NACA TN 2062

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2062

DYNAMIC SIMILITUDE BETWEEN A MODEL AND A FULL-SCALE BODY FOR MODEL INVESTIGATION AT FULL-SCALE MACH NUMBER

By Anshal I. Neihouse and Philip W. Pepoon

Langley Aeronautical Laboratory
Langley Air Force Base, Va.

Washington
March 1950

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2062

DYNAMIC SIMILITUDE BETWEEN A MODEL AND A FULL-SCALE BODY FOR MODEL INVESTIGATION AT FULL-SCALE MACH NUMBER

By Anshal I. Neihouse and Philip W. Pepoon

SUMMARY

An analysis is given for interpreting results of dynamic tests of a model investigated at full-scale Mach number in terms of the corresponding full-scale body. This analysis shows that dynamic similarity for such a condition can be closely approximated although the effect of gravity is not to scale. Thus an error is introduced which, however, should be small if the time period of model action is short.

INTRODUCTION

When geometrically similar systems move under the action of forces in such a manner that the relative positions of the parts of one system after a certain time are geometrically similar to those of another system after a proportional period of time, the systems are said to be dynamically similar. When geometrical similarity of the paths of motion of corresponding points is associated with geometrical similarity of the parts, constant scale ratios of force, mass, and time must be maintained in addition to that of length.

In model testing, full-scale conditions cannot be completely duplicated in every respect and some compromise is generally necessary, the nature of the compromise depending upon the end result required. In simulating equal flow patterns, the Reynolds number, or the ratio of inertia to frictional or viscous forces, is maintained constant between the model and its counterpart. For dynamic testing such as model spin tests, the Froude number, or the ratio of inertia to gravity forces, is maintained constant. If compressibility is believed to be involved in the flow, the Mach number, or the ratio of inertia to elastic forces, is kept constant.

With the advent of high-speed missiles and rocket-propelled airplanes, an increasing amount of dynamic testing is conducted at full-scale Mach numbers on scale models of flight vehicles. Proper ballasting of the model and corresponding intepretation of the results should be of interest for such tests as well as for tests in which parts of the airplane, such as pilot-escape capsules or bombs, are jettisoned at high speeds.

This paper presents a special application of dynamic similarity for an investigation in which a scale model would be tested dynamically at the actual Mach number of the corresponding full-scale body.

SYMBOLS

V	velocity
V_c	velocity of sound
Ω	angular velocity
l	linear dimension
F	force
m	mass
W	weight (mg)
g	acceleration due to gravity
I	moment of inertia
a	linear acceleration
ρ	density of air surrounding test vehicle
S	area of surface
C	coefficient
α	angular acceleration
t	time

$$K = \frac{V_{c_{alt}}}{V_{c_{SL}}}$$

$$R = \frac{l_{fs}}{l_m}$$

Subscripts:

fs	full scale
m	model
alt	altitude
SL	sea level

ANALYSIS

If the Mach number for the model tests and that for the airplane are to be equal, the ratio of the velocity of the airplane to the velocity of sound at the flight altitude must be equal to the ratio of the velocity of the model to the velocity of sound at its test altitude. If it is assumed that the model tests made at sea level represent flight of the airplane at altitude,

$$\frac{V_{fs}}{V_{c_{alt}}} = \frac{V_m}{V_{c_{SL}}}$$

Then the ratio of the velocity of the full-scale body to the velocity of the model is

$$\frac{V_{fs}}{V_m} = \frac{V_{c_{alt}}}{V_{c_{SL}}} = K \tag{1}$$

For dynamic similitude between airplane and model, the helix angles of corresponding points due to any rotary motion must be equal (reference 1); that is,

$$\frac{\Omega_{fs} l_{fs}}{V_{fs}} = \frac{\Omega_m l_m}{V_m}$$

Therefore,

$$\frac{\Omega_{fs}}{\Omega_m} = \frac{V_{fs}}{V_m} \frac{l_m}{l_{fs}}$$

Since $\frac{l_{fs}}{l_m} = R$, then the ratio of the angular velocity for the full-scale body to the angular velocity for the model can be written as

$$\frac{\Omega_{fs}}{\Omega_m} = \frac{K}{R} \tag{2}$$

As pointed out in reference 1, the ratio of inertia forces to aerodynamic forces must also be equal for airplane and model. Since

$$F = ma = C \frac{1}{2} \rho V^2 S$$

then

$$\frac{m_{fs} a_{fs}}{C_{fs} \frac{1}{2} \rho_{fs} {V_{fs}}^2 S_{fs}} = \frac{m_m a_m}{C_m \frac{1}{2} \rho_m {V_m}^2 S_m}$$

If the assumption is made that $C_{fs} = C_m$,

$$\frac{a_{fs}}{a_m} = \frac{m_m}{m_{fs}} \left(\frac{\rho_{fs}}{\rho_m} \frac{V_{fs}^2}{V_m^2} \frac{l_{fs}^2}{l_m^2} \right)$$

$$= \frac{m_m}{m_{fs}} \left(\frac{\rho_{fs}}{\rho_m} K^2 R^2 \right)$$

(The term ρ_{fs}/ρ_m was omitted in reference 1 because it was assumed that $\frac{\rho_{fs}}{\rho_m} = 1$.)

If a difference in altitude between airplane and model tests is considered, it follows from reference 1 that

$$\frac{m_{fs}}{m_m} = R^3 \frac{\rho_{fs}}{\rho_m}$$

and

$$\frac{m_{fs}}{\rho_{fs} l_{fs}^3} = \frac{m_m}{\rho_m l_m^3}$$

Therefore the ratio of the linear acceleration for the full-scale body to the linear acceleration for the model can be written as

$$\frac{a_{fs}}{a_m} = \frac{\frac{\rho_{fs}}{\rho_m} K^2 R^2}{R^3 \frac{\rho_{fs}}{\rho_m}} = \frac{K^2}{R} \tag{3}$$

Since $t = \frac{V}{a}$ and $\frac{t_{fs}}{t_m} = \frac{V_{fs}}{a_{fs}} \frac{a_m}{V_m} = \frac{R}{K^2} K$, the relation between t_{fs} and t_m is

$$\frac{t_{fs}}{t_m} = \frac{R}{K} \tag{4}$$

Also, inasmuch as $a = l\alpha$, the necessary relation between the angular acceleration for the full-scale body and for the model is

$$\frac{\alpha_{fs}}{\alpha_m} = \frac{a_{fs}}{a_m} \frac{l_m}{l_{fs}}$$

$$= \frac{K^2}{R} \frac{1}{R} = \frac{K^2}{R^2} \tag{5}$$

Also, in reference 1 the ratio of the moments of inertia is shown to be

$$\frac{I_{fs}}{I_m} = R^5 \frac{\rho_{fs}}{\rho_m} \tag{6}$$

DISCUSSION

When the Mach number for the model equals the Mach number for the full-scale body and the model undergoes rotation, ballasting of the model as indicated in the foregoing analysis leads to an interpretation of the measured values in the manner indicated. (See equations (1) to (6).) Because the acceleration due to gravity cannot be altered, the vertical acceleration (due to gravity) for the model tests is too small for dynamic similarity by the scale ratio R. The acceleration of the model vertically downward, therefore, is not sufficient, or in other words, the vertical downward acceleration of the flow is too great, for dynamic similarity. Thus an error is introduced that does not permit complete dynamic similarity, but if the time period of model action is short, the deviation from dynamic similarity should be small. For longer time periods, this effect can be computed and added to the model motion.

For a condition in which the model does not undergo rotation or is rotating at constant speed about one principal axis only, dynamic similarity could be obtained by varying the ratio m_{fs}/m_m in such a manner that $a_{fs} = a_m$. Since

$$\frac{m_{fs}a_{fs}}{m_m a_m} = \frac{\rho_{fs}{V_{fs}}^2{l_{fs}}^2}{\rho_m {V_m}^2{l_m}^2}$$

and since $a_{fs} = a_m$ and $V_{fs} = V_m K$,

$$\frac{m_{fs}}{m_m} = \frac{\rho_{fs}}{\rho_m} K^2 R^2$$

This equation can be written

$$\frac{W_{fs}/g{l_{fs}}^2}{W_m/g{l_m}^2} = \frac{\rho_{fs}}{\rho_m} K^2$$

If both the model and the airplane are considered to be operating at sea level, their sea-level wing loadings would therefore be the same.

CONCLUDING REMARKS

An analysis is given for interpreting results of dynamic tests of a model investigated at full-scale Mach number in terms of the corresponding full-scale body. This analysis shows that dynamic similarity for such a condition can be closely approximated although the effect of gravity is not to scale. Thus an error is introduced which, however, should be small if the time period of model action is short.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va., January 23, 1950

REFERENCE

1. Scherberg, Max, and Rhode, R. V.: Mass Distribution and Performance of Free Flight Models. NACA TN 268, 1927.

NACA-Langley - 3-24-50 -1000