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ON INTERNAL DAMPING OF ROTATING BEAMS

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SUMMARY

For a beam rotating about a transverse axis and harmonically vibrating in a direction normal to its plane of rotation, it is assumed that an internal damping force exists inversely proportional to the vibration frequency and directly proportional to the time rate of change of the elastic load. It is shown that, with such a force, the internal logarithmic decrement of the beam increases asymptotically with the principal mode of vibration to the value πg, which it would have in any mode if the same beam were not rotating. The term g in this value is a dimensionless internal-damping coefficient depending on the beam material. If the beam performs vibrations in the plane of rotation, then the internal logarithmic decrement in the fundamental mode will be either equal to, or slightly less than, πg, but will decrease in the second mode and will then increase asymptotically with the higher modes to the value πg. Thus, rotation of a beam in general diminishes the internal logarithmic decrements.

INTRODUCTION

The purpose of this investigation is to derive the implications, for rotating beams, of the assumption $p_d = \frac{g}{\omega}\frac{\partial}{\partial t}(EIy'')''$ which was originally made for nonrotating beams and which has been found to agree with experimental results for such beams. Such an investigation may be of particular interest in the design of helicopter blades, propellers, or turbine blades, which often act like beams. (See reference 1.) Internal damping is especially important in flutter phenomena, where it increases the stability, and in vibrations of fixed-ended helicopter blades in the plane of rotation, where it is of the same magnitude as the aerodynamic load. Experimental data on the internal logarithmic decrements of beams rotating about a transverse axis appear to be lacking, and there is therefore no conclusive empirical check at present on the basic assumption used here for rotating beams.[1]

The mechanism of internal damping in structures appears as yet to be not entirely understood. It is possible, nevertheless, to take internal damping mathematically into account in vibration phenomena by making simple assumptions such that the implications will be in accord with experimental evidence.

Numerous experiments (references 2 to 5) have been made on non-rotating vibrating beams, all leading to the conclusion that the internal logarithmic decrements of such beams are independent of the frequency, and therefore of the mode, of vibration. In view of these results Theodorsen and Garrick (reference 6) have, for purposes of flutter investigation, introduced a damping force in bending of the form $p_d = ig\omega_e^2 m'y$. Reissner (reference 7) more recently has suggested a damping force of the form $p_d = d_s g \frac{\partial}{\partial t}(EIy'')''$, where the coefficient d_s is to be chosen so that the logarithmic decrements will be independent of the frequency.[2]

It will be observed that in both of the expressions for p_d the damping force is assumed to be in phase with the velocities, although the second expression does not contain an imaginary factor. It should be observed, moreover, that the term $(EIy'')''$ represents the unit elastic load of a rotating, as well as of a stationary, beam, while the term $\omega_e^2 m'y$ represents exactly the unit elastic load of a stationary beam but only approximately[3] the unit elastic load of a rotating beam. Although the two basic assumptions discussed here are in this sense not quite equivalent for rotating beams, it will be found that in actual cases they lead to virtually the same results for the logarithmic decrements. It is therefore necessary to treat only one of these assumptions in detail.

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[1]It may be remarked here that in a rotating beam aerodynamic as well as internal damping forces exist. Consequently, the effect of aerodynamic damping must be taken into account in the interpretation of any experimental data on the logarithmic decrements of rotating beams.

[2]Previous investigators, such as Sezawa (reference 8), also used an internal damping force proportional to $\partial/\partial t\,(EIy'')''$, but for simplicity it was assumed that the proportionality factor was a constant, independent of the frequency of vibration. This led to results which were not in agreement with experiments.

[3]It can be easily shown that if the centrifugal loads had no effect on the modes of deflection of a rotating beam this term would represent the unit elastic load of a rotating beam exactly.

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SYMBOLS

A cross-sectional area

A_o cross-sectional area at root of beam

E modulus of elasticity of beam material

$f_{cn} = \frac{{\omega_{cn}}^2}{\Omega^2}$

$f_{en} = \frac{{\omega_{en}}^2}{\Omega^2 K}$

g dimensionless internal-damping coefficient depending on beam material

$i = \sqrt{-1}$

I moment of inertia of structural cross section of beam

I_o value of I at root of beam

K dimensionless bending-stiffness parameter $\left(\frac{EI_o}{\rho A_o \Omega^2 l^4}\right)$

l length of beam

m' mass per unit length of beam

n mode of vibration

p complex frequency; if $p = -R \pm i\omega$ (R and ω real) then $\omega/2\pi$ is the natural frequency in cycles per second, while $2\pi R/\omega$ is the logarithmic decrement

$q = p/\Omega$

r radial distance of a beam element from axis of rotation

R radial distance of beam tip from axis of rotation

t time

$\bar{y} = y/l$

y vibrational bending deflection of beam at any point x

x distance along beam

δ internal logarithmic decrement; logarithm of ratio of amplitude of vibration at one time to amplitude one period later

$\xi = x/l$

ρ density of beam material

τ dimensionless centrifugal-force parameter $\left(\int_{\xi}^{1} \frac{A}{A_0} \frac{r}{l} d\xi\right)$

Ω angular speed of rotating beam, radians per second

ω natural frequency of vibration of rotating beam, cycles per 2π seconds

ω_{en} natural frequency of vibration of beam in "nth" mode when it is stationary $(\Omega = 0)$, cycles per 2π seconds

ω_{cn} natural frequency of vibration of rotating beam in nth mode if it had no bending stiffness $(K = 0)$, cycles per 2π seconds

$' = \partial/\partial x$ at first; $' = \partial/\partial \xi$ in equation (3) and thereafter

$\cdot = \partial/\partial t$

THEORY

Let the internal or structural damping force in a rotating, as well as a stationary, vibrating beam be proportional to the elastic load per unit length of the beam, and let this force be in phase with the vibrational velocities. Then the unit damping force p_d can be expressed in the form (reference 7)

$$p_d = d_s g \frac{\partial}{\partial t} (EIy'')'' \tag{1}$$

where g is a dimensionless internal-damping coefficient characteristic of the material of the beam, and where d_s is a factor which, in accordance with empirical data, can be chosen so that the internal logarithmic decrement of a stationary beam performing harmonic motion will be independent of the frequency ω of vibration. It can be shown that d_s, thus chosen, must have the value

$$d_s = \frac{1}{\omega} \tag{2}$$

(In reference 7, d_s was incorrectly expressed as $d_s = \omega$.)

The equation of the small free vibrations of a beam rotating with angular velocity Ω about a transverse axis passing through the root of the beam and parallel to the vibrational displacements y of the beam can then be written in the following dimensionless form:

$$K\left(\frac{I}{I_o} \overline{y}''\right)'' + gK \frac{\Omega}{\omega}\left(\frac{I}{I_o} \frac{\dot{\overline{y}}''}{\Omega}\right)'' - (\tau\overline{y}')' + \frac{A}{A_o} \frac{\ddot{\overline{y}}}{\Omega^2} = 0 \tag{3}$$

where, and henceforth, $' \equiv \frac{\partial}{\partial \xi}$. The term $(\tau\overline{y}')'$ represents the unit centrifugal load.

For harmonic motion, let

$$\overline{y} = \overline{y}(\xi)e^{pt} \tag{4}$$

Then, with the assumption (justified a posteriori, as is shown subsequently) that $1 + (pg/\omega) \approx 1 + ig$, the following equation is obtained for $\bar{y}(\xi)$ and $q \equiv p/\Omega$:

$$K(1 + ig)\left(\frac{I}{I_o}\bar{y}''\right)'' - (\tau\bar{y}')' + \frac{A}{A_o}\left(\frac{p}{\Omega}\right)^2 \bar{y} = 0 \qquad (5)$$

An expression for the internal damping in any mode can be obtained from equation (5) in the following manner. The natural frequency of any undamped mode n of vibration, as determined by equation (5) with $g = 0$, can be expressed, to a satisfactory approximation, by (reference 9)

$$q_{on}^2 = -f_{en}K - f_{cn} \qquad (6)$$

where f_{en} and f_{cn} are positive nondimensional quantities representing, respectively, the contribution of the elastic resistance $(\Omega = 0)$ and the contribution of the centrifugal load $(K = 0)$ to the natural frequency of the nth mode.

From equation (5) it is seen that the mathematical effect of internal damping here is to replace the constant K by the constant $K(1 + ig)$. It follows from equation (6), then, that the complex frequency ratio q_{dn} for the nth mode with internal damping will be given by:

$$q_{dn}^2 = -f_{en}K(1 + ig) - f_{cn} \qquad (7)$$

Assuming, as is actually the case, that $g \ll 1$, equation (7) leads to the following expression for q_{dn}:

$$q_{dn} = i\,\frac{\omega_{on}}{\Omega} - \frac{1}{2}\,gK\,\frac{f_{en}}{\omega_{on}/\Omega} \qquad (8)$$

where ω_{on} is the natural frequency of the rotating beam in the nth undamped mode and is given by:

$$\omega_{on} = \Omega\sqrt{f_{en}K + f_{cn}} \qquad (9)$$

Observing that

$$\frac{\omega_{en}^2}{\Omega^2} = Kf_{en}$$

where ω_{en} is the natural frequency of the nth mode when the beam is not rotating, it is seen that equation (8) can be written as:

$$q_{dn} = i\,\frac{\omega_{on}}{\Omega} - \frac{1}{2}\,g\,\frac{\omega_{en}^2/\Omega^2}{\omega_{on}/\Omega} \tag{10}$$

From equation (10), one finds

$$\frac{p}{\omega} = 1 - \frac{1}{2}\,g\,\frac{\omega_{en}^2}{\omega_{on}^2}$$

or

$$1 + \frac{p}{\omega}\,g = \left(1 - \frac{1}{2}\,g^2\,\frac{\omega_{en}^2}{\omega_{on}^2}\right) + ig$$

Since $g \ll 1$, it is evident that

$$1 + \frac{p}{\omega}\,g \approx 1 + ig$$

as was originally assumed.

Equation (10) implies that to a first (and for practical purposes, sufficient) approximation, internal damping does not affect the natural frequency of any mode, since the imaginary part of the complex frequency ratio q remains unchanged by the damping. However, the damping adds a negative real part to the value of q, which implies the following value for the logarithmic decrement δ_n of any principal mode:

$$\delta_n = \pi g\,\frac{\omega_{en}^2}{\omega_{on}^2} = \pi g\,\frac{Kf_{en}}{Kf_{en} + f_{cn}} \tag{11}$$

where $\omega_{en}^2/\omega_{on}^2$ is the ratio of the square of the natural frequency of the beam in any mode n when it is not rotating to the square of the natural frequency in that mode when it is rotating.

Since in general $\omega_{en} \leqq \omega_{on}$ for any mode ($\omega_{en} = \omega_{on}$ only if $\Omega = 0$), and since the ratio ω_{en}/ω_{on} approaches unity as the mode of vibration increases, it follows from equation (11) that the internal logarithmic decrement of a rotating beam increases asymptotically with the mode to the value $\delta = \pi g$, which it would have in any mode if the beam were not rotating. Rotation of a beam evidently decreases the logarithmic decrement in any mode of vibration of the beam, but this decrease will in actual cases be found appreciable mainly in the lower modes. As can be seen from equation (11), the decrease in the logarithmic decrements is due to the increase in the natural frequencies caused by the rotation of the beam.

If the beam is rotating about a transverse axis perpendicular, instead of parallel, to its vibrational displacements, so that the vibrations are in the plane of rotation, then the basic differential equation remains the same as equation (3) except that the unit centrifugal load is now represented by an additional term $-A/A_o\overline{y}$ on the left side of the equation. The approximate relation, equation (6), remains valid, although the numerical values of f_{cn} will now be different.[4] All of the previous reasoning leading to equations (7) to (11) thus remains valid and, therefore, the general equations which have been derived here for a rotating beam performing vibrations in a direction perpendicular to its plane of rotation are valid also for a beam vibrating in its plane of rotation.

In the latter case, however, the value of f_{cn} in the fundamental mode is usually either zero or very small. Consequently, according to equation (11), the internal logarithmic decrement of a beam vibrating in the plane of rotation will have a value of almost πg in the fundamental mode, will decrease in the second mode, and will then increase asymptotically with the higher modes to the value πg.

It may be noted that all the general relations which have been developed here are valid regardless of the boundary conditions of the beam, although the numerical values of f_{en} depend on the boundary conditions.

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[4]It can be shown that if the rotational axis passes through the root of the blade, then $(f_{cn})_2 = (f_{cn})_1 - 1$, where the subscripts 1 and 2 refer, respectively, to vibrations normal to, and in, the plane of rotation.

NUMERICAL EXAMPLE

A numerical example may be given to illustrate the nature of the results derived here. Consider a uniform fixed-ended beam vibrating normally to its plane of rotation. Then the values of f_{en} and f_{cn} for the various modes of vibration are given by:

$$f_{en}^{1/4} = 1.874,\ 4.71,\ 7.85,\ .\ .\ .,\ \pi(2n - 1)/2 \text{ (approx.)} \qquad n \geqq 2$$

$$f_{cn} = 1,\ 6,\ 15,\ .\ .\ .,\ n(2n - 1) \qquad n \geqq 1$$

A typical value of K for a fixed-ended single-tubular-spar helicopter blade is $K = 0.004$. For propellers and turbine blades, the value of K may be higher. Figures 1 and 2 show the variation of δ_n with the principal mode of vibration and with the vibration frequency, respectively, for the uniform fixed-ended beam with $K = 0.005$, $K = 0.05$, and $K = 1$.

If a uniform fixed-ended beam is vibrating in its plane of rotation, and if the axis of rotation passes through its root, then the values of f_{en} remain as given, but the values of f_{cn} are now:

$$f_{cn} = 0,\ 5,\ 14,\ .\ .\ .,\ (2n + 1)(n - 1)$$

Figure 3 shows the variation of δ_n with the principal modes of vibration of such a beam.

CONCLUSIONS

If a beam rotating about a transverse axis is harmonically vibrating in a direction normal to its plane of rotation and if it is assumed that an internal damping force exists inversely proportional to the vibration frequency and directly proportional to the time rate of change of the elastic load, then it follows that the internal logarithmic decrement of such a beam will increase asymptotically with the mode of vibration and will approach the value πg which it would have in all modes if the beam were not rotating. The term g in this value is a dimensionless internal-damping coefficient depending on the beam material. If the beam is vibrating in, instead or normally to, its plane of rotation, then the internal logarithmic decrement will be equal to, or will be slightly less than, the value πg in the fundamental mode, will decrease in the second mode, and will then increase asymptotically with the higher modes to the value πg. Thus, rotation of a beam in general diminishes the structural logarithmic decrements. The greatest variation of the logarithmic decrement with the mode will in general occur in the lower modes.

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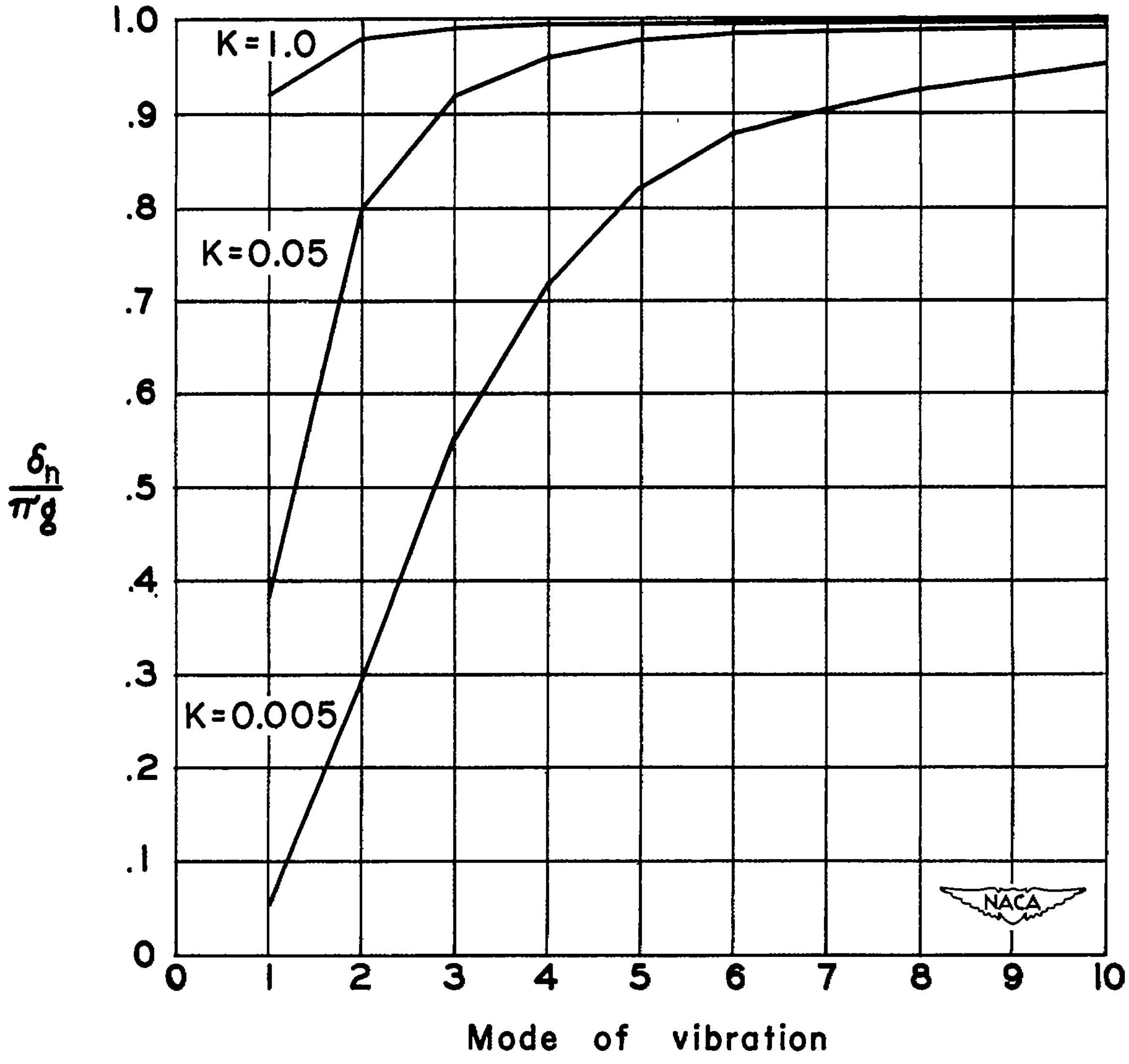

Figure 1.- Variation of structural logarithmic decrements with principal mode of vibration for a uniform fixed-ended beam rotating at angular speed Ω about a transverse axis and vibrating normally to the plane of rotation.

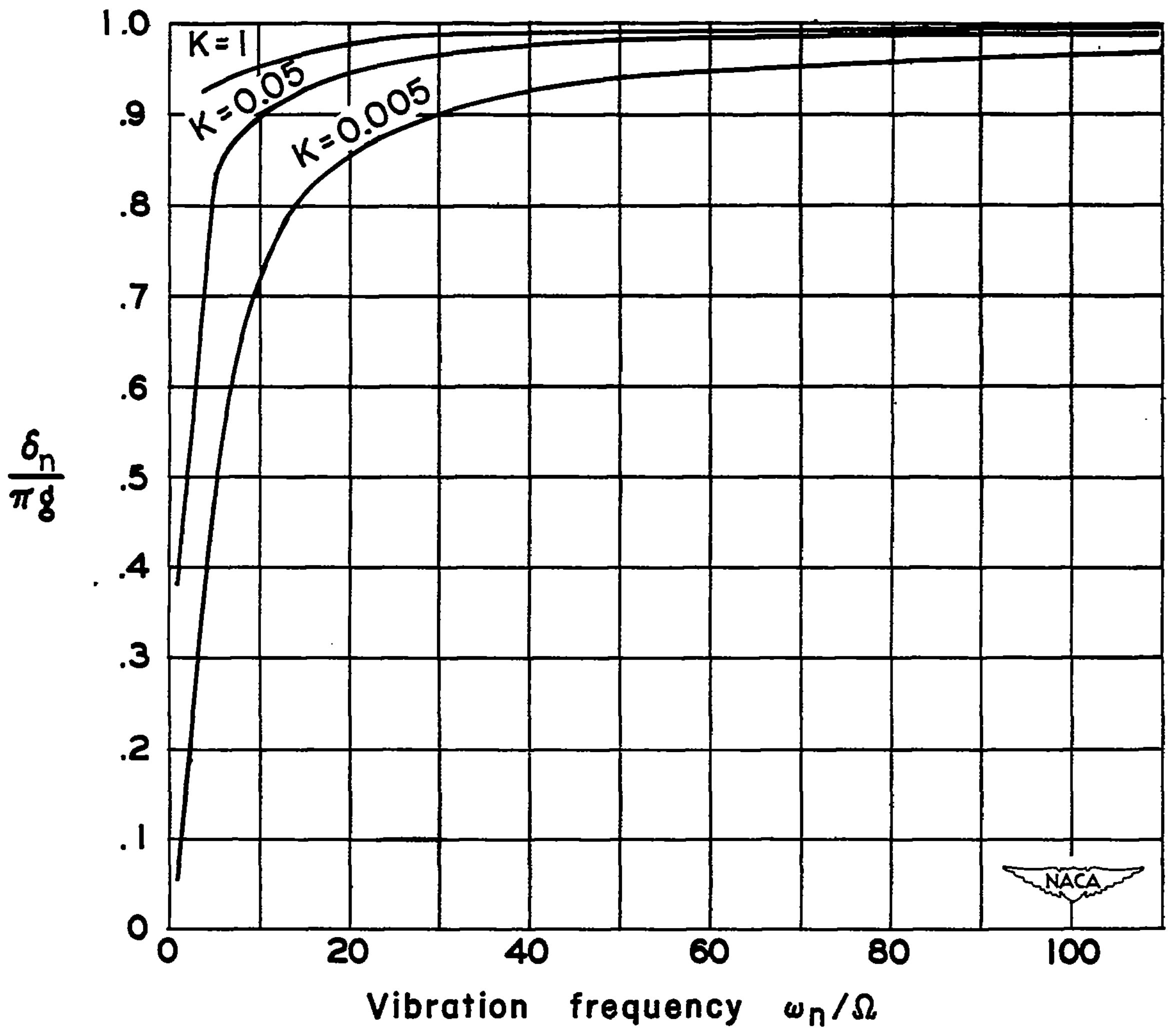

Figure 2.- Variation of structural logarithmic decrements with vibration frequency for a uniform fixed-ended beam rotating at angular speed Ω about a transverse axis and vibrating normally to the plane of rotation.

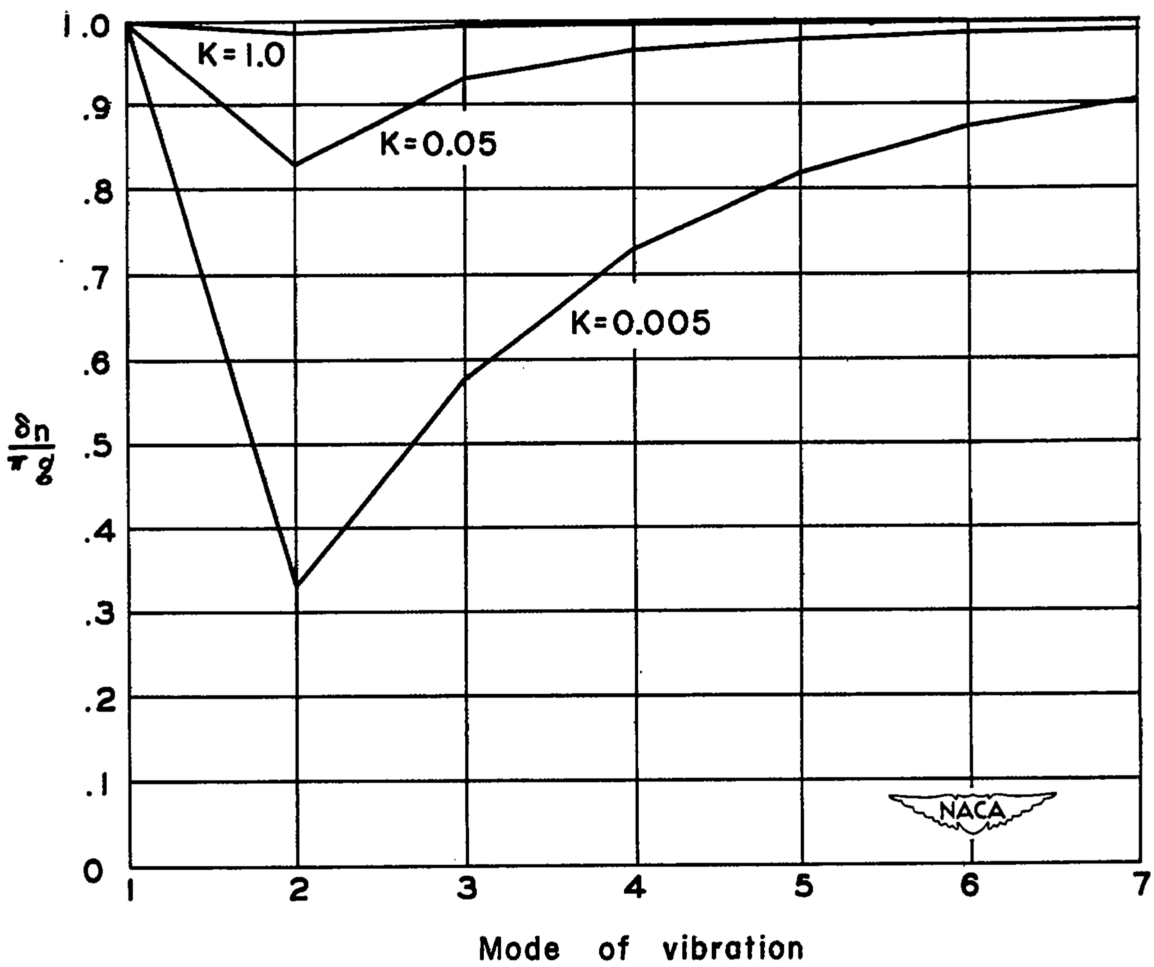

Figure 3.- Variation of internal logarithmic decrements with principal mode of vibration for a uniform fixed-ended beam rotating at angular speed Ω about a transverse axis through its root and vibrating in the plane of rotation.

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