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STABILITY OF ALCLAD PLATES

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SUMMARY

On the basis of plasticity theory a theoretical solution for the buckling of Alclad plates has been developed. Both the differential equation of equilibrium of the buckled plate and the energy expression are derived. Results are presented for the buckling of long simply supported plates under longitudinal compression and under shear, and for a plate-column. Good agreement is shown between the theoretical values for a simply supported plate and available experimental results, which are for channel and Z-sections in compression. A comparison of the theoretical and experimental results for a strip or narrow column shows good agreement in the high stress region and shows that in the low stress region the theoretical results tend to overestimate the buckling stress.

INTRODUCTION

Most of the sheet used in aircraft is Alclad sheet. An Alclad or aluminum-covered sheet has a high-strength aluminum-alloy core, which is covered on each side with a coating of almost pure aluminum with a high resistance to corrosion but a very low strength. (See fig. 1.) Although the combined thickness of this soft coating on the two sides is generally only from 5 to 10 percent of the total thickness of the plate, its location at the outer fibers causes the buckling stress of an Alclad plate to be considerably less than that of a plate of the same thickness made of all core material.

Because the elastic limit of the cladding material occurs at a relatively low stress, elastic theory is generally inadequate for the prediction of buckling stresses and a plasticity theory should be used. In the present paper, Ilyushin's general relations (reference 1) for the plastic state of stress have been used to derive the differential equation of equilibrium and the energy expression of a plate under combined loads.

THEORETICAL RESULTS

Equation for Critical Stress

The compressive stress σ_{cr} or shear stress τ_{cr}, which is the average stress on the gross cross section, at which buckling occurs is given by the expression

$$\sigma_{cr} \text{ or } \tau_{cr} = \frac{\eta k \pi^2 E}{12(1 - \mu^2)} \frac{t^2}{b^2}$$

where

η plasticity reduction factor that takes into account the reduction of the modulus of elasticity for stresses above the elastic range of the cladding material; depends upon the stress and the type of plate

k nondimensional critical-stress coefficient as used in the elastic range

E initial tangent modulus of core and cladding material in combination, ksi

t total plate thickness, inches

μ value of Poisson's ratio of the composite sheet at the buckling stress

b width of plate, inches

By use of the symbols in appendix A and the theoretical derivations in appendix B, expressions for η are derived in appendix C for both compression and shear for Alclad plates with different edge conditions and are given in table I.

Calculation of η

In order to calculate the value of η for a given buckling stress, values of the tangent and secant moduli of both the core and the cladding as functions of stress must be known. Since stress-strain curves of both the core and the cladding are necessary for calculating these values, two methods of arriving at these curves are discussed in the following paragraphs.

One method is to assume approximate curves for both the core and the cladding material. For most engineering purposes, satisfactory results can be obtained by this method. As an illustration, the curve for the core of Alclad 24S-T sheet can be assumed to be the same as the stress-strain curve of bare 24S-T sheet. The cladding stress-strain curve can be assumed to be the same as the stress-strain curve for one-half hard aluminum. (See solid-line curve in fig. 2.)

A second and generally more accurate method of obtaining the stress-strain curves of the core and the cladding is to adopt an approximate stress-strain curve of the cladding only, and with a stress-strain curve of the composite sheet determined from a simple compression or tension test and the percent of cladding given, calculate the stress-strain curve of the core. Since the cladding is a small percentage of the total plate thickness, errors in the selection of the cladding stress-strain curve will have a very small effect on the calculated stress-strain curve of the core. For example, the dotted-line conservative stress-strain curve for the cladding shown in figure 2 gives practically the same stress-strain curve for the core material of Alclad 24S-T84 sheet with 5.7 percent cladding as the solid-line curve. (See fig. 3 for calculated stress-strain curve of core.)

The error in the calculated value of η for a given error in the assumed cladding stress-strain curve will generally be small for plates but may be appreciable for columns. To illustrate, the solid-line and dash-line η curves for plates shown in figure 4 and for columns shown in figure 5 were calculated by using the solid- and dash-line stress-strain curves, respectively, for the cladding (2S-H14) given in figure 2. The large difference in the theoretical values of η for columns is due to the fact that η varies almost directly as the tangent modulus and, for strains between 0.001 and 0.002, the tangent moduli of the two assumed stress-strain curves of the cladding differ considerably.

For the evaluation of η by the formulas presented, the percentage of cladding is needed. If the percentage of cladding is not known, it can be determined from the stress-strain curve of the composite sheet by the approximate equation

$$\text{Percent Cladding} = 100\left(1 - \frac{E_1}{E}\right)$$

where E is the primary modulus of elasticity, or the initial slope of the composite stress-strain curve, and E_1 is the secondary modulus of elasticity or the slope of the composite stress-strain curve after the cladding material has become plastic and the core material is still elastic.

COMPARISON OF THEORY AND EXPERIMENT

Experimental data available for the buckling of flat Alclad plates in compression and for Alclad strip or narrow columns are compared in the following paragraphs with the corresponding theoretical values calculated by the formulas derived in appendix C.

Flat plates in compression.- Studies of the plastic buckling of flat plates in either compression or shear (references 2 and 3) show that the curve of η plotted against plastic buckling stress is almost independent of the amount of restraint against rotation at the side edges of the plate. The same relationship might reasonably be expected to be true for an Alclad plate. Compression tests were therefore made on channel and Z-sections formed from Alclad 24S-T84 with 5.7 percent cladding in the manner described in reference 4 for the determination of plate buckling strength. The results are given in table II and plotted in figure 4 where theoretically computed curves are also plotted. The experimental points follow the trend and agree fairly well with the theoretical curves based upon estimates that are believed to be reasonable estimates for the properties of the cladding material (stress-strain curves of fig. 2).

Strip columns.- The results of the strip-column tests reported in reference 5 are plotted in figure 5 where theoretical curves for this case based upon the composite stress-strain curves of reference 6 are also plotted. For values of σ_{cr} above about 20 ksi the experimental points agree well with the theoretical curves based upon the cladding stress-strain properties of figure 2. For stresses below this point the separation of the solid and dash-line theoretical curves and the trend of the plotted experimental data suggest the importance of knowing accurately the stress-strain curve for the cladding if accurate predictions are to be made of the strip-column strength in this stress range.

CONCLUSIONS

The theoretical results of this paper, two of which were checked experimentally, show that the buckling stress of an Alclad plate can be calculated by using the same type formula as that which was used in NACA Rep. 898 to find the plastic buckling stress of a plate made of one material. The buckling-stress formula contains a plasticity reduction factor that takes into account the reduction of the modulus of elasticity for stresses above the elastic range of the cladding material. This factor is a function of the percentage of cladding and the tangent and secant moduli of both the core and the cladding. If the stress-strain curves of both the core and the cladding are known, therefore, the critical buckling stress of the Alclad plate may be calculated.

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APPENDIX A

SYMBOLS

a	ratio of cladding thickness to core thickness (see fig. 1)
b	width of plate or column
b_F	width of flange of channel or Z-section
b_W	width of web of channel or Z-section
$C_1, C_2, \ldots C_7$	plasticity coefficients (See pp. 15 and 16.)
$C_{1_\sigma}, C_{2_\sigma}, \ldots C_{7_\sigma}$	plasticity coefficients for pure compression (See p. 18.)
$C_{1_\tau}, C_{2_\tau}, \ldots C_{7_\tau}$	plasticity coefficients for pure shear (See p. 24.)
$D_c' = \frac{E_s h^3}{9}$	flexural rigidity of the core using Poisson's ratio as $\frac{1}{2}$
$e_i = \frac{2}{\sqrt{3}}\sqrt{\epsilon_x^2 + \epsilon_y^2 + \epsilon_{xy} + \frac{\gamma^2}{4}}$	strain intensity
E	initial tangent modulus of core and cladding material in combination
E_t	tangent modulus of core material at a stress equal to σ_i
E_s	secant modulus of core material at a stress equal to σ_i
$\bar{E}_t$	tangent modulus of cladding material at a stress equal to $\bar{\sigma}_i$
$\bar{E}_s$	secant modulus of cladding material at a stress equal to $\bar{\sigma}_i$
E_1	secondary modulus of elasticity of the composite Alclad sheet

$g = (1 + 2a)^3 - 1$

h	thickness of core
k	nondimensional critical-stress coefficient as used in elastic range
k_W	nondimensional critical-stress coefficient for channel or Z-section
l	length of column or plate
M_x	bending moment per inch about y-axis
M_y	bending moment per inch about x-axis
M_{xy}	twisting moment per inch
N_x	force per inch in x-direction at buckling
N_y	force per inch in y-direction at buckling
N_{xy}	shear force per inch at buckling
$S_x = \sigma_x - \frac{1}{2}\sigma_y$	
$S_y = \sigma_y - \frac{1}{2}\sigma_x$	
t	total plate thickness
V	strain energy in plate during buckling
w	deflection of plate normal to xy-plane
x,y	rectangular coordinates
z	distance measured normal to plate from middle surface
z_0	distance from middle surface of plate to neutral axis
γ	shear strain in xy-plane
δ	used as variation in parameter at buckling
ϵ_x	strain in x direction

ϵ_y	strain in y-direction
ϵ_1, ϵ_2	middle-surface strain variations in x- and y-directions, respectively, at buckling
ϵ_3	middle-surface shear-strain variation at buckling
η	plasticity reduction factor that takes into account reduction of modulus of elasticity for stresses above elastic range of cladding material; depends upon stress and type of plate
λ	half wave length of buckle
μ	value of Poisson's ratio of composite sheet at buckling stress
$\rho = \frac{h}{\sqrt{12}}$	
σ_x	stress in x-direction in core material
$\bar{\sigma}_x$	stress in x-direction in cladding material
σ_y	stress in y-direction in core material
$\bar{\sigma}_y$	stress in y-direction in cladding material
$\sigma_i = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau^2}$	stress intensity in core material
$\bar{\sigma}_i = \sqrt{\bar{\sigma}_x^2 + \bar{\sigma}_y^2 - \bar{\sigma}_x\bar{\sigma}_y + 3\tau^2}$	stress intensity in cladding material
$\sigma_{x_{el}}, \sigma_{y_{el}}, \sigma_{el}, \tau_{el}$	critical stresses if both core and cladding material are elastic
$\sigma_{x_{cr}}, \sigma_{y_{cr}}, \sigma_{cr}, \tau_{cr}$	critical stresses in Alclad sheet at buckling
τ	shear stress in xy-plane in core material
$\bar{\tau}$	shear stress in xy-plane in cladding material
χ_1, χ_2	curvatures developed in x- and y-directions, respectively, at buckling
χ_3	twist developed in buckling

APPENDIX B

THEORETICAL DERIVATIONS

The same procedure may be used to derive the plastic buckling stress of a plate as that which was used in deriving the elastic buckling stress if suitable polyaxial stress-strain relations are used. These stress-strain relations replace Hooke's law in the plastic stress region and reduce to Hooke's law in the elastic stress region. In this appendix the two-dimensional plastic polyaxial stress-strain relations of the Hencky theory are described and then used to derive the differential equation of equilibrium and the energy expression of a buckled Alclad plate. These stress-strain relations are the same as those used in references 2 and 3.

Stress-strain relations.- The Hencky deformation theory of plasticity is based on the hypothesis that if a certain function of the stresses at a point (the stress intensity)

$$\sigma_i = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau^2} \tag{1}$$

is plotted against a certain function of the strains at the same point (the strain intensity)

$$e_i = \frac{2}{\sqrt{3}}\sqrt{\epsilon_x^2 + \epsilon_y^2 + \epsilon_x\epsilon_y + \frac{\gamma^2}{4}} \tag{2}$$

a unique curve for all combinations of stresses and compatible strains is obtained. This relation as used in the present derivations is assumed to hold for both loading (increasing σ_i) and unloading (decreasing σ_i). The material is assumed to be incompressible, for which case Poisson's ratio is 1/2.

The relations between the individual stresses and strains are

$$\left.\begin{aligned} \epsilon_x &= \frac{\sigma_x - \frac{1}{2}\sigma_y}{E_s} = \frac{S_x}{E_s} \\ \epsilon_y &= \frac{\sigma_y - \frac{1}{2}\sigma_x}{E_s} = \frac{S_y}{E_s} \\ \gamma &= \frac{3\tau}{E_s} \\ E_s &= \frac{\sigma_i}{e_i} \end{aligned}\right\} \qquad (3)$$

where E_s is the secant modulus taken from the uniaxial compressive stress-strain curve at a stress equal to σ_i.

When buckling occurs let ϵ_x, ϵ_y, and γ vary slightly from their values before buckling. These variations $\delta\epsilon_x$, $\delta\epsilon_y$, and $\delta\gamma$ will arise partly from the variations of middle-surface strains and partly from strains due to bending. If ϵ_1 and ϵ_2 are middle-surface strain variations, ϵ_3 is the middle-surface shear-strain variation, χ_1 and χ_2 are the changes in curvature, χ_3 is the change in twist, and z is the distance out from the middle of the plate, then

$$\left.\begin{aligned} \delta\epsilon_x &= \epsilon_1 - z\chi_1 \\ \delta\epsilon_y &= \epsilon_2 - z\chi_2 \\ \delta\gamma &= 2\epsilon_3 - 2z\chi_3 \end{aligned}\right\} \qquad (4)$$

As buckling occurs, these variations in strain $\delta\epsilon_x$, $\delta\epsilon_y$, and $\delta\gamma$ cause corresponding variations in the stress quantities S_x, S_y, and τ.

The calculation of the variation in S_x will be shown in detail. From equations (3)

$$S_x = E_s\epsilon_x$$

and the variation in S_x therefore becomes

$$\delta S_x = E_s \, \delta \epsilon_x + \epsilon_x \, \delta\left(\frac{\sigma_i}{e_i}\right)$$

$$= E_s \, \delta \epsilon_x - \frac{\epsilon_x}{e_i}\left(\frac{\sigma_i}{e_i} - \frac{d\sigma_i}{de_i}\right)\delta e_i$$

Since $\frac{\sigma_i}{e_i}$ is the secant modulus E_s and $\frac{d\sigma_i}{de_i}$ is the tangent modulus E_t,

$$\delta S_x = E_s \, \delta \epsilon_x - \frac{\epsilon_x}{e_i}\left(E_s - E_t\right)\delta e_i \tag{5}$$

In this equation for δS_x it is convenient to express δe_i in terms of the coordinate z and also z_0, the distance from the middle surface of the plate to the neutral axis. From equations (1), (2), and (3) it can be shown that

$$\sigma_i e_i = \sigma_x \epsilon_x + \sigma_y \epsilon_y + \tau\gamma \tag{6}$$

and

$$(\sigma_i + \delta\sigma_i)(e_i + \delta\epsilon_i) = (\sigma_x + \delta\sigma_x)(\epsilon_x + \delta\epsilon_x) + (\sigma_y + \delta\sigma_y)(\epsilon_y + \delta\epsilon_y) + (\tau + \delta\tau)(\gamma + \delta\gamma) \tag{7}$$

Subtracting equation (6) from equation (7), substituting for the variations $\delta\sigma_i$, $\delta\sigma_x$, $\delta\sigma_y$, and $\delta\tau$, their values obtained from equation (3), and neglecting second order terms gives

$$\delta e_i = \frac{\sigma_x \, \delta\epsilon_x + \sigma_y \, \delta\epsilon_y + \tau \, \delta\gamma}{\sigma_i}$$

Substituting the values of the variations $\delta\epsilon_x$, $\delta\epsilon_y$, and $\delta\gamma$ from equation (4) into this equation gives

$$\delta e_i = \frac{\sigma_x \epsilon_1 + \sigma_y \epsilon_2 + 2\tau\epsilon_3 - z(\sigma_x \chi_1 + \sigma_y \chi_2 + 2\tau\chi_3)}{\sigma_i} \tag{8}$$

If $\delta e_i = 0$, this equation will give the coordinate of the neutral surface

$$z_0 = \frac{\sigma_x \epsilon_1 + \sigma_y \epsilon_2 + 2\tau \epsilon_3}{\sigma_x \chi_1 + \sigma_y \chi_2 + 2\tau \chi_3}$$

and equation (8) may be written

$$\delta e_i = \frac{\left(\sigma_x \chi_1 + \sigma_y \chi_2 + 2\tau \chi_3\right)\left(z_0 - z\right)}{\sigma_i}$$

Substituting this value for δe_i and the value of $\delta \epsilon_x$ given in equation (4) into equation (5) gives

$$\delta S_x = E_s\left(\epsilon_1 - z\chi_1\right) + \frac{\epsilon_x}{\sigma_i e_i}\left(E_s - E_t\right)\left(\sigma_x \chi_1 + \sigma_y \chi_2 + 2\tau \chi_3\right)\left(z - z_0\right) \quad (9)$$

Similarly the values of the variation of the stress quantities S_y and τ are

$$\delta S_y = E_s\left(\epsilon_2 - z\chi_2\right) + \frac{\epsilon_y}{\sigma_i e_i}\left(E_s - E_t\right)\left(\sigma_x \chi_1 + \sigma_y \chi_2 + 2\tau \chi_3\right)\left(z - z_0\right) \quad (10)$$

and

$$\delta \tau = \frac{2}{3}E_s\left(\epsilon_3 - z\chi_3\right) + \frac{\gamma}{3\sigma_i e_i}\left(E_s - E_t\right)\left(\sigma_x \chi_1 + \sigma_y \chi_2 + 2\tau \chi_3\right)\left(z - z_0\right) \quad (11)$$

The variations in the stresses σ_x and σ_y in terms of the variations in the stress quantities S_x and S_y are, from equations (3)

$$\left.\begin{aligned} \delta\sigma_x &= \frac{4}{3}\left(\delta S_x + \frac{1}{2}\right)\delta S_y \\ &\text{and} \\ \delta\sigma_y &= \frac{4}{3}\left(\delta S_y + \frac{1}{2}\right)\delta S_x \end{aligned}\right\} \quad (12)$$

In order to use these stress-strain relations to derive the differential equation and the energy expression for an Alclad plate, the quantities σ_i, σ_x, σ_y, τ, E_s, and E_t designate the quantities that apply to the core material, whereas $\bar{\sigma}_i$, $\bar{\sigma}_x$, $\bar{\sigma}_y$, $\bar{\tau}$, $\bar{E}_s$, and $\bar{E}_t$ designate the corresponding quantities that apply to the cladding material.

Differential equation of buckled plate.- In order to derive the differential equation of equilibrium of the plate, the values of the variations of the moments M_x, M_y, and M_{xy} due to the strain variations $\delta\epsilon_x$, $\delta\epsilon_y$, and $\delta\gamma$, which are obtained by integration of the product of stress and moment arm over the thickness of the plate, are substituted into the equilibrium equation (reference 2, equation (15))

$$\frac{\partial^2(\delta M_x)}{\partial x^2} + 2\frac{\partial^2(\delta M_{xy})}{\partial x\,\partial y} + \frac{\partial^2(\delta M_y)}{\partial y^2} = N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y} + N_y\frac{\partial^2 w}{\partial y^2} \tag{13}$$

In this equation δM_x, δM_y, and δM_{xy} are the variations in M_x, M_y, and M_{xy} during buckling and w is the deflection of the plate.

Each integration for a variation of moment consists of three integrals: one over the thickness of the core, and one over each thickness of the cladding. The variations of the moments M_x, M_y, and M_{xy} are

$$\left.\begin{aligned}
\delta M_x &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \delta\sigma_x z\,dz + \int_{\frac{h}{2}}^{\frac{h}{2}+ah} \delta\bar{\sigma}_x z\,dz + \int_{-\left(\frac{h}{2}+ah\right)}^{-\frac{h}{2}} \delta\bar{\sigma}_x z\,dz \\
\delta M_y &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \delta\sigma_y z\,dz + \int_{\frac{h}{2}}^{\frac{h}{2}+ah} \delta\bar{\sigma}_y z\,dz + \int_{-\left(\frac{h}{2}+ah\right)}^{-\frac{h}{2}} \delta\bar{\sigma}_y z\,dz \\
\delta M_{xy} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \delta\tau z\,dz + \int_{\frac{h}{2}}^{\frac{h}{2}+ah} \delta\bar{\tau} z\,dz + \int_{-\left(\frac{h}{2}+ah\right)}^{-\frac{h}{2}} \delta\bar{\tau} z\,dz
\end{aligned}\right\} \tag{14}$$

Substituting the values of the stress variations given by equations (11) and (12) into the equations for the moment variations given by equations (14) gives, with the aid of equations (9) and (10),

$$\delta M_x = -D_c'\left(\left[1 - \frac{3}{4}\left(\frac{\sigma_x}{\sigma_i}\right)^2\left(1 - \frac{E_t}{E_s}\right)\right]\chi_1 + \frac{1}{2}\left[1 - \frac{3}{2}\,\frac{\sigma_x\sigma_y}{\sigma_i^2}\left(1 - \frac{E_t}{E_s}\right)\right]\chi_2\right.$$

$$- \frac{3}{2}\,\frac{\sigma_x\tau}{\sigma_i^2}\left(1 - \frac{E_t}{E_s}\right)\chi_3 + \frac{\bar{E}_s}{E_s}\,g\left\{\left[1 - \frac{3}{4}\left(\frac{\bar{\sigma}_x}{\bar{\sigma}_i}\right)^2\left(1 - \frac{\bar{E}_t}{\bar{E}_s}\right)\right]\chi_1\right.$$

$$\left.\left. + \frac{1}{2}\left[1 - \frac{3}{2}\,\frac{\bar{\sigma}_x\bar{\sigma}_y}{\bar{\sigma}_i^2}\left(1 - \frac{\bar{E}_t}{\bar{E}_s}\right)\right]\chi_2 - \frac{3}{2}\,\frac{\bar{\sigma}_x\bar{\tau}}{\bar{\sigma}_i^2}\left(1 - \frac{\bar{E}_t}{\bar{E}_s}\right)\chi_3\right\}\right) \tag{15}$$

$$\delta M_y = -D_c'\left(\left[1 - \frac{3}{4}\left(\frac{\sigma_y}{\sigma_i}\right)^2\left(1 - \frac{E_t}{E_s}\right)\right]\chi_2 + \frac{1}{2}\left[1 - \frac{3}{2}\,\frac{\sigma_x\sigma_y}{\sigma_i^2}\left(1 - \frac{E_t}{E_s}\right)\right]\chi_1\right.$$

$$- \frac{3}{2}\,\frac{\sigma_y\tau}{\sigma_i^2}\left(1 - \frac{E_t}{E_s}\right)\chi_3 + \frac{\bar{E}_s}{E_s}\,g\left\{\left[1 - \frac{3}{4}\left(\frac{\bar{\sigma}_y}{\bar{\sigma}_i}\right)^2\left(1 - \frac{\bar{E}_t}{\bar{E}_s}\right)\right]\chi_2\right.$$

$$\left.\left. + \frac{1}{2}\left[1 - \frac{3}{2}\,\frac{\bar{\sigma}_x\bar{\sigma}_y}{\bar{\sigma}_i^2}\left(1 - \frac{\bar{E}_t}{\bar{E}_s}\right)\right]\chi_1 - \frac{3}{2}\,\frac{\bar{\sigma}_y\bar{\tau}}{\bar{\sigma}_i^2}\left(1 - \frac{\bar{E}_t}{\bar{E}_s}\right)\chi_3\right\}\right) \tag{16}$$

$$\delta M_{xy} = -\frac{D_c'}{2}\left(\left[1 - \frac{3\tau^2}{\sigma_i^2}\left(1 - \frac{E_t}{E_s}\right)\right]\chi_3 - \frac{3}{2}\left(\frac{\sigma_x\tau}{\sigma_i^2}\,\chi_1 + \frac{\sigma_y\tau}{\sigma_i^2}\,\chi_2\right)\left(1 - \frac{E_t}{E_s}\right)\right.$$

$$\left. + \frac{\bar{E}_s}{E_s}\,g\left\{\left[1 - \frac{3\bar{\tau}^2}{\bar{\sigma}_i^2}\left(1 - \frac{\bar{E}_t}{\bar{E}_s}\right)\right]\chi_3 - \frac{3}{2}\left(\frac{\bar{\sigma}_x\bar{\tau}}{\bar{\sigma}_i^2}\,\chi_1 + \frac{\bar{\sigma}_y\bar{\tau}}{\bar{\sigma}_i^2}\,\chi_2\right)\left(1 - \frac{\bar{E}_t}{\bar{E}_s}\right)\right\}\right) \tag{17}$$

where $D_c' = \dfrac{E_s h^3}{9}$

The differential equation of equilibrium can now be written by substituting the moment variations given by equations (15), (16), and (17) into equation (13) and recognizing that the changes in curvature χ_1 and χ_2 and the change in twist χ_3 are, respectively,

$$\chi_1 = \frac{\partial^2 w}{\partial x^2} \tag{18}$$

$$\chi_2 = \frac{\partial^2 w}{\partial y^2} \tag{19}$$

$$\chi_3 = \frac{\partial^2 w}{\partial x\, \partial y} \tag{20}$$

Thus the differential equation of equilibrium becomes

$$C_1 \frac{\partial^4 w}{\partial x^4} - C_2 \frac{\partial^2 w}{\partial x^3 \partial y} + 2C_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} - C_4 \frac{\partial^4 w}{\partial x\, \partial y^3} + C_5 \frac{\partial^4 w}{\partial y^4} =$$

$$- \frac{1}{D_c'}\left(N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x\, \partial y} + N_y \frac{\partial^2 w}{\partial y^2}\right) \tag{21}$$

where

$$C_1 = 1 - \frac{3}{4}\left(\frac{\sigma_x}{\sigma_i}\right)^2\left(1 - \frac{E_t}{E_s}\right) + \frac{\overline{E}_s}{E_s}\, g\left[1 - \frac{3}{4}\left(\frac{\overline{\sigma}_x}{\overline{\sigma}_i}\right)^2\left(1 - \frac{\overline{E}_t}{\overline{E}_s}\right)\right]$$

$$C_2 = 3\left[\frac{\sigma_x \tau}{\sigma_i^2}\left(1 - \frac{E_t}{E_s}\right) + \frac{\overline{E}_s}{E_s}\, g\, \frac{\overline{\sigma}_x \overline{\tau}}{\overline{\sigma}_i^2}\left(1 - \frac{\overline{E}_t}{\overline{E}_s}\right)\right]$$

$$C_3 = 1 - \frac{3}{4}\, \frac{\sigma_x \sigma_y + 2\tau^2}{\sigma_i^2}\left(1 - \frac{E_t}{E_s}\right) + \frac{\overline{E}_s}{E_s}\, g\left[1 - \frac{3}{4}\, \frac{\overline{\sigma}_x \overline{\sigma}_y + 2\overline{\tau}^2}{\overline{\sigma}_i^2}\left(1 - \frac{\overline{E}_t}{\overline{E}_s}\right)\right]$$

$$C_4 = 3\left[\frac{\sigma_y \tau}{\sigma_i^2}\left(1 - \frac{E_t}{E_s}\right) + \frac{\overline{E}_s}{E_s}\, g\, \frac{\overline{\sigma}_y \overline{\tau}}{\overline{\sigma}_i^2}\left(1 - \frac{\overline{E}_t}{\overline{E}_s}\right)\right]$$

$$C_5 = 1 - \frac{3}{4}\left(\frac{\sigma_y}{\sigma_i}\right)^2\left(1 - \frac{E_t}{E_s}\right) + \frac{\overline{E}_s}{E_s}\, g\left[1 - \frac{3}{4}\left(\frac{\overline{\sigma}_y}{\overline{\sigma}_i}\right)^2\left(1 - \frac{\overline{E}_t}{\overline{E}_s}\right)\right]$$

Energy expression.- The strain energy in the plate during buckling, given in reference 7, is

$$V = -\frac{1}{2}\iint\left(\delta M_x \frac{\partial^2 w}{\partial x^2} + 2\, \delta M_{xy} \frac{\partial^2 w}{\partial x\, \partial y} + \delta M_y \frac{\partial^2 w}{\partial y^2}\right) dx\, dy$$

Substituting the values of δM_x, δM_y, and δM_{xy} given in equations (15), (16), and (17) into this equation gives

$$V = \iint \frac{D_c'}{2}\left[C_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - C_2 \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x\,\partial y} + C_6 \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + C_7\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 - C_4 \frac{\partial^2 w}{\partial x\,\partial y}\frac{\partial^2 w}{\partial y^2} + C_5\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right]dx\,dy$$

where

$$C_6 = 1 - \frac{3}{2}\frac{\sigma_x\sigma_y}{\sigma_i^2}\left(1 - \frac{E_t}{E_s}\right) + \frac{\bar{E}_s}{E_s}\,g\left[1 - \frac{3}{2}\frac{\bar{\sigma}_x\bar{\sigma}_y}{\bar{\sigma}_i^2}\left(1 - \frac{\bar{E}_t}{\bar{E}_s}\right)\right]$$

$$C_7 = 1 - \frac{3\tau^2}{\sigma_i^2}\left(1 - \frac{E_t}{E_s}\right) + \frac{\bar{E}_s}{E_s}\,g\left[1 - \frac{3\bar{\tau}^2}{\bar{\sigma}_i^2}\left(1 - \frac{\bar{E}_t}{\bar{E}_s}\right)\right]$$

$$C_6 + C_7 = 2C_3$$

The difference between the strain energy in the plate and the work done on the plate by the external forces, provided that external restraining forces do no work during buckling, is

$$\iint\left\{\frac{D_c'}{2}\left[C_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - C_2 \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x\,\partial y} + C_6 \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + C_7\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 - C_4 \frac{\partial^2 w}{\partial x\,\partial y}\frac{\partial^2 w}{\partial y^2} + C_5\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right] - \frac{1}{2}\left[N_x\left(\frac{\partial w}{\partial x}\right)^2 + 2N_{xy}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} + N_y\left(\frac{\partial w}{\partial y}\right)^2\right]\right\}dx\,dy \tag{22}$$

If no cladding is used, this expression differs slightly from that given in references 2 and 3. The difference between the two expressions is a quantity proportional to

$$\iint\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right]dx\,dy$$

which can be shown to be identically zero for plates with supported edges. The two expressions will therefore give identical results for such plates. Other cases in which the two expressions give identical results occur when C_6 and C_7 are equal to C_3 (for example, single compressive loading), in which cases the coefficient of the above quantity is zero.

If the integral (22) is set equal to zero the resulting equation may be solved for N_x, N_y, or N_{xy} and minimized to obtain the critical force per inch at buckling. This minimization is equivalent to minimizing the integral (22). The average critical stresses are

$$\left.\begin{aligned} \sigma_{cr_x} &= \frac{N_x}{(1 + 2a)h} \\ \sigma_{cr_y} &= \frac{N_y}{(1 + 2a)h} \\ \tau_{cr} &= \frac{N_{xy}}{(1 + 2a)h} \end{aligned}\right\} \tag{23}$$

APPENDIX C

APPLICATIONS

The theory will now be applied to the following cases: long simply supported plate in compression, plate-column, and long simply supported plate in shear.

Long simply supported plate in compression.- The plastic buckling of a long Alclad plate subject to longitudinal compression with the edges hinged is solved by using the energy method. If the plate is compressed in the x-direction only

$$\sigma_y = \bar{\sigma}_y = \tau = \bar{\tau} = N_y = N_{xy} = 0$$

$$\sigma_i = \sigma_x$$

$$\bar{\sigma}_i = \bar{\sigma}_x$$

and the coefficients reduce to

$$C_{1_\sigma} = \frac{1}{4} + \frac{3}{4}\frac{E_t}{E_s} + \frac{\bar{E}_s}{E_s} g\left[\frac{1}{4} + \frac{3}{4}\frac{\bar{E}_t}{\bar{E}_s}\right]$$

$$C_{2_\sigma} = C_{4_\sigma} = 0$$

$$C_{3_\sigma} = C_{5_\sigma} = C_{6_\sigma} = C_{7_\sigma} = 1 + \frac{\bar{E}_s}{E_s} g$$

With these substitutions the energy expression when solved for N_x becomes

$$N_x = D_c' \frac{\iint \left\{C_{1\sigma}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + C_{3\sigma}\left[\left(\frac{\partial^2 w}{\partial x\, \partial y}\right)^2 + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] + C_{5\sigma}\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right\} dx\, dy}{\iint \left(\frac{\partial w}{\partial x}\right)^2 dx\, dy} \tag{24}$$

If the deflection surface is

$$w = \cos\frac{\pi y}{b}\cos\frac{\pi x}{\lambda}$$

where $y = \pm\frac{b}{2}$ are the unloaded edges which are simply supported and λ is the half wave length of buckle, equation (24) reduces to

$$N_x = \frac{\pi^2 D_c'}{b^2}\left[C_{1_\sigma}\left(\frac{b}{\lambda}\right)^2 + 2C_{3_\sigma} + C_{5_\sigma}\left(\frac{\lambda}{b}\right)^2\right] \tag{25}$$

The average critical stress is obtained by dividing the smallest value of the load per inch N_x by the total thickness $(1 + 2a)h$. In order to find the minimum value of N_x

$$\frac{\partial N_x}{\partial \frac{b}{\lambda}} = 0$$

which gives

$$\left(\frac{b}{\lambda}\right)^2 = \sqrt{\frac{C_{5_\sigma}}{C_{1_\sigma}}}$$

Substitution of this value of $\left(\frac{b}{\lambda}\right)^2$ into equation (25) and dividing by $(1 + 2a)h$ gives for the critical stress

$$\sigma_{x_{cr}} = \frac{2\pi^2 D_c'}{(1 + 2a)hb^2}\left\{\sqrt{C_{1_\sigma} C_{5_\sigma}} + C_{3_\sigma}\right\}$$

The corresponding critical stress if both core and cladding material are elastic is

$$\sigma_{x_{el}} = \frac{4\pi^2 E h^2}{9(1 + 2a)b^2}(g + 1)$$

If η is defined as the ratio of the actual critical stress to the critical stress obtained on the assumption of perfect elasticity $\sigma_{x_{el}}$ then

$$\eta = \frac{\sigma_{x_{cr}}}{\sigma_{x_{el}}} = \frac{\sqrt{C_{1_\sigma} C_{5_\sigma}} + C_{3_\sigma}}{2(g + 1)} \frac{E_s}{E} \tag{26}$$

Since Poisson's ratio is taken as 1/2 in the computation of each of these stresses, an error is ordinarily present in both stresses, but most of this error is eliminated in the process of division to obtain η. If both the core and the cladding material are elastic, $\eta = 1$.

Plate loaded as a column.- This problem of the plastic buckling of a rectangular Alclad plate in compression with the unloaded edges free is solved by using the differential-equation method. If the plate is compressed in the x-direction only, the differential equation given in equation (21) reduces to

$$C_{1_\sigma} \frac{\partial^4 w}{\partial x^4} + 2C_{3_\sigma} \frac{\partial^4 w}{\partial x^2 \partial y^2} + C_{5_\sigma} \frac{\partial^4 w}{\partial y^4} = -\frac{N_x}{D_c'} \frac{\partial^2 w}{\partial x^2}$$

where C_{1_σ}, C_{3_σ}, and C_{5_σ} are the same as in the preceeding example. The boundary conditions are

$$(\delta M_y)_{y=\pm\frac{b}{2}} = \left(\frac{\partial^2 w}{\partial y^2} + \frac{1}{2} \frac{\partial^2 w}{\partial x^2}\right)_{y=\pm\frac{b}{2}} = 0$$

$$\left[\frac{\partial(\delta M_y)}{\partial y} - 2 \frac{\partial(\delta M_{xy})}{\partial x}\right]_{y=\pm\frac{b}{2}} = \left(\frac{\partial^3 w}{\partial y^3} + \frac{3}{2} \frac{\partial^3 w}{\partial x^2 \partial y}\right)_{y=\pm\frac{b}{2}} = 0$$

$$(\delta M_x)_{x=0,l} = \left(C_{1_\sigma} \frac{\partial^2 w}{\partial x^2} + \frac{1}{2} C_{3_\sigma} \frac{\partial^2 w}{\partial y^2}\right)_{x=0,l} = 0$$

$$\left[\frac{\partial(\delta M_x)}{\partial x} - 2 \frac{\partial(\delta M_{xy})}{\partial y}\right]_{x=0,l} = \left(C_{1_\sigma} \frac{\partial^3 w}{\partial x^3} + \frac{3}{2} C_{3_\sigma} \frac{\partial^3 w}{\partial x\, \partial y^2}\right)_{x=0,l} = 0$$

where $y = \pm\frac{b}{2}$ are the unloaded edges that are free and $x = 0,l$ are the ends of the column where the bending moment and shear are zero.

A deflection function that satisfies the differential equation and satisfies the first, third, and fourth of these boundary conditions is

$$w = \left(q \cos\frac{\beta}{2} \cosh\frac{\alpha y}{b} + p \cosh\frac{\alpha}{2} \cos\frac{\beta y}{b}\right)\cos\frac{\pi x}{l}$$

where

$$\alpha = \pi\sqrt{\frac{b}{l}}\sqrt{\frac{b}{l} + \sqrt{\frac{k}{C_{3_\sigma}} + \left(\frac{b}{l}\right)^2\left(1 - \frac{C_{1_\sigma}}{C_{3_\sigma}}\right)}}$$

$$\beta = \pi\sqrt{\frac{b}{l}}\sqrt{-\frac{b}{l} + \sqrt{\frac{k}{C_3} + \left(\frac{b}{l}\right)^2\left(1 - \frac{C_{1_\sigma}}{C_{3_\sigma}}\right)}}$$

$$N_x = \frac{k\pi^2 D_c'}{b^2}$$

$$p = \alpha^2 - \frac{1}{2}\left(\frac{\pi b}{l}\right)^2$$

$$q = \beta^2 + \frac{1}{2}\left(\frac{\pi b}{l}\right)^2$$

In order to satisfy the second boundary condition it is required that

$$\alpha^2 q\left[\beta - \left(\frac{\pi b}{l}\right)^2\right]\frac{\tanh\frac{\alpha}{2}}{\alpha/2} + \beta^2 p\left[q + \left(\frac{\pi b}{l}\right)^2\right]\frac{\tan\frac{\beta}{2}}{\beta/2} = 0 \qquad (27)$$

which is the buckling criterion the solution of which gives the buckling stress.

In solving equation (27) it is convenient to let the quanity that appears in the definition of α and β

$$\frac{k}{C_{3_\sigma}} + \left(\frac{b}{l}\right)^2\left(1 - \frac{C_{1_\sigma}}{C_{3_\sigma}}\right) = \left(\frac{b}{l}\right)^2\left(1 - \xi^2\right) \qquad (28)$$

where ξ^2 is a quantity that will depend on the length-width ratio $\left(\frac{l}{b}\right)$ of the plate. Then

$$\alpha^2 = \left(\frac{\pi b}{l}\right)^2\left(1 + \sqrt{1 - \xi^2}\right)$$

$$\beta^2 = \left(\frac{\pi b}{l}\right)^2\left(-1 + \sqrt{1 - \xi^2}\right)$$

$$p = \left(\frac{\pi b}{l}\right)^2\left(\frac{1}{2} + \sqrt{1 - \xi^2}\right)$$

$$q = \left(\frac{\pi b}{l}\right)^2\left(-\frac{1}{2} + \sqrt{1 - \xi^2}\right)$$

$$p - q = \left(\frac{\pi b}{l}\right)^2$$

and the buckling criterion given by equation (27) becomes

$$\left[\xi^2 + \left(\frac{1}{4} - \xi^2\right)\left(1 + \sqrt{1 - \xi^2}\right)\right]\frac{\tanh\frac{\alpha}{2}}{\alpha/2} - \left[\xi^2 + \left(\frac{1}{4} - \xi^2\right)\left(1 - \sqrt{1 - \xi^2}\right)\right]\frac{\tan\frac{\beta}{2}}{\beta/2} = 0 \tag{29}$$

This stability criterion is the same as that obtained in reference 2, equation (33). Although a separate solution of equation (29) is required for each value of $\frac{l}{b}$, only the three solutions given in reference 2 are considered here: for short columns $\left(\frac{l}{b} \ll 1\right)$, $\xi^2 \approx 0$; for a square plate $\left(\frac{l}{b} = 1\right)$, $\xi^2 = 0.15375$; and for a long column $\left(\frac{l}{b} \gg 1\right)$, $\xi^2 = \frac{1}{4}$. With the values of ξ^2 known, the value of the nondimensional critical-stress coefficient k given by equation (28) is substituted into the expression for the load per inch N_x and the result divided by the thickness of the plate $(1 + 2a)h$ to give the critical stress in terms of ξ^2 as

$$\sigma_{x_{cr}} = \frac{N_x}{(1 + 2a)h} = \frac{C_{3_\sigma}\left(\frac{C_{1_\sigma}}{C_{3_\sigma}} - \xi^2\right)}{(1 + 2a)}\frac{\pi^2 E_s}{\frac{3}{4}\left(\frac{l}{\rho}\right)^2}$$

where $\rho = \frac{h}{\sqrt{12}}$

The corresponding critical stress if both core and cladding material are elastic is

$$\sigma_{x_{el}} = \frac{\pi^2 E(1 - \xi^2)}{\frac{3}{4}\left(\frac{l}{\rho}\right)^2} (1 + 2a)^2$$

The plasticity reduction factor η, which is the ratio of the actual critical stress to the critical stress $\sigma_{x_{el}}$ obtained on the assumption of perfect elasticity, is

$$\eta = \frac{\sigma_{x_{cr}}}{\sigma_{x_{el}}} = \frac{C_{3\sigma}\left(\frac{C_{1\sigma}}{C_{3\sigma}} - \xi^2\right)}{(1 - \xi^2)(g + 1)} \frac{E_s}{E}$$

For a short plate-column $\left(\frac{l}{b} \ll 1\right)$, $\xi^2 \approx 0$ and

$$\eta = \frac{C_{1\sigma}}{(g + 1)} \frac{E_s}{E}$$

For a square plate-column $\left(\frac{l}{b} = 1\right)$, $\xi^2 = 0.15375$ and

$$\eta = \frac{C_{3\sigma}\left(\frac{C_{1\sigma}}{C_{3\sigma}} - 0.15375\right)}{0.84625\ (g + 1)} \frac{E_s}{E}$$

For a long plate-column $\left(\frac{l}{b} \gg 1\right)$, $\xi^2 = \frac{1}{4}$ and

$$\eta = \frac{4C_{3\sigma}\left(\frac{C_{1\sigma}}{C_{3\sigma}} - \frac{1}{4}\right)}{3(g + 1)} \frac{E_s}{E}$$

Long simply supported plate in shear.- The plastic buckling of a long Alclad plate subject to shear with the edges hinged is solved by using the energy method. If the plate is under pure shear

$$\sigma_x = \sigma_y = \bar{\sigma}_x = \bar{\sigma}_y = N_x = N_y = 0$$

$$\sigma_i = \sqrt{3}\tau$$

$$\bar{\sigma}_i = \sqrt{3}\bar{\tau}$$

$$C_{1_\tau} = C_{5_\tau} = C_{6_\tau} = 1 + \frac{\bar{E}_s}{E_s} g$$

$$C_{2_\tau} = C_{4_\tau} = 0$$

$$C_{7_\tau} = \frac{E_t}{E_s} + g \frac{\bar{E}_t}{E_s}$$

$$C_{3_\tau} = \frac{C_{6_\tau} + C_{7_\tau}}{2} = \frac{1}{2} + \frac{1}{2}\frac{E_t}{E_s} + \frac{\bar{E}_s}{E_s} g \left[\frac{1}{2} + \frac{1}{2}\frac{\bar{E}_t}{\bar{E}_s}\right]$$

With these substitutions the energy expression when solved for N_{xy} becomes:

$$N_{xy} = \frac{D_c'}{2} \frac{\iint \left[C_{1_\tau}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + C_{7_\tau}\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 + C_{6_\tau}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + C_{5_\tau}\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right] dx\,dy}{\iint \left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right) dx\,dy} \tag{30}$$

If the approximate deflection surface which was used in the elastic stress region (see reference 7)

$$w = \sin\frac{\pi y}{b}\sin\frac{\pi(x - \alpha y)}{\lambda}$$

where $y = 0,b$ are the long sides of the plate, λ is the half wave length of buckle, and α is the slope of the nodal lines, is assumed satisfactory for the plastic stress region and is substituted into equation (30), the load per inch N_{xy} becomes

$$N_{xy} = \frac{\pi^2 D_c'}{2\alpha b^2}\left[C_{1_\tau}\left(\frac{b}{\lambda}\right)^2 + 2C_{3_\tau} + C_{5_\tau}\left(\frac{\lambda}{b}\right)^2 + C_{5_\tau}\left(\frac{b}{\lambda}\right)^2\left(\alpha^4 + 2\frac{C_{3_\tau}}{C_{5_\tau}}\alpha^2 + 1\right)\right] \tag{31}$$

The critical stress is obtained by dividing the smallest value of the load per inch N_{xy} by the total thickness $(1 + 2a)h$. In order to find the minimum value of N_{xy}

$$\frac{\partial N_{xy}}{\partial\left(\frac{\lambda}{b}\right)} = 0$$

and

$$\frac{\partial N_{xy}}{\partial \alpha} = 0$$

The first minimization gives

$$\left(\frac{\lambda}{b}\right)^2 = \sqrt{\alpha^4 + 2 \frac{C_{3_\tau}}{C_{5_\tau}} \alpha^2 + 1}$$

The second minimization gives

$$\sqrt{\alpha^4 + 2 \frac{C_{3_\tau}}{C_{5_\tau}} \alpha^2 + 1} = \frac{1 - \alpha^4}{3\alpha^2 - \frac{C_{3_\tau}}{C_{5_\tau}}} \tag{32}$$

Substitution of the value of $\left(\frac{\lambda}{b}\right)^2$ into equation (31) and dividing by $(1 + 2a)h$ gives for the critical stress

$$\tau_{cr} = \frac{\pi^2 D_c' C_{5_\tau}}{(1 + 2a)h\alpha b^2}\left(3\alpha^2 + \frac{C_{3_\tau}}{C_{5_\tau}} + \sqrt{\alpha^4 + 2 \frac{C_{3_\tau}}{C_{5_\tau}} \alpha^2 + 1}\right)$$

where α is determined by equation (32). The critical shear stress τ_{el} when both the core material and the cladding material are elastic is

$$\tau_{x_{el}} = \frac{4\sqrt{2}\pi^2 E h^2}{9b^2}(1 + 2a)^2$$

As in the preceding cases, the plasticity reduction factor η is the ratio of the actual critical stress to the critical stress obtained on the assumption of perfect elasticity or .

$$\eta = \frac{\tau_{cr}}{\tau_{el}} = \frac{C_{5_\tau}}{4\sqrt{2}(g + 1)}\left\{3\alpha^2 + \frac{C_{3_\tau}}{C_{5_\tau}} + \sqrt{\alpha^4 + 2\frac{C_{3_\tau}}{C_{5_\tau}}\alpha^2 + 1}\right\}\frac{E_s}{E}$$

where α is given by equation (32).

REFERENCES

1. Ilyushin, A. A.: The Elasto-Plastic Stability of Plates. NACA TM 1188, 1947.

2. Stowell, Elbridge Z.: A Unified Theory of Plastic Buckling of Columns and Plates. NACA Rep. 898, 1948.

3. Stowell, Elbridge Z.: Critical Shear Stress of an Infinitely Long Plate in the Plastic Region. NACA TN 1681, 1948.

4. Lundquist, Eugene E., Schuette, Evan H., Heimerl, George J., and Roy, J. Albert: Column and Plate Compressive Strengths of Aircraft Structural Materials. 24S-T Aluminum-Alloy Sheet. NACA ARR L5F01, 1945.

5. Moore, R. L.: Some Comparative Tests of Plain and Alclad 24S-T Sheet. NACA TN 821, 1941.

6. Templin, R. L., Hartmann, E. C., and Paul, D. A.: Typical Tensile and Compressive Stress-Strain Curves for Aluminum Alloy 24S-T, Alclad 24S-T, 24S-RT, and Alclad 24S-RT Products. Tech. Paper No. 6, Aluminum Res. Lab., Aluminum Co. of Am., 1942.

7. Timoshenko, S.: Theory of Elastic Stability. McGraw-Hill Book Co., Inc., 1936.

TABLE I

FORMULAS FOR η FOR ALCLAD PLATES WITH DIFFERENT

EDGE CONDITIONS IN BOTH COMPRESSION AND IN SHEAR

[Symbols are defined in appendix A]

Type of plate	Formula for η
Plates in compression	
Long plate, both unloaded edges simply supported	$\frac{\sqrt{C_{1_\sigma} C_{5_\sigma}} + C_{3_\sigma}}{2(g+1)} \frac{E_s}{E}$
Short plate-column $\left(\frac{l}{b} \ll 1\right)$	$\frac{C_{1_\sigma}}{g+1} \frac{E_s}{E}$
Square plate-column $\left(\frac{l}{b} = 1\right)$	$\frac{C_{3_\sigma}\left(\frac{C_{1_\sigma}}{C_{3_\sigma}} - 0.15375\right)}{0.84625\,(g+1)} \frac{E_s}{E}$
Long plate-column or strip column $\left(\frac{l}{b} \gg 1\right)$	$\frac{4C_{3_\sigma}\left(\frac{C_{1_\sigma}}{C_{3_\sigma}} - \frac{1}{4}\right)}{3(g+1)} \frac{E_s}{E}$
Plates in shear[a]	
Long plate, edges simply supported	$\frac{C_{5_\tau}}{(g+1)\,4(\sqrt{2}\alpha)}\left(3\alpha^2 + \frac{C_{3_\tau}}{C_{5_\tau}} + \sqrt{\alpha^4 + 2\frac{C_{3_\tau}}{C_{5_\tau}}\alpha^2 + 1}\right)\frac{E_s}{E}$

[a]Values of α determined by

$$\sqrt{\alpha^4 + 2\frac{C_{3_\tau}}{C_{5_\tau}}\alpha^2 + 1} = \frac{1 - \alpha^4}{3\alpha^2 - \frac{C_{3_\tau}}{C_{5_\tau}}}.$$

TABLE II

DIMENSIONS AND TEST RESULTS FOR 24S-T84 ALCLAD CHANNEL- AND Z-SECTION

COLUMNS THAT DEVELOP LOCAL INSTABILITY

Specimen	t (in.)	b_W (in.)	b_F (in.)	l (in.)	$\frac{l}{b_W}$	$\frac{b_W}{t}$	$\frac{b_F}{b_W}$	k_W	$\frac{b_W}{t}\sqrt{\frac{12(1-\mu^2)}{k_W}}$	σ_{cr}/η (ksi)	σ_{cr} (ksi)	η
Channel section												
1	0.127	1.84	1.067	6.67	3.63	14.49	0.580	2.33	31.37	110.0	64.5	0.585
2	.127	1.84	1.067	9.06	4.92	14.49	.580	2.33	31.37	110.0	61.7	.560
3	.091	1.39	.863	4.77	3.43	15.27	.621	2.08	34.99	86.2	63.7	.748
4	.091	1.39	.875	4.83	3.47	15.27	.628	2.04	35.33	84.6	63.0	.745
5	.091	1.39	.875	4.88	3.51	15.27	.629	2.04	35.33	84.6	60.8	.719
6	.064	1.41	.586	4.98	3.53	22.03	.416	3.62	38.26	71.9	56.3	.782
7	.064	1.41	.588	4.98	3.53	22.03	.417	3.61	38.31	71.9	54.3	.755
8	.064	1.41	.591	4.98	3.53	22.03	.419	3.54	38.42	71.5	53.7	.750
9	.063	1.41	.669	5.29	3.75	22.38	.474	3.12	41.87	60.3	51.1	.847
10	.064	1.41	.671	5.30	3.76	22.03	.476	3.09	41.41	61.5	49.4	.803
11	.063	1.41	.674	5.29	3.75	22.38	.478	3.07	42.21	59.2	48.6	.821
12	.063	1.41	.713	5.67	4.02	22.38	.506	2.85	43.81	54.9	46.7	.852
13	.063	1.41	.719	5.67	4.02	22.38	.510	2.82	44.04	54.4	48.4	.890
14	.062	1.92	.582	6.78	3.53	30.97	.303	4.38	48.90	44.2	39.1	.884
15	.062	1.92	.589	6.78	3.53	30.97	.307	4.35	49.07	43.9	41.1	.935
16	.062	1.93	.592	6.78	3.51	31.13	.307	4.35	49.32	43.4	40.3	.930
17	.062	1.92	.777	8.18	4.26	30.97	.405	3.70	53.21	37.3	34.5	.925
18	.062	1.92	.779	8.17	4.26	30.97	.406	3.70	53.21	37.3	33.3	.894
19	.062	1.92	.782	8.17	4.26	30.97	.407	3.70	53.21	37.3	35.5	.951
20	.041	1.41	.718	6.77	4.80	34.39	.509	2.82	67.55	23.1	20.2	.875
21	.040	1.44	.513	5.77	4.01	36.00	.356	4.06	59.04	30.3	27.0	.890
22	.040	1.43	.518	5.77	4.03	35.75	.362	4.02	58.92	30.4	26.3	.865
23	.040	1.44	.523	5.77	4.01	36.00	.363	4.02	59.33	30.0	26.8	.894
24	.040	1.43	.625	6.27	4.38	35.75	.437	3.43	63.79	26.0	24.3	.935
25	.040	1.44	.633	6.26	4.35	36.00	.440	3.41	64.42	25.4	24.5	.965
26	.040	1.44	.633	6.27	4.35	36.00	.440	3.41	64.42	25.5	23.9	.937
27	.040	1.42	.720	6.78	4.77	38.50	.507	2.83	69.73	21.7	20.0	.922
28	.040	1.42	.728	6.77	4.77	38.50	.513	2.83	69.98	21.6	20.9	.965
29	.049	1.92	.609	7.30	3.80	39.18	.317	4.31	62.36	27.2	27.1	.995
30	.049	1.91	.611	7.38	3.86	38.98	.320	4.30	62.12	27.4	24.0	.877
31	.049	1.91	.614	7.36	3.85	38.98	.321	4.30	62.12	27.4	23.6	.862
32	.049	1.91	.694	7.99	4.18	38.92	.363	4.01	64.32	25.6	21.9	.930
33	.049	1.90	.696	8.00	4.21	38.78	.366	4.00	64.08	25.7	23.9	.923
34	.049	1.91	.699	7.99	4.18	38.98	.366	4.00	64.41	25.4	21.1	.937
Z-section												
1	.092	1.91	.724	6.59	3.45	20.70	.379	3.90	34.74	87.5	60.1	.688
2	.091	1.90	.725	6.58	3.46	20.88	.382	3.88	35.03	86.0	62.1	.722
3	.091	1.90	.725	6.58	3.46	20.88	.382	3.88	35.03	86.0	62.3	.725
4	.069	1.43	.606	4.98	3.48	20.72	.424	3.53	36.44	79.5	59.0	.742
5	.069	1.43	.606	4.98	3.48	20.72	.424	3.53	36.44	79.5	60.1	.756
6	.041	1.45	.450	5.78	3.99	35.37	.310	4.34	56.10	33.6	31.3	.932
7	.040	1.46	.465	6.56	4.49	36.50	.318	4.28	58.10	31.2	27.8	.892
8	.040	1.45	.468	6.60	4.55	36.25	.323	4.30	57.90	31.5	31.1	.988

NACA

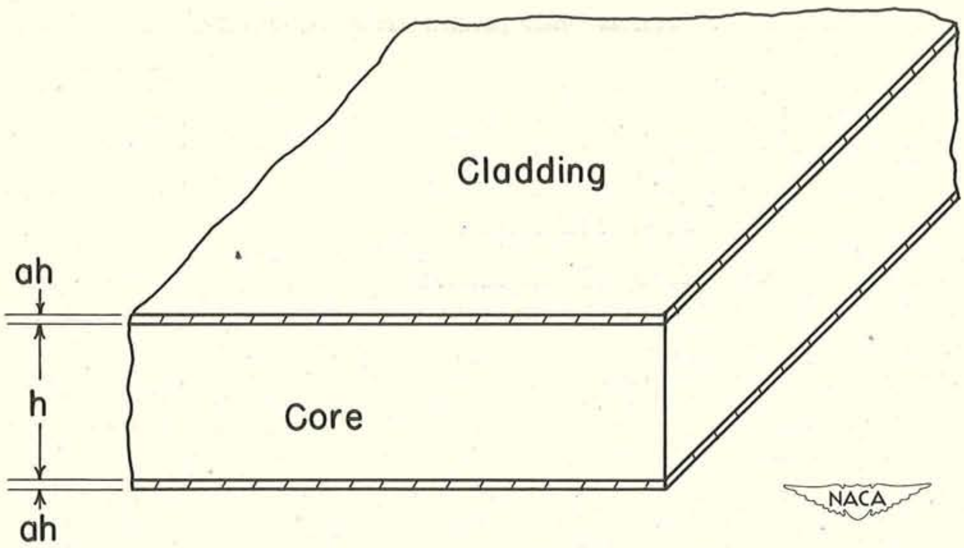


Figure 1.- Alclad sheet.

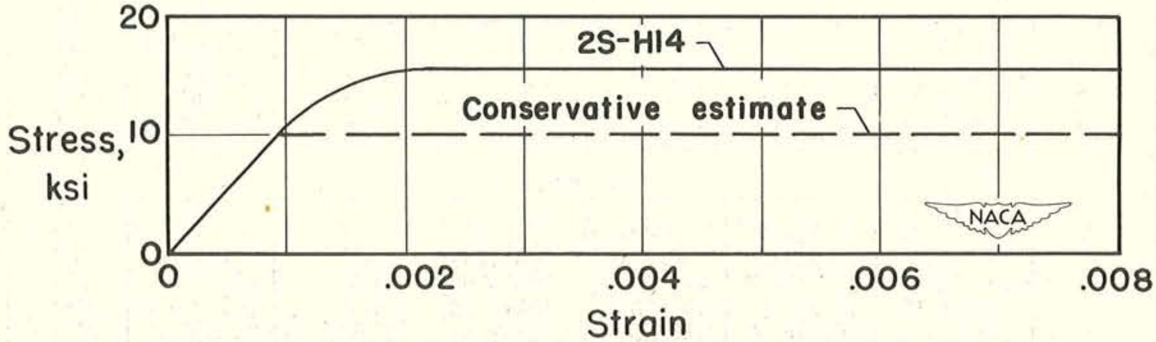
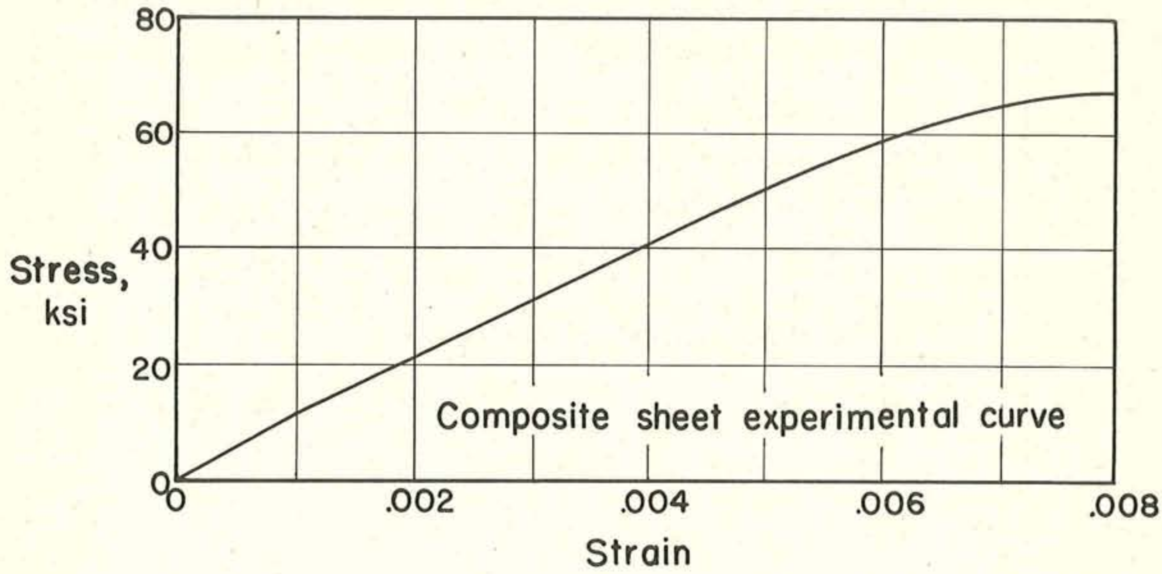


Figure 2.- Assumed stress-strain curves of cladding.



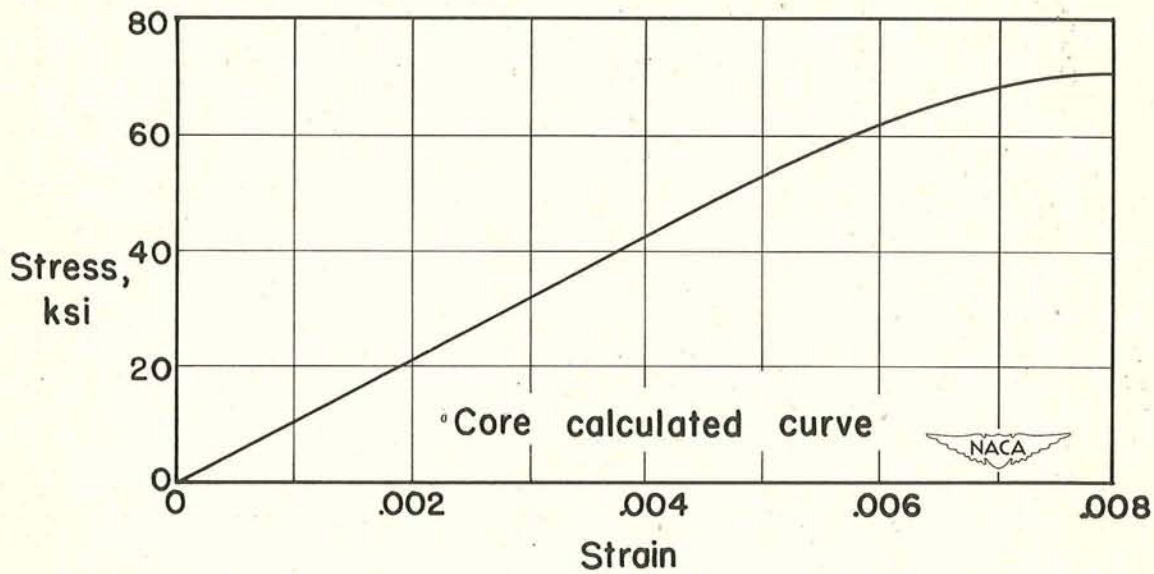


Figure 3.- Stress-strain curves of Alclad 24S-T84 aluminum-alloy sheet with 5.7 percent cladding.

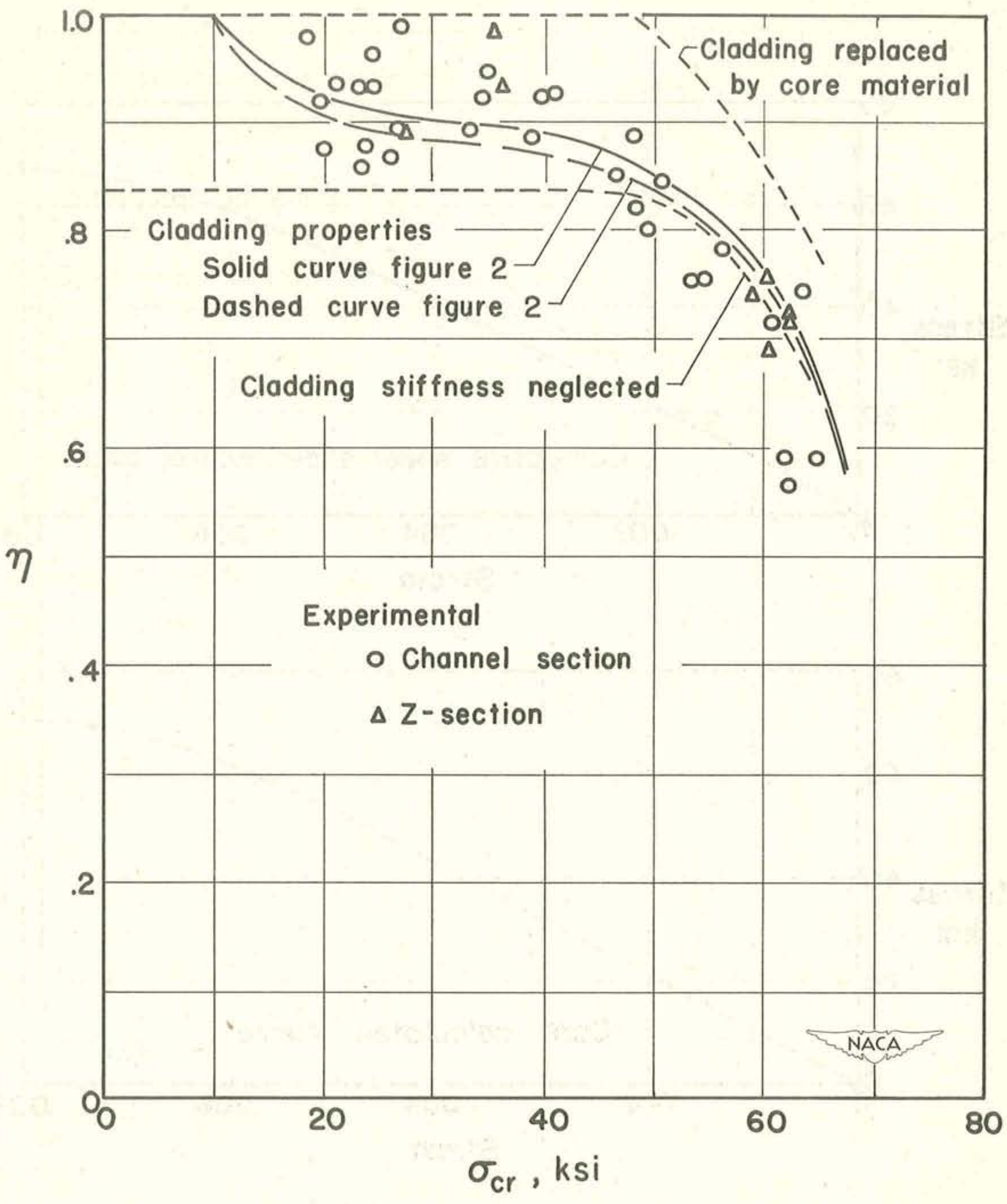


Figure 4.- Theoretical values of η for a simply supported plate in compression and experimental points for Z- and channel sections made of Alclad 24S-T84 aluminum-alloy sheet with 5.7 percent cladding.

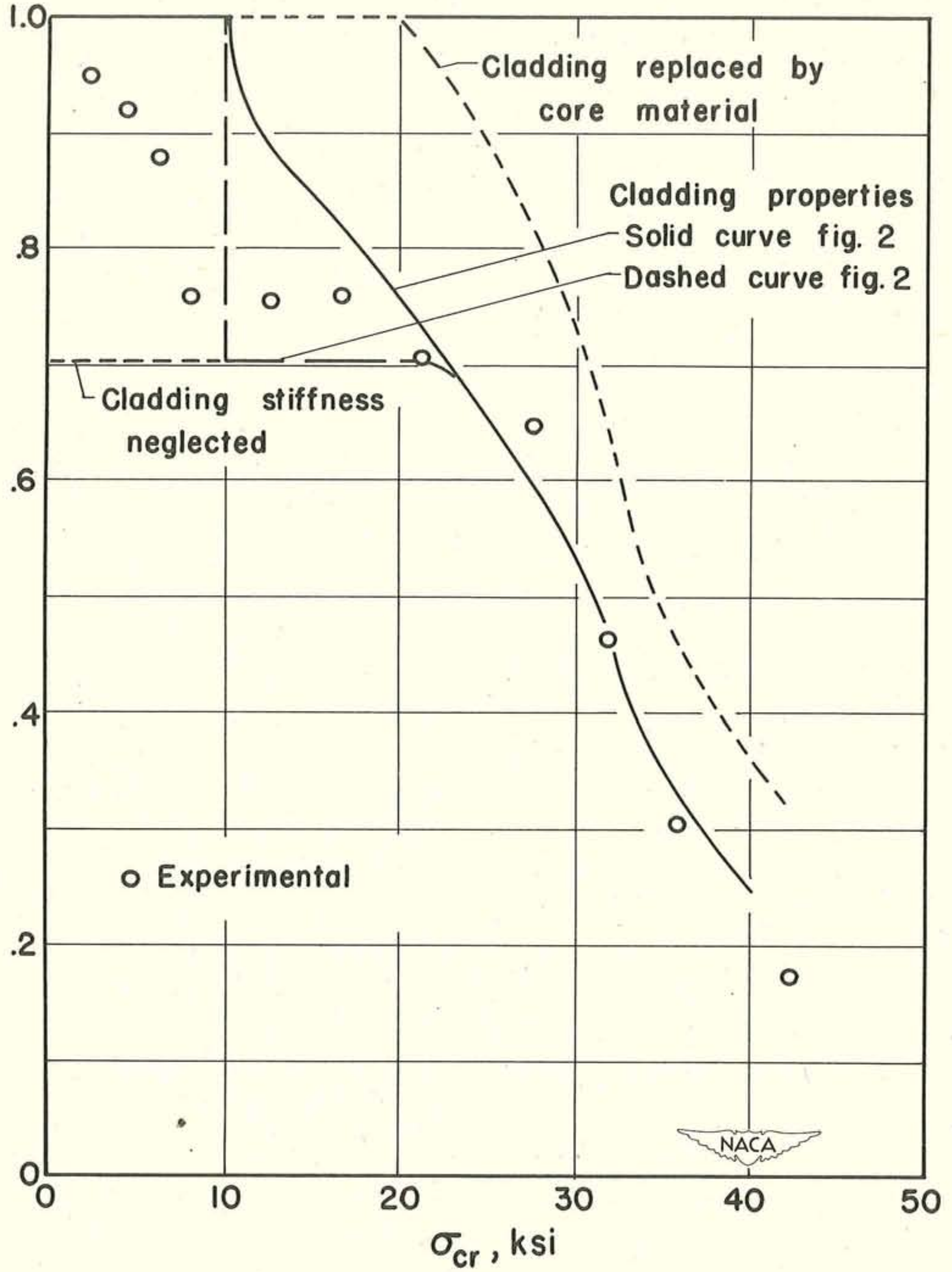


Figure 5.-Theoretical and experimental values of η for a long column made of Alclad 24S-T sheet with 11 percent cladding.