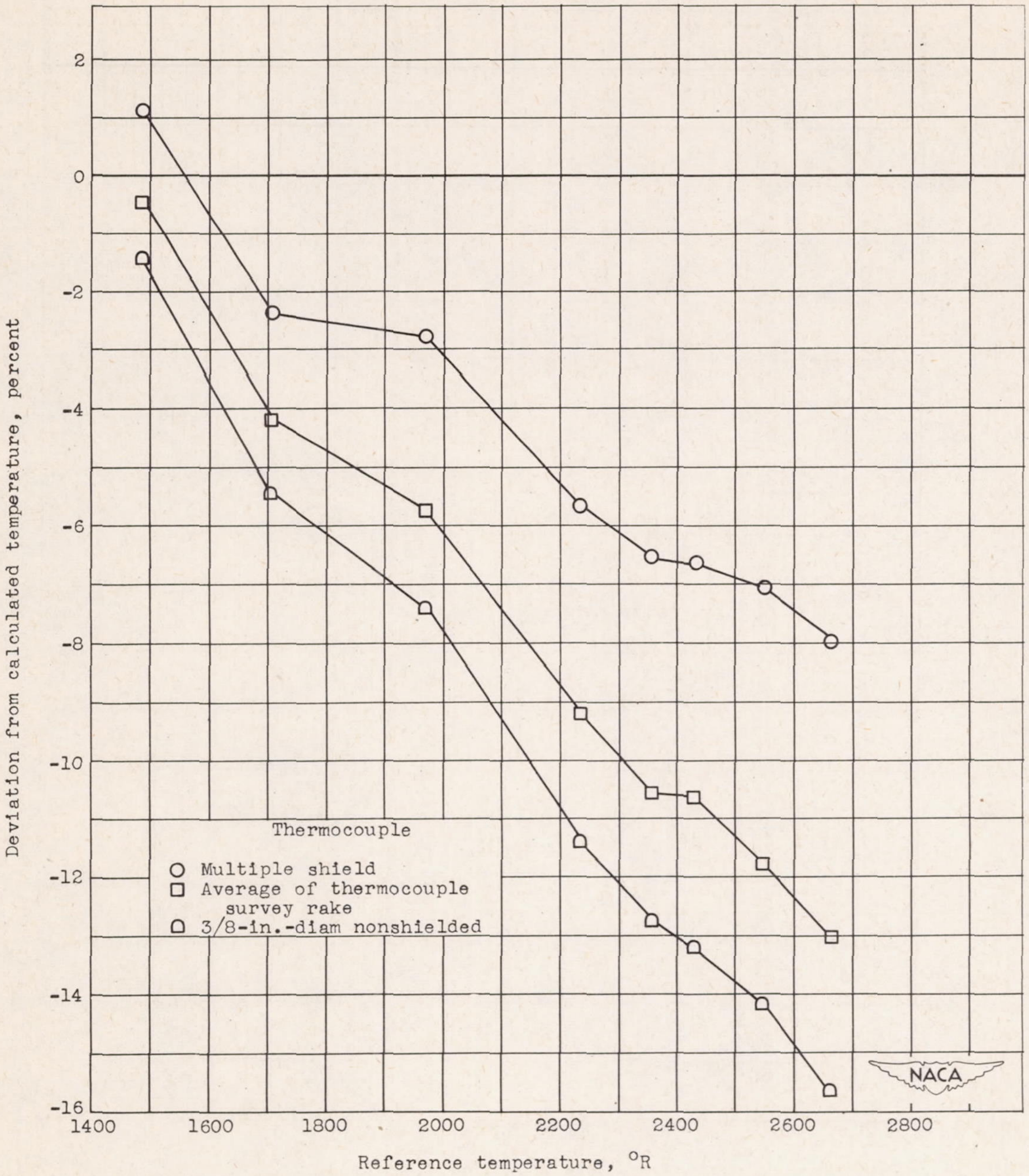
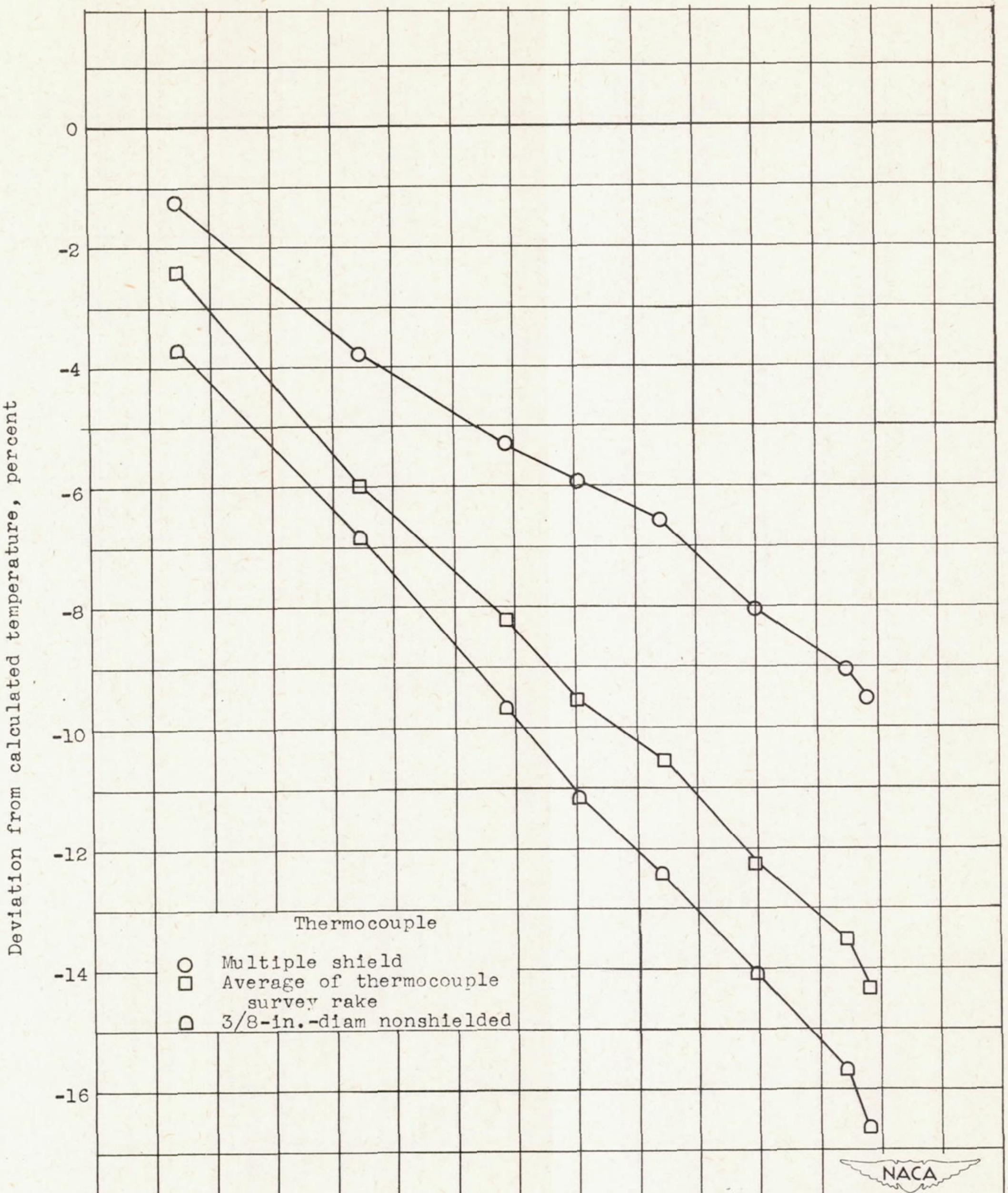


Figure 20. - Temperature deviation of conventional thermocouples from reference system with wall cooling. Weight flow, 6.6 pounds per second; wall temperature, 710° R.



(a) Weight flow, 4.6 pounds per second.

Figure 21. - Percentage deviations of conventional-thermocouple temperature indications from reference system with water cooling on walls. Wall temperature, 710° R.



(b) Weight flow, 6.6 pounds per second.

Figure 21. - Concluded. Percentage deviations of conventional-thermocouple temperature indications from reference system with water cooling on walls. Wall temperature, 710° R.